

# An eclipsing binary black hole candidate system in the blazar Mrk 421

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Removing strong outbursts from multiwavelength light curves of the blazar Mrk 421, we construct outburstless time series for this system. A model-independent power spectrum light curve analysis in the optical, hard X-ray and  $\gamma$ -rays of this outburstless state shows clear evidence for a periodicity of  $\approx 400$  days. A subsequent full maximum likelihood analysis fitting an eclipse model confirms a periodicity of 387.16 days. The power spectrum of the signal in the outburstless state of the source does not follow a flicker noise behaviour and so, the system producing it is not self-organised. This means that the periodicity is not produced by any internal physical processes associated to the central engine. The simplest physical mechanism to which this periodicity could be ascribed is a dynamical effect produced by an orbiting supermassive black hole companion of mass  $\sim 10^7 M_\odot$  eclipsing the central black hole, which has a mass  $\sim 10^8 M_\odot$ . The optimal model restricts the physics of the eclipsing binary black hole candidate system to have an eclipse fraction of 0.36, occurring over approximately 30% of the orbital period.

## Introduction

Blazars are known to be the most powerful radio-loud Active Galactic Nuclei (AGN). They include the BL Lacertae objects and the Flat Spectrum Radio Quasars (FSRQ). The observed emission in blazars is produced by a relativistic jet viewed at a small angle relative to the observer's line of sight (see e.g. Scheuer, 1974; Blandford & Königl, 1979; Angel & Stockman, 1980; Ulrich et al., 1997). Among AGN, blazars are considered the most variable sources since they exhibit the most rapid and largest amplitude continuum variations extending from radio to X-rays, and many of them up to the  $\gamma$ -rays. The detection of variability in blazars is considered a valuable tool since it enables us to constrain properties associated with their central emission regions and set upper limits on the physical scale of the system through causality arguments.

Several variability studies in blazars have been done with the aim of finding periodic or quasi-periodic variations, which could establish the presence of supermassive binary black hole systems (cf. Begelman et al., 1980). To date, the best known example of a binary black hole candidate associated with a blazar is found in OJ 287, an object which shows a variability with period  $\sim 12$  years. The periodicity was discovered by analysing its historical light curve comprising data for more than 100 years in the optical V band (see Lehto & Valtonen, 1996; Hudec et al., 2013). This result led to the proposal of a precessing binary black hole model for OJ 287, which can reproduce the periodic outbursts displayed by this object. Furthermore, another two blazars: 3C 273 and PKS 2155-304 have been proposed as candidates for harbouring supermassive binary black hole systems (Wiita, 2011). These claims do not provide fully convincing evidence, due to a lack of theoretical models able to accurately describe the light curves, although some developments in this direction have been recently done by some of us in Cabrera et al. (2013) using the model by Mendoza et al. (2009). Nevertheless, direct imaging of binary supermassive black hole systems in AGNs have been performed with great success in very few cases (see e.g. Deane et al., 2014, and references therein), ranging from the kiloparsec to the extreme  $\sim 5$  pc separation scales. This technique is useful if members of the system are strongly emitting radiation, particularly in the radio bands. The small outburst to quiescent AGN fraction raises the possibility of a large population of binary black holes remaining undetected, and highlights the importance of developing search techniques for such objects in the absence of outbursts.

Mrk 421 is a BL Lac object hosted by an elliptical galaxy and one of the most well studied blazars due to its proximity, with a redshift  $z \approx 0.03$  (Ulrich et al., 1975). It has been intensively studied since it was discovered as a TeV source by Punch et al. (1992). The TeV flux is found to vary by a factor of  $\sim 5$ , on time-scales of  $\sim 30$  min (Gaidos et al., 1996), with outbursts increasing the flux up to factors of about  $\sim 16$  (Tluczyk et al., 2010). Although several multiwavelength variability studies have been done on Mrk 421 (see Pian et al., 2014, and references therein), the light-curves obtained for this blazar (e.g. Liu et al., 1997; Chen et al., 2014) have not shown any convincing evidence of periodic or quasi-periodic variations, potentially due to shortcomings in the statistical methods employed (Balcev, 2015).

In this work we model multiwavelength light curves of Mrk 421 using the free software R-package RobPer (Thieler et al., 2014), to analyse the associated periodograms on time scales of several years. The method is applied and tested in what we define as “outburstless” light-curves, i.e. those curves where the signal coming from flares or outbursts displayed by the source have been removed. More precisely, we analyse the signal below the  $3\text{-}\sigma$  noise level. In section we present the multiwavelength database used for the analysis. Section deals with the periodogram analysis obtained using RobPer. Once the existence of a  $\sim 400$  days period was found through the periodogram analysis, a full maximum likelihood model is introduced to recover the optimal frequency, phase, eclipse fraction and duration of the eclipse for the X-ray data. Finally, we present a discussion of our results in Section .

## Observational data

The outburstless multifrequency light curves of the Blazar Mrk 421 were constructed using average fluxes to find the  $3\text{-}\sigma$  level, using the following databases:

- **AAVSO Optical data**

The optical long-term light curves analysed in this work were built using the database of the *American Association of Variable Star Observers (AAVSO)*, from 1981 April 11 to 2014 July 21 ( $\sim 33$  years). Average V-band magnitudes were obtained with their corresponding standard deviations  $\sigma$ .

- **Swift-BAT data**

X ray data from 15-50 KeV were obtained using the database of the *SWIFT-BAT* hard X ray transient monitor (Stroh & Falcone, 2013)<sup>a</sup> from 2004 December 22 to 2014 May 3 ( $\sim 10$  years).

- **Fermi-LAT data**

The  $\gamma$ -ray fluxes were obtained in the range 0.2-300 GeV using the public raw-data. The data used covers the interval 2008 August 08 up to 2014 May 31 ( $\sim 6$  years), and were reduced using the Fermi science tool package (see e.g. Atwood et al., 2009) in the same energy range, taking into account the diffuse Galactic background radiation, the instrument response matrix p7v6, and considering a zenith angle  $< 105^\circ$ . Furthermore, we also calculated the active time of the detector and the point spread function (PSF). The  $\gamma$ -ray light curve was constructed modelling the flux with a simple power law of the form  $dN/dE = N_0(E/E_0)^{-\gamma}$ , with  $\gamma = 2-3$  following Abdo et al. (2009). The fluxes and errors obtained with this package are given in photons  $\times \text{cm}^{-2} \text{s}^{-1}$ . For further physical interpretation of the data, we have converted these fluxes and errors to  $\text{MeV cm}^{-2} \text{s}^{-1}$ . The photons considered for analysis were taken from a region centred on the coordinates of Mrk 421 with a radius of  $15^\circ$ .

Once all the outbursts and any signal above the  $3\text{-}\sigma$  level have been identified, they are replaced by blank intervals in the light curve. This generates the outburstless curves we further analyse. Thus, we study the signal which is usually discarded in studies of AGN variability.

## Data analysis

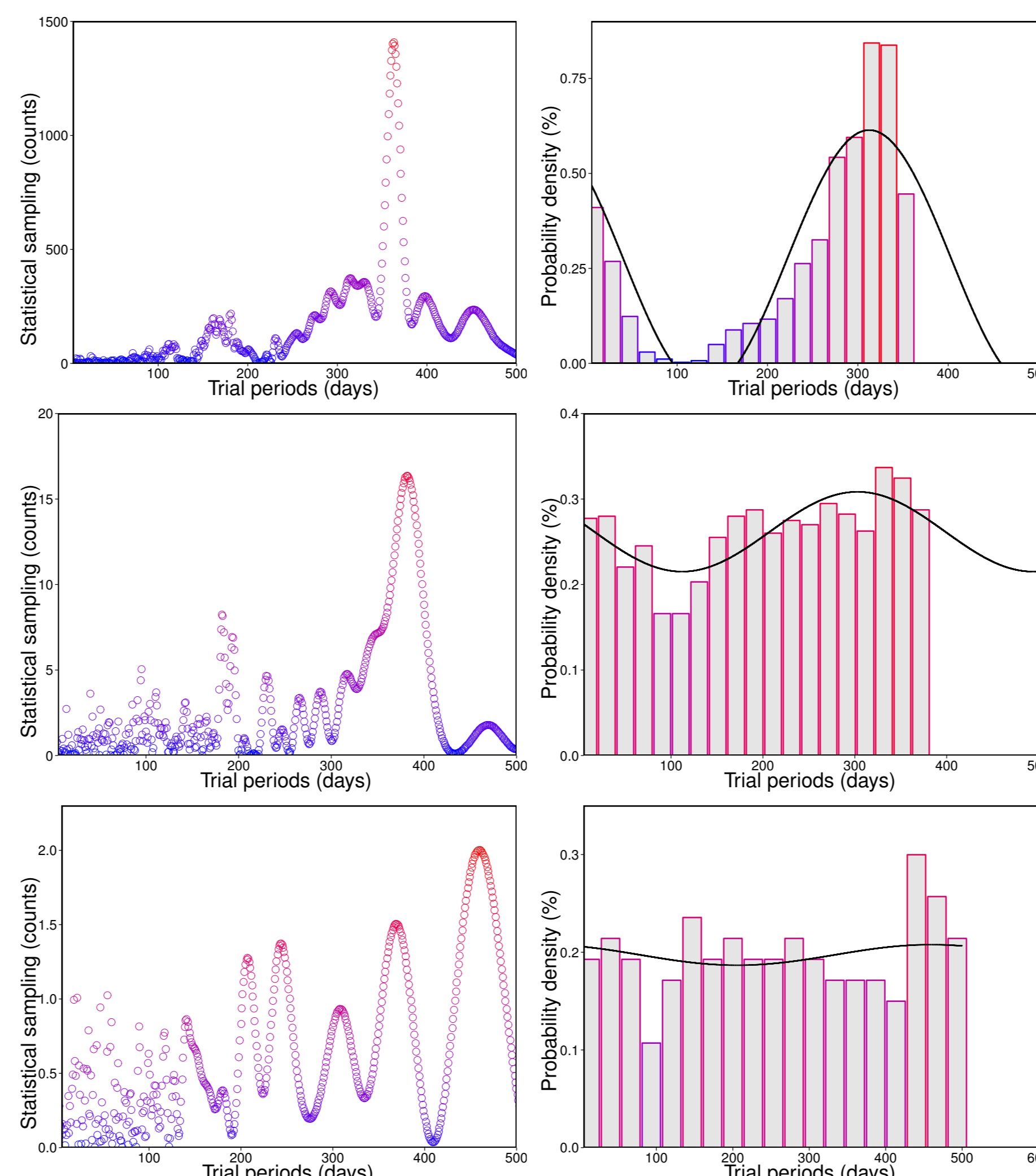
An important task in several astrophysical process is the detection of periodicities in irregularly sampled time series. The main problem associated with these measurements is that the classic Fourier periodogram analysis cannot be applied to irregularly sampled databases with the expected accuracy. Hence, periodicity searches in this kind of time series has become a very active research field. In the case of light curves, the Lomb-Scargle periodogram (Scargle, 1982) or the phase dispersion minimisation periodogram (Stellingwerf, 1978), have been widely used by the astronomical community. However, in recent years, interest has turned to methods that allow to fit periodic functions using weighted regressions. One of these new developments is RobPer (Thieler et al., 2013a), an R package that combines different periodic functions and regression techniques to calculate periodograms. It applies an outlier search on the periodogram, instead of using fixed critical values that are theoretically only justified in case of least squares regression. Among other special features, RobPer has a very complete pool of regression techniques, consisting of the classic least squares, least absolute deviations, least trimmed squares, M-, S- and  $\tau$ -regressions. Due to the advance capabilities of the package and the power and generality of the R project for Statistical Computing, we have analysed the multiwavelength light curves of Mrk 421 for the first time using this software.

<sup>a</sup><http://swift.gsfc.nasa.gov/results/transients/>

The Figure presents the number counts and the probability distributions of the best trial period in a heuristic search for periodicity in the outburstless state of the optical, hard X-ray and  $\gamma$ -ray light curves of Mrk 421. The analysis reveals that a period of  $\sim 4 \times 10^2$  days is associated with the observed signal. Note that the high energy  $\gamma$ -ray data does not exhibit a high probability distribution on its peak, since observations only cover a range of  $\sim 6$  years.

In general terms, periodogram analysis not always yields straightforward confidence intervals. This is due to the fact that there is not a well established technique that considers the non-linear transformation of uncertainties when converting from the time to the frequency domains. This is also true when a periodicity is calculated directly in the time domain with a low signal to noise ratio.

The power spectrum shows that the multiwavelength signal does not follow a flicker  $1/f$  noise behaviour. Instead, a more correlated brown-like noise signal is detected; meaning that the physical processes responsible of this periodicity are not related to the central engine, which is typically associated with a self-organised  $1/f$  signal (Per Bak & Wiesenfeld, 1987; Sole et al., 1997; Press, 1978; Kataoka et al., 2001). The most parsimonious interpretation of our results is provided by taking them as evidence suggesting an eclipsing supermassive binary black hole system.



Statistical sampling (left) and probability distributions histograms (right) -measured in counts and probability density respectively- as a function of time (measured in days) obtained from the light curves of the blazar Mrk 421 in the outburstless state. From top to bottom  $\approx 33$  years V band optical data from AAVSO,  $\approx 10$  years hard X-ray (15 - 50 keV) using Swift-BAT, and  $\approx 6$  years  $\geq 200$  MeV of  $\gamma$ -ray using Fermi-LAT. From top to bottom the peak frequencies correspond to values of: 363, 382, 459 days respectively, which is  $\sim 4 \times 10^2$  days. The observed light curves have a time resolution  $\geq 1$  day.

## Modelling the outburstless light curves

From the RobPer analysis of Section we have found a consistent periodicity in the outburstless signal with period of order  $\sim 400$  days. In order to verify this result independently, we implemented a maximum likelihood analysis for a Gaussian noise observed through a window eclipse function  $e(t)$  for the high quality semi-continuous hard X-ray data. Whenever  $\sin(\omega t + \phi)$ , being  $\omega$ ,  $t$  and  $\phi$  the frequency, time and phase respectively, exceeds a critical threshold parameter  $\Theta$  such that  $-1 \leq \Theta \leq 1$ , the signal is reduced by an eclipse fraction  $f$ , otherwise the inherent Gaussian noise is unaffected, i.e.:

$$e(t) = \left[ 1 - \frac{|\Theta - \sin(\omega t + \phi)| f}{|1 - \Theta|} \right]. \quad (1)$$

The log-likelihood function is now given by:

$$\ln \mathcal{L}(T, \phi, f, \Theta; \{S_i, \Delta S_i\}) = \sum_i \frac{(S_i/e(t) - 1)^2}{2\Delta S_i^2}, \quad (2)$$

where  $S_i$  and  $\Delta S_i$  is the normalised  $i$ -th measured signal value and its uncertainty, respectively, with a period  $T := 2\pi/\omega$ . A dense exploration of parameter space returns the following optimal parameters:  $T = 387.16$ ,  $\phi = 302.78$ ,  $f = 0.36$  and  $\Theta = 0.6$ . The resulting likelihood hypersurface is extremely noisy, making the determination of confidence intervals uncertain, but smaller than 10% of the inferred optimal values. This lack of smoothness on the hypersurface suggests that the underlying physics being observed, although clearly presenting the periodicity detected through two independent statistical methods, is more complex than the simple eclipsing model presented.

## Order of magnitude physics of the binary system

In the previous section, we found evidence suggesting the presence of a supermassive black hole binary system in the centre of Mrk 421. The periodic oscillations inferred by the statistical analysis and the model described support the idea that a second black hole orbits about the central engine. The orbit is such that the radiation coming from the most massive one (the primary supermassive black hole) is eclipsed by the orbital motion of a less massive (secondary) about the primary.

Let us now analyse some zeroth-order physical properties of this eclipsing binary system. To do so, we first note that the integrated central mass of Mrk 421 has been estimated to be  $\sim 10^8 M_\odot$  (Woo & Urry, 2002; Barth et al., 2003; Gorham et al., 2000). Using this value and the obtained periodicity of  $4 \times 10^2$  days, it follows that the centre of mass of the binary system is very close to the primary black hole, which is thus almost at rest. Therefore, the period  $p_1$  of the primary's orbit is  $p_1 \lesssim 1$  day and so, undetectable as the temporal resolution of the observations is typically  $\gtrsim 1$  day.

Let the primary source have a mass  $m_1 \sim 10^8 M_\odot$ . The secondary black hole has a mass  $m_2$  and an orbital period  $p_2 \sim 10^2$  days according to the results of Section . Kepler's third law of motion and momentum conservation for a binary system in circular orbits with radii  $r_1$  and  $r_2$  yield  $p_1^2/p_2^2 = r_2/r_1 = m_1/m_2$  and so, the mass of the companion is  $m_2 \sim 10^7 M_\odot$ . A lower limit to the orbit can be calculated since the maximum orbital velocity of the secondary is  $\lesssim$  than the velocity of light  $c$  and so,  $cp_2 \sim r_2$ . With this, it follows that the minimum possible orbital radius  $r_2 \sim 0.1$  pc. Since there is no gravitational restriction as to the maximum separation of the two masses, we deal only with the lower separation limit in what follows.

Let us now describe the circular orbit of the binary system in the frame of reference of the centre of mass corrotating with both supermassive black holes. In this non-inertial frame, both centrifugal and gravitational forces are needed for the description of the orbits. The Roche lobe equipotential shared by the binary system intersects at the Lagrangian point L1, which is separated by a distance  $r$  from the secondary and is given by (see e.g. Eggleton, 1983; Seidov, 2004):

$$\frac{m_1}{(R-r)^2} = \frac{m_2}{r^2} + \left( \frac{m_1}{m_1+m_2} R - r \right) \frac{m_1+m_2}{R^3},$$

where  $R$  and  $r$  are the separation distance between the two objects and the distance from the L1 point to the secondary black hole respectively. Since  $r < R$ , the previous equation can be simplified to yield:

$$r \approx R \left( \frac{m_2}{3m_1+m_2} \right)^{1/3}.$$

The end result of this calculation is that the Lagrange L1 point is less than  $\sim 10\%$  of the separation distance of the binary system. Since the minimum separation distance of both objects is  $\sim 0.1$  pc, this means that the Lagrange L1 point lies at a distance of  $\sim 0.01$  pc away from the secondary black hole. Under the assumption that both black holes have an accretion disc with radius between 10 - 100 of their respective gravitational radii, the approximate radius of each of the accretion discs is  $\sim 10^{-6}$  pc and  $10^{-5}$  pc. These last values are 3-4 orders of magnitude smaller than the distance from the location of the Lagrange L1 point to the smaller black hole and so, there is no disc Roche lobe accretion induced by the orbiting combined black hole and accretion disc system. This excludes a possible binary mass accretion mechanism which could induce oscillations on the observed radiation and enhances even more the fact that an eclipsing dynamical effect is the simplest explanation for the observed periodicity.

The separation distance between both black holes is so large that the gravitational radiation emitted (Peters, 1964; Landau & Lifshitz, 1975) by the system is  $(32/5)G^4 m_1^2 m_2^2 (m_1 + m_2) / (c^5 r^5) \sim 10^{29}$  W. This value yields a coalescent time  $c^5 r^4 / (G^2 256 (m_1 + m_2) m_1 m_2)$  of a few hundred Hubble times, making the orbit extremely stable.

## Discussion

An established fact of Fourier periodogram analysis is that it cannot deal with irregularly sampled time series or with highly variable measurement accuracies (Thieler et al., 2013b), such as the optical light curves used in this work. To avoid the use of a Fourier frequency based analysis on the data, we used the R-package RobPer (Thieler et al., 2014).

The high frequency oscillation modes in all the light curves analysed in this work follow a white-like noise pattern and are thus not related to any self-organised phenomenon (Landa et al., 2011). This behaviour corresponds to microphysical activity of the source, producing non-correlated variability, typical of random stochastic processes. When taking into account both active (with outbursts) and outburstless states of the light curves, the low frequency oscillation modes show a flicker pink-like noise for the active state and a brown-like noise pattern in the outburstless state. The idea of analysing the outburstless signal in the search for periodicities is inspired by the fact that Mrk 421 has a self-organised flicker noise in the low frequency regime. This is quite probably associated to the emissions from the inner central engine, contrary to what is found in the quasar PG 1302-102 which exhibits a W-damped random walk behaviour (Graham et al., 2015) associated with a brown-like noise. The  $1/f$  noise is the result of an infinite superposition of stochastic signals (Eliazar & Klafter, 2009a,b) and so, it has no main periodicities associated. In order not to confuse our calculations with any false periodicity in this source, we removed any signal above the  $3\text{-}\sigma$  noise level. With this, a brown-like noise in the resulting outburstless state is obtained. Thus, the existence of a highly correlated process such as the inferred eclipsing supermassive binary black hole system appears natural, as in the case of the PG 1302-102 quasar mentioned above.

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