Non linear evolution of BAO and IR resummation

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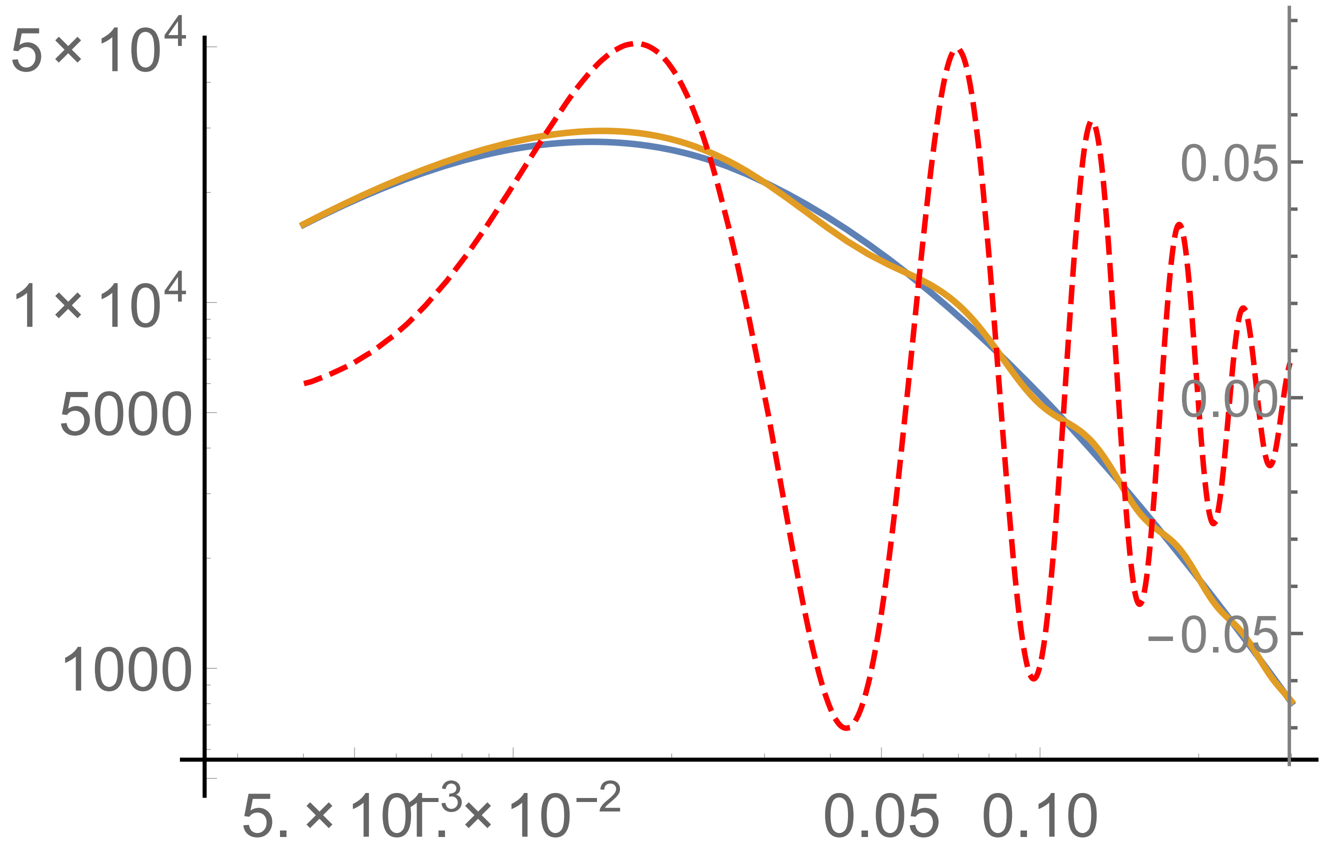
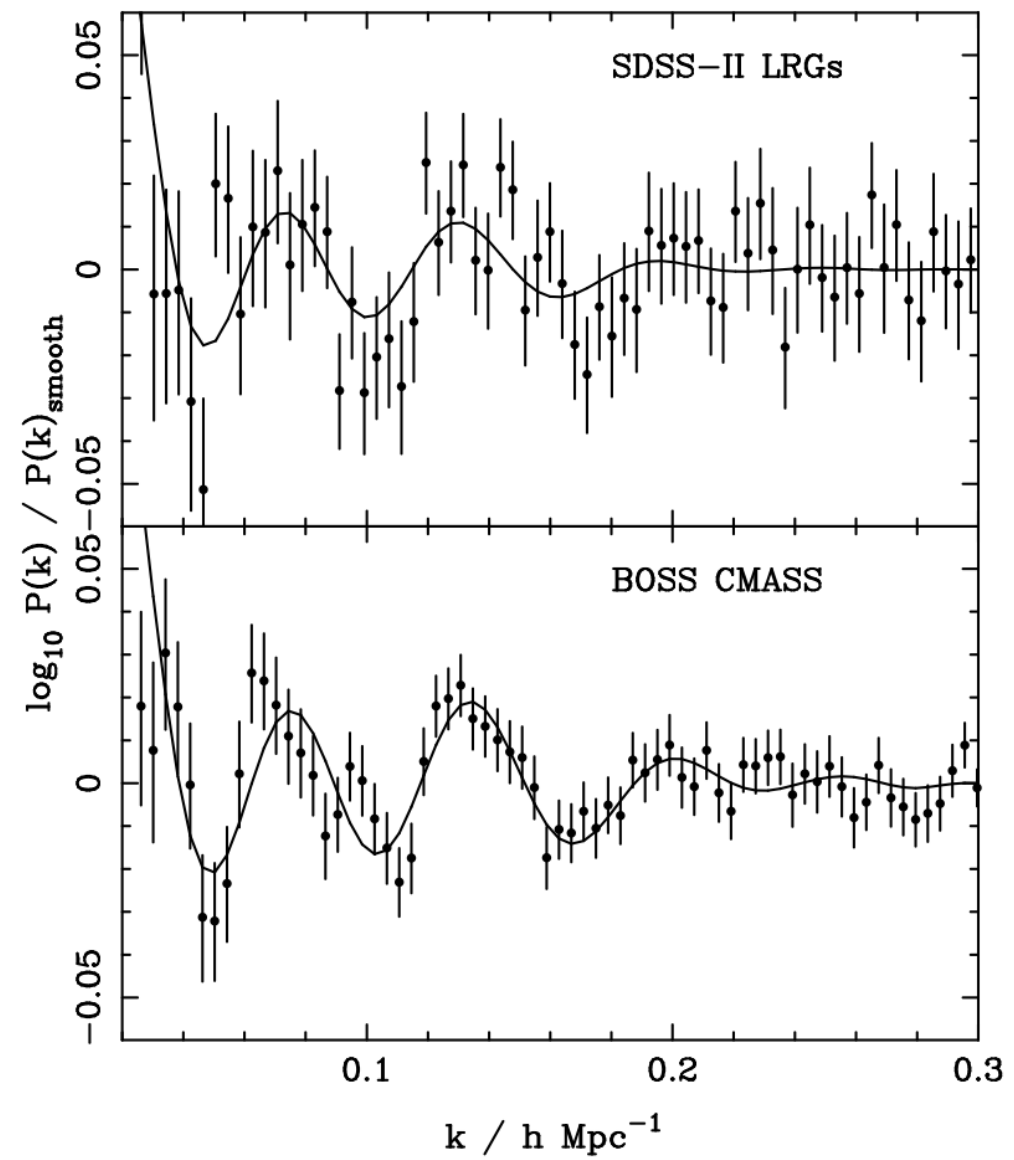
Baryon acoustic oscillations

$$P_0 = P_{wiggles} + P_{smooth}$$





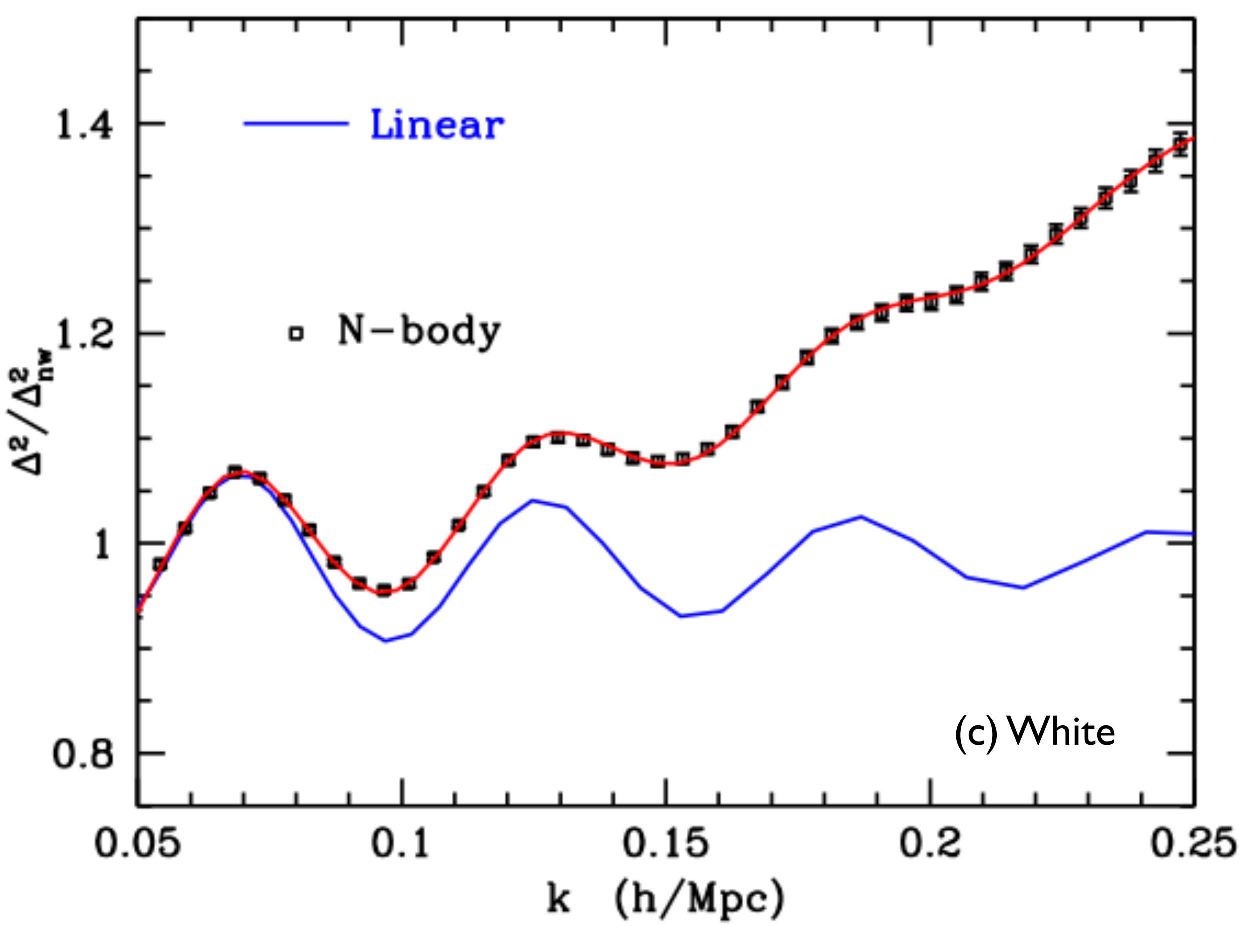
- Standard ruler - extremely powerful probe
- Essential to understand properties of the Universe
- Theoretical control with good accuracy needed

Non-linearities come into play

Power spectrum

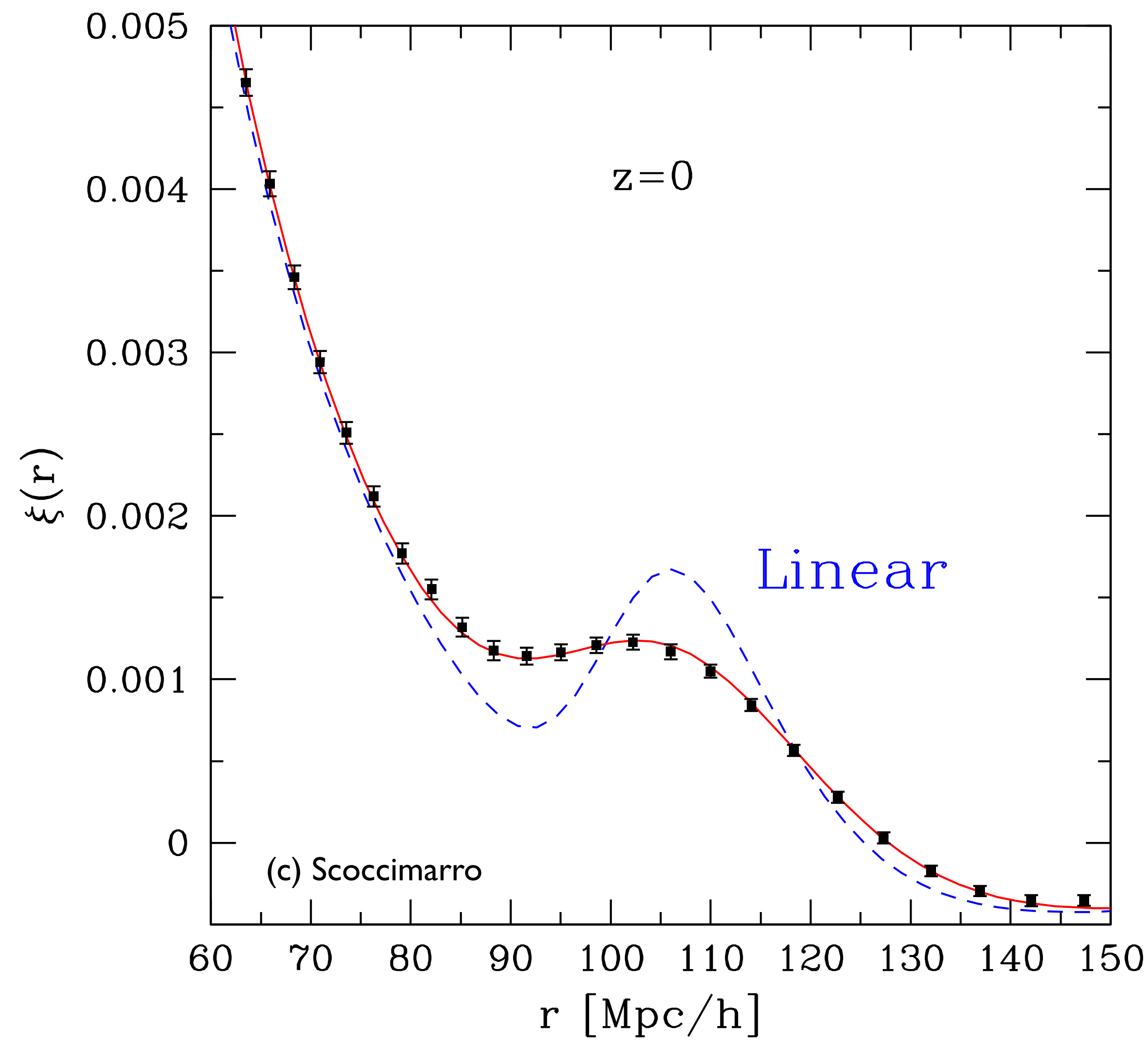
$$\delta = \frac{\delta\rho(\mathbf{x},\tau)}{\bar{\rho}(\tau)}$$

$$\langle \delta(\mathbf{k})\delta(\mathbf{k}')\rangle = \delta_D^{(3)}(\mathbf{k}'+\mathbf{k})P(k)$$

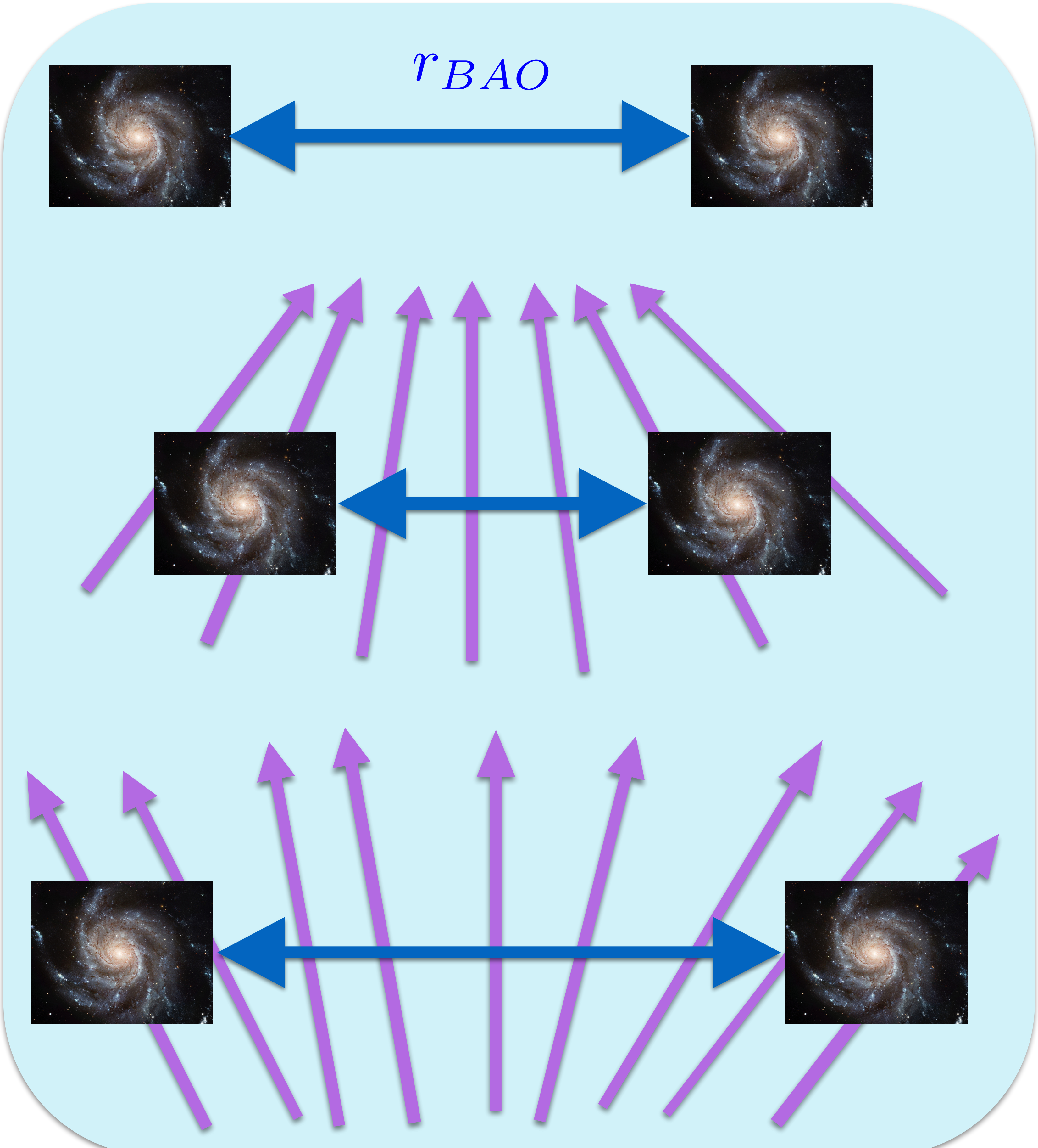


Correlation function

$$\xi(r) = \int d^3k e^{i\vec{k}\cdot\vec{r}} P(k)$$



Physical picture



- large scale bulk flows = correlation degrades

BAO peak broadening

- Clustering = BAO scale shrinks

Shift of the BAO peak

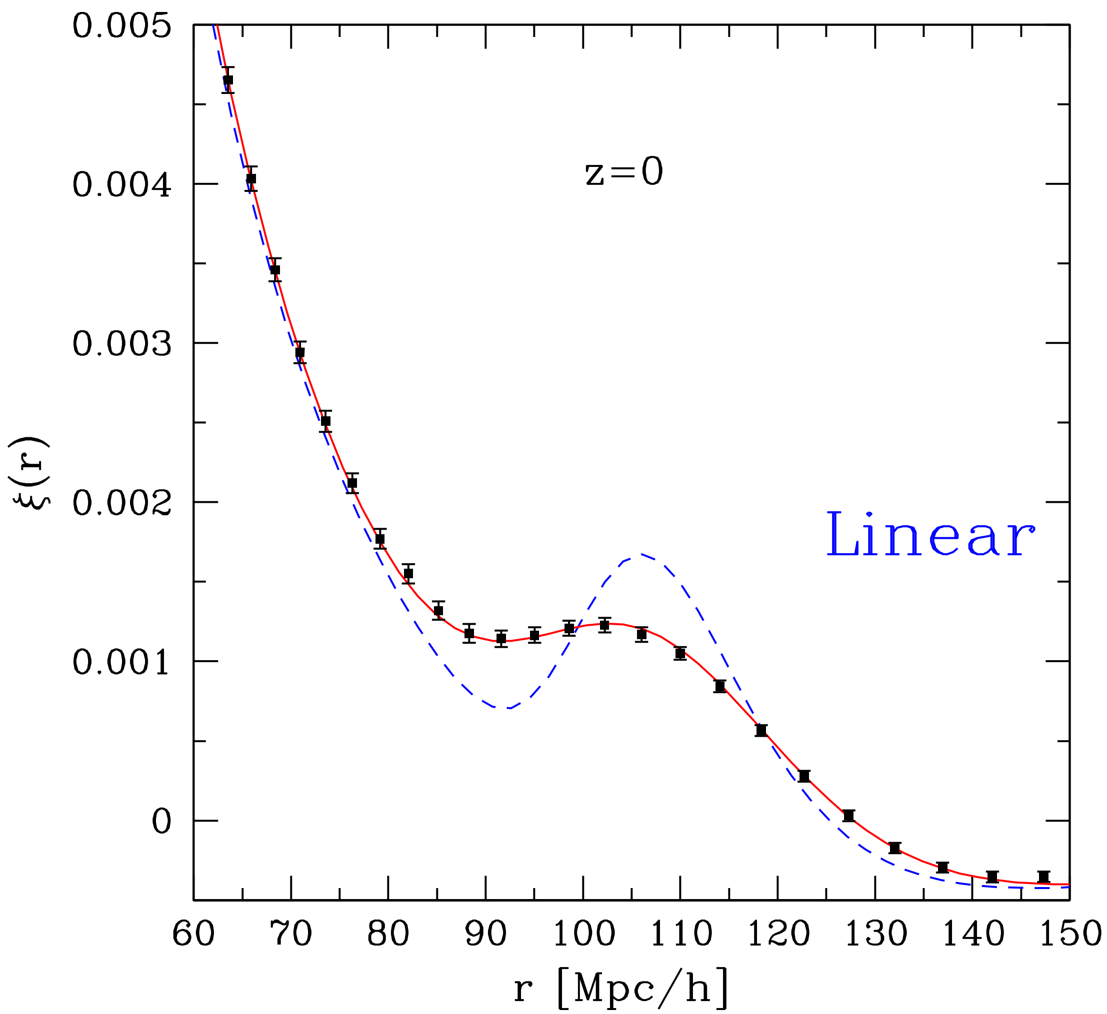
- Displacement can be big => Resummation to all orders

IR resummation

How to get through ?

RPT

Crocce, Scoccimarro'07

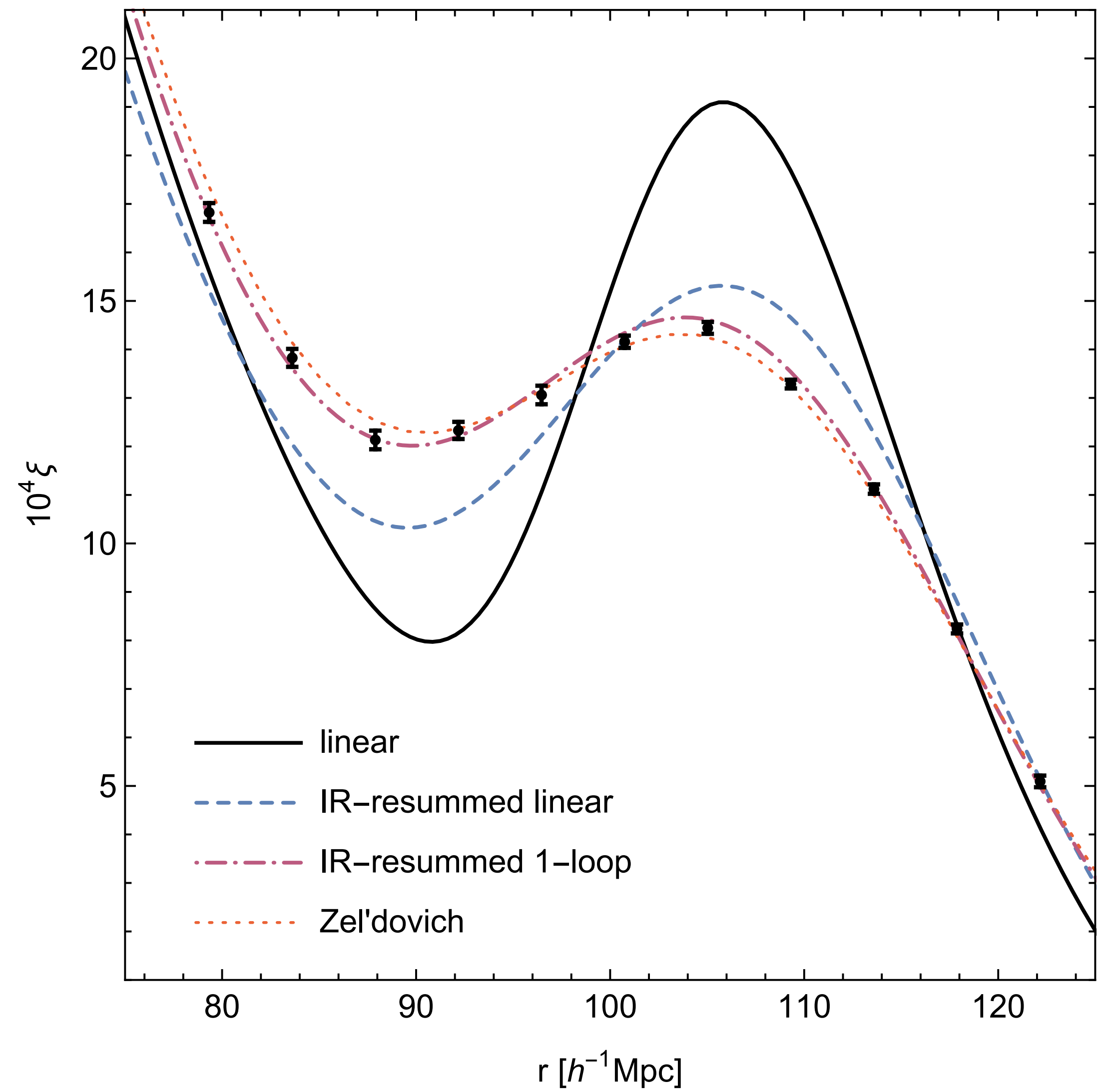


I) ~~Galilean Invariance~~, Equiv. pr

II) IR unsafe -
non-physical IR corrections

IR - resummed EFT

Zaldarriaga, Senatore'15



I) Equivalence principle

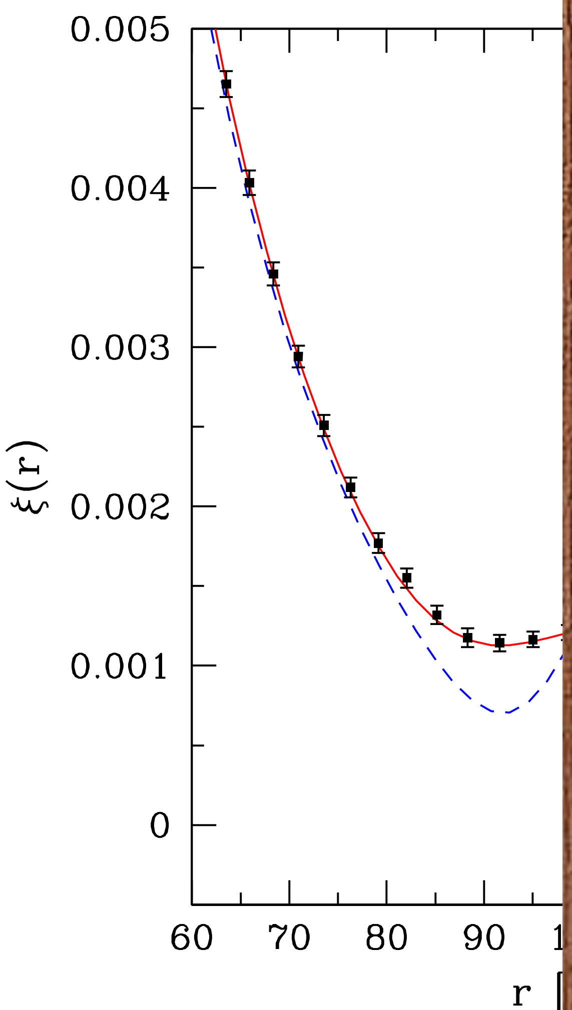
II) Accuracy not under control

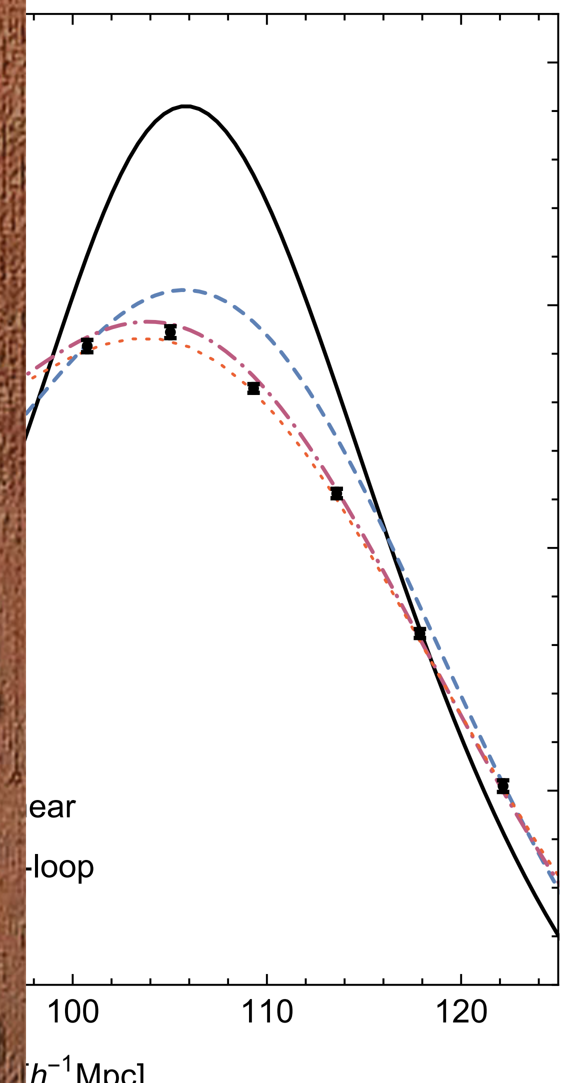
III) Extensions - ?

How to get through ?

R ... mmed EFT

Crocce, S ... ga, Senatore’15





I) Galilean In ... principle

II) IR unsafe ... ot under control

non-physical ... - ?

Time - sliced perturbation theory (TSPT)

Fields + initial PDF ⟷ Time-dependent PDF

ex: SPT,…

$$\frac{\partial \delta}{\partial \tau} + \nabla \cdot (1+\delta)\vec{v} = 0$$

$$\frac{\partial \vec{v}}{\partial \tau} + H\vec{v} + \vec{v}\cdot\nabla\vec{v} = -\nabla\Phi$$

$$\nabla^2\Phi = 4\pi G a^2 \bar{\rho}\delta$$

$$\partial_t \mathcal{P} + \frac{\partial}{\partial \psi}(\dot{\psi}\mathcal{P}) = 0$$

$$\mathcal{P}[\psi]\Big|_{t=0} = \mathcal{P}[\psi_0]$$

Liouville eqn.

Gaussian:

$$\langle \delta_0(\mathbf{k})\delta_0(\mathbf{k}')\rangle = P_0(k)\delta^{(3)}(\mathbf{k}'+\mathbf{k})$$

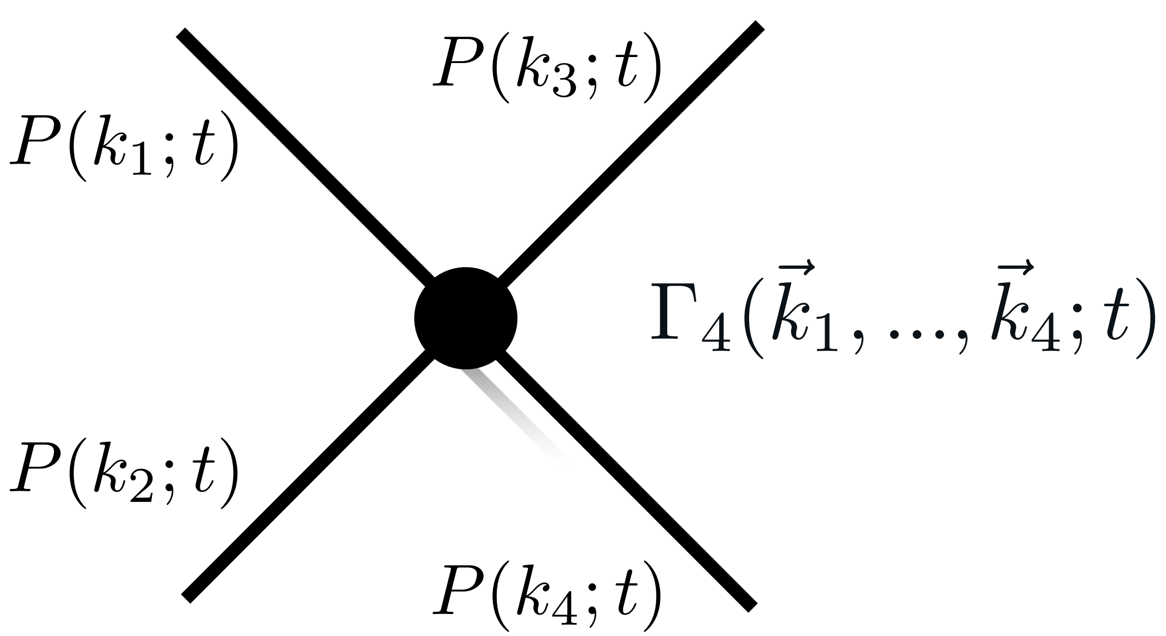
$$Z_t[J] = \mathcal{N}^{-1}\int \mathcal{D}\psi \mathcal{P}[\psi] \exp\left\{\int J\psi\right\}$$

Time - sliced perturbation theory (TSPT)

$$\mathcal{P} = \exp\{-W[\psi]\} \qquad W = \sum_{n=2}^{\infty} \int \Gamma_n \psi^n$$

All vertices can be computed exactly!

$$\langle \psi(\mathbf{k}_1, t) ... \psi(\mathbf{k}_n, t)) \rangle^{tree, 1PI} = \Gamma_n(\mathbf{k}_1, ..., \mathbf{k}_n) \prod_{i=1}^{n} P^L(t, \mathbf{k}_i)$$



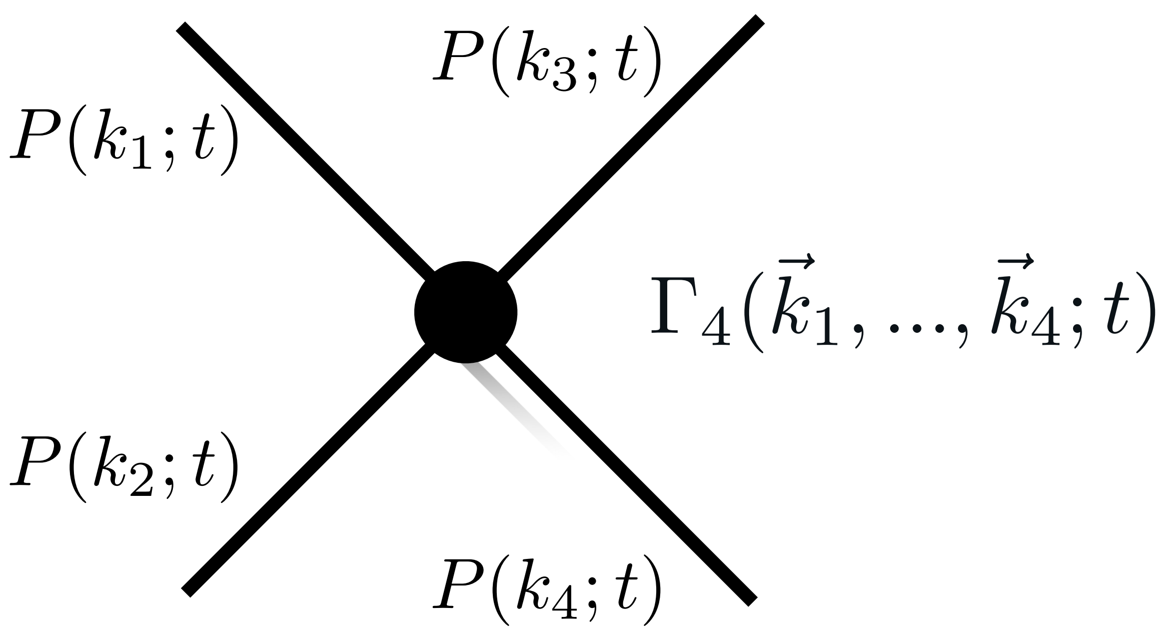
$1PI$ - Trispectrum

Time - sliced perturbation theory (TSPT)

$$\mathcal{P} = \exp\{-W[\psi]\} \qquad W = \sum_{n=2}^{\infty} \int \Gamma_n \psi^n$$

All vertices can be computed exactly!

I) All integrands are IR safe (consequence of EP)
II) Simplified diagrammatics (Euclidean QFT)



$1PI$ - Trispectrum

BAO IR - resummation

$$P^L(k) = P_{smooth}(k) + P_{wiggly}(k) \qquad P_w \sim \cos(k/k_{osc})$$

$$\Gamma_n = \Gamma_n^{smooth} + \Gamma_n^{wiggly}$$

$$\Gamma_n^{smooth}(\mathbf{k}, -\mathbf{k}, \mathbf{q}_1, ..., \mathbf{q}_{n-2}) = \mathcal{O}(1)$$

$$\Gamma_n^{wiggly}(\mathbf{k}, -\mathbf{k}, \mathbf{q}_1, ..., \mathbf{q}_{n-2}) = \mathcal{O}(1) \cdot \left(\frac{k}{k_{osc}}\right)^{n-2} \gg 1$$

$$q_i \to 0 \qquad \frac{k}{k_{osc}} \sim 10$$

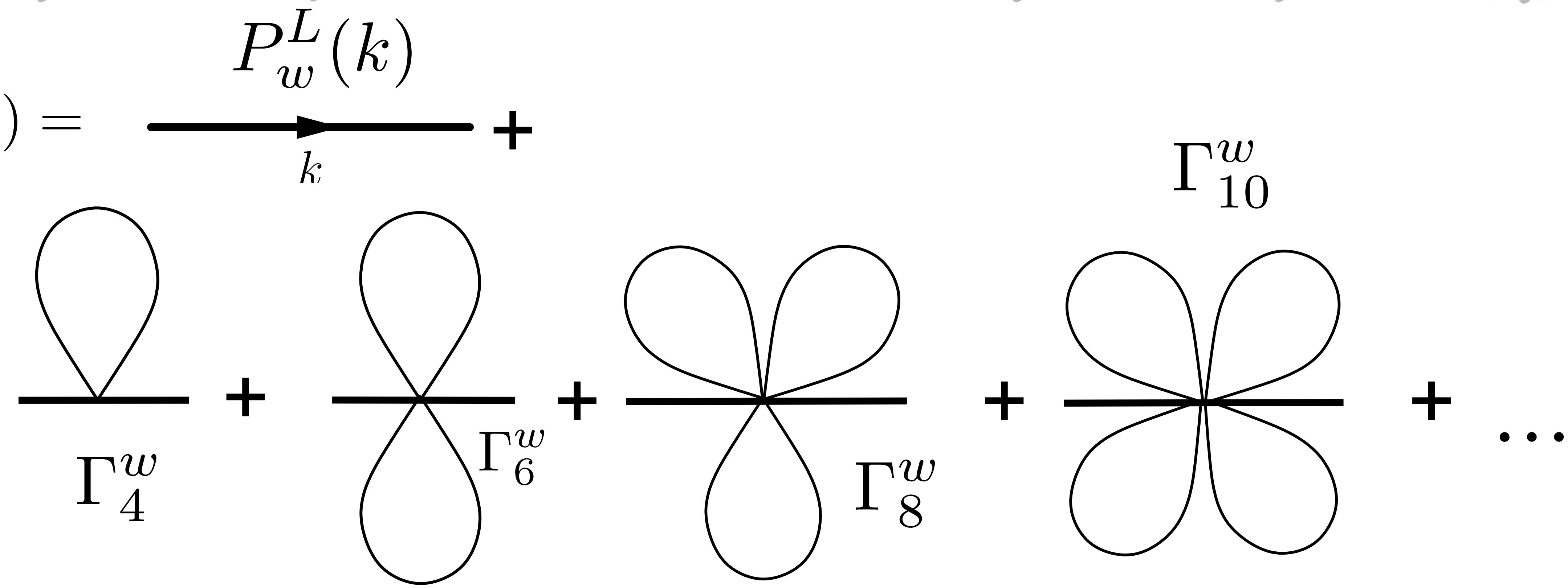
Must be resummed!

At each given loop order we should take only
the graph with the bigger n in Gn !

BAO IR - resummation

Daisy graphs

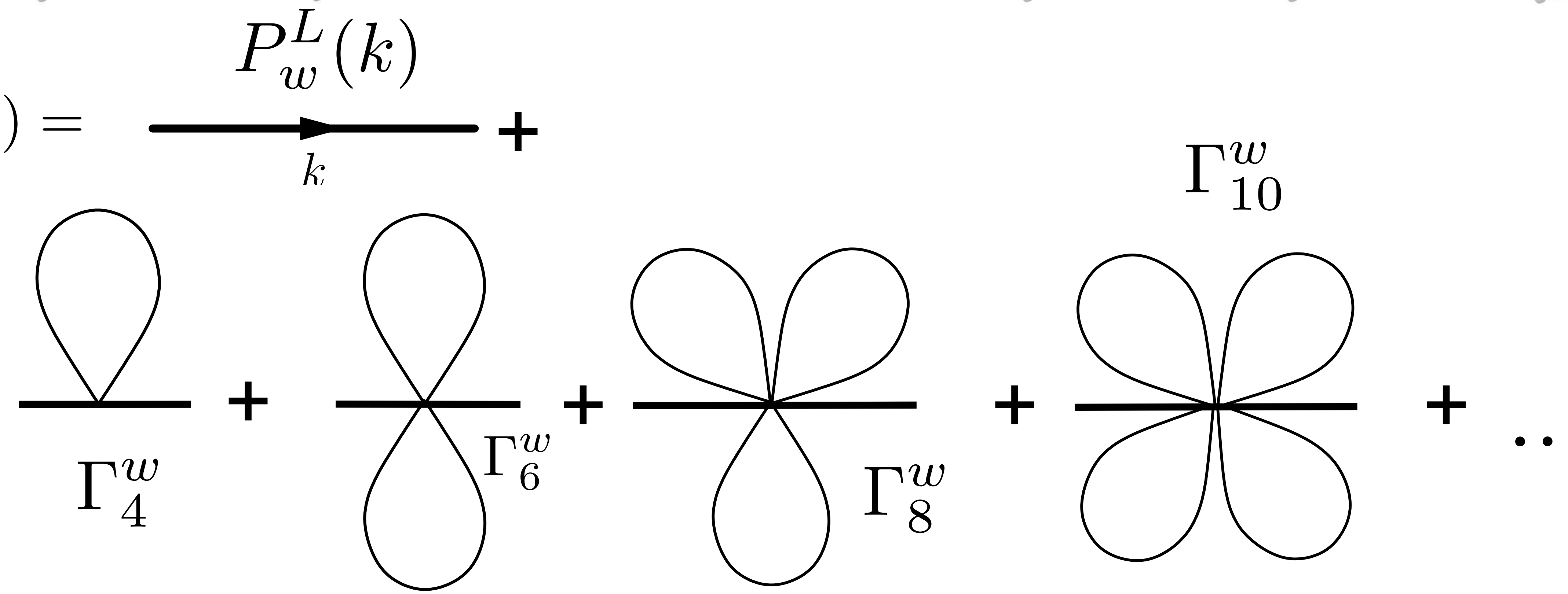
$$P_w^{IR-res.}(k) =$$



BAO IR - resummation

Daisy graphs

$$P_w^{IR-res.}(k) =$$

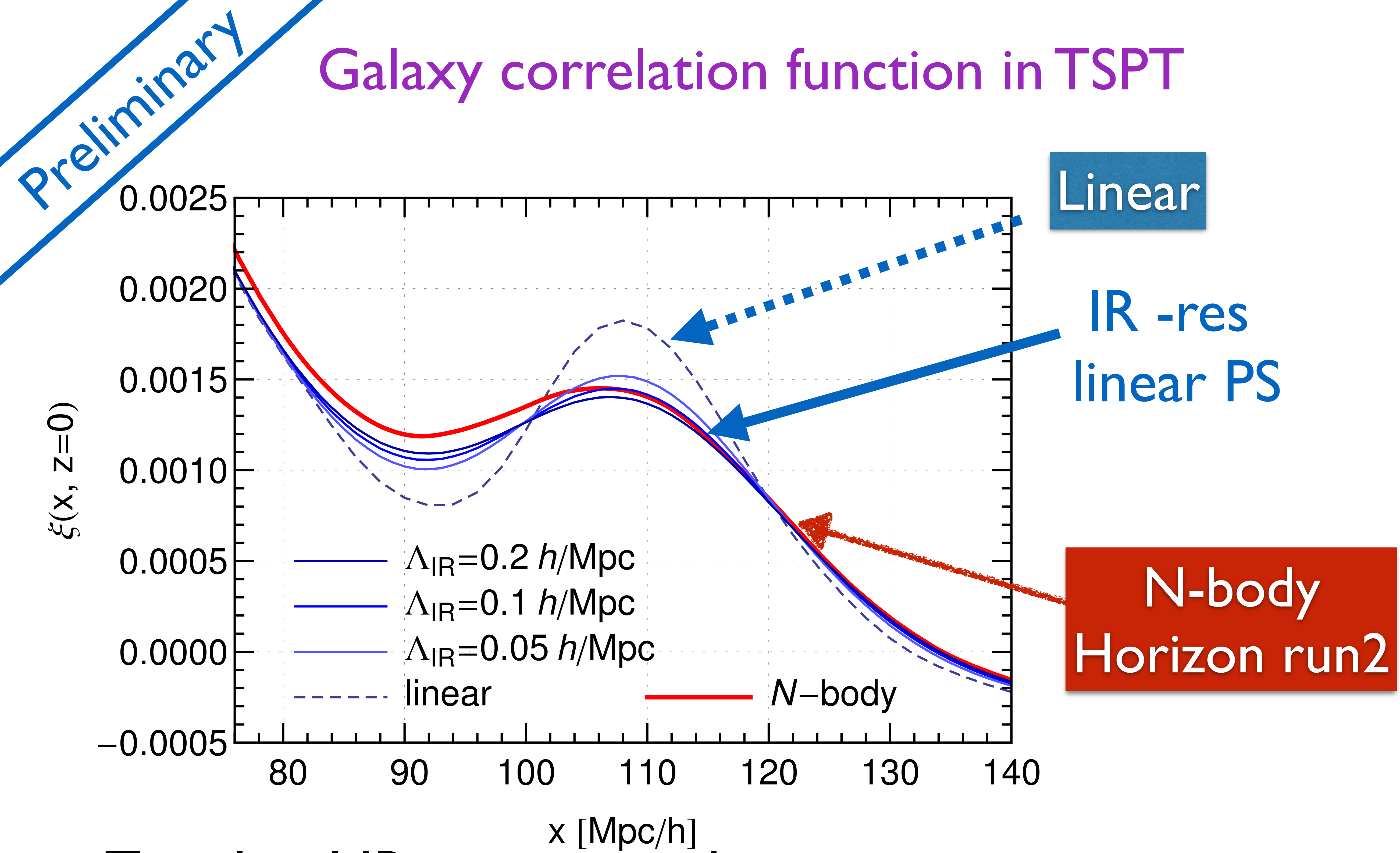


$$= e^{-\Sigma^2 k^2} P_w(k)$$

$$= \exp\left\{ -\frac{k^2}{3} \cdot 4\pi \int_0^{\Lambda_{IR}} dq P_s^L(q)(1 - j_0(qr_{BAO}) + 2j_2(qr_{BAO})) \right\} P_w(k)$$

BAO scale

Galaxy correlation function in TSPT

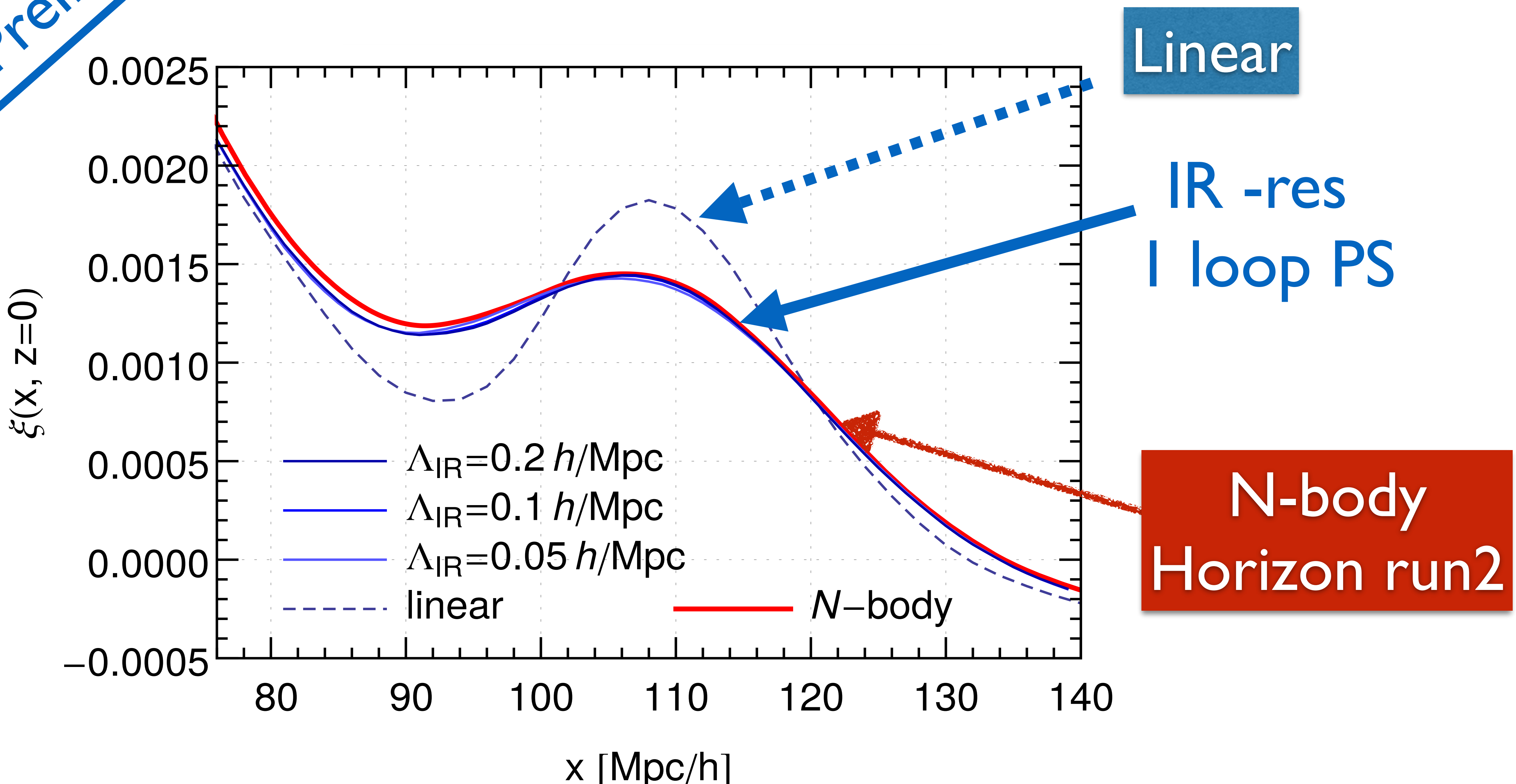


Tree-level IR - resummed

$$P_w^{tree,res.}(k) = e^{-\Sigma^2 k^2} P_w(k)$$

Preliminary

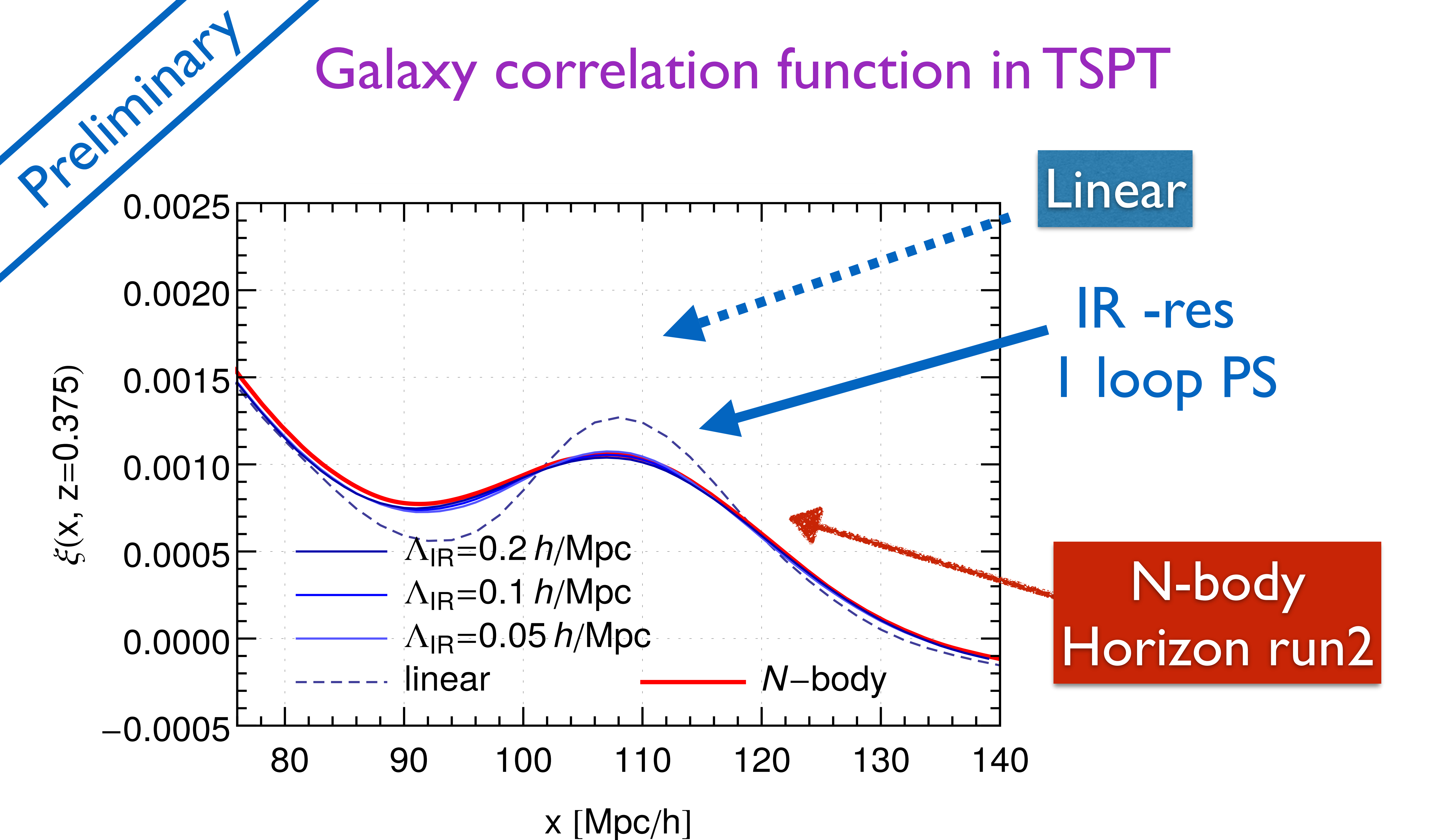
Galaxy correlation function in TSPT



1-loop IR - resummed

$$P_w^{1loop,res.}(k) = e^{-\Sigma^2 k^2} P_w(k)(1+\Sigma^2 k^2) + P_w^{1loop}[e^{-\Sigma^2 k^2} P_w(k)]$$

Galaxy correlation function in TSPT

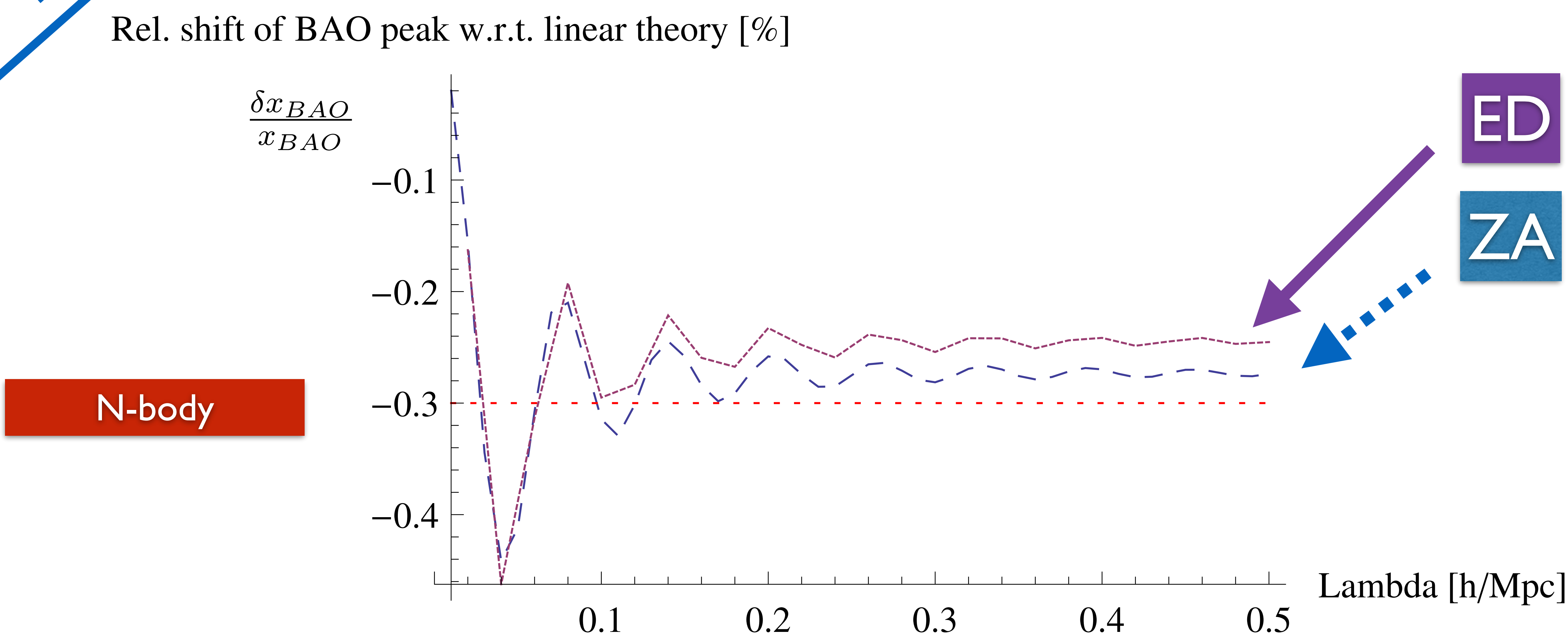


1-loop IR - resummed

$$P_w^{1loop,res.}(k) = e^{-\Sigma^2 k^2} P_w(k)(1+\Sigma^2 k^2) + P_w^{1loop}[e^{-\Sigma^2 k^2} P_w(k)]$$

Preliminary

Shift of the BAO peak



Small in LCDM - few per mille!

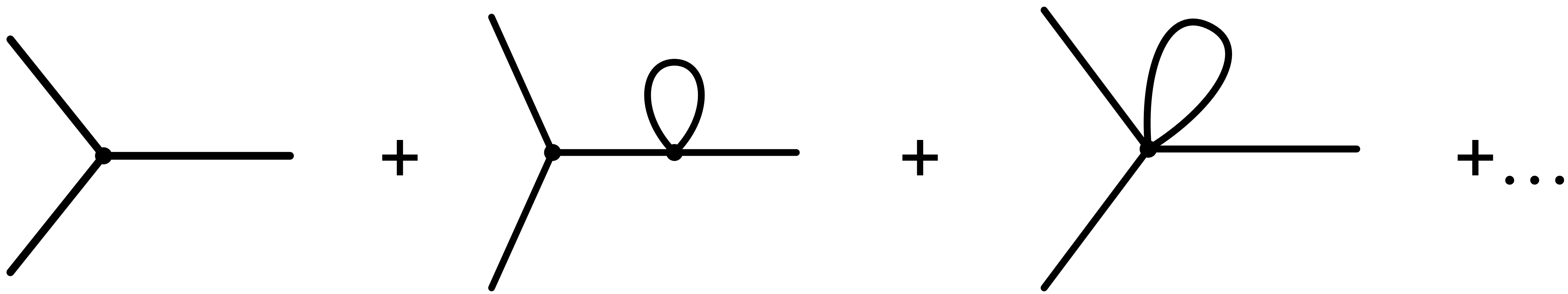
Big in Modified Gravity - Galileons

(Bellini, Zumalacárregui'15)

Preliminary

IR - resummed bispectrum

$B^w(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) =$ dressed with daisies

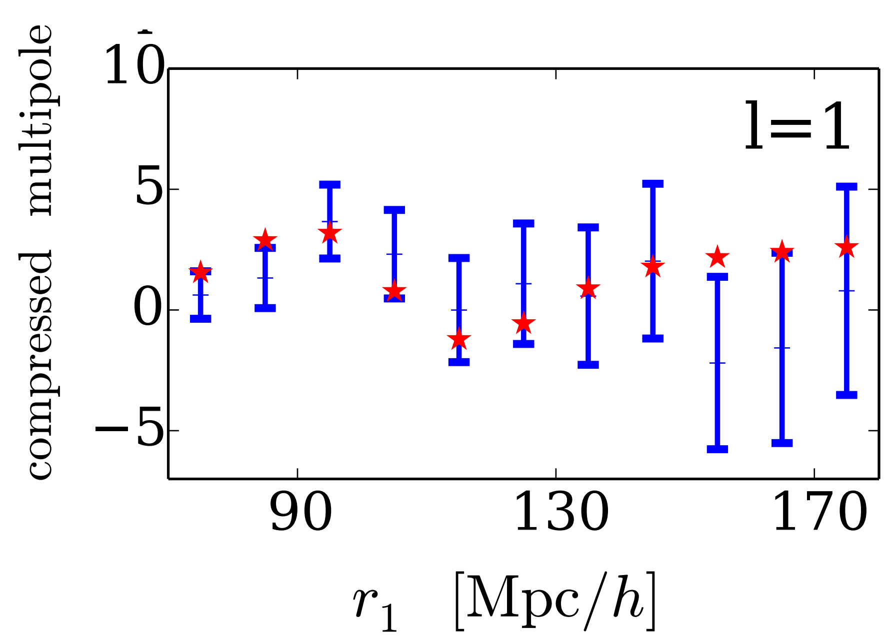


$$= 2F_2(\mathbf{k}_1, \mathbf{k}_2)(e^{-\Sigma^2 k_1^2} P_w(k_1) P_s(k_2) + e^{-\Sigma^2 k_2^2} P_w(k_2) P_s(k_1))$$

+symm.

$$F_2(\mathbf{k}_1, \mathbf{k}_2) = \frac{5}{7} + \frac{1}{2}\frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k_1^2} + \frac{1}{2}\frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k_2^2} + \frac{2}{7}\frac{(\mathbf{k}_1 \cdot \mathbf{k}_2)^2}{k_1^2\, k_2^2}$$

BOSS 1512.02231



detected @ 2.8σ

BAO in bispectrum

Summary:

- BAO as a key probe in cosmology - non linearities !
- Systematic IR resummation in TSPT:
  = accurate description of BAO peak to all orders in PT
- Leading order IR resummed Bispectrum

Outlook:

- Higher point statistics,
  effects in Bispectrum, Trispectrum
- Non-minimal models: massive neutrinos, DE, MG
- Bias, redshift space, etc.

Thank you for your attention !

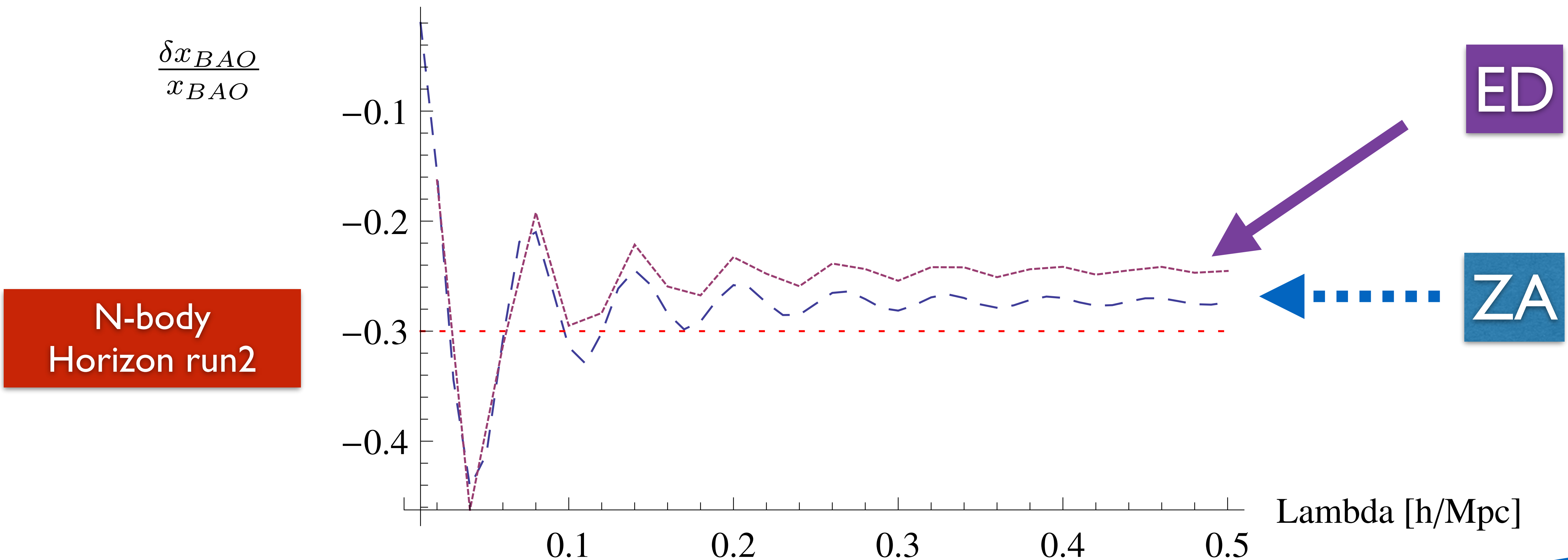
Backup slides

Shift of the BAO peak

$$\xi_{BAO}(x) \propto e^{-\frac{(x-s)^2}{4\Sigma^2(\Lambda)}} \left(1 - \frac{\Sigma_{sub-leading} - \frac{3}{2}\Sigma^B_{sub-leading}}{2\Sigma^2}(x-s) - \frac{\Sigma^B_{sub-leading}}{8\Sigma^4}(x-s)^3\right)$$

$$s \equiv r_{BAO}$$

Rel. shift of BAO peak w.r.t. linear theory [%]



Big in Modified Gravity - Galileons

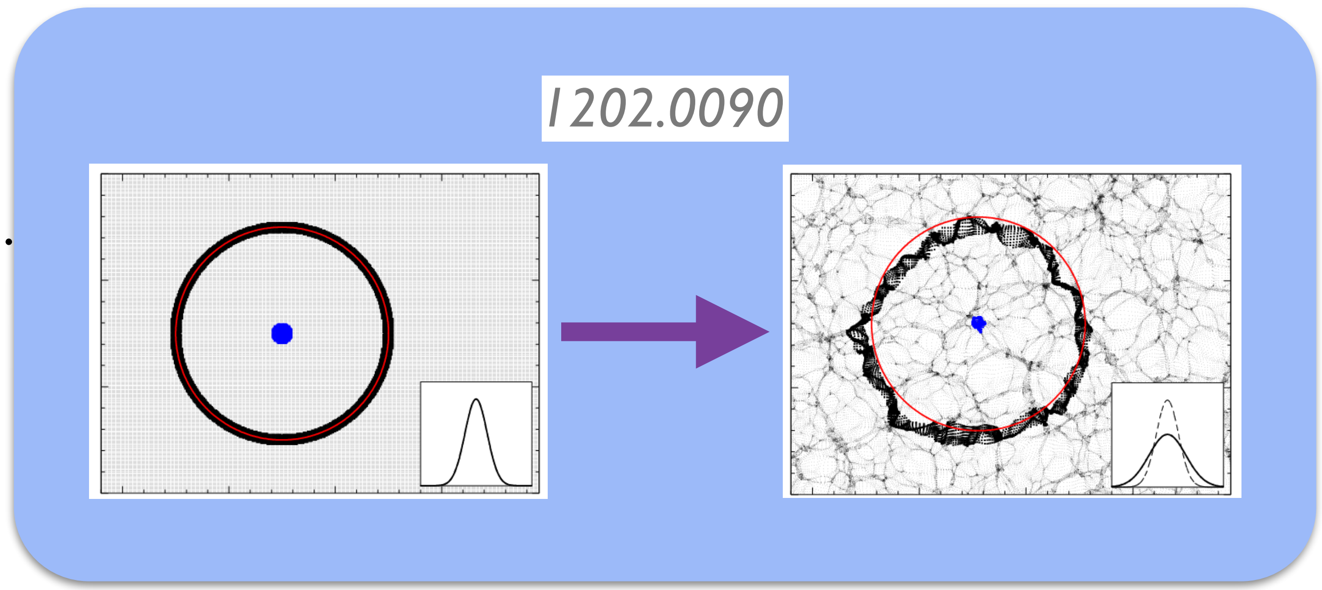
(Bellini, Zumalacárregui'15)

Preliminary

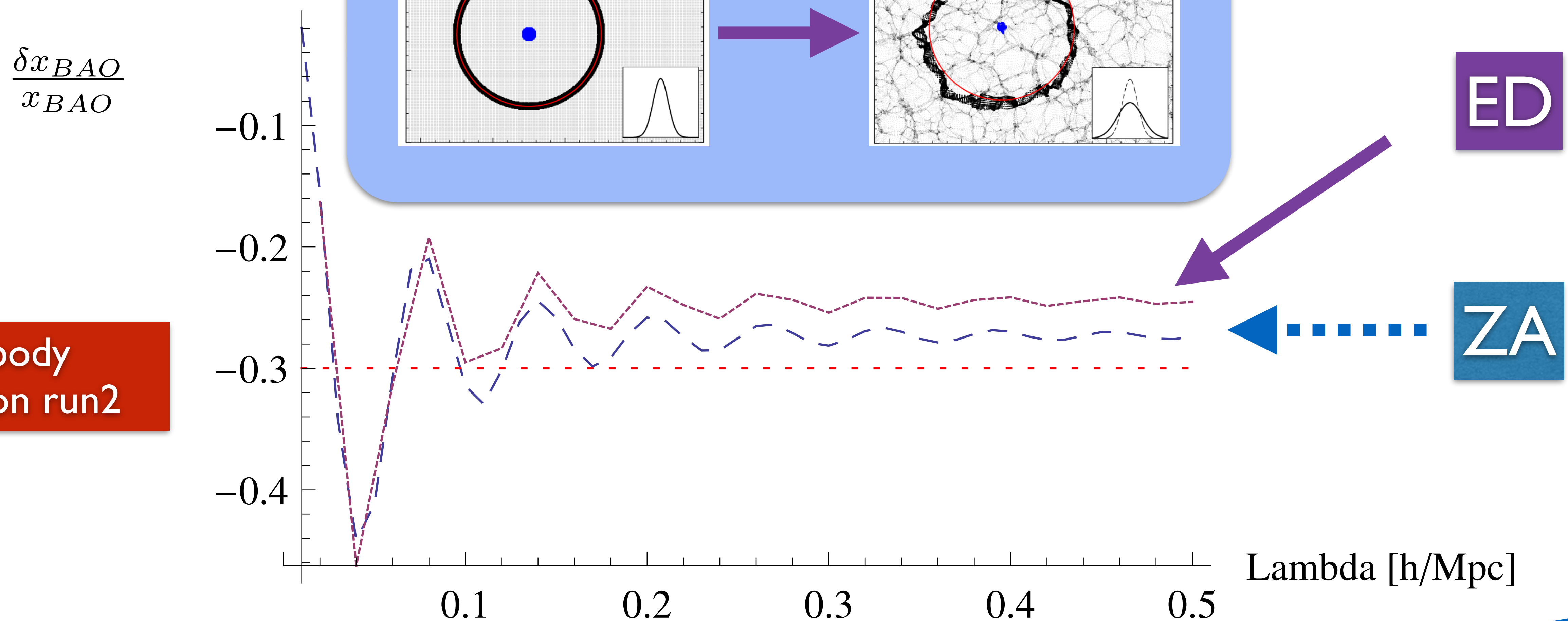
Shift of the BAO peak

$$\xi_{BAO}(x) \propto e^{-\frac{(x-s)^2}{4\Sigma^2(\Lambda)}} \left(1 - \frac{\Sigma_{sub-leading} - \frac{3}{2}\Sigma^B_{sub-leading}}{2\Sigma^2}(x-s) - \frac{\Sigma^B_{sub-leading}}{8\Sigma^4}(x-s)^3\right)$$

$$s \equiv r_{BAO}$$



Rel. shift of BAO peak w.



N-body
Horizon run2

Big in Modified Gravity - Galileons
(Bellini, Zumalacárregui'15)

Preliminary

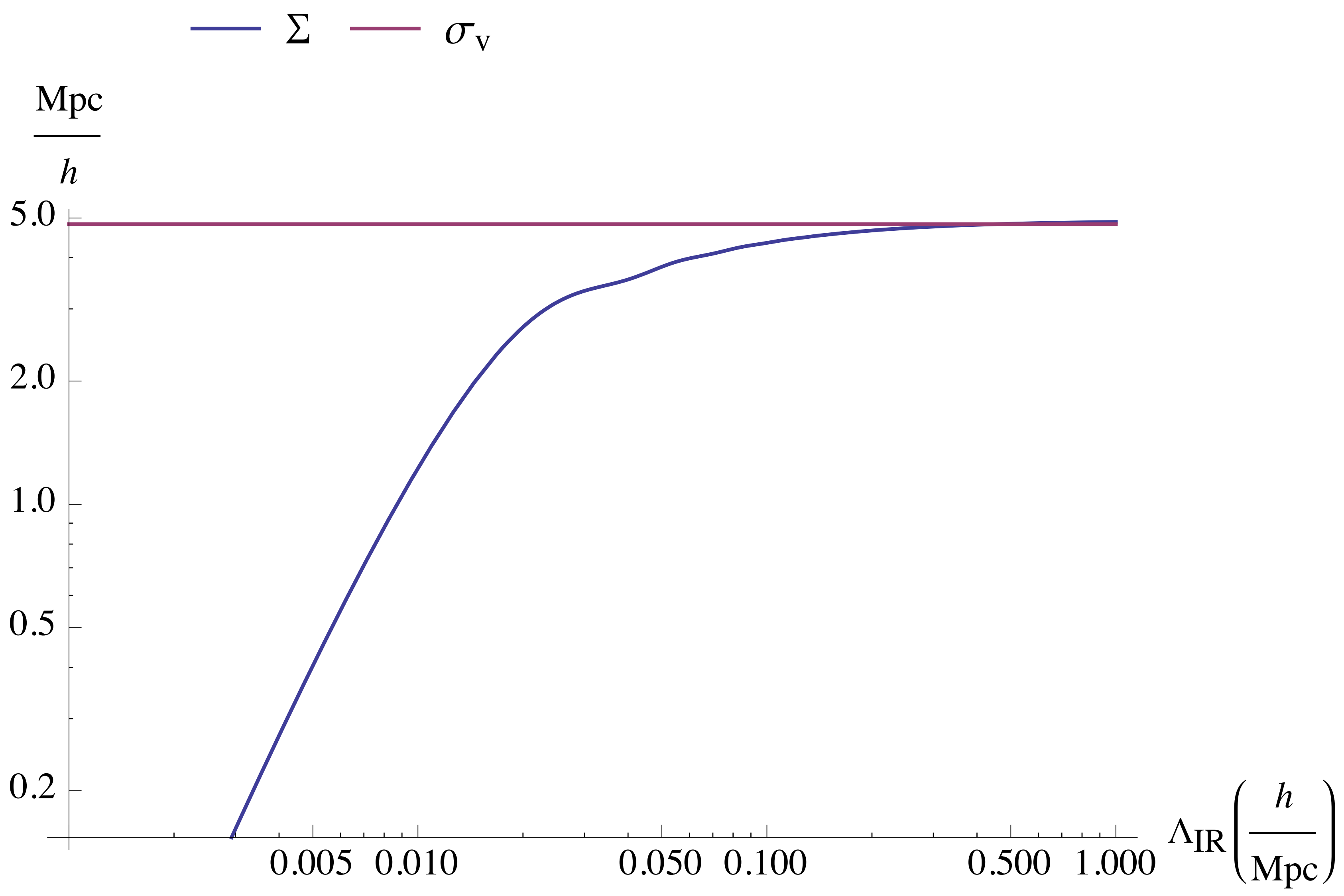
BAO IR - resummation

$$P_w^{IR-res}(k) = \exp\left\{ -\Sigma^2(\Lambda_{IR})k^2 \right\} P_w(k)$$

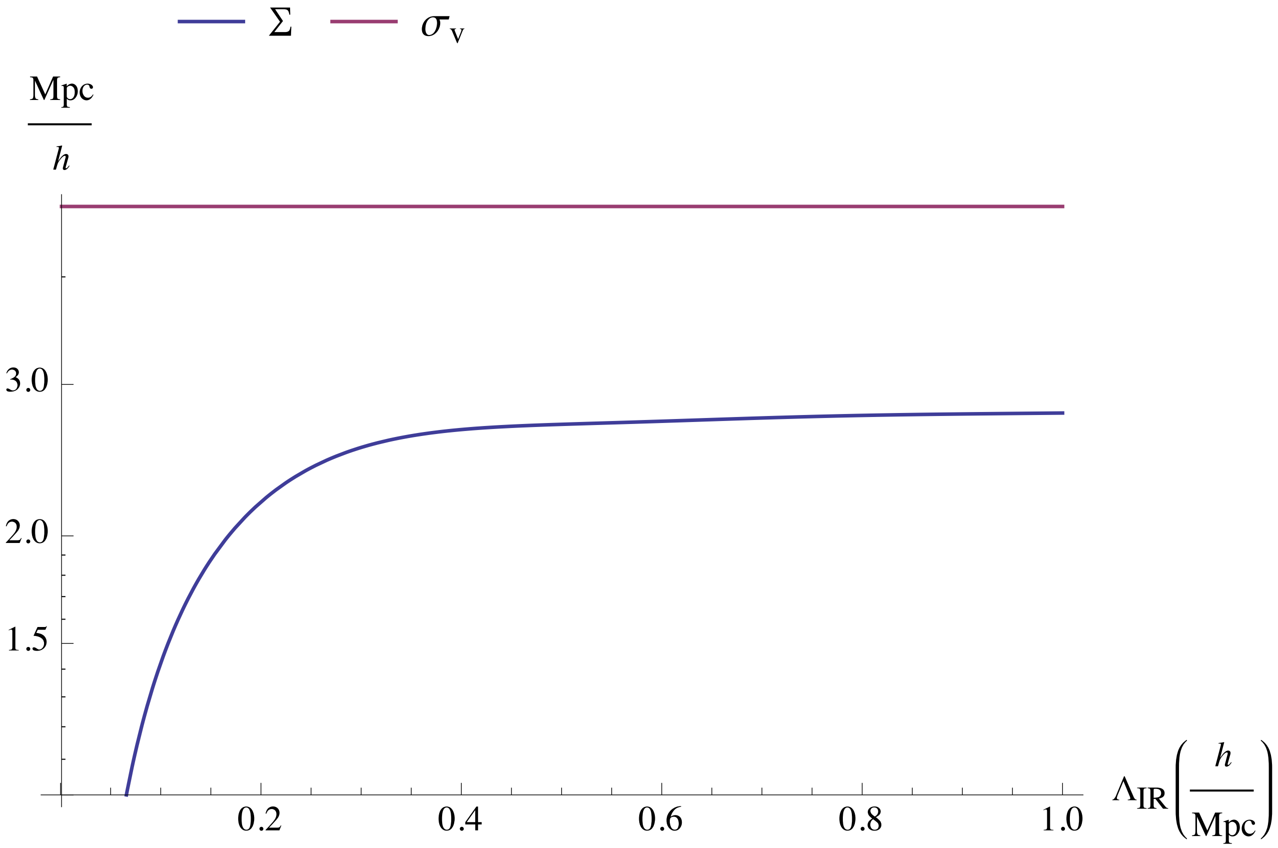
We, Zaldarriaga et al'15

$$P_w^{RPT}(k) = \exp\left\{ -\sigma_v^2(\Lambda = \infty)k^2 \right\} P_w(k)$$

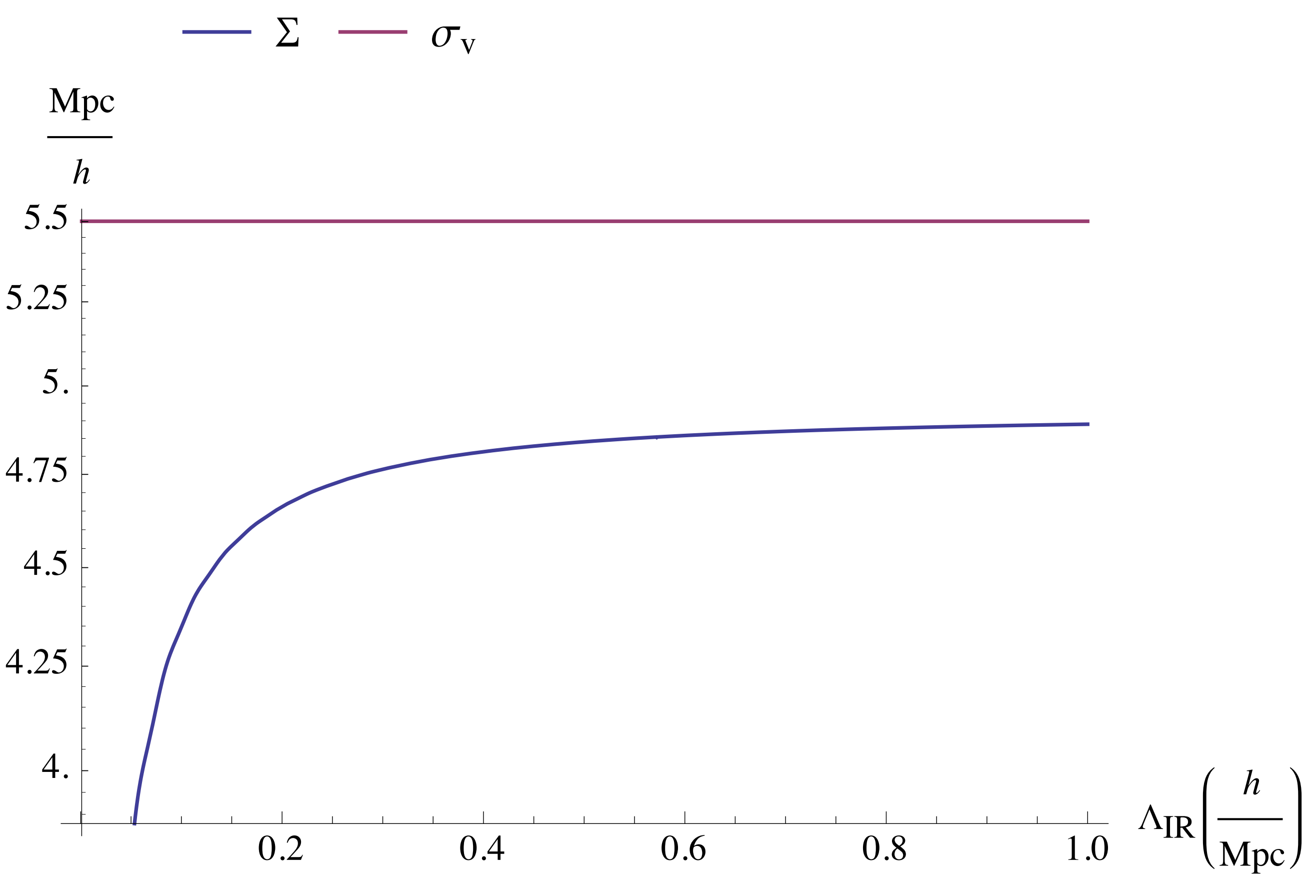
Crocce, Scoccimarro'06

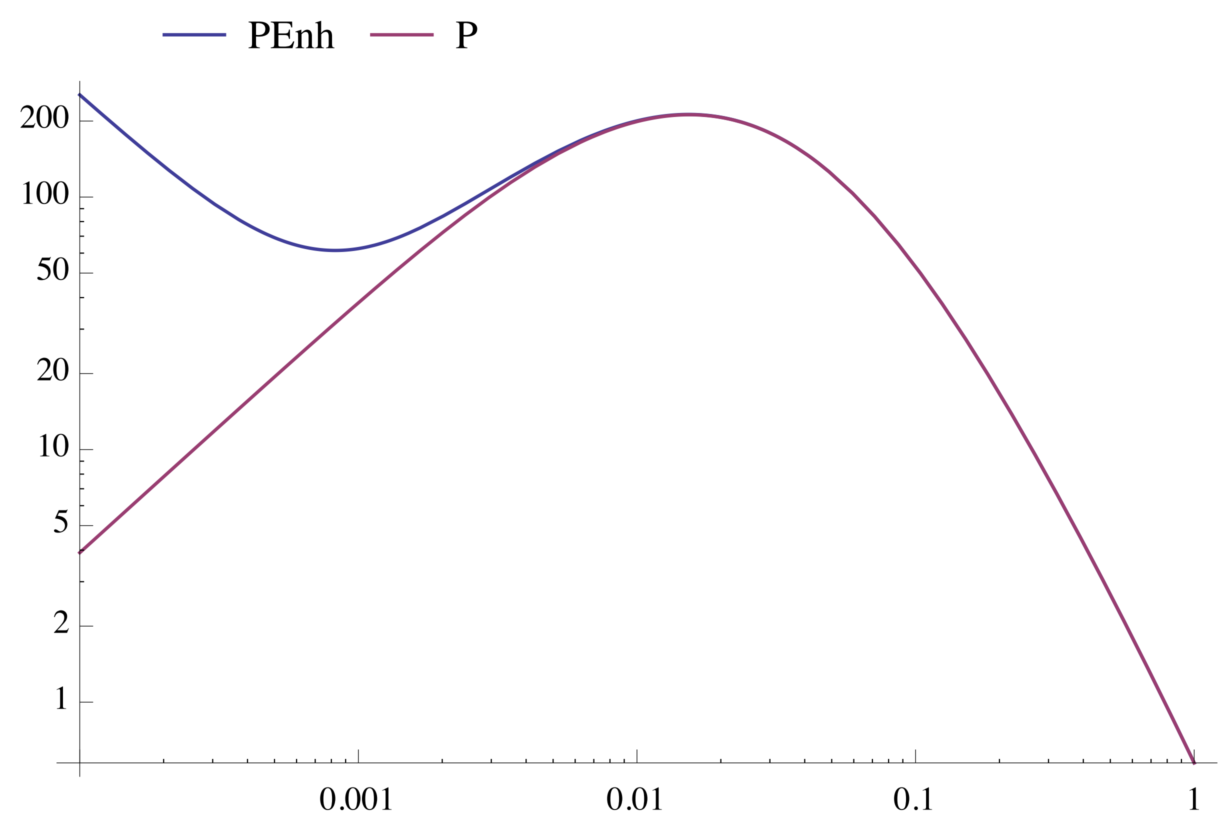


Success of other approximations



$$r_{BAO} = 7\ Mpc/h$$

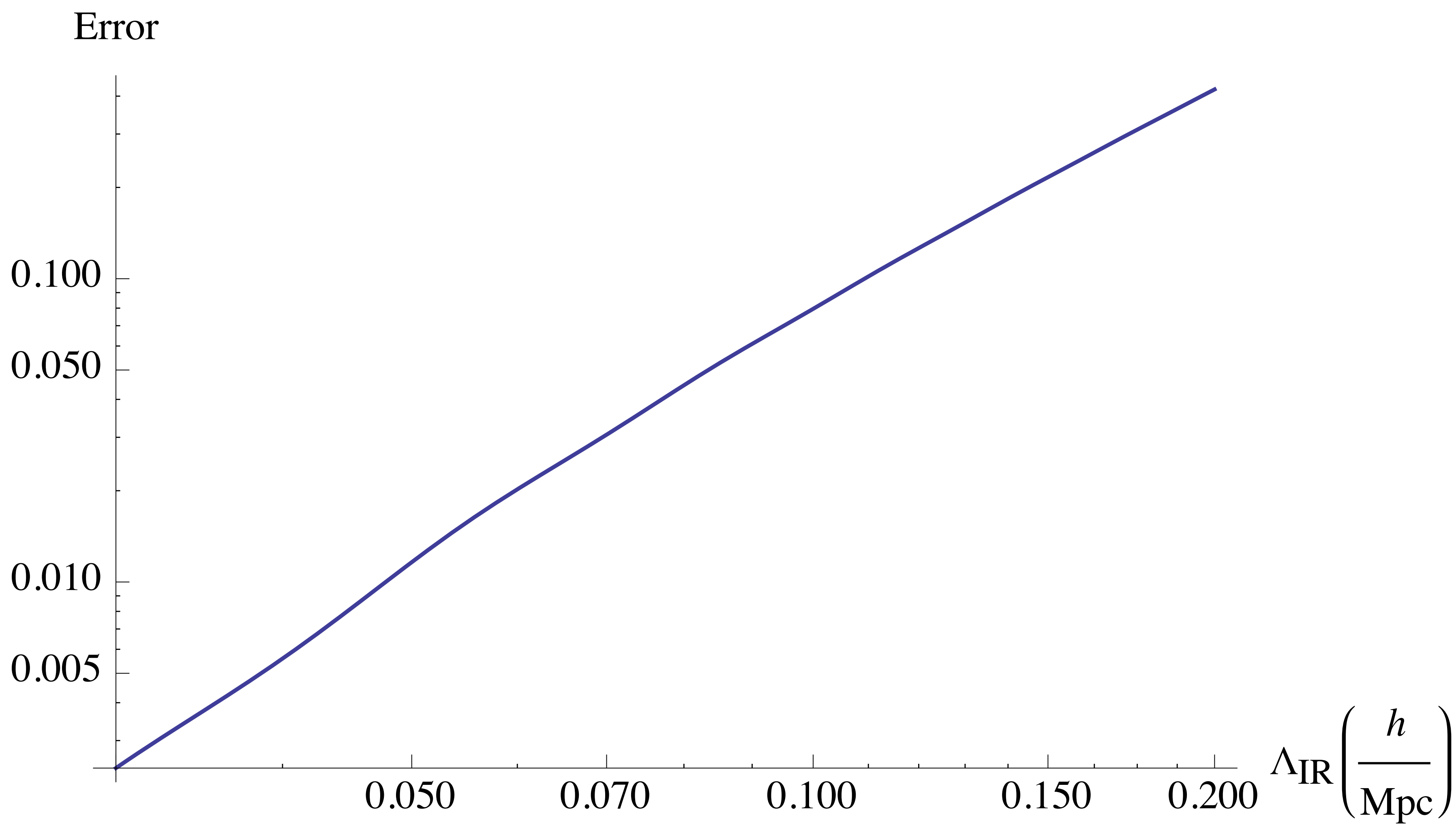




NLO IR - resummation

Theoretical accuracy $\sim \mathcal{O}(\Sigma^4(\Lambda_{IR})k^2\Lambda_{IR}^2)$

$k = 0.1h/Mpc$



NLO IR - resummation

$$P_w^{1loop}(k) = P_{1loop}[P_s(q) + P_w(q)] - P_{1loop}[P_s(q)]$$

ZA:

$$P_{\delta\delta}^{NLO-res}(k) = e^{-\Sigma^2(\Lambda_{IR})k^2}(1 + \Sigma^2 k^2)P_w(k)$$
$$+ P_{1-loop}[P_s(q) + e^{-\Sigma^2 q^2} P_w(q)] - P_{1-loop}[P_s(q)]$$

ED:

$$P_{\delta\delta}^{NLO-res}(k) = e^{-\Sigma^2(\Lambda_{IR})k^2}(1 + \Sigma^2 k^2)P_w(k)$$
$$+ P_{1-loop}[P_s(q) + e^{-\Sigma^2 q^2} P_w(q)] - P_{1-loop}[P_s(q)]$$
$$+ e^{-\Sigma^2 k^2}\delta_B$$

$$\delta_B = \frac{6}{7}\int_q \int_{q'} P_s(q)P_s(q')\sin^2(\mathbf{q},\mathbf{q}')\frac{(\mathbf{k}\cdot(\mathbf{q}+\mathbf{q}'))}{(\mathbf{q}+\mathbf{q}')^2}\frac{(\mathbf{k}\cdot\mathbf{q})}{q^2}\frac{(\mathbf{k}\cdot\mathbf{q}')}{q'^2}\sinh(\mathbf{q}\cdot\nabla)(\cosh(\mathbf{q}'\cdot\nabla)-1)P_w(k)$$

Shift of the BAO peak

$$P_{wiggly}(k) = A_{osc}\frac{e^{i\frac{k}{k_{osc}}}}{k^m}$$

$$\Sigma_{sub-leading}(\Lambda) \equiv \Sigma^{ZA}_{sub-leading}(\Lambda) + \Sigma^{A}_{sub-leading}(\Lambda)$$

$$\Sigma^{A}_{sub-leading}(\Lambda) = 4\pi\frac{6}{7}\int_0^\Lambda dq q P_L(q,\eta)\int_0^1 d\mu\,(1-\mu^2)\mu\sin(\mu q/k_{osc})$$

$$\Sigma^{B}_{sub-leading}(\Lambda) = \frac{1}{k^3\Sigma^2(\Lambda)}\frac{6}{7}\int_{|q|<\Lambda} d^3q P_L(q,\eta)\int_{|p|<\Lambda} d^3p P_L(p,\eta)\frac{k\cdot q}{q^2}\frac{k\cdot p}{p^2}$$
$$\times\frac{k\cdot(p+q)}{(p+q)^2}\sin^2(p,q)\sin[\cos(k,q)\,q/k_{osc}]\,\{1-\cos[\cos(k,p)\,p/k_{osc}]\}$$

$$\Sigma^{ZA}_{sub-leading}(\Lambda) \equiv 4\pi\int_0^\Lambda dq q P_L(q,\eta)\int_0^1 d\mu\,\mu^2\Big((2+m)\mu\sin(\mu q/k_{osc})$$
$$-\frac{q}{2k_{osc}}(1-\mu^2)\cos(q\mu/k_{osc})\Big)$$

$$\xi_{BAO}(x) \propto e^{-\frac{(x-s)^2}{4\Sigma^2(\Lambda)}}\left(1-\frac{\Sigma_{sub-leading}-\frac{3}{2}\Sigma^B_{sub-leading}}{2\Sigma^2}(x-s)-\frac{\Sigma^B_{sub-leading}}{8\Sigma^4}(x-s)^3\right)$$

Standard perturbation theory

density var.

eff. coupl. const

$$k/k_{osc} \sim 10 \qquad \frac{k^2}{k_{osc}^2}\sigma_l^2 = \mathcal{O}(1)$$

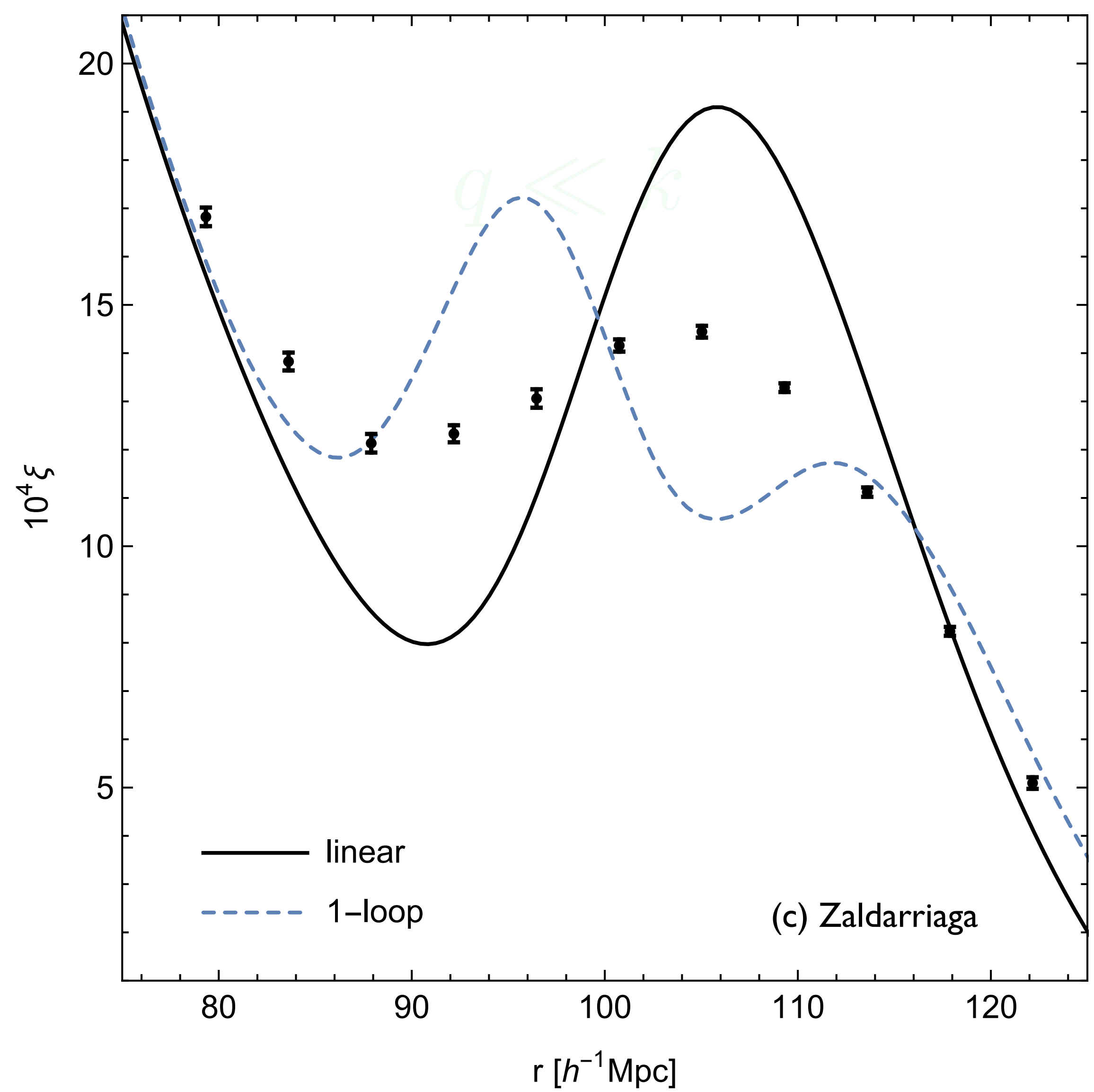
Advantages

1) very straightforward

2) gave us a lot of intuition

Disadvantages

1) Spurious IR enhancements
2) Difficulties in the correct description of IR effects - BAO



BAO IR - resummation

$$\Gamma_n \propto \frac{1}{P^L(k)} \qquad P^L(k) = P_s(k) + P_w(k)$$

$$\Gamma_n = \Gamma_n^s + \Gamma_n^w + \mathcal{O}\left((P_w/P_s)^2\right)$$

$$\Gamma_n^w(\vec{k}, -\vec{k}, \vec{q}_1, ..., \vec{q}_{n-2}) = \prod_{j=1}^{n-2} \frac{(\vec{k}\cdot\vec{q}_j)}{q_j^2}(1 - e^{-q_j\cdot\nabla_k})\frac{P_w(k)}{P_s^2(k)}$$

$$\frac{q_j}{k} \to 0$$

$$q \ll k \qquad\qquad q \ll k_{osc}$$

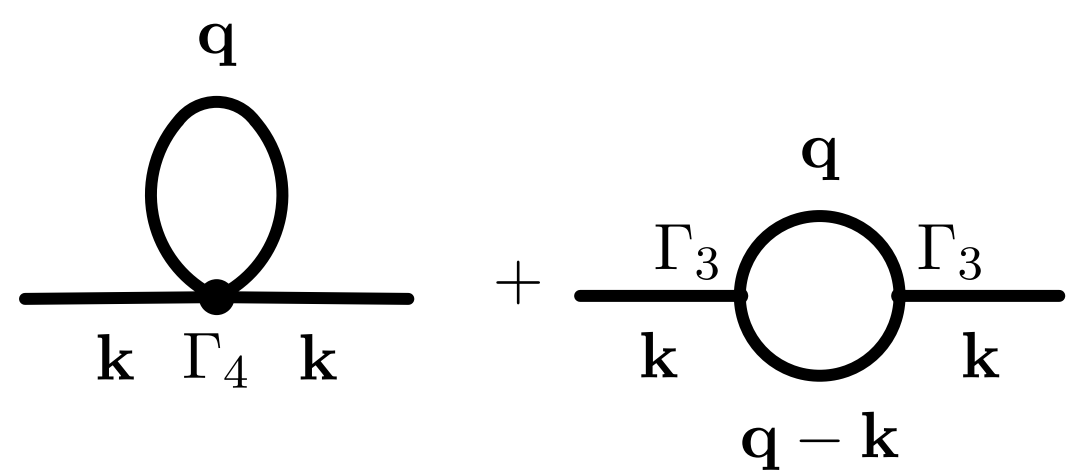
$$\left(\frac{k}{q}\right)^{n-2} \sin^{n-2}\left(\frac{q}{k_{osc}}\right) \qquad \sim \left(\frac{k}{k_{osc}}\right)^{n-2} \frac{P_w(k)}{P_s^2(k)}$$

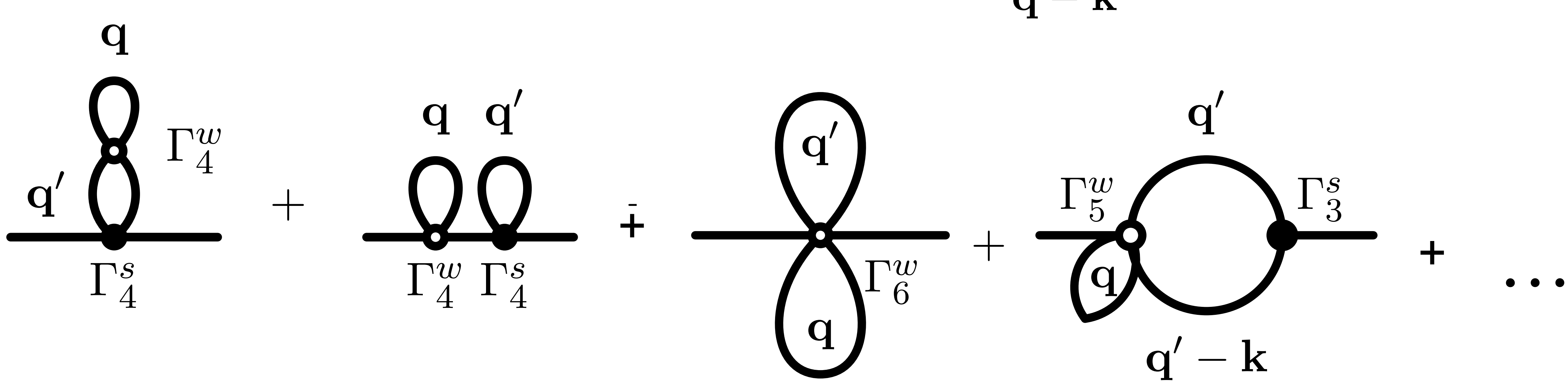
At each given loop order we should take only the graph with the bigger n in Gn !

NLO IR - resummation

1) Let's take 1-loop 'wiggly' PS and take on top of it higher loop corrections enhanced in the IR

Exact:





Leading order in $\frac{k^2}{k_{osc}^2} P_w^{1loop}(k)$

Large scale structure: basics

Two cornerstones:

I) Initial probability distribution

$$\langle \delta_0(\mathbf{k})\delta_0(\mathbf{k}')\rangle = P_0(k)\delta^{(3)}(\mathbf{k}'+\mathbf{k})$$

δ is a random gaussian-distributed variable

II) Time evolution

$$\frac{\partial \delta}{\partial \tau} + \nabla \cdot (1+\delta)\vec{v} = 0$$

$$\frac{\partial \vec{v}}{\partial \tau} + H\vec{v} + \vec{v}\cdot\nabla\vec{v} = -\nabla\Phi$$

Poisson equation:

$$\nabla^2\Phi = 4\pi G a^2 \bar{\rho}\delta$$

$\delta = \rho/\bar{\rho} - 1$ density contrast

$\vec{v}$ fluid velocity

H Hubble parameter

Φ grav. potential

Time - sliced perturbation theory (TSPT)

Time -evol. fields: $$\partial_t \theta = \sum_{n=1}^{\infty} \int I_n \theta^n$$

$$\mathcal{P} = \exp\{-W[\theta]\} \qquad W = \sum_{n=2}^{\infty} \int [dq]^n \Gamma_n \theta^n$$

$$\partial_t \Gamma_n + \sum_{k=1}^{n} \Gamma_k I_{n-k+1} = 0$$

NB. Contact with 3d Euclidean QFT: W is 1PI effective action
Cosmic time t is an external parameter

TSPT vertices

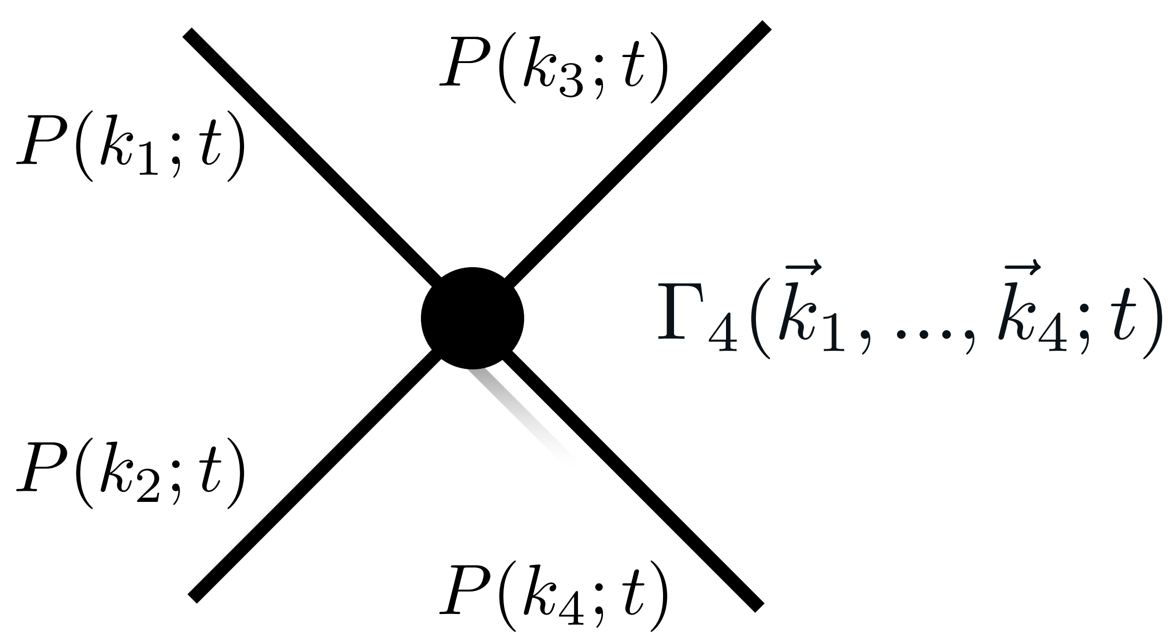
$$\Gamma_n = -\frac{1}{n-2}\sum_{k=2}^{n-1}\Gamma_k I_{n-k+1}$$

$$\Gamma_2(t) = \frac{1}{P^L(k,t)}$$

NB. Exact result!

$$\Gamma_n(t) \propto \frac{1}{P^L(t)}$$

$$\langle\theta(\mathbf{k}_1,t)...\theta(\mathbf{k}_n,t)\rangle^{tree,1PI} = \Gamma_n(\mathbf{k}_1,...,\mathbf{k}_n)\prod_{i=1}^{n}P^L(t,k_i)$$



$1PI$ - Trispectrum

Time - sliced perturbation theory (TSPT)

1) Compute all the statistical weights

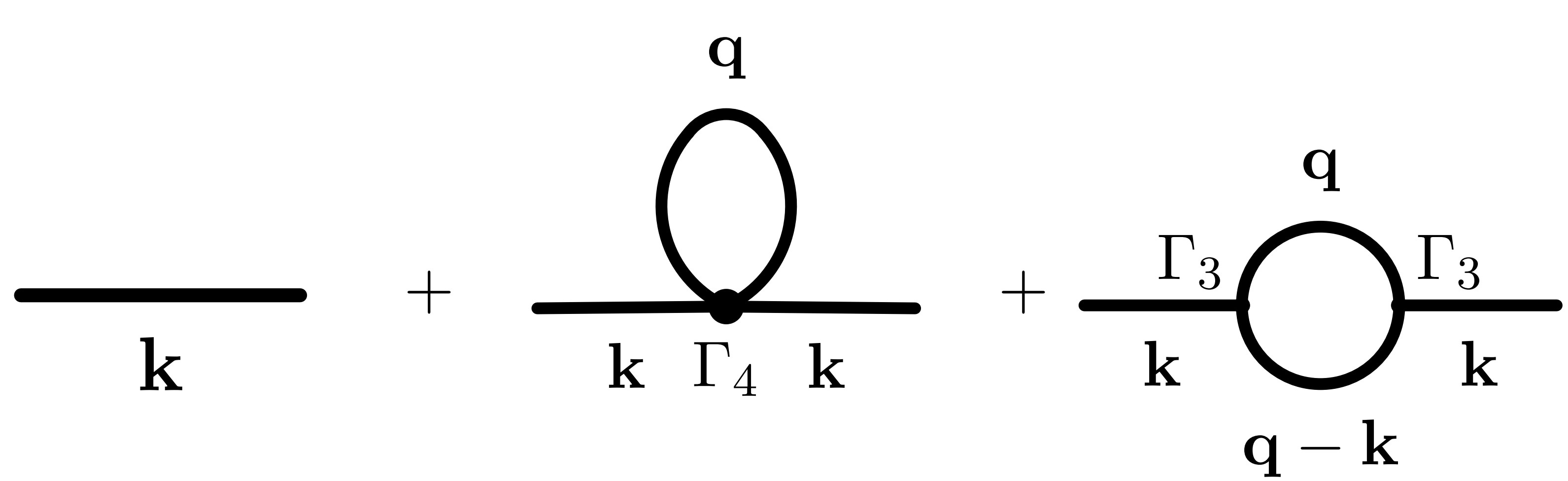
$$\Gamma_n = -\frac{1}{n-2}\sum_{k=2}^{n-1}\Gamma_k I_{n-k+1}$$

2) Insert them into the partition function

$$Z[J] = \mathcal{N}^{-1}\int \mathcal{D}\theta \left(1 - \frac{1}{3!}\int \Gamma_3\,\theta^3 - \frac{1}{4!}\int \Gamma_4\,\theta^4 + \frac{1}{2}\left(\frac{1}{3!}\int \Gamma_3\,\theta^3\right)^2 + \dots\right) e^{-\frac{\Gamma_2\theta^2}{2} + J\theta}$$

3) Compute correlation functions like in QFT
(Time evol. already solved!)

$$P^L_{\theta\theta}(k) + P^{1-loop}_{\theta\theta}(k) =$$

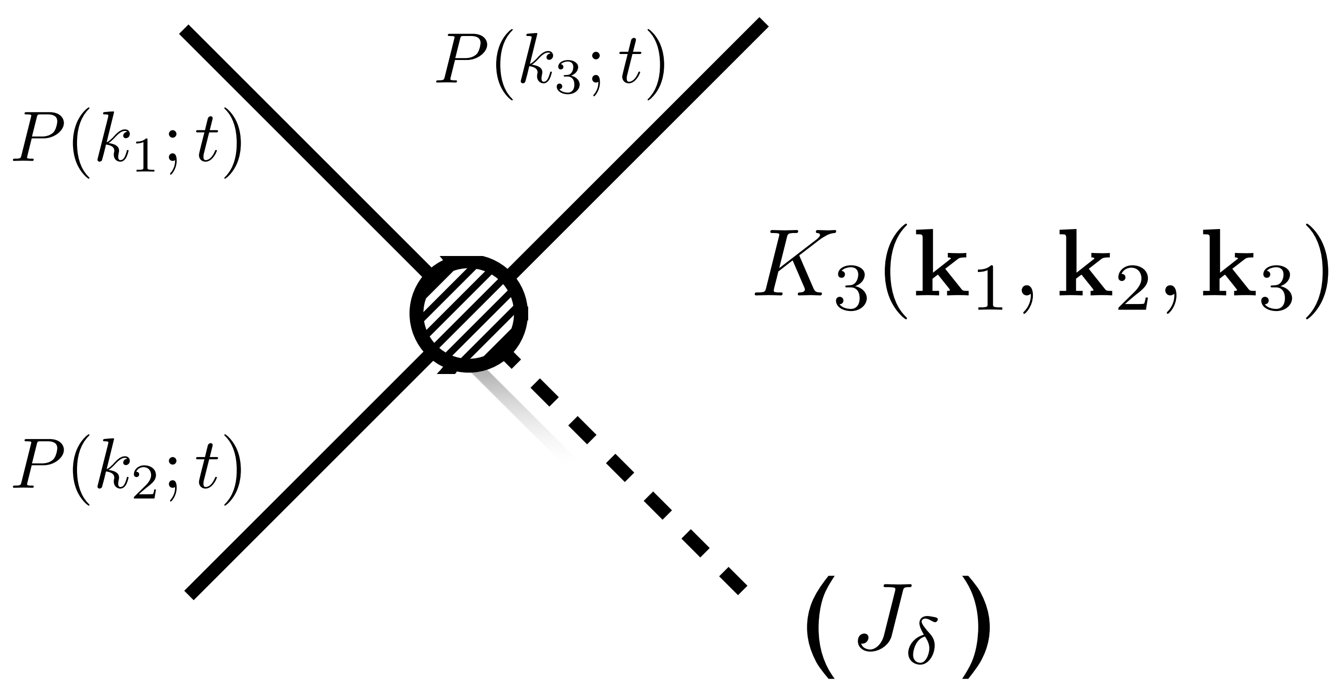


The density field

$$\delta = \sum_{n=1} \int [d\mathbf{q}] \delta_D^{(3)}(\mathbf{k} - \mathbf{q}_1 - ...) K_n(\mathbf{q}_1, ..., \mathbf{q}_n) \theta(\mathbf{q}_1) ... \theta(\mathbf{q}_n)$$

$$Z_t[J_\delta, J_\theta] \propto \int \mathcal{D}\theta \exp\left\{ - W[\theta] + \theta J_\theta + J_\delta \sum K_n \theta^n \right\}$$

Composite source



can have more than one leg !

The density field

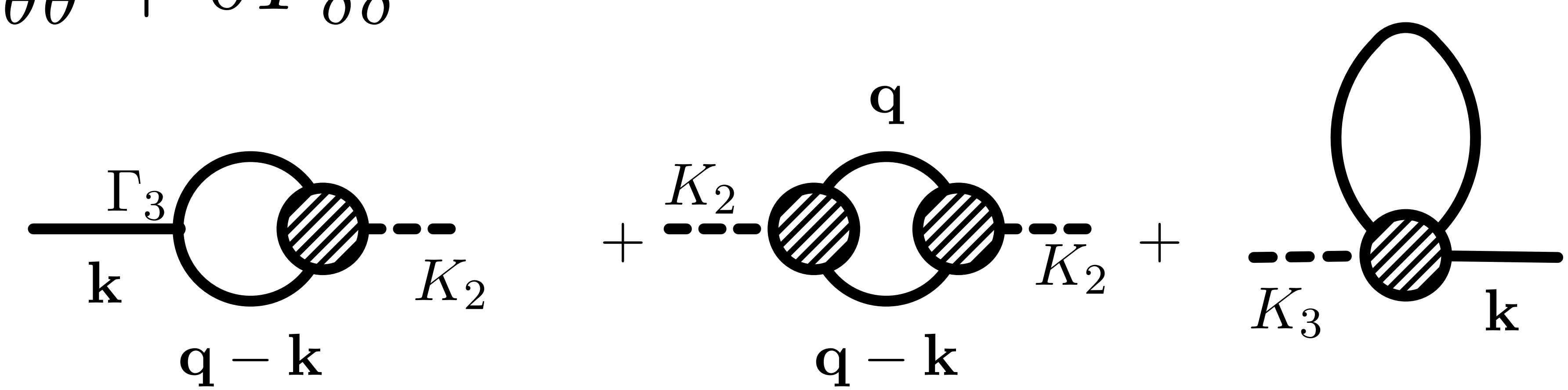
$$\delta = \sum_{n=1} \int [d\mathbf{q}] \delta_D^{(3)}(\mathbf{k} - \mathbf{q}_1 - \ldots) K_n(\mathbf{q}_1, \ldots, \mathbf{q}_n) \theta(\mathbf{q}_1) \ldots \theta(\mathbf{q}_n)$$

$$Z_t[J_\delta, J_\theta] \propto \int \mathcal{D}\theta \exp\left\{ - W[\theta] + \theta J_\theta + J_\delta \sum K_n \theta^n \right\}$$

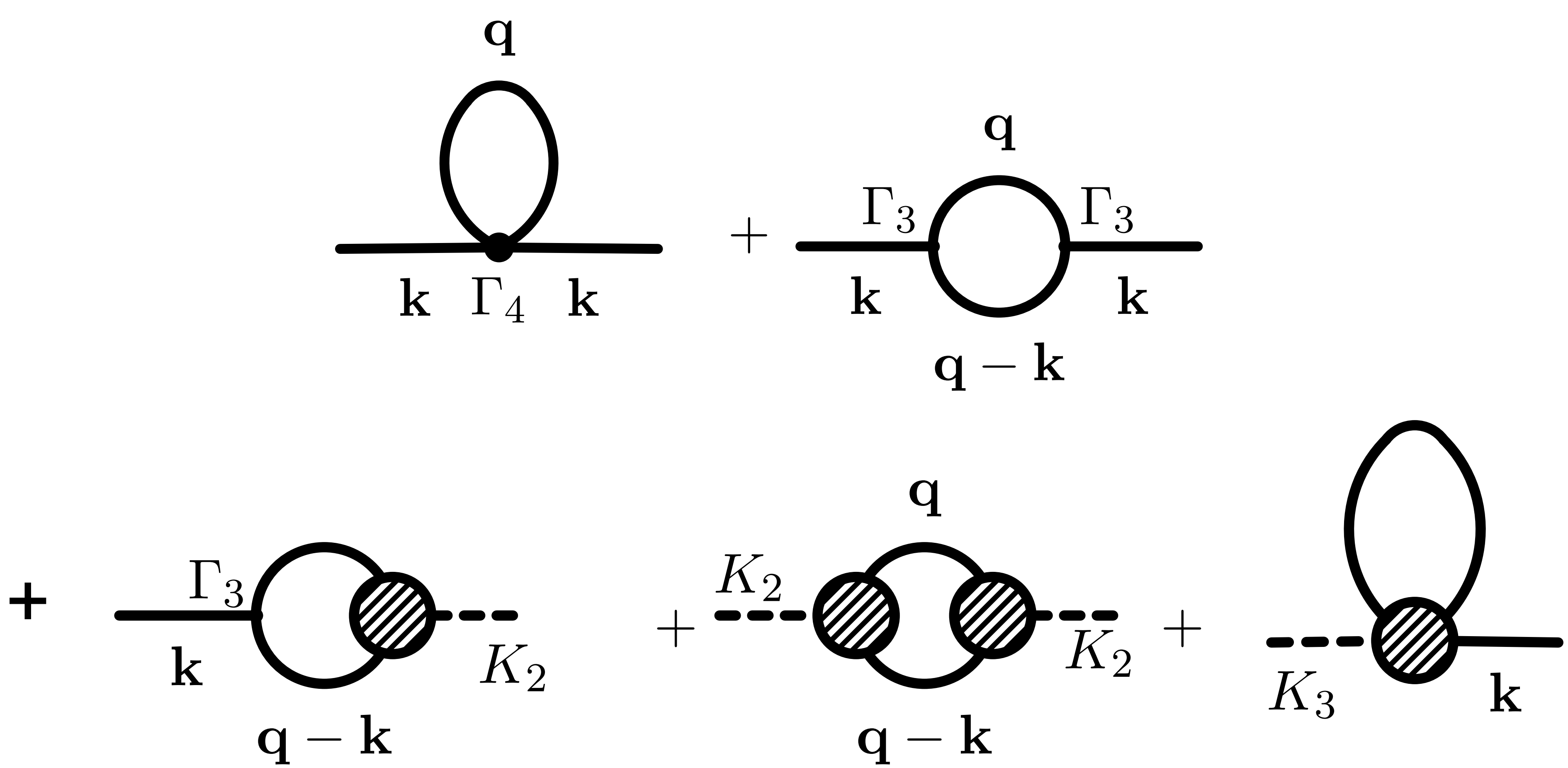
Composite source

$$P_{\delta\delta} = P_{\theta\theta} + \delta P_{\delta\delta}$$

$$\delta P_{\delta\delta} =$$



Comparison with SPT



$$= P_{13} + P_{22} = P_{\delta\delta}^{1loop}$$

IR safety

I) Loop integrants are not IR safe in SPT
=> IR divergences

II) in TSPT

$$\Gamma_{n+1}(\vec{k}_1, ..., \vec{k}_n, \vec{q})|_{q\to 0} \simeq \delta^{(3)}\left(\sum_{i=1}^{n} \vec{k}_i\right) \sum_{i=1}^{n} \frac{(\vec{k}_i \cdot \vec{q})}{q^2} \Gamma_n(\vec{k}_1, ..., \vec{k}_n)$$

NB. Consequence of the equivalence principle,
according to which all equal time correlators must be IR - safe

(cf. Consistency conditions - *Criminelli, Noreña, Simonovic, Vernizzi'14 Valages'13, Kehagias et al.'13)*

Loop integrants are IR safe in TSPT
=> no IR divergences

Towards UV - renormalisation

- SPT (EFT of LSS):
  infinite amount of UV counter-terms with
  an arbitrary non-local time dependence

Pajer et al'15

- TSPT: infinite amount of UV counter terms with
  fixed local time dependence

$$\Gamma_n = \frac{1}{a^2(\tau)} \hat{\Gamma}_n$$

effective coupl. const

time = μ in QFT MS, $\bar{M}S$

nb. gaussian i.c.

a) Necessary set of counter-terms: $\Gamma_n^{ctr} = \frac{1}{a^2(\tau)} \hat{\Gamma}_n^{ctr}$

b) Full set of counter-terms: $\Gamma_n^{ctr} \supset \frac{1}{a^m(\tau)} \hat{\Gamma}_n^{ctr}$

c) Local in time!

NB. QFT methods, renormalisation group

IR safety and Ward identities

non-rel. diff of FRW $\quad \eta \to \eta\,, \qquad x^i \to x^i + \frac{1}{6}\eta^2 \partial_i \Phi_L$

Physical solution in the limit $\quad q \to 0$

$$\frac{\partial \delta}{\partial \tau} + \nabla \cdot (1+\delta)\vec{v} = 0$$

$$\frac{\partial \vec{v}}{\partial \tau} + H\vec{v} + \vec{v}\cdot\nabla\vec{v} = -\nabla\Phi$$

$$\delta_S \to \delta_S + \frac{2}{3\mathcal{H}^2}\partial_i \delta_S \partial_i \Phi_L + \frac{2}{3\mathcal{H}^2}\Delta\Phi_L$$

$$\theta_s \to \theta_s + \frac{2}{3\mathcal{H}^2}\partial_i \theta \partial_i \Phi_L + \frac{2}{3\mathcal{H}^2}\Delta\Phi_L$$

$$\delta W\Big|_{q\to 0} = 0$$

$$\lim_{q\to 0}\, q \cdot W_{n+1}(\mathbf{k}_1, ..., \mathbf{k}_n, q) = 0$$

$$W_{n+1}(\mathbf{k}_1, ..., \mathbf{k}_n, q) = \mathcal{O}(q^0)$$