Observational constraints in nonlocally modified gravity

Yves DIRIAN

with S. Foffa, M. Kunz, M. Maggiore, V. Pettorino

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Université de Genève
Faculté des Sciences
Département de Physique Théorique

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Introduction: Theory

- Inspiration (Arkani-Hamed et al. 2002, Dvali 2006)

$$\mathcal{L}_{\text{proca}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{2}A_\mu A^\mu - A_\mu j^\mu \quad \Leftrightarrow \quad \mathcal{L}_{\text{nl}} = -\frac{1}{4}F_{\mu\nu}\left(1 - \frac{m^2}{\Box}\right)F^{\mu\nu} - A_\mu j^\mu$$

where $(\Box^{-1}\phi)(x) = \int d^4y\ G(x,y)\phi(y)$

- Applying the same idea to Fierz-Pauli massive gravity

$$\mathcal{L}_{\text{nl}} = \frac{1}{2}h_{\mu\nu}\left(1 - \frac{m^2}{\Box}\right)\mathcal{E}^{\mu\nu\rho\sigma}h_{\rho\sigma} - 2m^2\chi\frac{1}{\Box}\partial_\mu\partial_\nu(h^{\mu\nu} - \eta^{\mu\nu}h) + \frac{\kappa}{2}h_{\mu\nu}T^{\mu\nu}$$

$\longrightarrow$ Obstruction: covariantization $\Rightarrow g^{\mu\nu}R_{\mu\nu} = 0$ "Covariant vDVZ discontinuity"

$$\left[\left(1 - \frac{m^2}{\Box_g}\right)G_{\mu\nu}\right]^T = 8\pi G T_{\mu\nu}$$

(Porrati 2002; Jaccard, Maggiore, Mitsou 2013)

- ▷ Unviable background cosmology
- ▷ $\Box^{-1}R_{\mu\nu} \subset \Box^{-1}G_{\mu\nu}$ generates instabilities (Ferreira, Maroto 2013)
- ▷ $g_{\mu\nu}\Box^{-1}R \subset \Box^{-1}G_{\mu\nu}$ stable (Foffa, Maggiore, Mitsou 2013)

Introduction: Phenomenology

Model RT : $G_{\mu\nu} - m^2 \left(g_{\mu\nu}\Box^{-1}_{ret} R\right)^T = 8\pi G\, T_{\mu\nu}$ (Maggiore 2013)

- Two models modifying General Relativity nonlocally – in the infrared
  - m^2 sets a new reference energy scale

    $\longrightarrow$ Nonlocal terms contributes for $\Box_g \ll m^2$ and vice versa
- Phenomenological approach
- Interesting cosmology:
  - FRW background/linear perturbations
  - Observational constraints and model comparison

Model RR : $S_{\text{RR}} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,\left[R - \frac{1}{2}m^2 R \frac{1}{\Box^2} R\right]$ (Maggiore, Mancarella 2013)

Application to Cosmology

Model RT

$$G_{\mu\nu} - m^2(g_{\mu\nu}\Box_{ret}^{-1}R)^T = 8\pi G T_{\mu\nu}$$

Model RR

$$G_{\mu\nu} - m^2 K_{\mu\nu}(\Box_{ret}^{-1}R, \Box_{ret}^{-2}R) = 8\pi G T_{\mu\nu}$$

- Resolution method: Localisation

$$\Box V = R \quad \Rightarrow \quad V = \Box^{-1}R + V^{(hom)}$$

  - Auxiliary fields with *vanishing initial conditions*
  - They are not genuine (*freely* propagating) degrees of freedom

$$G_{\mu\nu} + m^2\left[U g_{\mu\nu} - \frac{1}{2}(\nabla_\mu S_\nu + \nabla_\nu S_\mu)\right] = 8\pi G T_{\mu\nu}$$

$$\Box_g U = -R, \qquad \partial_\mu U = \frac{1}{2}\nabla_\nu(\nabla_\mu S^\nu + \nabla^\nu S_\mu)$$

$$G_{\mu\nu} - m^2 K_{\mu\nu}(V, S) = 8\pi G T_{\mu\nu}$$

$$\Box_g V = R \quad , \quad \Box_g S = V$$

- Specialisation to flat FRW

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$$

Background evolution

- Modified Friedmann equations :

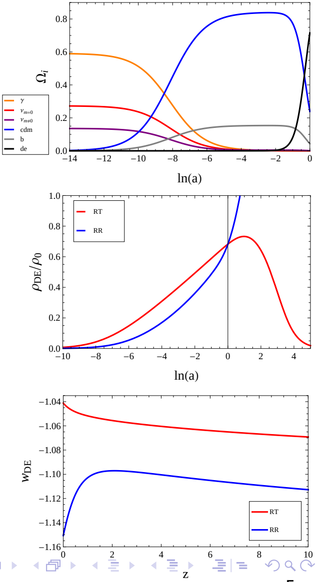
$$H^2(t) = 8\pi G \sum_i \bar{\rho}_i(t) + m^2 Y\left(\{\bar{V}_k\}, H(t)\right)$$
$$+ \quad \text{auxiliary EoM for } \{\bar{V}_k\}$$

- $m^2 Y \equiv \bar{\rho}_{\rm DE}(t)$: Dynamical dark energy
- $\Box^{-1} R\big|_{\rm RD} = 0$: Late-time effectiveness
- Flatness today: $m_{\rm RT} \simeq 0.67 H_0$, $m_{\rm RR} \simeq 0.28 H_0$
- From $\dot{\bar{\rho}}_{\rm DE} = -3H(1 + w_{\rm DE})\bar{\rho}_{\rm DE}$

Fit : $\qquad w(t) = w_0 + \big(1 - a(t)\big) w_a$

RT: $w_0 \simeq -1.04$, $w_a \simeq -0.02$
RR: $w_0 \simeq -1.15$, $w_a \simeq 0.08$

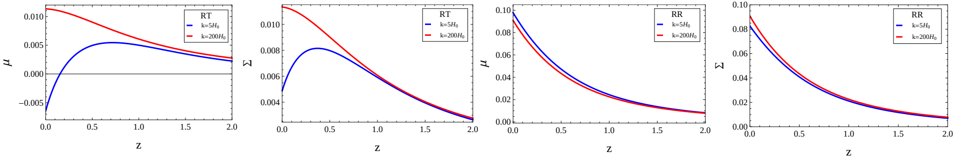
$\longrightarrow$ On the phantom side: $w_{\rm DE} < -1$



Scalar perturbations and Structure Formation

- Gravitational Ψ and lensing potential $(\Psi - \Phi)$ (YD, Foffa, Khosravi, Kunz, Maggiore 2014)

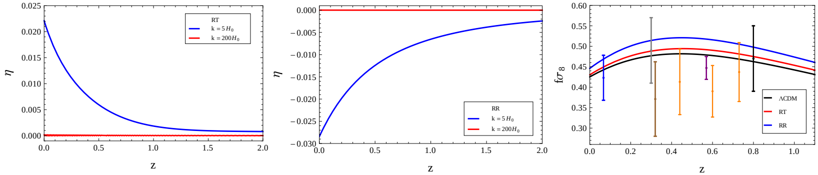
$$\Psi = \left[1 + \mu(z,k)\right]\Psi_{\Lambda\mathrm{CDM}}, \qquad (\Psi - \Phi) = \left[1 + \Sigma(z,k)\right](\Psi - \Phi)_{\Lambda\mathrm{CDM}}$$



▷ Fit: $\mu(t) = \mu_s a(t)^s$ RT: $\mu_s = 0.01$, $s = 0.8$, RR: $\mu_s = 0.09$, $s = 2$ (EUCLID: $\Delta\mu_s = 0.01$)

- Gravitational slip and RSD (6dF, SDSS LRG, BOSS LOWZ+CMASS, WiggleZ, VIPERS)

$$\eta = (\Psi + \Phi)/\Phi, \qquad f \equiv \frac{d\ln D}{d\ln a} \ \text{with} \ D(a) \sim \delta_M(a)$$



▷ Consistency with structure formation

▷ Nonlinear structure formation for RR: N-body simulation (Barreira et al. 2014)

Boltzmann Code and Parameter Inference

- Implementation in CLASS: Computation of CMB and LSS observables
- Observational constraints and model comparison with MONTEPYTHON

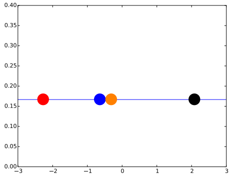
(Lesgourgues, Audren et al.)

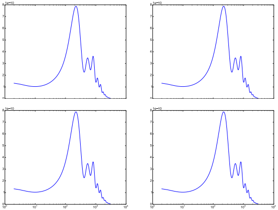
- Cosmological scenario: *Planck* baseline
  - ▷ 6 cosmo parameters varied: $\{\omega_b, \omega_c, H_0, A_s, n_s, z_{reio}\}$
  - ▷ Neutrino: Two massless species $N_{eff} = 2.03351$, one massive $m_\nu = 0.06eV$
- Datasets:
  - ▷ CMB: Planck 2013 (TT+lens.), Planck 2015 (TT+TE+EE+lens.)
  - ▷ Supernovae: SDSS-II/SNLS3 Joint Light-Curve Analysis (JLA 2014)
  - ▷ BAO: BOSS LOWZ+CMASS DR10&11 (iso., aniso.), 6dF and SDSS MGS
  - ▷ H_0: HST $(70.6 \pm 3.3, 73.0 \pm 2.4, 73.8 \pm 2.4)$

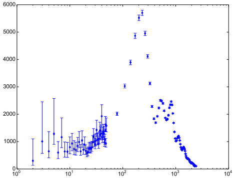
(YD, Foffa, Kunz, Maggiore, Pettorino, 2014)
(YD, Foffa, Kunz, Maggiore, Pettorino, in prep.)

Observational constraints and parameter inference

- Bayesian inference:
  - ▷ Observed datasets: Planck 2013/2015, JLA, BAO, HST, etc
  - ▷ Statistical models: ΛCDM, RT and RR with $\{\omega_{\rm b}, \omega_{\rm c}, H_0, A_{\rm s}, n_{\rm s}, z_{\rm reio}\}$
  - ▷ Parameter estimation: Update our degree of belief through observations

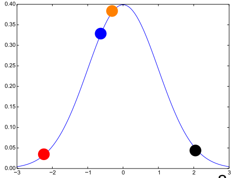


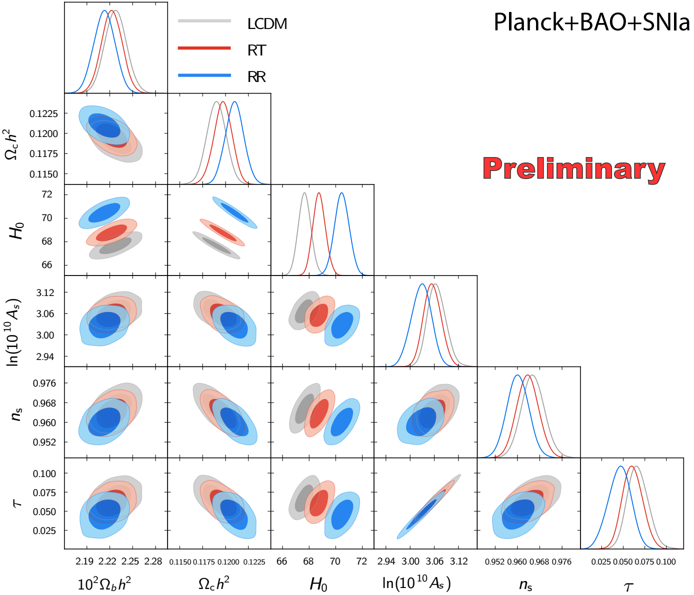




  - ▷ Minimum χ^2 estimation:

$$\chi^2 = \sum_{\rm dataset} \chi^2_{\rm dataset} \quad \text{with} \quad \chi^2_{\rm dataset} = \sum_i \frac{(\theta^i_{\rm theo} - \theta^i_{\rm obs})^2}{(\sigma^i_{\rm obs})^2}$$



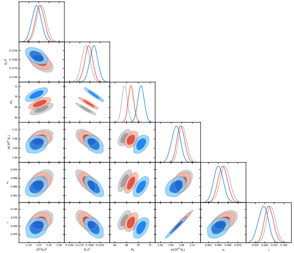


(YD, Foffa, Kunz, Maggiore, Pettorino in prep.)

Observational constraints and parameter inference

	Planck			BAO+Planck+JLA		
Param	ΛCDM	$g_{\mu\nu}\Box^{-1}R$	$R\Box^{-2}R$	ΛCDM	$g_{\mu\nu}\Box^{-1}R$	$R\Box^{-2}R$
ω_c	$0.1194^{+0.0015}_{-0.0014}$	$0.1195^{+0.0015}_{-0.0014}$	$0.1191^{+0.0015}_{-0.0014}$	$0.119^{+0.001}_{-0.001}$	$0.1197^{+0.001}_{-0.001}$	$0.121^{+0.001}_{-0.001}$
H_0	$67.5^{+0.65}_{-0.66}$	$68.86^{+0.69}_{-0.7}$	$71.51^{+0.81}_{-0.84}$	$67.67^{+0.47}_{-0.5}$	$68.76^{+0.51}_{-0.46}$	$70.44^{+0.56}_{-0.56}$
$\chi^2_{\rm min}$	12943.3	12943.2	12941.7	13631.0	13631.6	13637.74

	BAO+Planck+JLA+$H_0 = 73.8 \pm 2.4$		
Param	ΛCDM	$g_{\mu\nu}\Box^{-1}R$	$R\Box^{-2}R$
ω_c	$0.1185^{+0.001}_{-0.001}$	$0.1194^{+0.001}_{-0.001}$	$0.1207^{+0.001}_{-0.001}$
H_0	$67.93^{+0.48}_{-0.43}$	$68.91^{+0.49}_{-0.5}$	$70.65^{+0.52}_{-0.54}$
$\chi^2_{\rm min}$	13637.5	13636.1	13638.9



- Few parameters with $\gtrsim 1\sigma$ deviation from ΛCDM
  $\longrightarrow$ Bigger H_0 in nonlocal models
- Nonlocal vs ΛCDM: Overall $|\Delta\chi^2| \lesssim 2$
  $\longrightarrow$ Mostly statistically equivalent to ΛCDM
- Planck: RR fits slightly better C_l^{TT} at low-l
- BAO+Planck+JLA: RR creates a Planck-JLA 1σ-tension

Observational constraints and parameter inference

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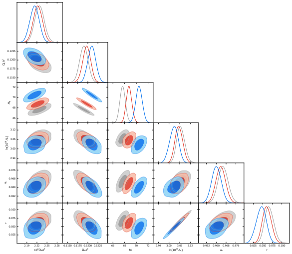
$\Delta\chi^2_{\rm min}$	1.6	1.5	0	0	0.6	6.7

	BAO+Planck+JLA+$H_0 = 73.0 \pm 2.4$		
Param	ΛCDM	$g_{\mu\nu}\Box^{-1}R$	$R\Box^{-2}R$
ω_c	$0.117^{+0.0014}_{-0.0014}$	$0.1182^{+0.0013}_{-0.0014}$	$0.1201^{+0.0013}_{-0.0013}$
H_0	$68.72^{+0.61}_{-0.63}$	$69.60^{+0.66}_{-0.63}$	$71.14^{+0.72}_{-0.69}$
$\Delta\chi^2_{\rm min}$	1.6	0	2



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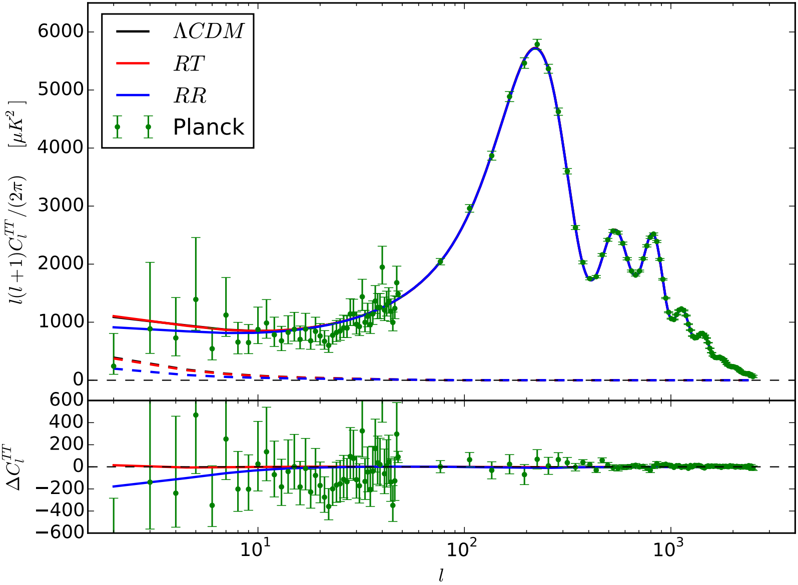

ΛCDM
RT
RR
Planck
$l(l+1)C_l^{TT}/(2\pi)$ $[\mu K^2]$
ΔC_l^{TT}
l
6000
5000
4000
3000
2000
1000
0
600
400
200
0
−200
−400
−600
10^1
10^2
10^3

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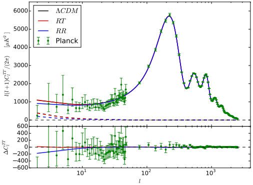

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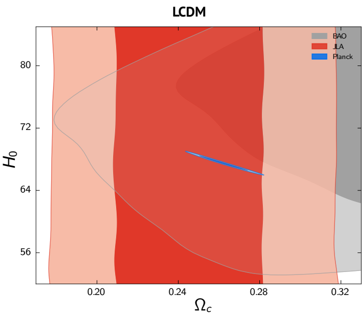
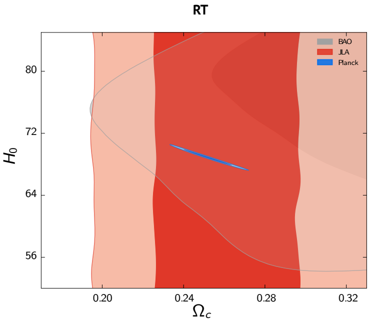
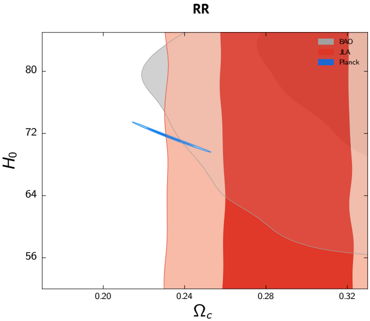




$$\Delta\omega_c = \Delta(\Omega_c h^2) \sim 1\%$$



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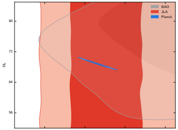

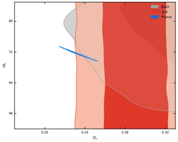

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Bayesian model selection

- Computation of the Bayes factor: done by considering the nested models

$$G_{\mu\nu} - m^2\left(g_{\mu\nu}\Box_{\rm ret}^{-1}R\right)^T - g_{\mu\nu}\Lambda = 8\pi G T_{\mu\nu}$$

$$\mathcal{L} = \frac{1}{16\pi G}\left[R - 2\Lambda - m^2 R\Box^{-2}R\right] + \mathcal{L}_m$$

with cosmological parameter space $\{\omega_{\rm b}, H_0, A_{\rm s}, n_{\rm s}, z_{\rm reio}, \Omega_\Lambda, \Omega_{de}\}$

$\longrightarrow$ Non-informative priors are flat on Ω_Λ and Ω_{de}

- Three statistical models in each case: $\mathcal{M}_{\Lambda+de}$, $\mathcal{M}_\Lambda$, $\mathcal{M}_{de}$
- Bayes theorem

$$P(\theta|d,\mathcal{M}) = \frac{P(d,\mathcal{M}|\theta)P(\theta|\mathcal{M})}{P(d,\mathcal{M})}$$

- Savage-Dickey density ratio:

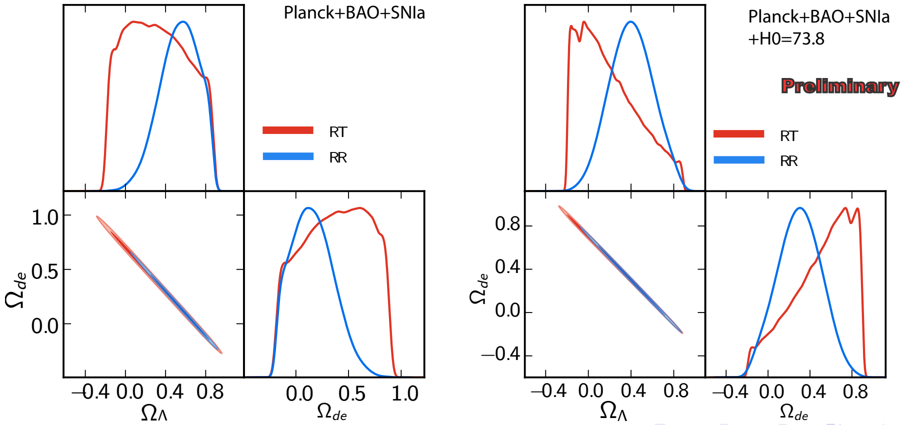
$$B_{\Lambda/(\Lambda+de)} = \frac{P(d,\mathcal{M}_\Lambda)}{P(d,\mathcal{M}_{\Lambda+de})} = \left.\frac{P(\Omega_{de}|d,\mathcal{M}_{\Lambda+de})}{P(\Omega_{de}|\mathcal{M}_{\Lambda+de})}\right|_{\Omega_{de}=0}$$

$\longrightarrow$ Model Λ (dis)favored with betting odds $B_{\Lambda/(\Lambda+de)}$: 1 wrt $\Lambda + de$

Bayesian model selection

- We are interested in,

$$B_{de/\Lambda} = \frac{P(d,\mathcal{M}_{de})}{P(d,\mathcal{M}_{\Lambda})} = \frac{P(d,\mathcal{M}_{de})}{P(d,\mathcal{M}_{\Lambda+de})}\frac{P(d,\mathcal{M}_{\Lambda+de})}{P(d,\mathcal{M}_{\Lambda})} = \frac{B_{de/(\Lambda+de)}}{B_{\Lambda/(\Lambda+de)}} = \frac{P(\Omega_\Lambda|d,\mathcal{M}_{\Lambda+de})|_{\Omega_\Lambda=0}}{P(\Omega_{de}|d,\mathcal{M}_{\Lambda+de})|_{\Omega_{de}=0}}$$

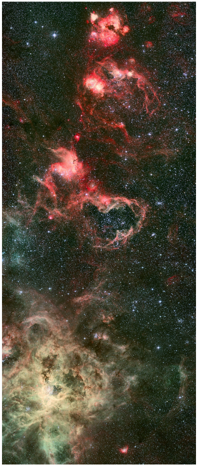


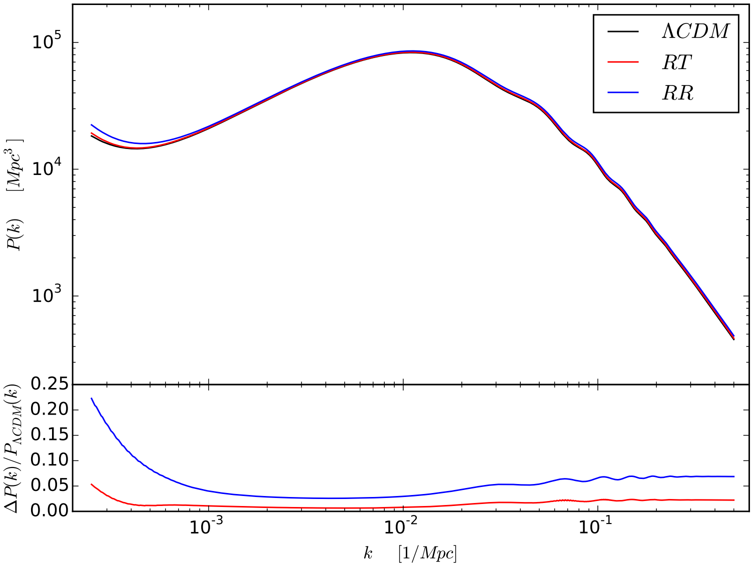
Conclusion

$$G_{\mu\nu} - m^2 (g_{\mu\nu} \Box_{\text{ret}}^{-1} R)^T = 8\pi G T_{\mu\nu}$$

$$\mathcal{L} = \frac{1}{16\pi G} \left[R - m^2 R \Box^{-2} R \right] + \mathcal{L}_m$$

- Two observationally viable models of gravity (JCAP 1504 (2015) 04, 044, arXiv:1411.7692)
- Phenomenological side
  - Well behaved dynamical dark energy
  - Same number of free parameters than ΛCDM
  - Fit the data as well as ΛCDM
  - $\longrightarrow$ Provide observationally consistent alternatives to ΛCDM
- Theoretical side: Effective models/terms
  - Suggest effects/mechanisms for dynamical dark energy generation
  - Dimensional transmutation, conformal anomaly (Maggiore 2015)





ΛCDM
RT
RR
10^5
10^4
10^3
$P(k) \quad [Mpc^3]$
0.25
0.20
0.15
0.10
0.05
0.00
$\Delta P(k)/P_{\Lambda CDM}(k)$
10^{-3}
10^{-2}
10^{-1}
$k \quad [1/Mpc]$

Structural aspects

Model RT : $G_{\mu\nu} \; - \; m^2 \left(g_{\mu\nu}\Box^{-1}_{ret} R\right)^{\mathrm{T}} \; = \; 8\pi G \; T_{\mu\nu}$

$\downarrow$

$$\mathcal{L}_{lin} = \frac{1}{2} h_{\mu\nu}\mathcal{E}^{\mu\nu\alpha\beta} h_{\alpha\beta} - \frac{1}{2} m^2 h_{\mu\nu} P^{\mu\nu} P^{\alpha\beta} h_{\alpha\beta}$$

$\downarrow$ Covariantisation

$$S_{\mathrm{RR}} = M^2 \int d^{d+1}x \sqrt{-g} \left[R - \frac{1}{2} m^2 R \frac{1}{\Box^2} R \right]$$

$\downarrow$

Model RR : $G_{\mu\nu} - m^2 K_{\mu\nu}\left[\Box^{-1}_{ret} R, \Box^{-2}_{ret} R\right] = 8\pi G \; T_{\mu\nu}$

Model RT : $G_{\mu\nu} - m^2 \left(g_{\mu\nu}\Box_{ret}^{-1}R\right)^{\mathrm{T}} = 8\pi G\, T_{\mu\nu}$

$$\mathcal{L}_{lin} = \frac{1}{2}h_{\mu\nu}\mathcal{E}^{\mu\nu\alpha\beta}h_{\alpha\beta} - \frac{1}{2}m^2 h_{\mu\nu}P^{\mu\nu}P^{\alpha\beta}h_{\alpha\beta}$$

Covariantisation

$$S_{\mathrm{RR}} = M^2 \int d^{d+1}x\sqrt{-g}\left[R - \frac{1}{2}m^2 R\frac{1}{\Box^2}R\right]$$

Propagator

$$\tilde{D}_{\mathrm{GR}}(k) + \frac{-i}{k^2} + \frac{-i}{-k^2+m^2}$$

→ No vDVZ discontinuity

→ Scalars are not genuine DoF

Model RR : $G_{\mu\nu} - m^2 K_{\mu\nu}\left[\Box_{ret}^{-1}R, \Box_{ret}^{-2}R\right] = 8\pi G\, T_{\mu\nu}$

Model RT : $G_{\mu\nu} - m^2 \left(g_{\mu\nu}\Box^{-1}_{ret}R\right)^{\mathrm{T}} = 8\pi G\, T_{\mu\nu}$

↓

$$\mathcal{L}_{lin} = \frac{1}{2} h_{\mu\nu}\mathcal{E}^{\mu\nu\alpha\beta}h_{\alpha\beta} - \frac{1}{2} m^2 h_{\mu\nu}P^{\mu\nu}P^{\alpha\beta}h_{\alpha\beta}$$

↓ Covariantisation

$$S_{\mathrm{RR}} = M^2 \int d^{d+1}x\sqrt{-g}\left[R - \frac{1}{2}m^2 R\frac{1}{\Box^2}R\right]$$

Propagator ↓

$$\tilde{D}_{\mathrm{GR}}(k) + \frac{-i}{k^2} + \frac{-i}{-k^2+m^2}$$

→ No vDVZ discontinuity

→ Scalars are not genuine DoF

↓ Localisation

$$S^{\mathrm{loc}}_{\mathrm{RR}} = \int d^{d+1}x\sqrt{-g}\left[MR\Phi + \frac{1}{2m^2}(\Box\Phi)^2\right]$$

↓ Einstein Frame

$$S^{\mathrm{loc}}_{\mathrm{RR}} = \int d^{d+1}x\sqrt{-\bar{g}}\left[M^2\bar{R} - \frac{1}{2}\bar{\nabla}_\mu\phi\bar{\nabla}^\mu\phi + \frac{1}{2}\bar{\nabla}_\mu\psi\bar{\nabla}^\mu\psi - \frac{m^2}{2}e^{-(\phi+\psi)/\tilde{M}}\psi^2\right]$$

(YD, Mitsou 2014)

↓ Jordan Frame + Var. Principle + Solving for scalars w/ vanishing IC

Model RR : $G_{\mu\nu} - m^2 K_{\mu\nu}\left[\Box^{-1}_{ret}R, \Box^{-2}_{ret}R\right] = 8\pi G\, T_{\mu\nu}$

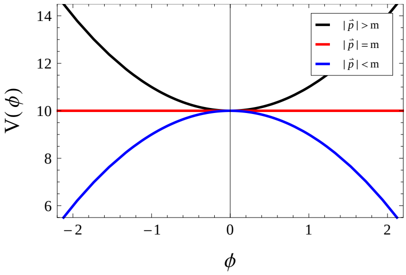
Not ghost-free but ghost not free

- Scalars have vanishing initial conditions
- Do not contribute to the DoF count
- Do not propagate *freely*

$$\left(\Box - m^2\right)\Phi = 0 \quad \text{and vanishing IC}$$

$$\Rightarrow \quad \Phi = 0$$

▷ But propagate once sourced



- On Minkowski massive ghosts are unstable for $|\vec{p}| < m$

$$E_{\vec{p}} = \sqrt{\vec{p}^2 - m^2} \quad \Rightarrow \quad \Phi(x) \quad \sim \quad e^{-i\sqrt{\vec{p}^2 - m^2}\,t}$$

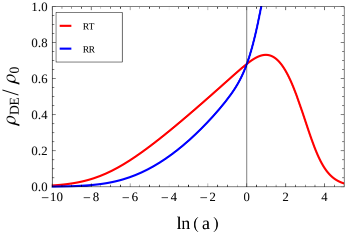
$$\overset{|\vec{p}|<m}{=} \quad e^{\sqrt{|\vec{p}^2 - m^2|}\,t}$$

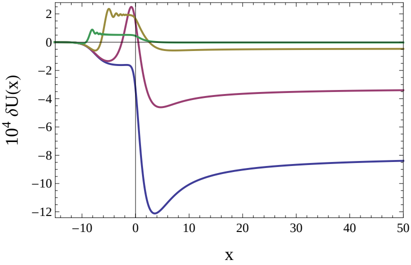
$$\overset{|\vec{p}|=0}{=} \quad e^{mt}$$

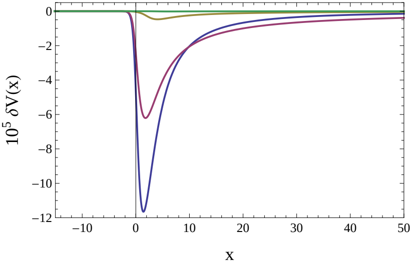
Big Rip scenario

- In our (phenomenological) case $m \sim H_0$
  $\longrightarrow$ FRW solution that makes sense
- RR: Perturbations tamed by Hubble friction
  - ▷ Big rip scenario:

$$\lim_{t \to t_{\mathbf{rip}}} a(t) = \infty$$







(YD, Mitsou 2014)

$$G_{\mu\nu} - m^2(g_{\mu\nu}\Box^{-1}R)^T = 8\pi G T_{\mu\nu}$$

(Maggiore 2013; Foffa, Maggiore, Mitsou 2013)

- Structural features
  - No ghost-like DoF (Foffa, Maggiore, Mitsou 2013)
  - Degrees of freedom: Massless graviton
  - No vDVZ discontinuity (Kehagias, Maggiore 2014)
  - Correction to GR of $\mathcal{O}(m^2r^2)$ on Schwarzchild in $r_s \ll r \ll m^{-1}$
- At linear level is equivalent to

$$\mathcal{L} = \frac{1}{16\pi G}\left[R - m^2 R \frac{1}{\Box^2} R\right] + \mathcal{L}_m$$

(Maggiore, Mancarella 2013)

- Inspiration (Arkani-Hamed et al. 2002, Dvali 2006)

$$\mathcal{L}_{\text{proca}} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \tfrac{m^2}{2}A_\mu A^\mu - A_\mu j^\mu \quad \Leftrightarrow \quad \mathcal{L}_{\text{nl}} = -\tfrac{1}{4}F_{\mu\nu}\left(1 - \frac{m^2}{\Box}\right)F^{\mu\nu} - A_\mu j^\mu$$

where $(\Box^{-1}\phi)(x) = \int d^4y\ G(x,y)\phi(y)$

- Applying the same idea to Fierz-Pauli massive gravity

$$\mathcal{L}_{\text{nl}} = \tfrac{1}{2}h_{\mu\nu}\left(1 - \frac{m^2}{\Box}\right)\mathcal{E}^{\mu\nu\rho\sigma}h_{\rho\sigma} - 2m^2\chi\frac{1}{\Box}\partial_\mu\partial_\nu(h^{\mu\nu} - \eta^{\mu\nu}h)$$

$\longrightarrow$ Obstruction: covariantization $\Rightarrow g^{\mu\nu}R_{\mu\nu} = 0$ "Covariant vDVZ discontinuity"

$$\left[\left(1 - \frac{m^2}{\Box_g}\right)G_{\mu\nu}\right]^T = 8\pi G T_{\mu\nu}$$

(Porrati 2002; Jaccard, Maggiore, Mitsou 2013)

- ▷ Unviable background cosmology
- ▷ $\Box^{-1}R_{\mu\nu} \subset \Box^{-1}G_{\mu\nu}$ generates instabilities (Ferreira, Maroto 2013)
- ▷ $g_{\mu\nu}\Box^{-1}R \subset \Box^{-1}G_{\mu\nu}$ stable (Foffa, Maggiore, Mitsou 2013)

Model RT : $G_{\mu\nu} - m^2 \left(g_{\mu\nu}\Box^{-1}_{ret} R\right)^{\mathrm{T}} = 8\pi G\, T_{\mu\nu}$

$\downarrow$ $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$\mathcal{E}^{\mu\nu\alpha\beta} h_{\alpha\beta} - m^2 P^{\mu\nu} P^{\alpha\beta} h_{\alpha\beta} = 8\pi G\, t^{\mu\nu}$$

with $P^{\alpha\beta} h_{\alpha\beta} \equiv \Box^{-1}\left(\Box\eta^{\alpha\beta} - \partial^\alpha\partial^\beta\right) h_{\alpha\beta}$

$\downarrow$

$$\mathcal{L}_{lin} = \frac{1}{2} h_{\mu\nu}\mathcal{E}^{\mu\nu\alpha\beta} h_{\alpha\beta} - \frac{1}{2} m^2 h_{\mu\nu} P^{\mu\nu} P^{\alpha\beta} h_{\alpha\beta}$$

$\downarrow$ Covariantisation

$$S_{\mathrm{RR}} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R - \frac{1}{2} m^2 R \frac{1}{\Box^2} R\right]$$

$\downarrow$ Var. Principle + Causality: $\Box^{-1} \to \Box^{-1}_{ret}$

Model RR : $G_{\mu\nu} - m^2 K_{\mu\nu}\left[\Box^{-1}_{ret} R, \Box^{-2}_{ret} R\right] = 8\pi G\, T_{\mu\nu}$

- Phenomenological approach
  - ▷ Ad hoc construction
  - ▷ Causality imposed by hand
- Other nonlocal models
  - ▷ $Rf(\Box^{-1}R)$ (Deser, Woodard 2007)

  where

  $f(X) \approx 0.245\tanh(0.350X + \dots)$
- Effective models motivated by
  - ▷ Loop quantum corrections
  - ▷ Dissipative effects

  $\longrightarrow$ Schwinger-Keldysh or in-in formalism

▷ Equations of motion for $m^2 R\Box^{-2}R$:

$$G_{ab}\left(1-\frac{\tilde{m}^2}{\Box_g^2}R\right)+\tilde{m}^2\left[\left(\nabla_a\nabla_b-g_{ab}\Box_g\right)\left(\frac{1}{\Box_g^2}R\right)+\frac{1}{4}g_{ab}\left(\frac{1}{\Box_g}R\right)^2 -\frac{1}{2}\left(g_{ac}g_{bd}+g_{ad}g_{bc}-g_{ab}g_{cd}\right)\nabla^c\left(\frac{1}{\Box_g}R\right)\nabla^d\left(\frac{1}{\Box_g^2}R\right)\right]=0.$$

$$G_{\mu\nu}-m^2K_{\mu\nu}(V,S)=8\pi G T_{\mu\nu}$$
$$\Box V = R \quad , \quad \Box S = V$$

- Specialized to
  $ds^2=-(1+2\Psi)dt^2+a^2(t)(1+2\Phi)d\vec{x}^2$

$$f_\delta\left(\Psi^{(k)},\Phi^{(l)},H,\vec{k}^2\right)=\delta_{\mathrm{R}}+\delta_{\mathrm{M}}+\delta_{\mathrm{DE}}\left(m^2,\ldots,\bar{U}_i^{(k)},\delta U_i^{(j)}\right)$$

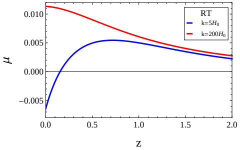
where $\delta_i\equiv\delta\rho_i/\bar{\rho}_i$ and the same for θ_i, σ_i and $c_{s,i}^2$

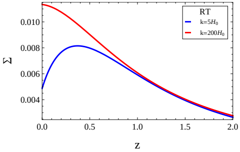
$$f_m\left(\delta_i^{(k)},\theta_i^{(l)},\Phi^{(k)},\Psi^{(l)},\vec{k}^2\right)=0$$
$$f_k\left(\Phi^{(k)},\Psi^{(l)},\bar{U}_i^{(l)},\delta U_i^{(l)},H,\vec{k}^2\right)=0$$

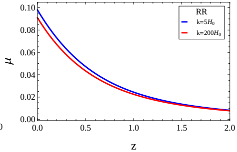
Scalar perturbations and Structure Formation

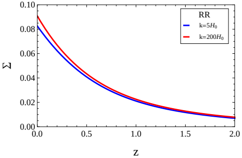
- Gravitational Ψ and lensing potential $(\Psi - \Phi)$ (YD, Foffa, Khosravi, Kunz, Maggiore 2014)

$$\Psi = \left[1 + \mu(z,k)\right]\Psi_{\Lambda\mathrm{CDM}}, \qquad (\Psi - \Phi) = \left[1 + \Sigma(z,k)\right](\Psi - \Phi)_{\Lambda\mathrm{CDM}}$$





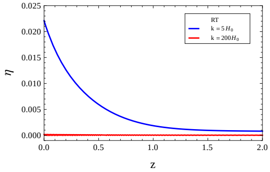


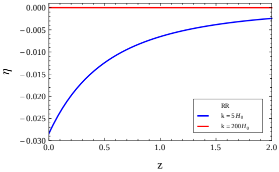


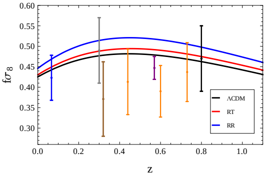
▷ Fit: $\mu(t) = \mu_s a(t)^s$ RT: $\mu_s = 0.01$, $s = 0.8$, RR: $\mu_s = 0.09$, $s = 2$ (EUCLID: $\Delta\mu_s = 0.01$)

- Gravitational slip and RSD (6dF, SDSS LRG, BOSS LOWZ+CMASS, WiggleZ, VIPERS)

$$\eta = (\Psi + \Phi)/\Phi, \qquad f \equiv \frac{d\ln D}{d\ln a} \;\text{ with }\; D(a) \sim \delta_M(a)$$







▷ Consistency with structure formation: $Rf(\Box^{-1}R)$ did not pass the test (Dodelson, Park 2013)

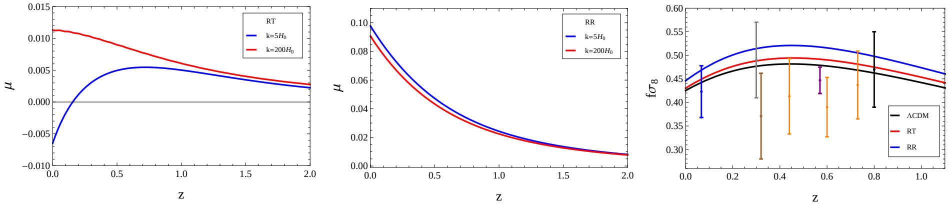
▷ Nonlinear structure formationfor RR: N-body simulation (Barreira et al. 2014)

Scalar perturbations and linear structure formation

- Gravitational pot. Ψ and growth $f\sigma_8$ (YD, Foffa, Khosravi, Kunz, Maggiore 2014)

$$\Psi = \left[1+\mu(z,k)\right]\Psi_{\Lambda\mathrm{CDM}}, \qquad f \equiv \frac{d\ln D}{d\ln a} \quad \text{with} \quad D(a)\sim\delta_M(a)$$

(6dF, SDSS LRG, BOSS LOWZ+CMASS, WiggleZ, VIPERS)



▷ Fit: $\mu(t)=\mu_s a(t)^s$, RT: $\mu_s=0.01$, $s=0.8$, RR: $\mu_s=0.09$, $s=2$

$\longrightarrow$ EUCLID: $\Delta\mu_s=0.01$

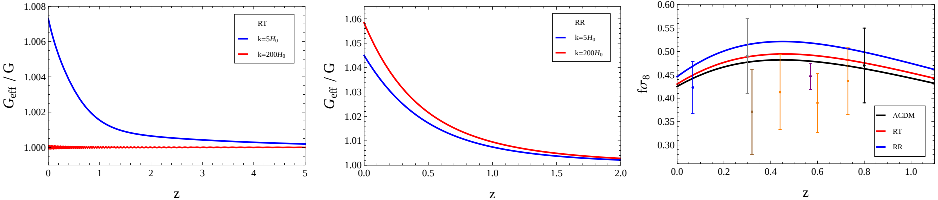
▷ Consistency with structure formation

$\longrightarrow$ Deser-Woodard's "$R\,f(\Box^{-1}R)$" did not pass the test (Dodelson, Park 2013)

▷ Nonlinear structure formation for RR: N-body simulation (Barreira et al. 2014)

- Effective Newton's constant and RSD (6dF, SDSS LRG, BOSS LOWZ+CMASS, WiggleZ, VIPERS)

$$k^2\Phi = 4\pi G_{\rm eff}(z;k)\, a^2 \rho_m \delta_m, \qquad f \equiv \frac{d\ln D}{d\ln a} \text{ with } D(a) \sim \delta_M(a)$$

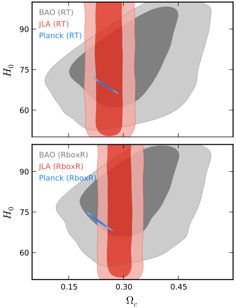


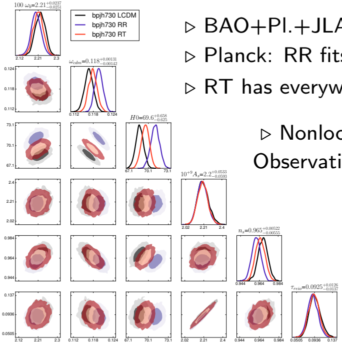
- Nonlinear structure formation (Barreira et al. 2014)
  - Massive haloes slightly more abundant: $\sim 10\%$ at $M \sim 10^{14} M_\odot/h$ and more concentrated: $\sim 8\%$
  - Linear biais almost unchanged
  - Amplitude of nonlinear matter and velocity div. power spectra enhanced by $\sim 12-15\%$ w.r.t ΛCDM at $z=0$

Observational constraints

	Planck			BAO+Planck+JLA			BAO+Planck+JLA+$H_0 = 73.0 \pm 2.4$		
Param	ΛCDM	$g_{\mu\nu}\Box^{-1}R$	$R\Box^{-2}R$	ΛCDM	$g_{\mu\nu}\Box^{-1}R$	$R\Box^{-2}R$	ΛCDM	$g_{\mu\nu}\Box^{-1}R$	$R\Box^{-2}R$
ω_c	$0.1194^{+0.0027}_{-0.0026}$	$0.1195^{+0.0026}_{-0.0028}$	$0.1191^{+0.0027}_{-0.0028}$	$0.1175^{+0.0015}_{-0.0014}$	$0.1188^{+0.0014}_{-0.0014}$	$0.1204^{+0.0014}_{-0.0013}$	$0.117^{+0.0014}_{-0.0014}$	$0.1182^{+0.0013}_{-0.0014}$	$0.1201^{+0.0013}_{-0.0013}$
H_0	$67.56^{+1.2}_{-1.3}$	$68.95^{+1.3}_{-1.3}$	$71.67^{+1.5}_{-1.5}$	$68.43^{+0.61}_{-0.69}$	$69.3^{+0.68}_{-0.66}$	$70.94^{+0.74}_{-0.7}$	$68.72^{+0.61}_{-0.63}$	$69.60^{+0.66}_{-0.63}$	$71.14^{+0.72}_{-0.69}$
$\chi^2_{\rm min}$	9801.7	9801.3	9800.1	10485.5	10485.0	10488.7	10488.9	10487.3	10489.3

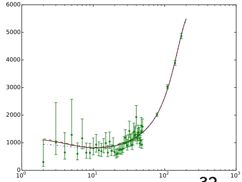
- ▷ Few parameters with $\gtrsim 1\sigma$ deviation from ΛCDM
  - $\longrightarrow$ Nonlocal models prefer a bigger H_0
- ▷ Nonlocal vs ΛCDM: Overall $|\Delta\chi^2| \lesssim 2$
  - $\longrightarrow$ Mostly statistically equivalent to ΛCDM





- ▷ BAO+Pl.+JLA: RR creates a Pl.-JLA 1σ-tension
- ▷ Planck: RR fits slightly better C_l^{TT} at low-l
- ▷ RT has everywhere a smaller χ^2 than ΛCDM

▷ Nonlocal models:
Observationally consistent alternatives to ΛCDM



Parameter inference

Param	ΛCDM	RT	RR
Planck			
$100\ \omega_b$	$2.201^{+0.028}_{-0.029}$	$2.204^{+0.028}_{-0.03}$	$2.207^{+0.029}_{-0.029}$
ω_c	$0.1194^{+0.0027}_{-0.0026}$	$0.1195^{+0.0026}_{-0.0028}$	$0.1191^{+0.0027}_{-0.0028}$
H_0	$67.56^{+1.2}_{-1.3}$	$68.95^{+1.3}_{-1.3}$	$71.67^{+1.5}_{-1.5}$
$10^9 A_s$	$2.193^{+0.052}_{-0.06}$	$2.194^{+0.048}_{-0.062}$	$2.198^{+0.053}_{-0.059}$
n_s	$0.9625^{+0.0072}_{-0.0074}$	$0.9622^{+0.007}_{-0.0081}$	$0.9628^{+0.0074}_{-0.0073}$
z_{re}	$11.1^{+1.1}_{-1.1}$	$11.1^{+1.1}_{-1.2}$	$11.16^{+1.2}_{-1.1}$
$\chi^2_{\rm min}$	9801.7	9801.3	9800.1
BAO+Planck+JLA			
$100\ \omega_b$	$2.215^{+0.025}_{-0.025}$	$2.207^{+0.024}_{-0.025}$	$2.197^{+0.024}_{-0.025}$
ω_c	$0.1175^{+0.0015}_{-0.0014}$	$0.1188^{+0.0014}_{-0.0014}$	$0.1204^{+0.0014}_{-0.0013}$
H_0	$68.43^{+0.61}_{-0.69}$	$69.3^{+0.68}_{-0.66}$	$70.94^{+0.74}_{-0.7}$
$10^9 A_s$	$2.199^{+0.055}_{-0.062}$	$2.196^{+0.052}_{-0.065}$	$2.192^{+0.051}_{-0.061}$
n_s	$0.9668^{+0.0055}_{-0.0054}$	$0.9636^{+0.0052}_{-0.0055}$	$0.9599^{+0.0052}_{-0.0051}$
z_{re}	$11.33^{+1.1}_{-1.1}$	$11.18^{+1.1}_{-1.2}$	$11.00^{+1.1}_{-1.2}$
$\chi^2_{\rm min}$	10485.5	10485.0	10488.7
BAO+Planck+JLA+$H_0 = 73.0 \pm 2.4$			
Param	ΛCDM	$g_{\mu\nu}\Box^{-1}R$	$R\Box^{-2}R$
$100\ \omega_b$	$2.222^{+0.025}_{-0.025}$	$2.212^{+0.024}_{-0.025}$	$2.202^{+0.023}_{-0.024}$
ω_c	$0.117^{+0.0014}_{-0.0014}$	$0.1182^{+0.0013}_{-0.0014}$	$0.1201^{+0.0013}_{-0.0013}$
H_0	$68.72^{+0.61}_{-0.63}$	$69.60^{+0.66}_{-0.63}$	$71.14^{+0.72}_{-0.69}$
$10^9 A_s$	$2.202^{+0.053}_{-0.067}$	$2.198^{+0.053}_{-0.059}$	$2.195^{+0.053}_{-0.058}$
n_s	$0.9679^{+0.0052}_{-0.0054}$	$0.9649^{+0.0052}_{-0.0056}$	$0.9607^{+0.0051}_{-0.0050}$
z_{re}	$11.39^{+1.1}_{-1.3}$	$11.25^{+1.1}_{-1.1}$	$11.05^{+1.1}_{-1.1}$
$\chi^2_{\rm min}$	10488.9	10487.3	10489.3

(YD, Foffa, Kunz, Maggiore, Pettorino 2014)