

Updated Constraints and Forecasts on Primordial Tensor Modes

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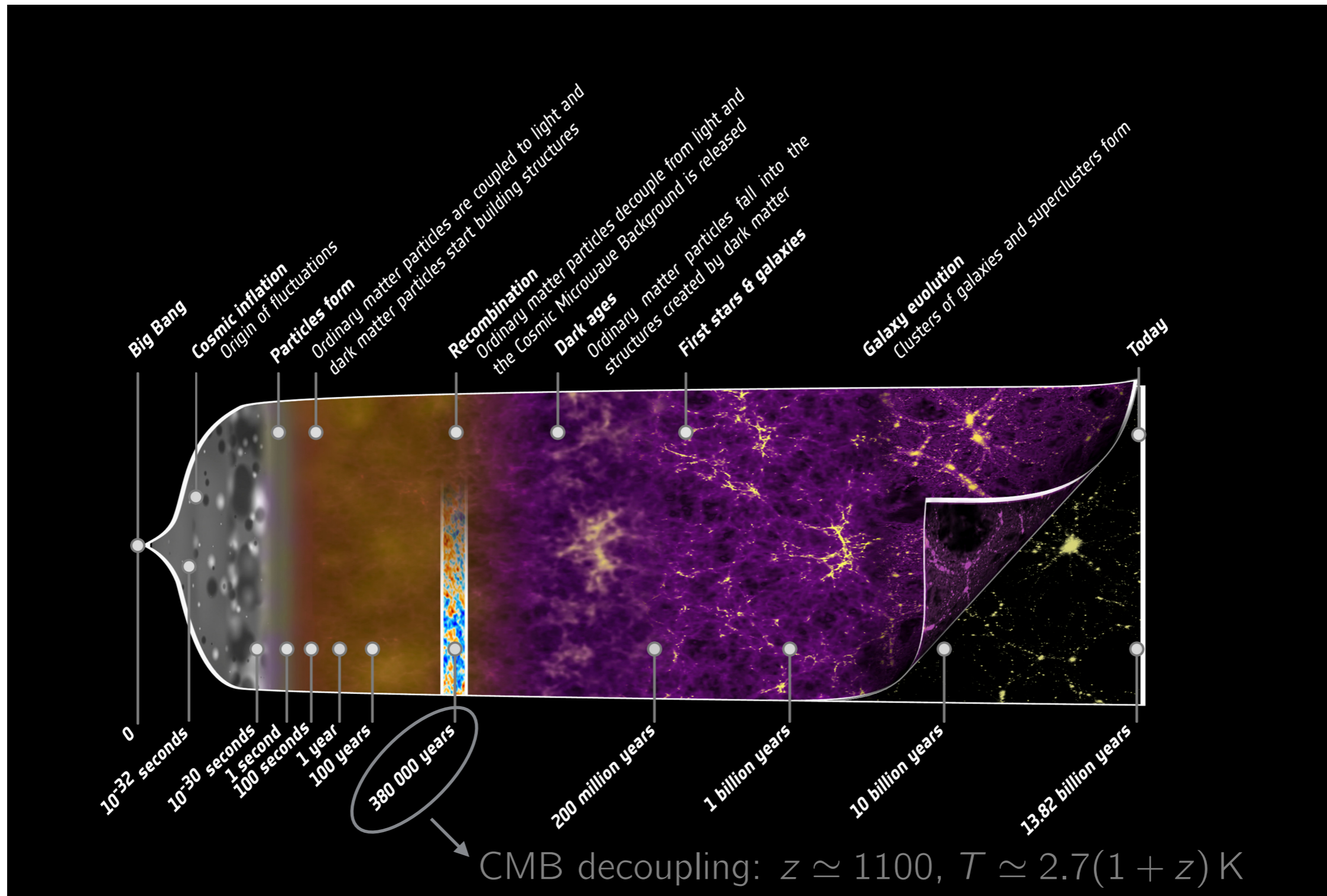
In collaboration with:

L. Pagano, L. Salvati, M. Gerbino, E. Giusarma, A. Melchiorri (arXiv: 1511.05146)

Outline

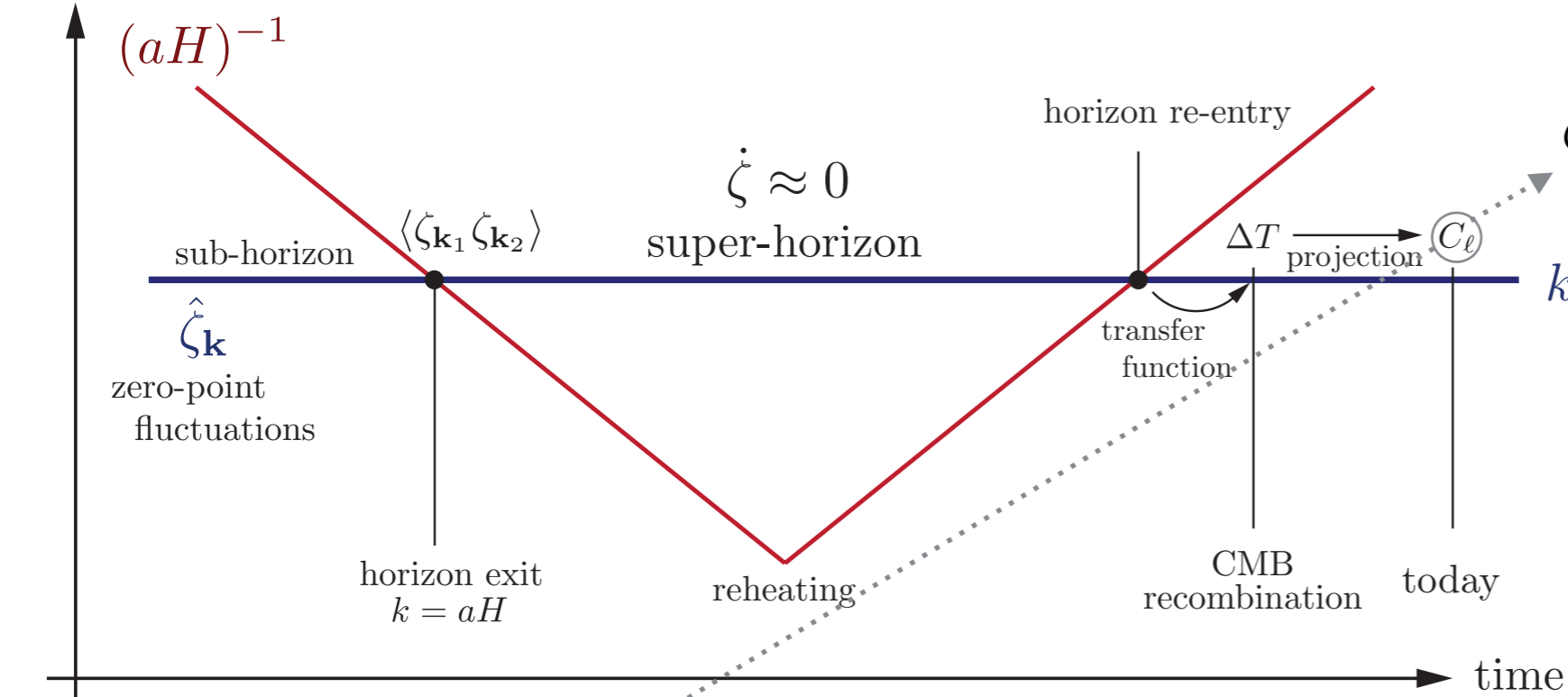
- Introduction
- Primordial gravitational waves – observable effects
 - CMB B -mode polarization
 - contribution to radiation energy density
- Constraints on primordial tensor spectrum, with and without BK14 data
- Forecasts
 - COrE-like mission
 - COrE + AdvLIGO

Introduction



CMB as probe of primordial perturbations

comoving scales



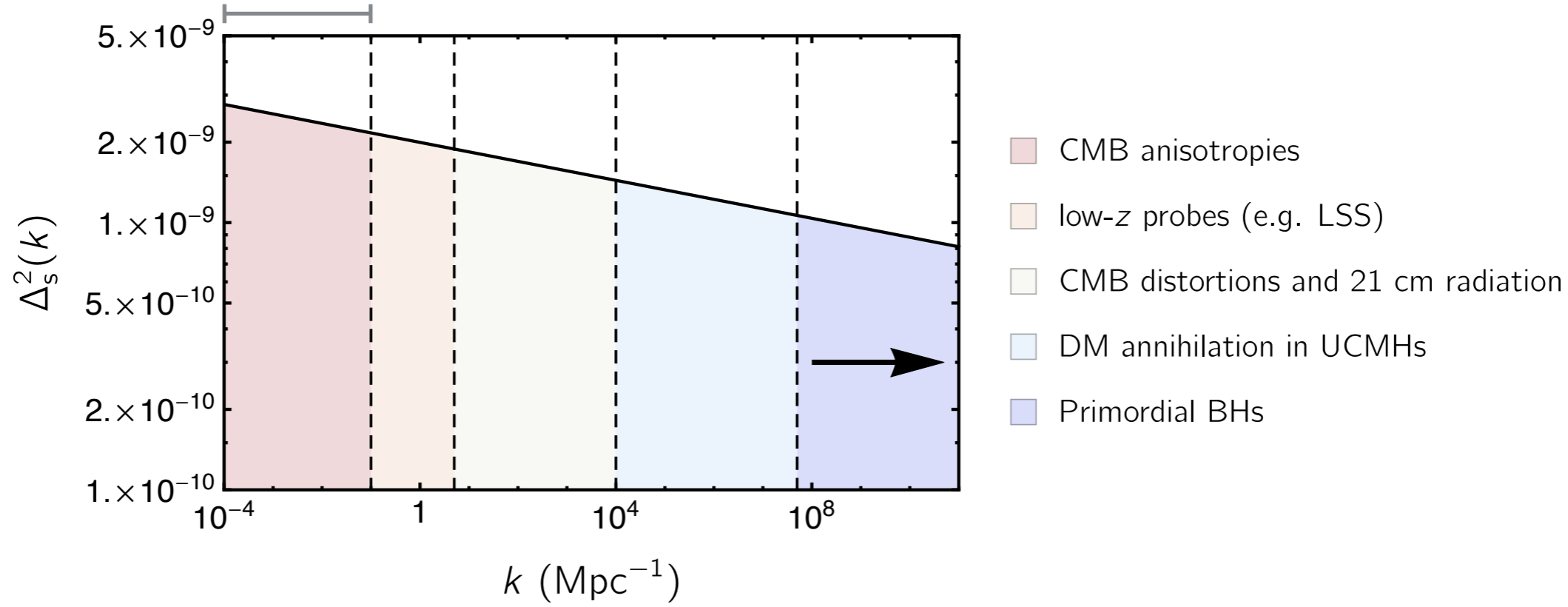
$$C_\ell \simeq \left\langle \frac{\delta T}{T}(\theta) \frac{\delta T}{T}(\theta') \right\rangle \longleftrightarrow P_\zeta(k)$$



$$\frac{k^3}{2\pi^2} P_\zeta(k) = A_s \left(\frac{k}{k_*} \right)^{n_s-1}$$

with $n_s = \underbrace{0.9645}_{\text{almost flat (red) spectrum}} \pm 0.0049$

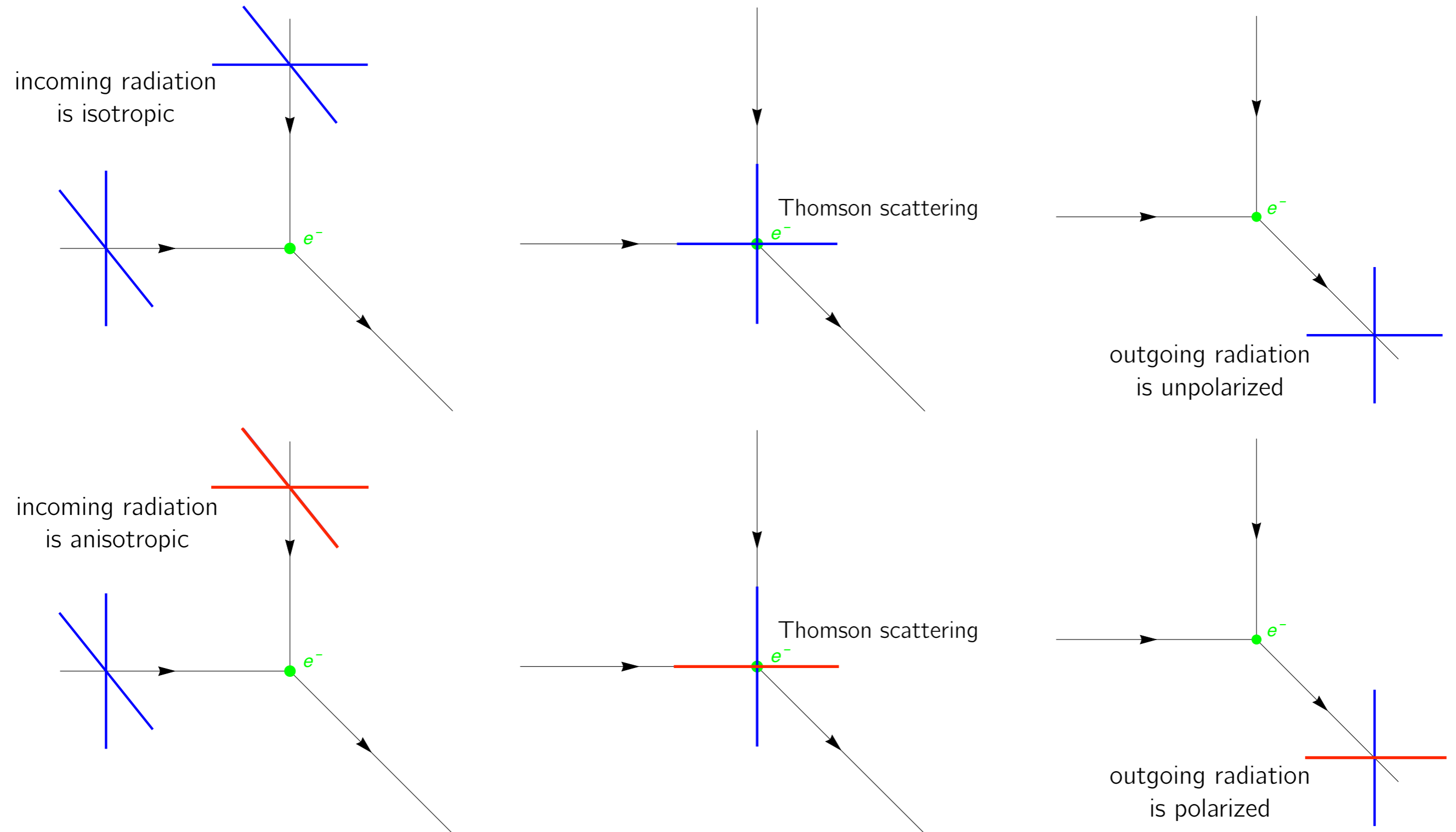
Baumann, 2009



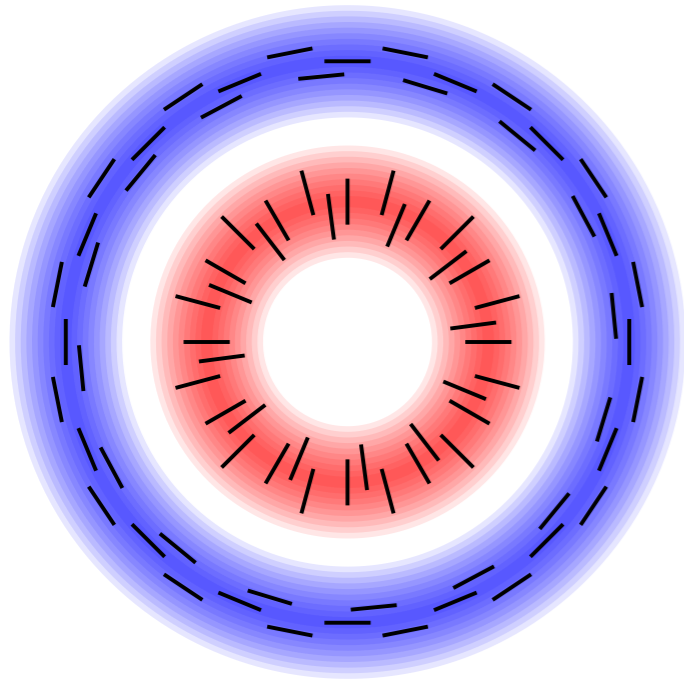
Observable Signatures of Primordial Tensor Modes

CMB polarization

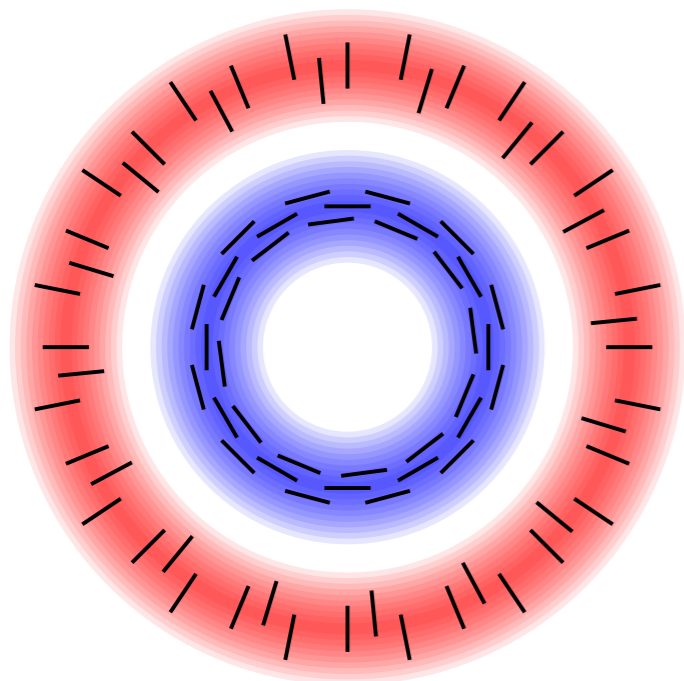
We expect the CMB to become polarized via Thomson scattering: $\frac{d\sigma}{d\Omega} \propto |\hat{\epsilon} \cdot \hat{\epsilon}'|$



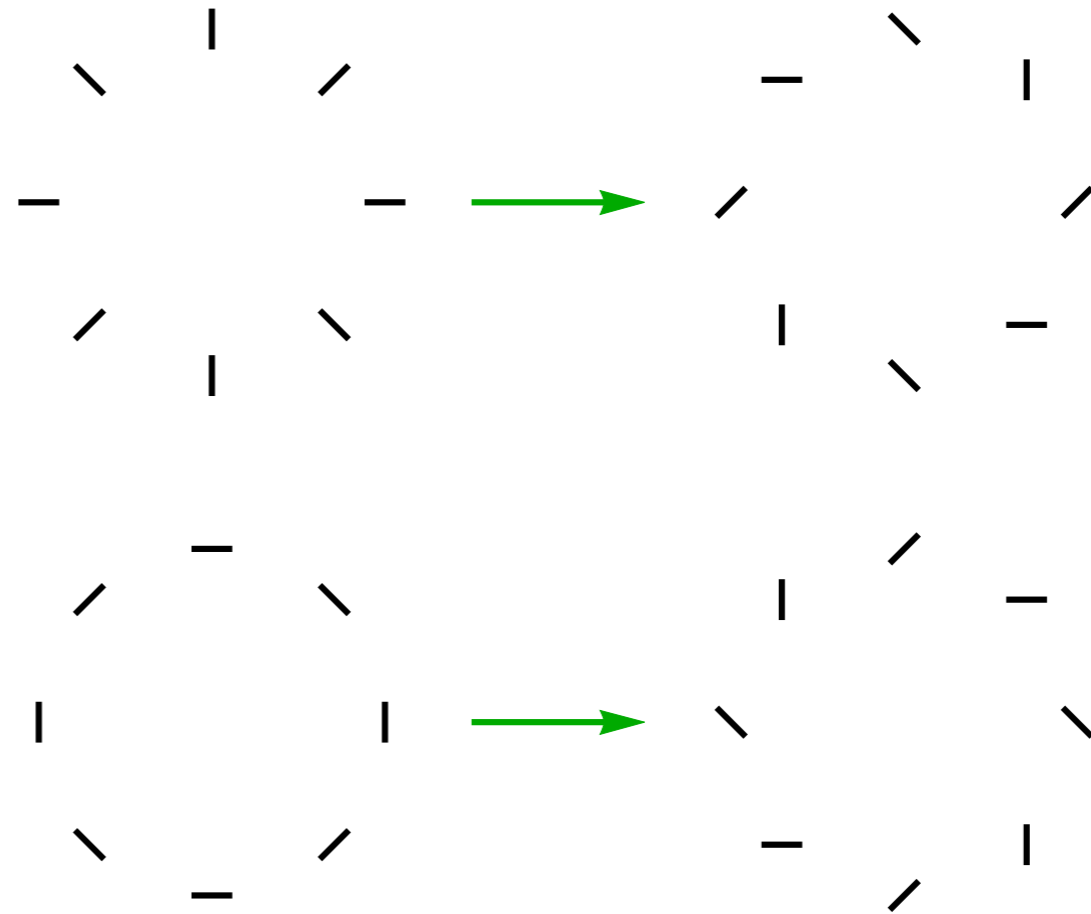
E - and B -mode polarization pattern



E -mode polarization around cold spot



E -mode polarization around hot spot



from E -mode to B -mode polarization

- E -mode pattern: invariant for $\hat{n} \rightarrow -\hat{n}$;
- B -mode pattern: odd under $\hat{n} \rightarrow -\hat{n}$.

Tensor perturbations and B -modes

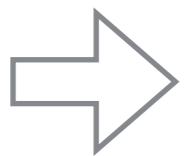
Quantum fluctuations of spatial metric $h_{ij} \rightarrow$ two-point function $P_t(k)$:

$$h_{ij}(x) = \int \frac{d\mathbf{k}}{(2\pi)^{3/2}} \sum_{p=+, \times} \epsilon_{ij}^p(\mathbf{k}) h_{\mathbf{k}}^p(t) e^{i\mathbf{k}\cdot\mathbf{x}} \longrightarrow \langle h_{\mathbf{k}}^p h_{\mathbf{k}'}^q \rangle' = P_t(k)$$

Need quadrupole anisotropy to polarize CMB photons. Both scalar and tensor modes source a quadrupole anisotropy \rightarrow

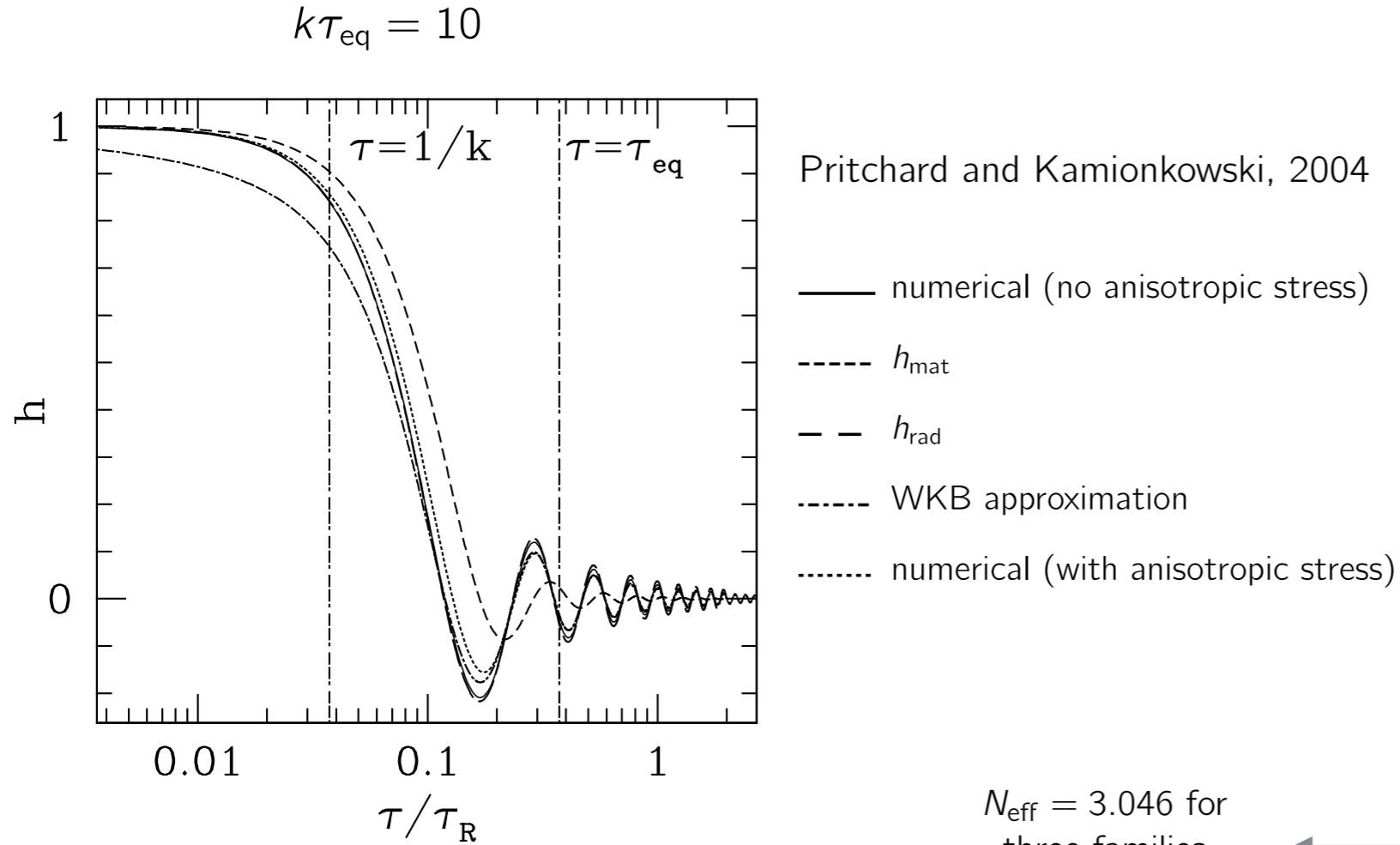
$$C_\ell^{XY} = \int d \log k \Delta_{s,\ell}^X(k) \Delta_{s,\ell}^Y(k) P_s(k) + \int d \log k \Delta_{t,\ell}^X(k) \Delta_{t,\ell}^Y(k) P_t(k)$$

- tensor perturbations $h_{\mathbf{k}}^p$ generate both E -modes and no B -modes.
- scalar perturbations $\zeta_{\mathbf{k}}$ generate E -modes, but no B -modes: transfer functions $\Delta_{s,\ell}^B$ are 0;

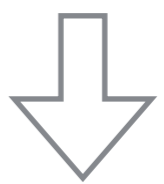


angular power spectra of B -mode polarization C_ℓ^{BB}
allow to probe the primordial tensor spectrum

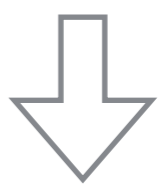
GW contribution to radiation energy density



tensor modes re-enter the horizon
 → they begin to oscillate



propagate as massless modes
 → contribute to ρ_{rad}



$N_{\text{eff}} = 3.046$ for
 three families
 of neutrinos

$$\Omega_{\text{rad}} = \Omega_{\gamma} \left[1 + \frac{7}{8} \left(\frac{4}{11} \right)^{\frac{4}{3}} N_{\text{eff}} \right]$$

Maggiore, 1999

$$N_{\text{eff}}^{\text{GW}} = \frac{h_0^2}{5.6 \times 10^{-6}} \int_{f_{\text{IR}}}^{f_{\text{UV}}} d \log f \Omega_{\text{GW}}(f)$$

with $\Omega_{\text{GW}}(f) = \frac{1}{\rho_c} \frac{\rho_{\text{GW}}}{d \log f}$

IR and UV cutoffs, primordial tensor spectrum

The integral for $N_{\text{eff}}^{\text{GW}}$ does not extend on all frequencies:

- at a given redshift, there is a IR cutoff equal to the horizon size at that redshift. **Energy of superhorizon modes is zero (they are frozen out)**;
- the UV cutoff is more arbitrary. In our analysis we take f_{UV} to be the horizon size at the end of inflation ($k_{UV} \approx 2 \times 10^{23} \text{ Mpc}^{-1}$).

$$\Omega_{\text{GW}}(f) = \frac{\Delta_{\text{t}}^2(f)}{24z_{\text{eq}}}, \text{ with } f/\text{Hz} = 1.6 \times 10^{-15} k/\text{Mpc}^{-1}$$

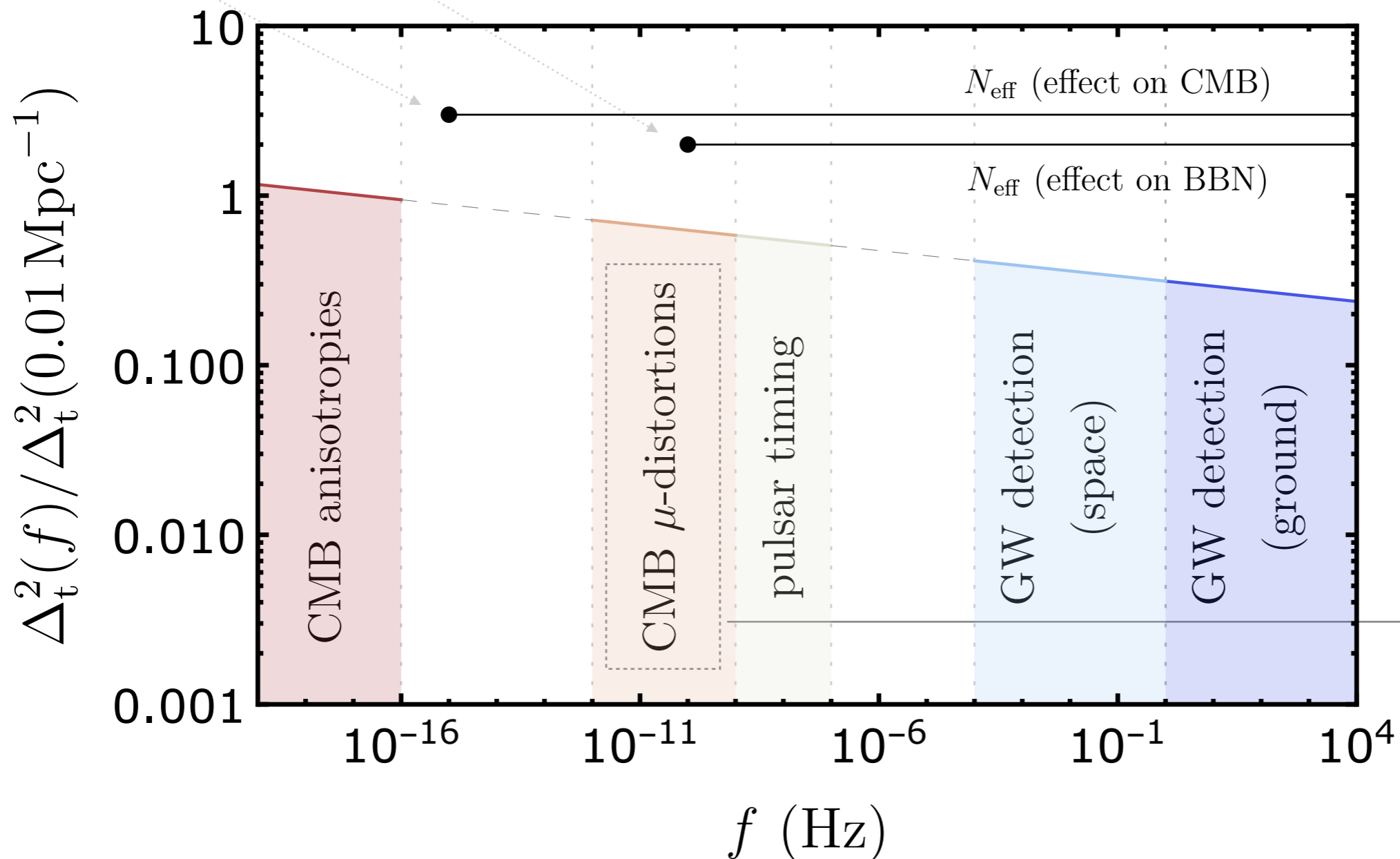
Ungarelli et al., 2005
Watanabe and Komatsu, 2006

$$\text{for } \Delta_{\text{t}}^2(k) = rA_s \left(\frac{k}{k_{\star}} \right)^{n_t} \Rightarrow N_{\text{eff}}^{\text{GW}} \approx 3 \times 10^{-6} \times \frac{rA_s}{n_t} \left[\left(\frac{f}{f_{\star}} \right)^{n_t} \right]_{f_{IR}}^{f_{UV}}$$

Constraints at different frequencies

(see also Meerburg et al., 2015)

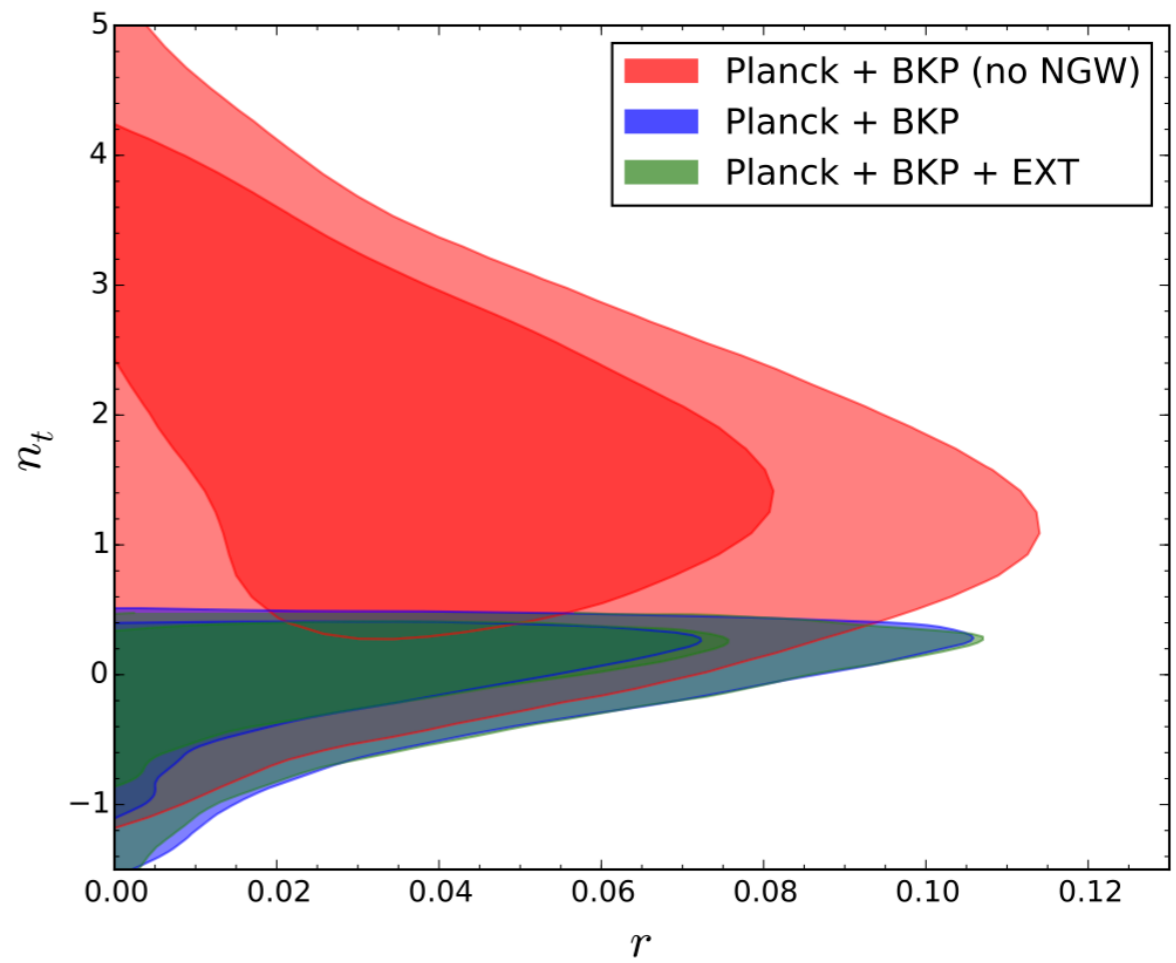
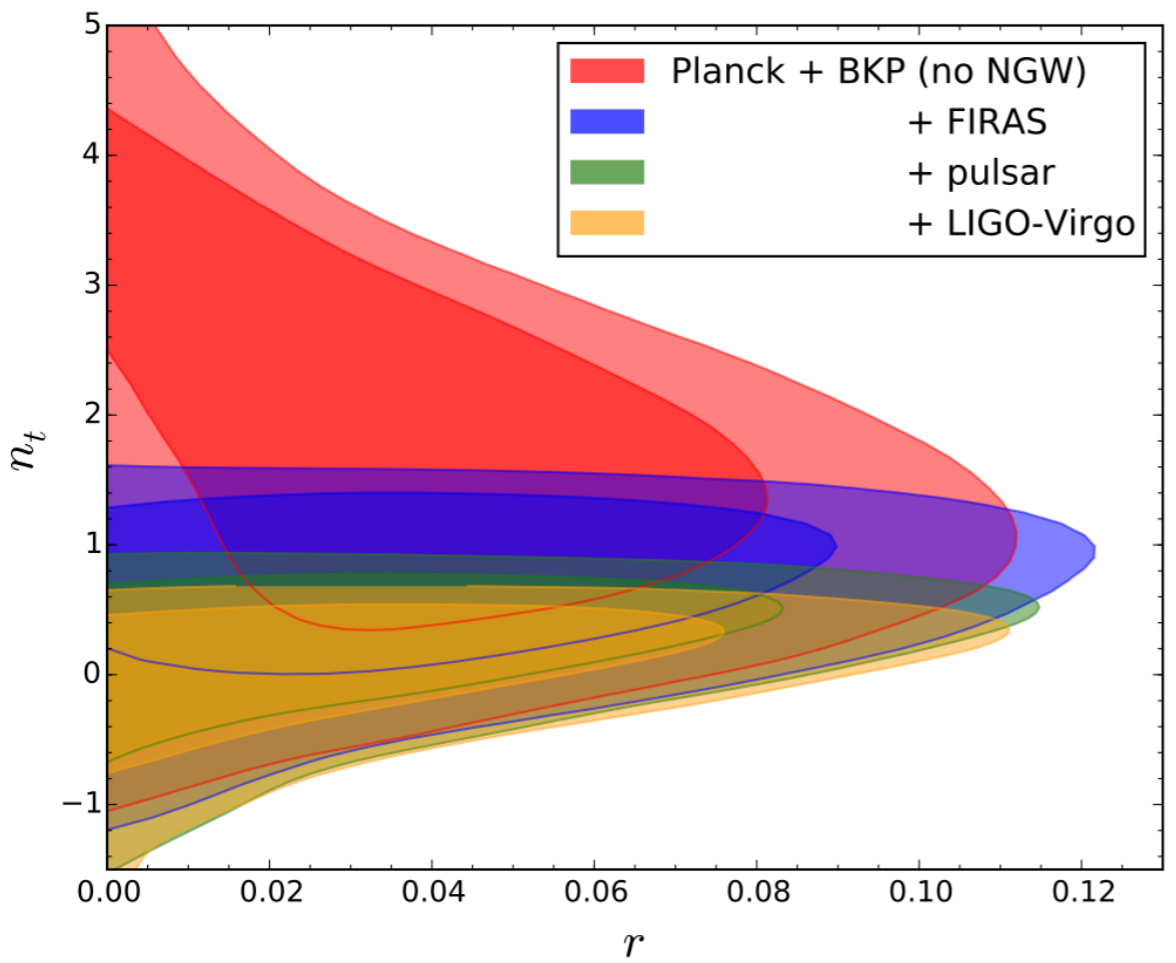
- $N_{\text{eff}}^{\text{BBN}}$: contribution of GWs to ρ_{rad} at BBN. A large $N_{\text{eff}}^{\text{BBN}}$ will result in an overproduction of ${}^4\text{He}$. The *IR* cutoff is the horizon size at BBN;
- $N_{\text{eff}}^{\text{CMB}}$: contribution of GWs to ρ_{rad} at decoupling. It affects CMB anisotropies. The *IR* cutoff is the horizon size at decoupling.



Ota et al., 2014
Chluba et al., 2014

Multi-Wavelength Constraints from Current Data

Constraints from (not so) current data



Dataset	r	n_t
<i>Planck</i> + BKP	< 0.089	$1.7^{+2.2}_{-2.0}$
<i>Planck</i> + BKP + FIRAS	< 0.098	$0.65^{+0.86}_{-1.1}$
<i>Planck</i> + BKP + pulsar	< 0.088	$0.20^{+0.69}_{-0.96}$
<i>Planck</i> + BKP + LIGO-Virgo	< 0.085	$0.04^{+0.61}_{-0.85}$
<i>Planck</i> + BKP, with $N_{\text{eff}}^{\text{GW}}$	< 0.082	$-0.05^{+0.58}_{-0.87}$
<i>Planck</i> + BKP + EXT, with $N_{\text{eff}}^{\text{GW}}$	< 0.080	$-0.05^{+0.57}_{-0.80}$
<i>Planck</i> + BKP + aLIGO	< 0.078	$-0.09^{+0.54}_{-0.78}$

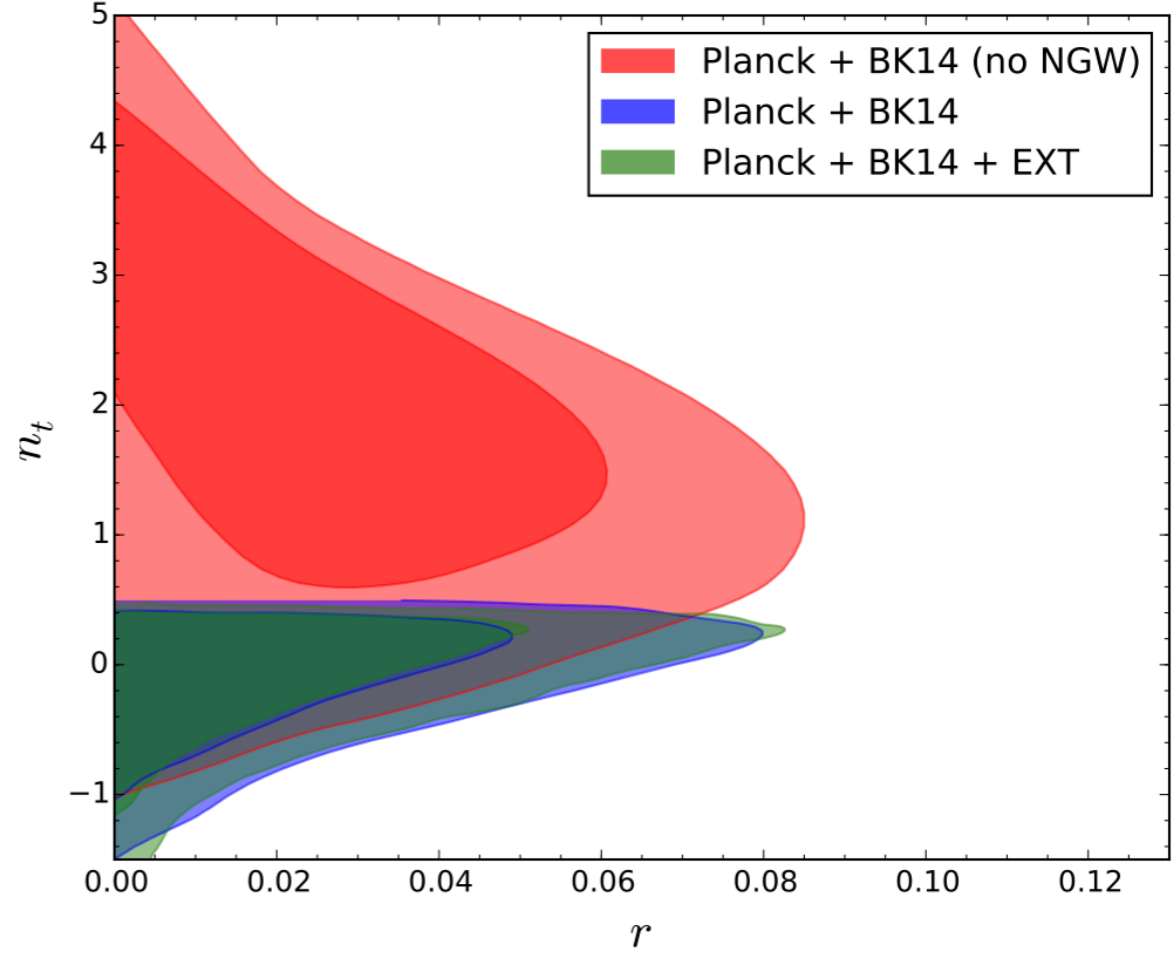
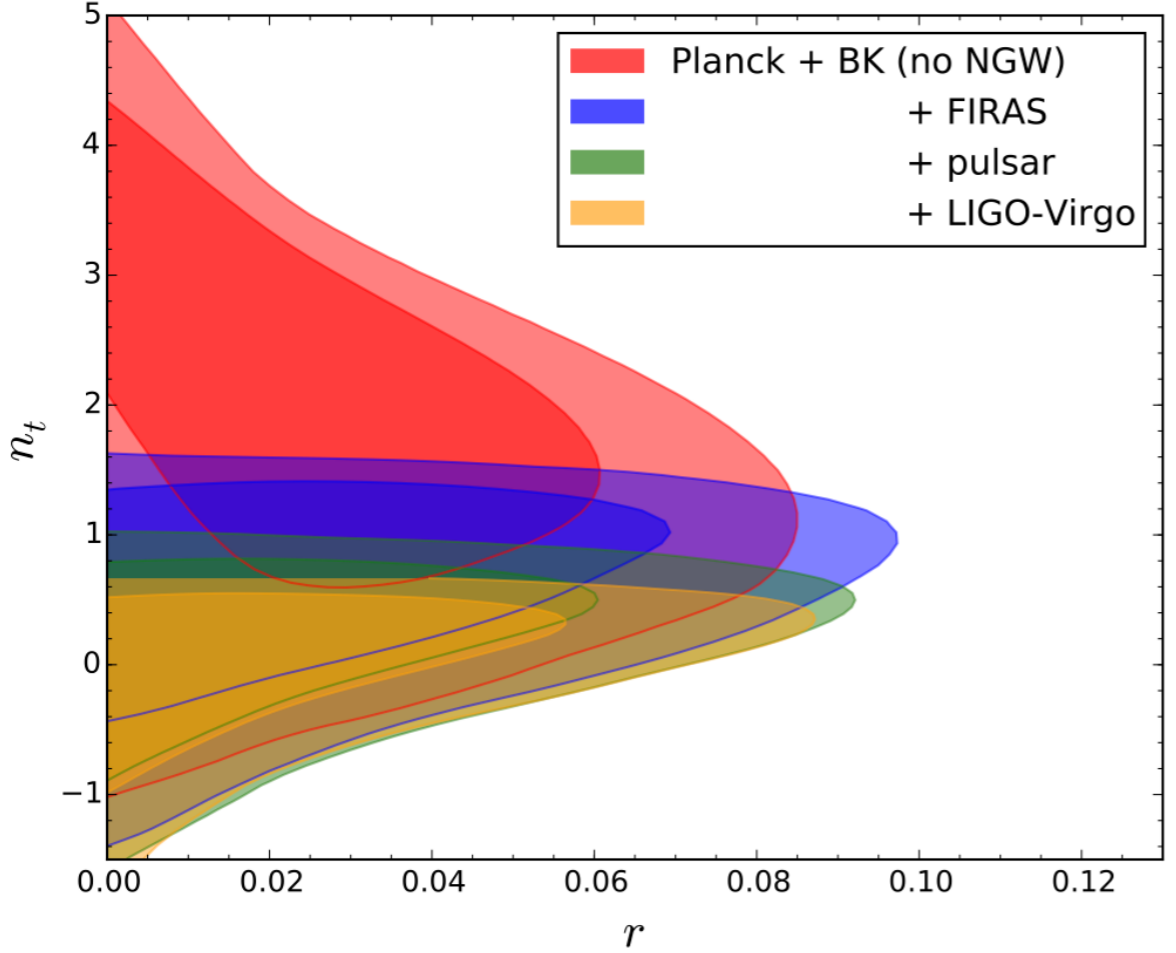
BAO + Deuterium

- ▶ $k_* = 0.01$: need blue n_t for low tensor power at small ℓ ;
- ▶ additional datasets: cut n_t exponentially $\Rightarrow r$ is lowered;
- ▶ adding N_{GW} : tighter constraints than external datasets.

AdvLIGO: 10x improvement of LIGO

still no detection from interferometers
but stronger bounds than CMB + $N_{\text{eff}}^{\text{GW}}$

Constraints from current data



95% CL constraints

Dataset	r	n_t
<i>Planck</i> + BK14	< 0.067	$1.8^{+2.0}_{-2.1}$
<i>Planck</i> + BK14 + LIGO-Virgo	< 0.067	$0.00^{+0.68}_{-0.91}$
<i>Planck</i> + BK14, with $N_{\text{eff}}^{\text{GW}}$	< 0.061	$-0.12^{+0.65}_{-0.84}$
<i>Planck</i> + BK14 + EXT, with $N_{\text{eff}}^{\text{GW}}$	< 0.061	$-0.10^{+0.63}_{-0.88}$
<i>Planck</i> + BK14 + aLIGO	< 0.060	$-0.16^{+0.63}_{-0.88}$

smaller error bars from BK14
 \Rightarrow tighter constraints on r



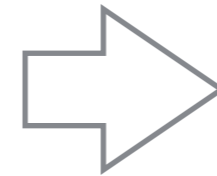
loss of sensitivity to n_t
 \Rightarrow constraints on n_t do not improve a lot

Forecasts: CMB and Direct Detection Experiments

Forecasts for COrE-like mission

COrE specifics (COrE Collab., 2011)

channel (GHz)	FWHM (arcmin)	$w^{-1/2} - T$ ($\mu\text{K} \cdot \text{arcmin}$)	$w^{-1/2} - Q, U$ ($\mu\text{K} \cdot \text{arcmin}$)
105	10	2.68	4.63
135	7.8	2.63	4.55
165	6.4	2.67	4.61
195	5.4	2.63	4.54
225	4.7	2.64	4.57



additional frequency channels
(from 45 GHz to 795 GHz):
used for foreground removal

Simulated likelihood (for simplicity consider only B -mode spectra) \rightarrow

$$L = \sum_{\ell} (2\ell + 1) \left[-1 + \frac{\hat{C}_{\ell}^{\text{tens}} + \hat{C}_{\ell}^{\text{lens}} + N_{\ell}}{C_{\ell}^{\text{tens}} + C_{\ell}^{\text{lens}} + N_{\ell}} + \log \left(\frac{C_{\ell}^{\text{tens}} + C_{\ell}^{\text{lens}} + N_{\ell}}{\hat{C}_{\ell}^{\text{tens}} + \hat{C}_{\ell}^{\text{lens}} + N_{\ell}} \right) \right]$$

- \hat{C}_{ℓ} : evaluated for cosmological parameters that describe the (assumed) true universe
- assuming a Gaussian beam, N_{ℓ} is

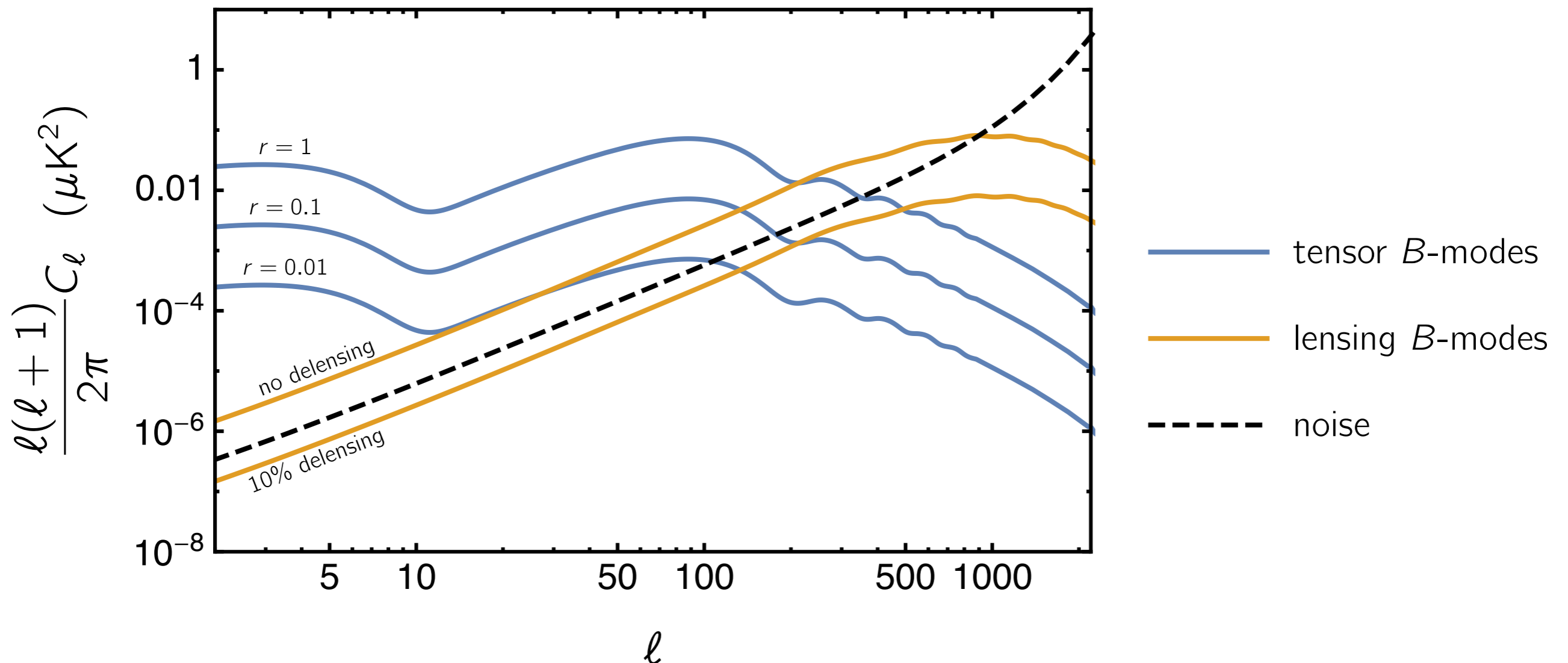
$$N_{\ell} = \left(\sum_i w_{(i)} e^{-\sigma_{(i)}^2 \ell(\ell+1)} \right)^{-1}$$

Delensing

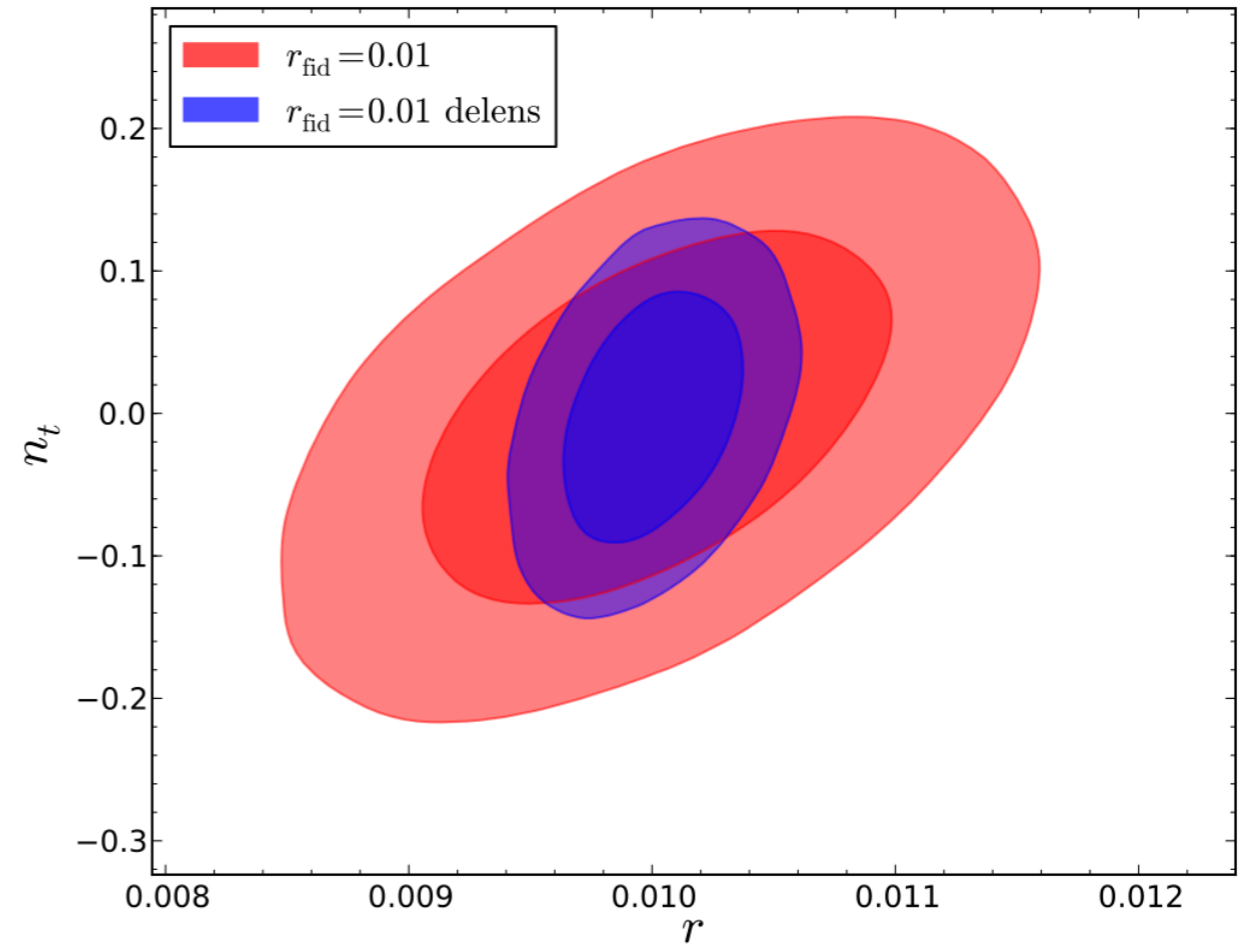
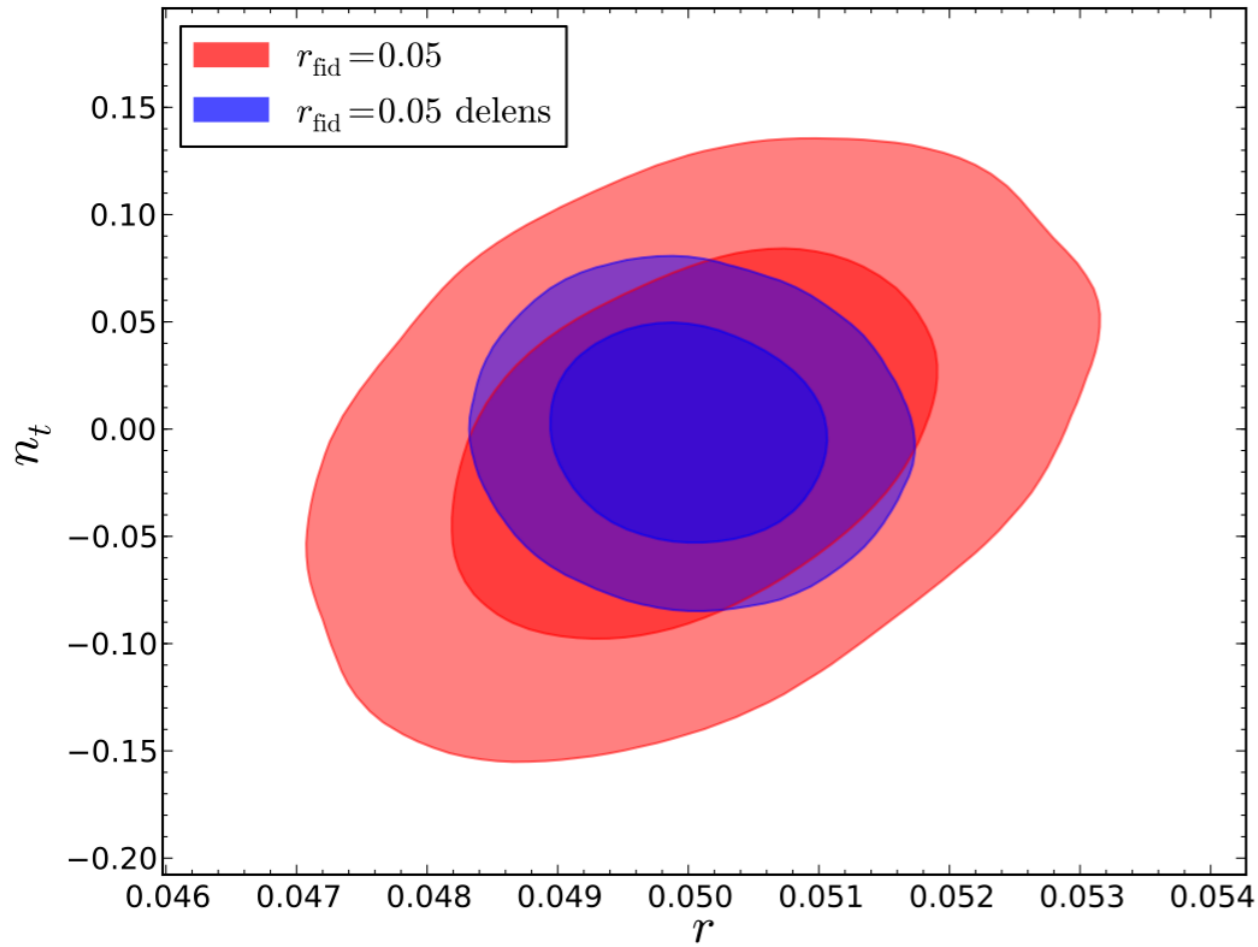
For an experiment with a noise level of order $\sim 1 \mu\text{K} \cdot \text{arcmin}$ post component separation, one can delens up to 10% (Errard et. al, 2015): **COrE can do it.**

$$C_{\ell}^{\text{lens}} \rightarrow 0.1 \times C_{\ell}^{\text{lens}}, \quad \hat{C}_{\ell}^{\text{lens}} \rightarrow 0.1 \times \hat{C}_{\ell}^{\text{lens}}$$

Creminelli et al., 2015



Results of COrE forecasts



95% CL constraints

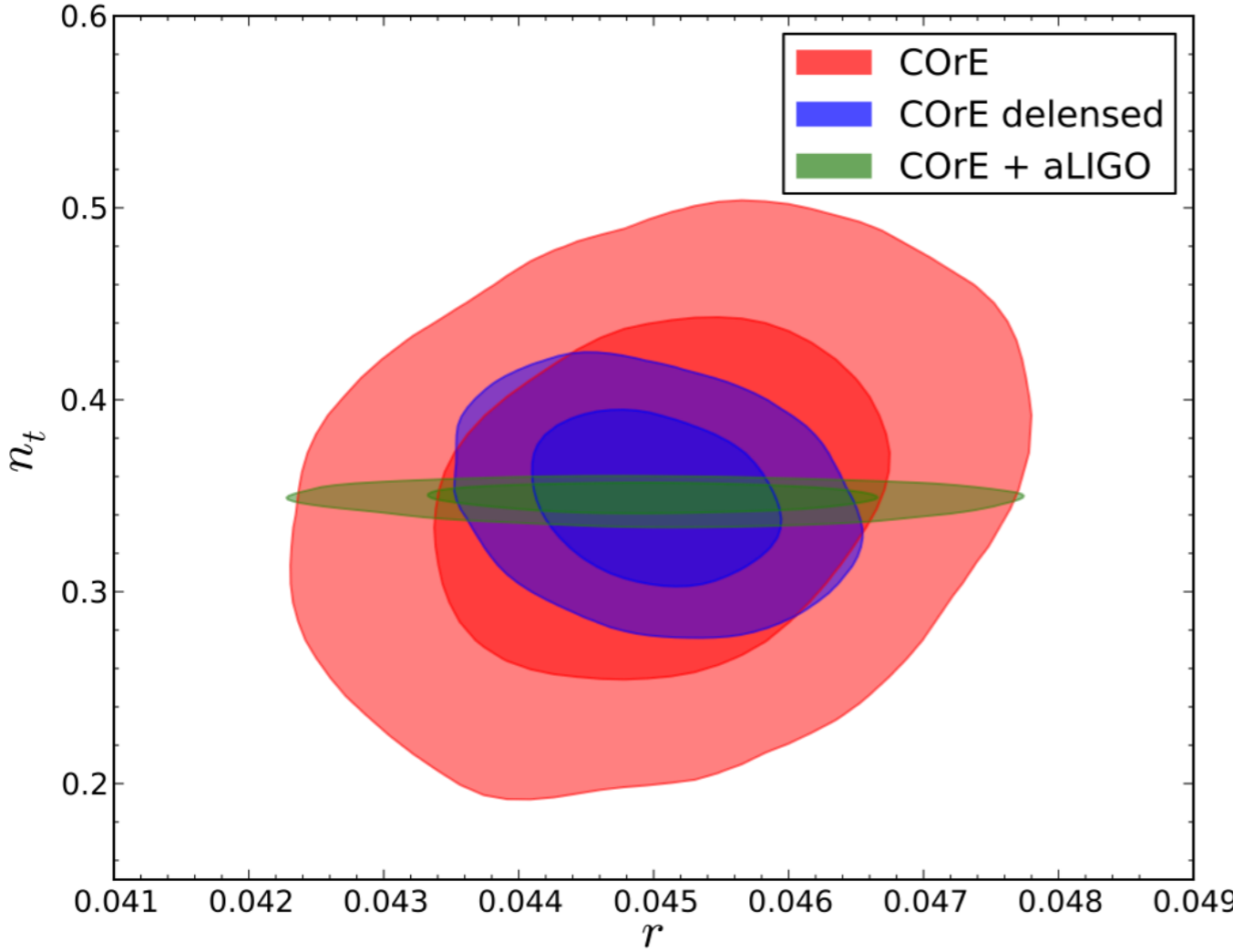
	r	n_t
fiducial	0.05	$-r/8 = -0.00625$
COrE	0.0500 ± 0.0012	$-0.0072^{+0.1108}_{-0.1143}$
COrE, delens.	0.05000 ± 0.00066	$-0.0023^{+0.0632}_{-0.0640}$
fiducial	0.01	$-r/8 = -0.00125$
COrE	0.01001 ± 0.00061	$-0.0024^{+0.1597}_{-0.1637}$
COrE, delens.	0.01000 ± 0.00024	$-0.0019^{+0.1074}_{-0.1088}$

- 10% delensing: break degeneracy between r and n_t ;
- r measured with a relative uncertainty of order 5×10^{-2} ;
- even with delensing, COrE cannot probe $n_t = -r/8$.

COrE + AdvLIGO forecast

	r	n_t
fiducial	0.045	0.35
$P + \text{BKP} + \text{aLIGO}$	< 0.095	0.354 ± 0.020
COrE	0.0450 ± 0.0011	0.348 ± 0.061
COrE, delens.	0.04500 ± 0.00060	0.350 ± 0.029
COrE + aLIGO	0.0450 ± 0.0010	0.3483 ± 0.0053

AdvLIGO will see Ω_{GW} ,
but *Planck* will not see r



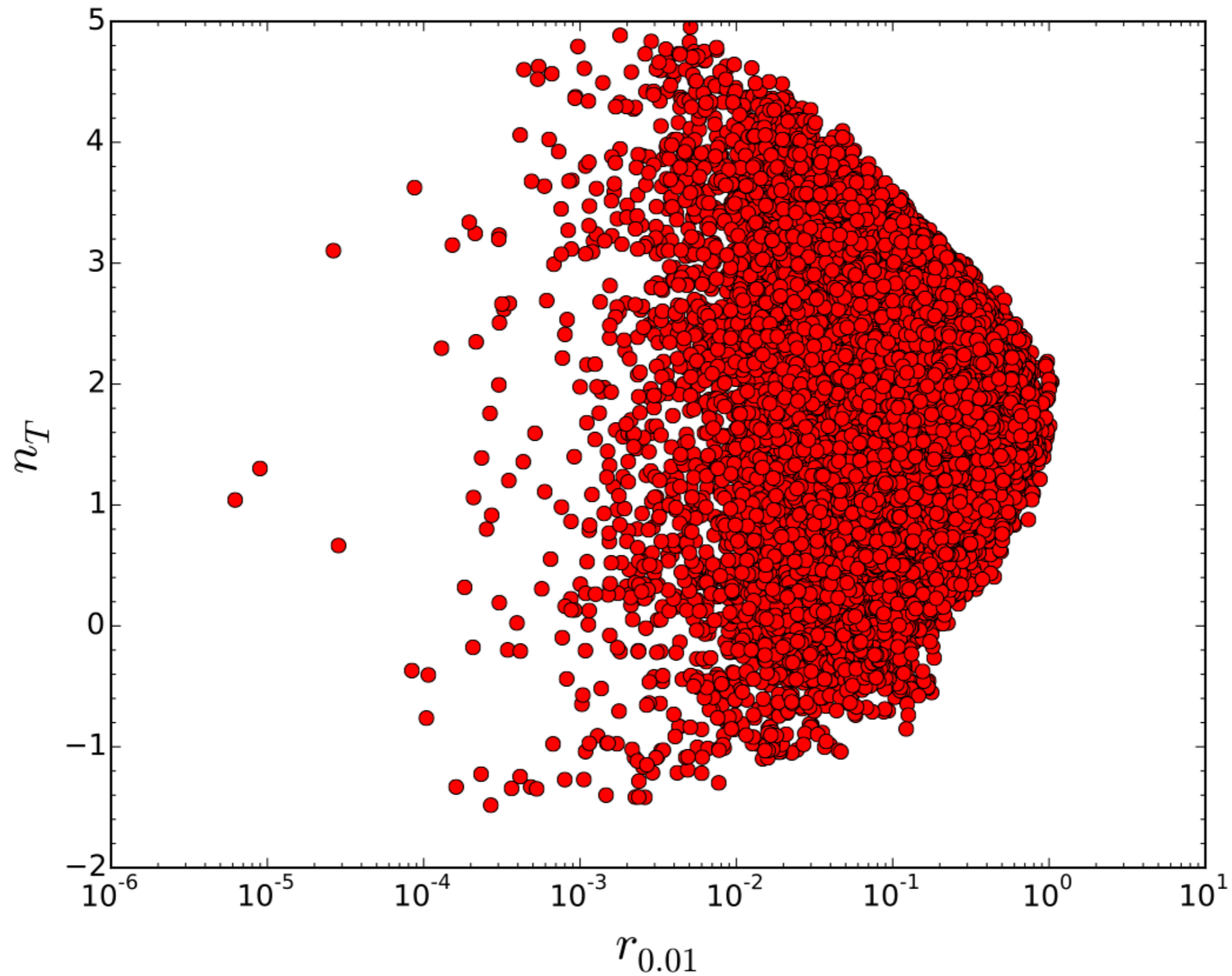
$\sim 10x$ improvement in σ_{n_t}
important to combine CMB
with high-frequency observables

Conclusions

- constraints from small-scale probes forbid very blue n_t , resulting in tighter bounds on r ;
- including contribution of GWs to N_{eff} gives the strongest bounds, comparable to AdvLIGO;
- new BK14 data improve constraints on r , but not on n_t ;
- forecasts for COrE-like mission – 5 channels (assuming that foregrounds are subtracted).
Fiducials with $n_t = -r/8$:
 - $\sigma_r/r \approx 10^{-2}$;
 - COrE will not be able to test $n_t = -r/8$ with high accuracy;
- combining CMB with small-scale probes will be necessary.

Backup slides

Linear sampling and priors on r



NEC and blue tilt

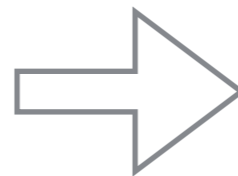
Consistency relation between $\epsilon_H \equiv -\dot{H}/H^2$ and $n_t \rightarrow$

$$n_t = -2\epsilon_H$$

EFT for π in decoupling limit (where the mixing between gravity and inflaton is negligible):

$$\mathcal{L}_\pi = -M_{\text{P}}^2 \dot{H} \underbrace{\left(\dot{\pi}^2 - \frac{(\partial_i \pi)^2}{a^2} \right)}_{\text{coefficient of } (\partial_i \pi)^2 \text{ is } M_{\text{P}}^2 \dot{H}} + 2M_2^4 \left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi} \frac{(\partial_i \pi)^2}{a^2} \right) + \dots ,$$

coefficient of
 $(\partial_i \pi)^2$ is $M_{\text{P}}^2 \dot{H}$



\dot{H} must be smaller than zero
to avoid instabilities

\rightarrow FLRW metric: $\dot{H} < 0$ is the Null Energy Condition, *i.e.* $T_{\mu\nu} k^\mu k^\nu > 0$ for all lightlike k^μ .

Suitable choice of higher derivative operators \rightarrow can have $\dot{H} > 0$ while maintaining stability (Creminelli et al., 2006).

Number of e-folds of inflation

Number N_* of e-folds of inflation after k_* has left the horizon

$$N_* \equiv \log \frac{a_{\text{end}}}{a_*} = -\log \frac{k_*}{H_0} + \log \frac{H_*}{H_0} + \log \frac{a_{\text{end}}}{a_{\text{reh}}} + \log \frac{a_{\text{reh}}}{a_0}$$

where t_{end} marks the transition to radiation dominance.

Standard assumption: **reheating is a period of matter domination**. Then \rightarrow

$$\frac{k_{\text{end}}}{\text{Mpc}^{-1}} = T_{\text{CMB}} \exp \left[\log \sqrt[3]{\beta} - \log \sqrt{3} + \log \sqrt[3]{\alpha^2} + \log \sqrt[3]{\frac{\pi^2}{45} g_*(T_{\text{CMB}})} \right]$$

- $E_{\text{end}} = (\alpha M_{\text{P}})^4$: energy density at the end of inflation;
- $T_{\text{reh}} = \beta M_{\text{P}}$: temperature at beginning of radiation dominance.

Assuming to have instant reheating ($\alpha = \beta$) at the GUT scale $E_{\text{end}} \approx 10^{16}$ GeV



$$k_{\text{end}} \approx 2 \times 10^{23} \text{ Mpc}^{-1}$$