Multi-scale modelling of Pulsar Glitches

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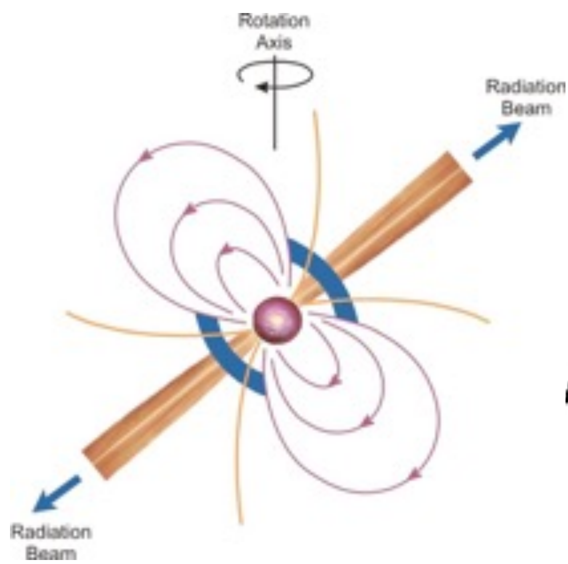
Pulsar glitches

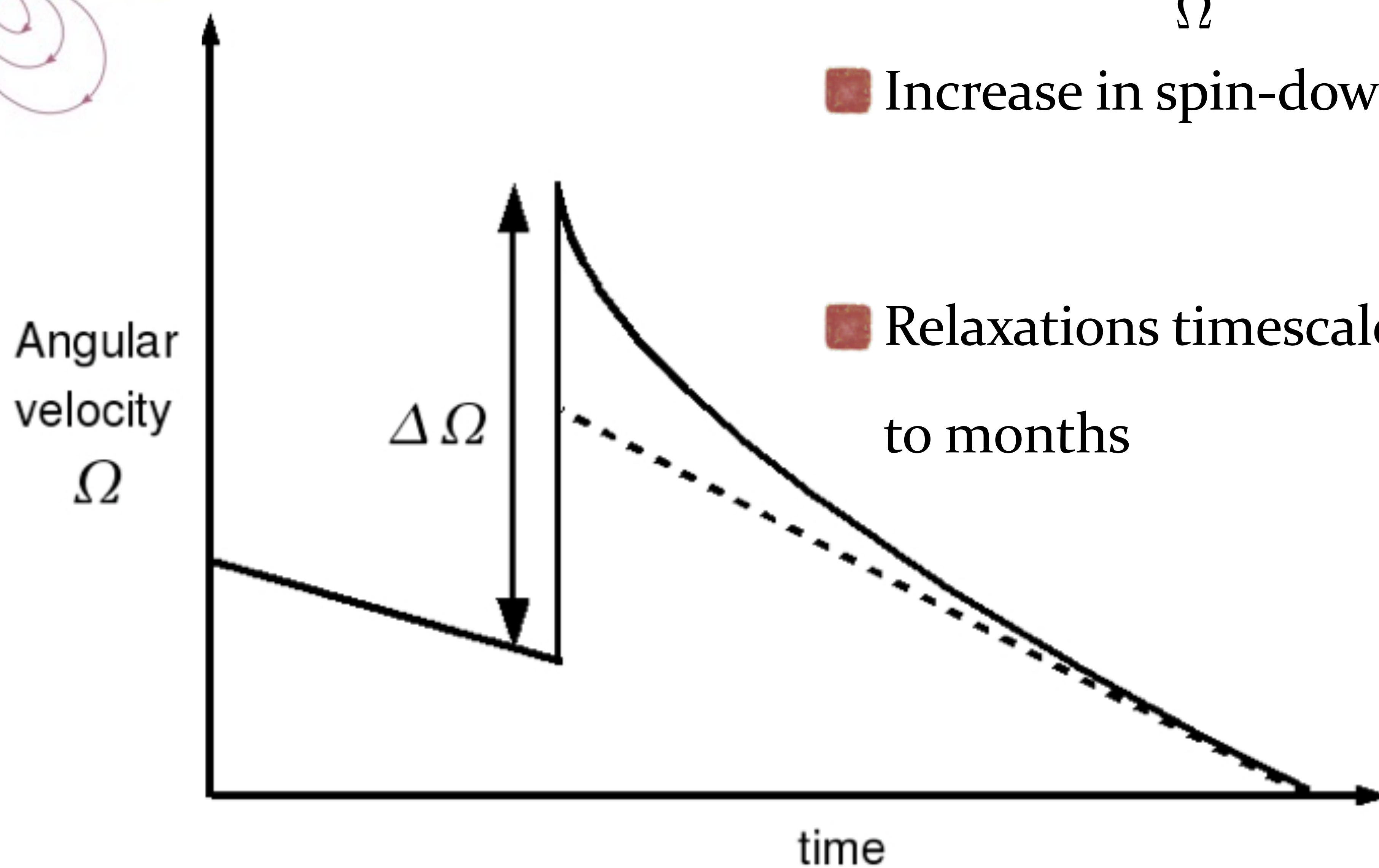


- Sudden 'jumps' in frequency, with

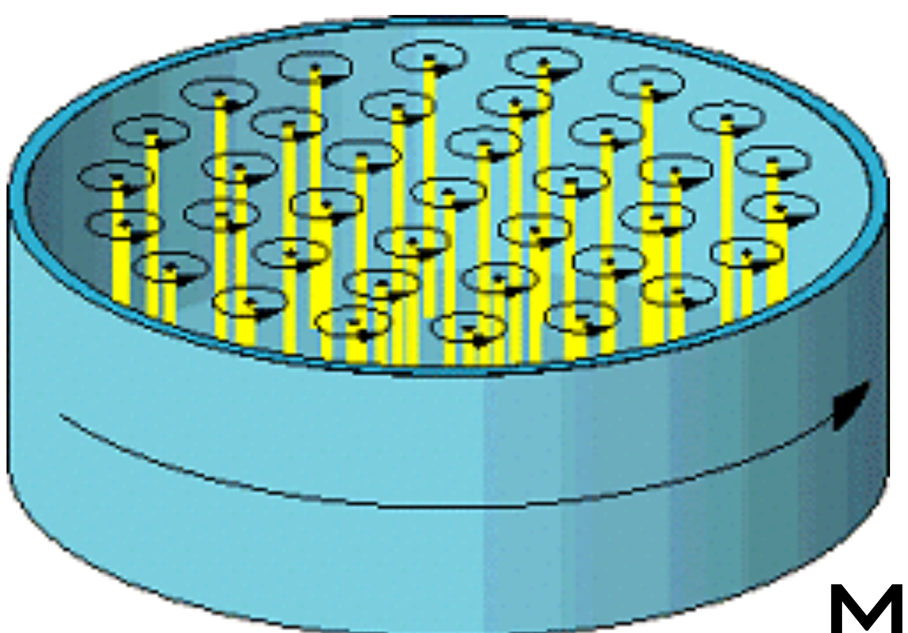
$$\frac{\Delta\Omega}{\Omega} \approx 10^{-10} - 10^{-5}$$

- Increase in spin-down rate

- Relaxations timescales from minutes to months

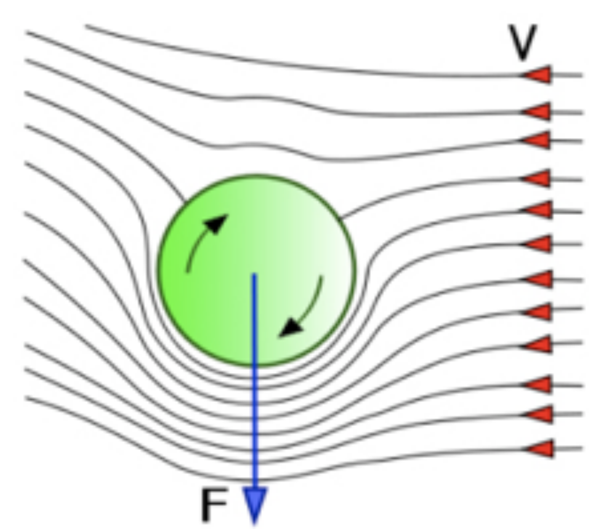


Vortex dynamics



- Superfluids rotate by forming quantised vortices
- Vortex density determines spin : vortices must move out to spin down the fluid!

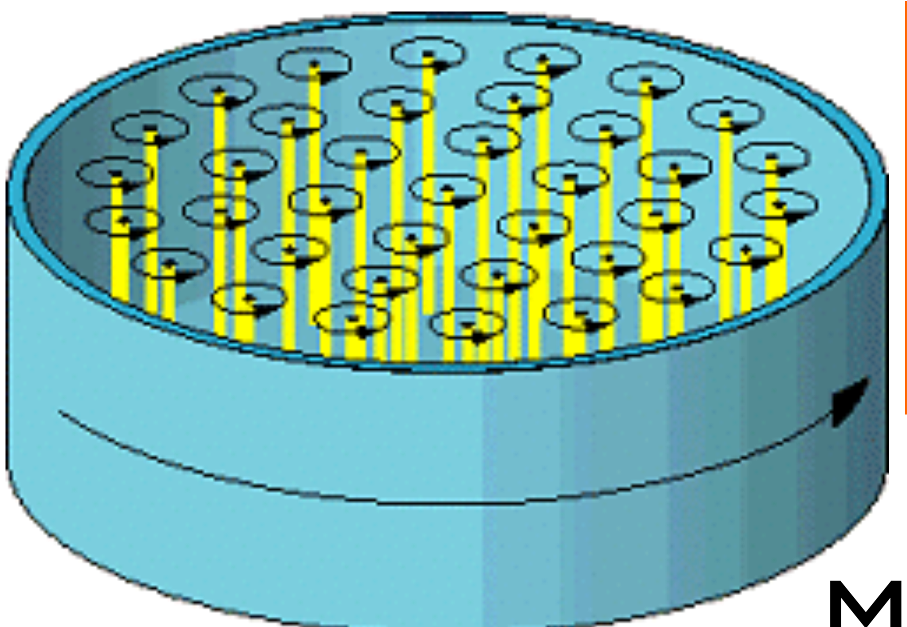
Magnus Force



$$\text{FREE}: \quad \epsilon^{ijk}\hat{k}_j(v_k^{\mathrm{v}} - v_k^{\mathrm{n}}) + \mathcal{R}(v_{\mathrm{c}}^i - v_{\mathrm{v}}^i) = 0$$

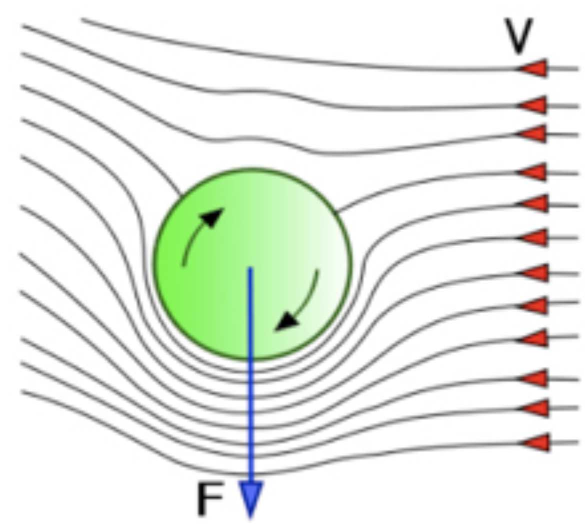
$$\text{PINNED}: \quad \epsilon^{ijk}\hat{k}_j(v_k^{\mathrm{v}} - v_k^{\mathrm{n}}) + F_{\mathrm{p}}^i = 0$$

Vortex dynamics



If vortices cannot move out the superfluid cannot spin down.

Magnus Force



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What triggers a glitch?

- Starquakes (Ruderman 69, 76)
- Hydrodynamical instabilities

  (Andersson et al. 2003, Glampedakis & Andersson 2009)
- Vortex avalanches (Cheng et al. 88, Alpar et al. 96)

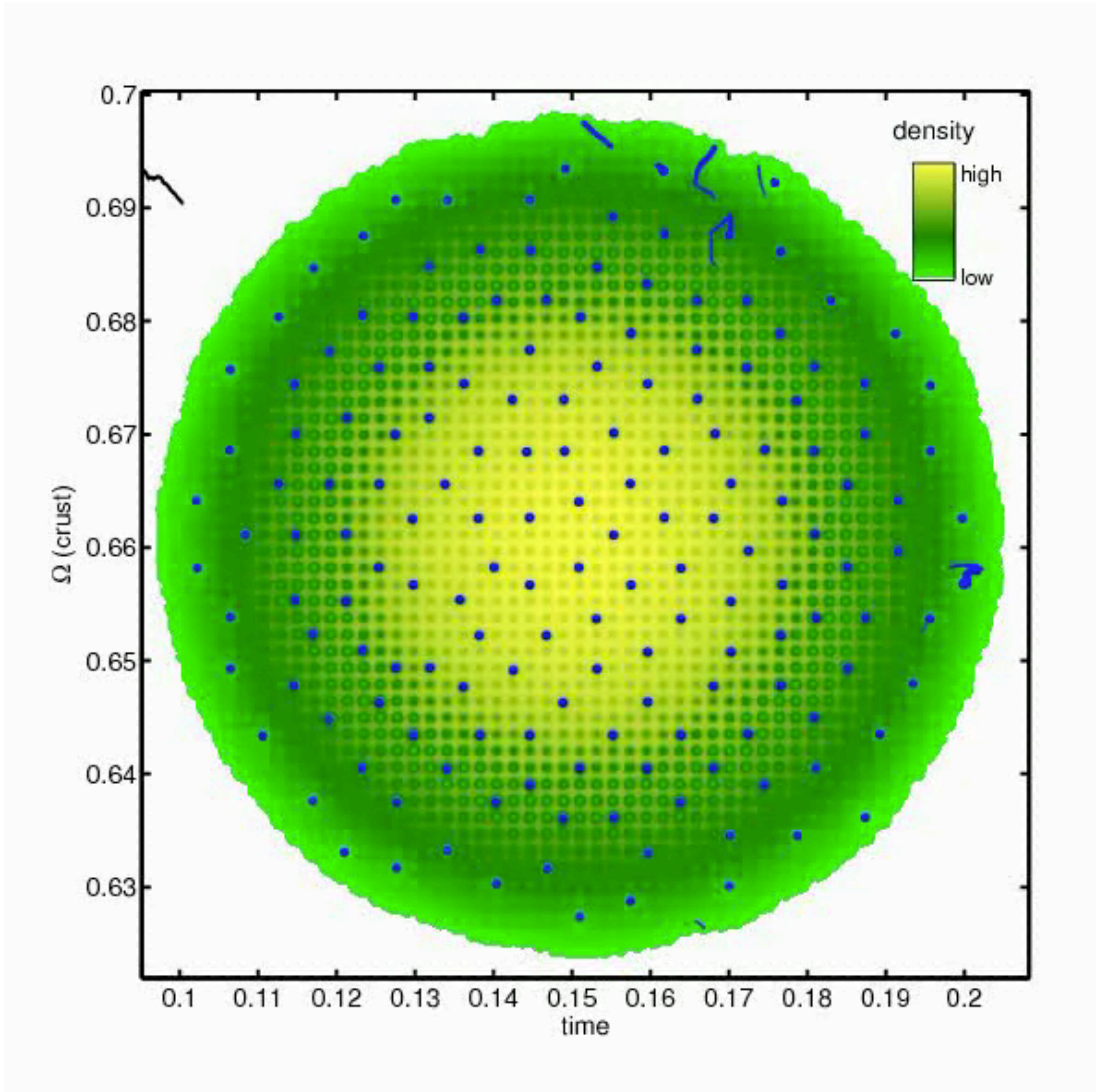
See Haskell & Melatos 2015 for a review

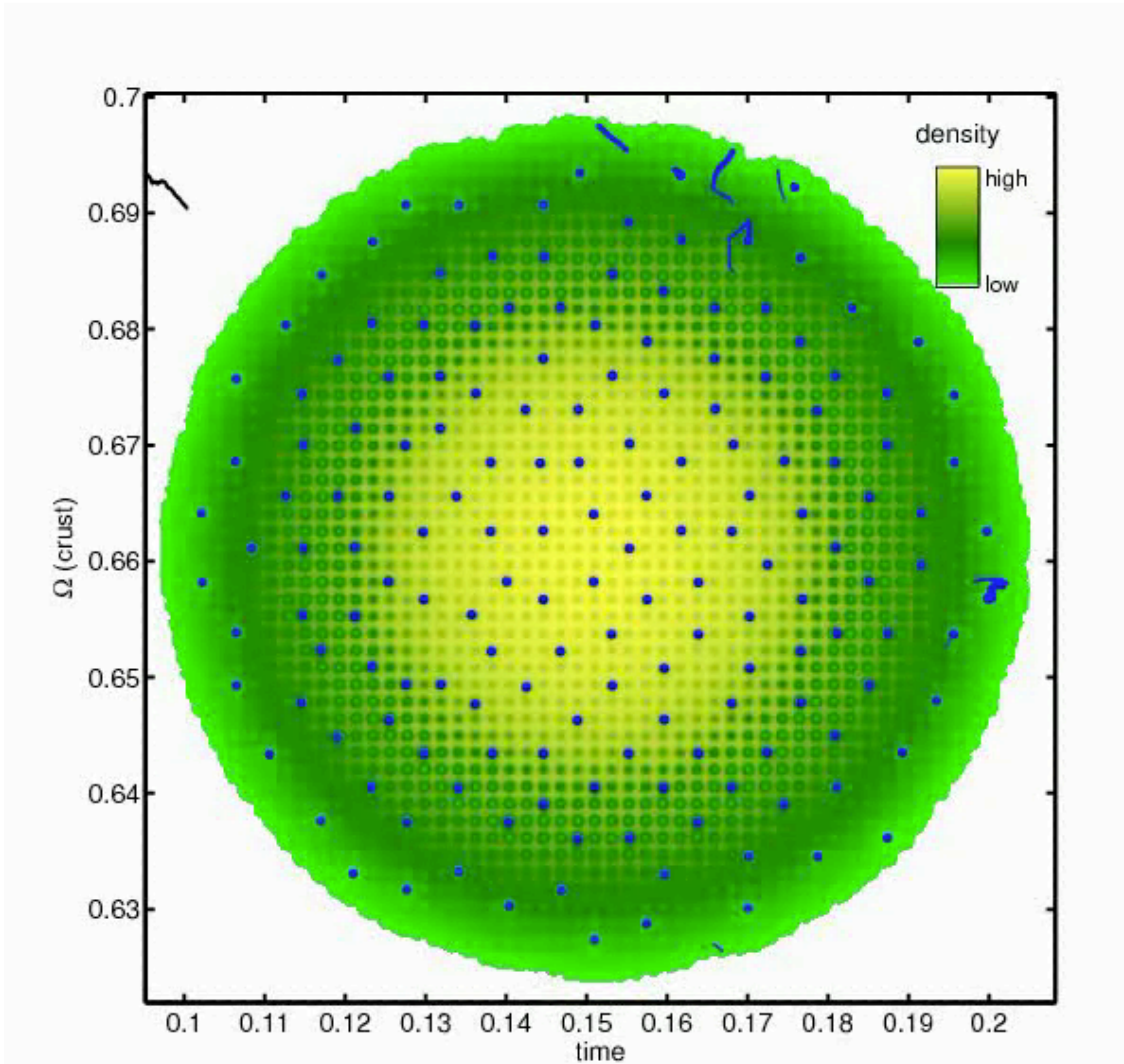
Gross Pitaevskii simulations:

$$(i-\gamma)\bar{h}\frac{\partial\psi}{\partial t} = -\frac{\bar{h}^2}{2m}\nabla^2\psi - (\mu - V - g|\psi|^2)\psi - \Omega\hat{L}_z\psi,$$

$$I_c\frac{d\Omega}{dt} = -\frac{d<\hat{L}_z>}{dt} + N_{EM}, \qquad V = V_{\rm trap} + \sum V_i[1+\tanh(\Theta(r-R_i)]$$

- Good description of BEC dynamics in which interactions are weak
- Predict power-law distributions for event sizes, and exponentials for waiting times (Warszawski & Melatos, 2008, 2013)
- Consistent with most pulsars (Melatos et al. 2008) but not the Crab! (Espinoza et al. 2014)


density
high
low
0.7
0.69
0.68
0.67
0.66
0.65
0.64
0.63
Ω (crust)
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18
0.19
0.2
time



(Courtesy of James Douglass)

Can an avalanche propagate in a NS?

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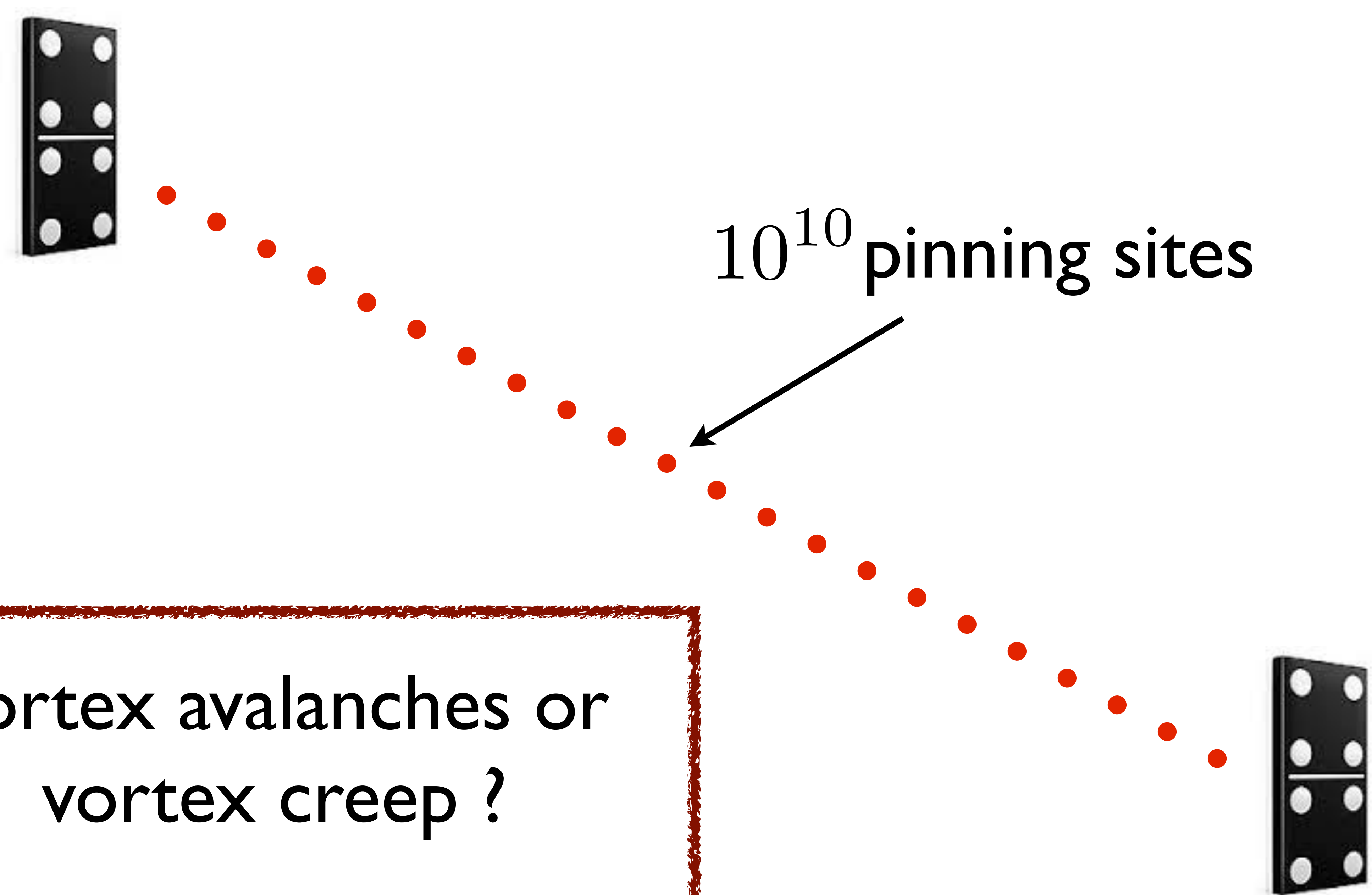


vortex avalanches or
vortex creep ?

Vortex Motion:

Magnus

Pinning

$$\epsilon^{ijk}\hat{k}_j(v_k^{\mathrm{L}} - v_k^{\mathrm{n}}) + \mathcal{R}(v_{\mathrm{p}}^i - v_{\mathrm{L}}^i) + \mathcal{F}^i + \sigma_i = 0$$

Drag

Vortex-Vortex

$$\mathcal{R} \approx 10^{-4}$$

Vortex Motion:

Magnus

Pinning

$$\epsilon^{ijk}\hat{k}_j(v^{\mathrm{L}}_k - v^{\mathrm{n}}_k) + \mathcal{R}(v^i_{\mathrm{p}} - v^i_{\mathrm{L}}) + \mathcal{F}^i + \sigma_i = 0$$

Drag

Vortex-Vortex

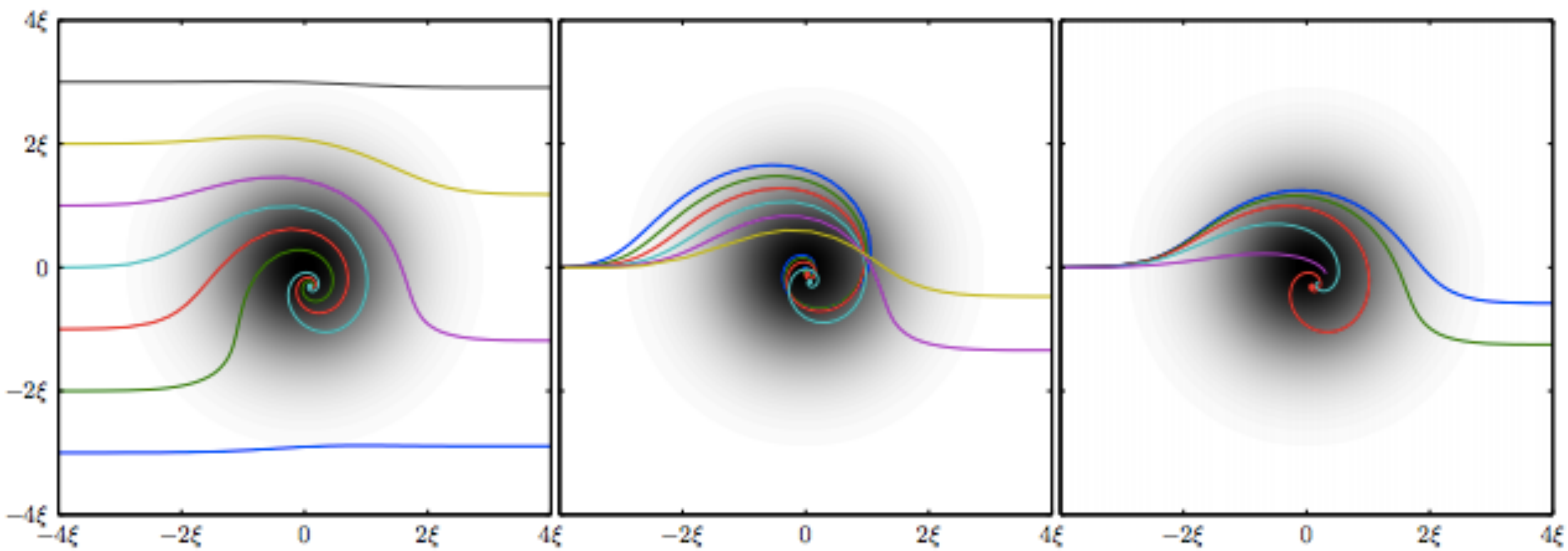
$$\mathcal{F}^i = -\nabla^i V$$

$$\mathcal{R} \approx 10^{-4}$$

in the core

$\mathcal{R} << 1$ Phonons in the crust

$\mathcal{R} \approx 1$ Kelvons in the crust



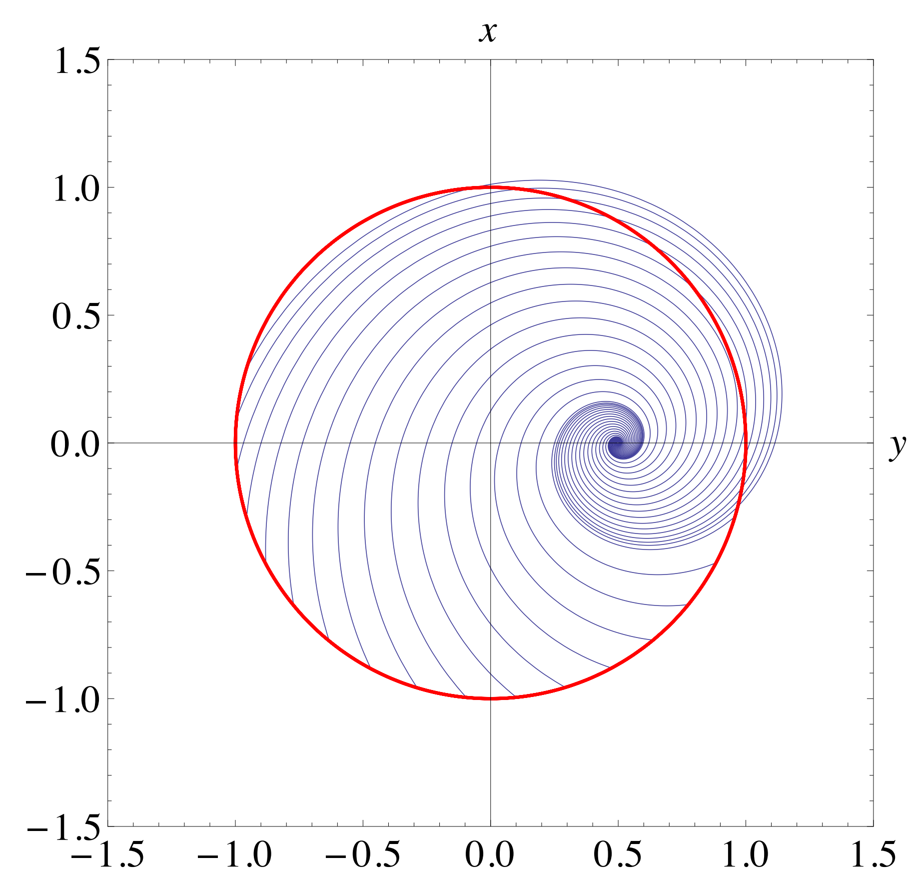
- N-body (Barnes-Hut) code [Douglass, Melatos & BH, in preparation]

$$V(r) = -E_p \exp\left(-\frac{|r-r_p|^2}{2\xi^2}\right)$$

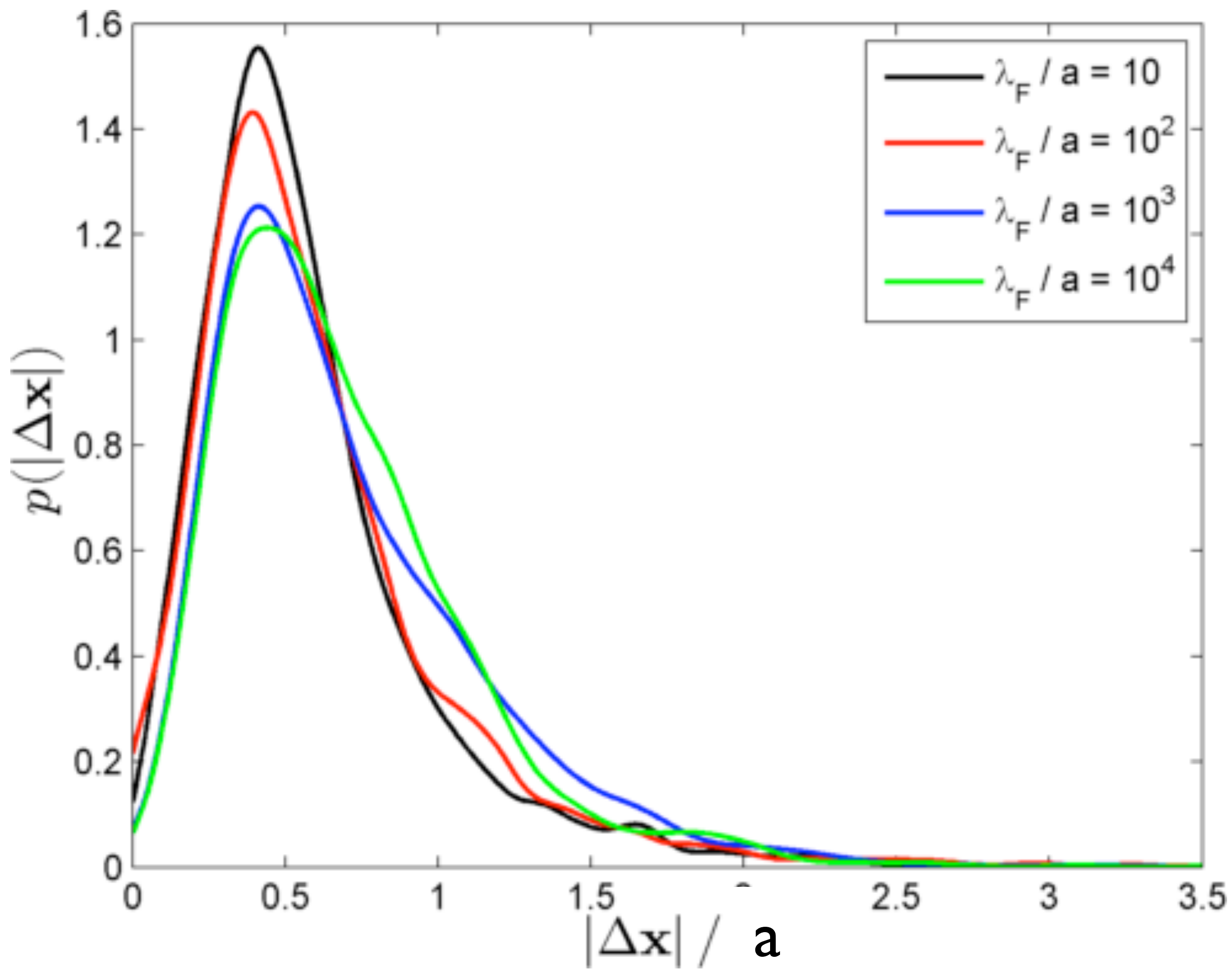
- Analytic cross section

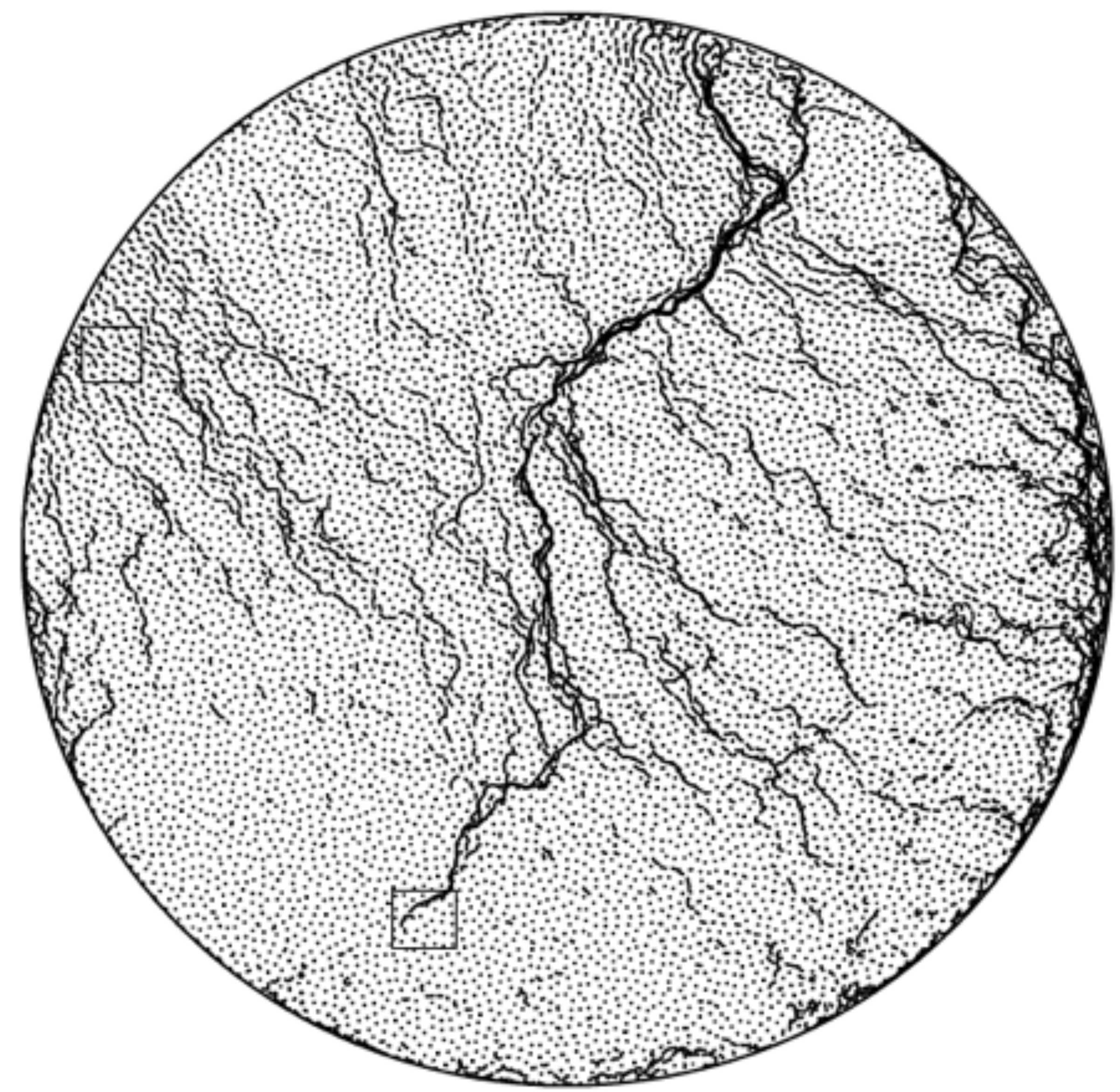
(Sedrakian 95, BH & Melatos, 2015)

$$V(r) = \frac{E_p}{2}(r-r_p)^2 \quad \text{for} \quad |r-r_p| < R_r$$



N-body results

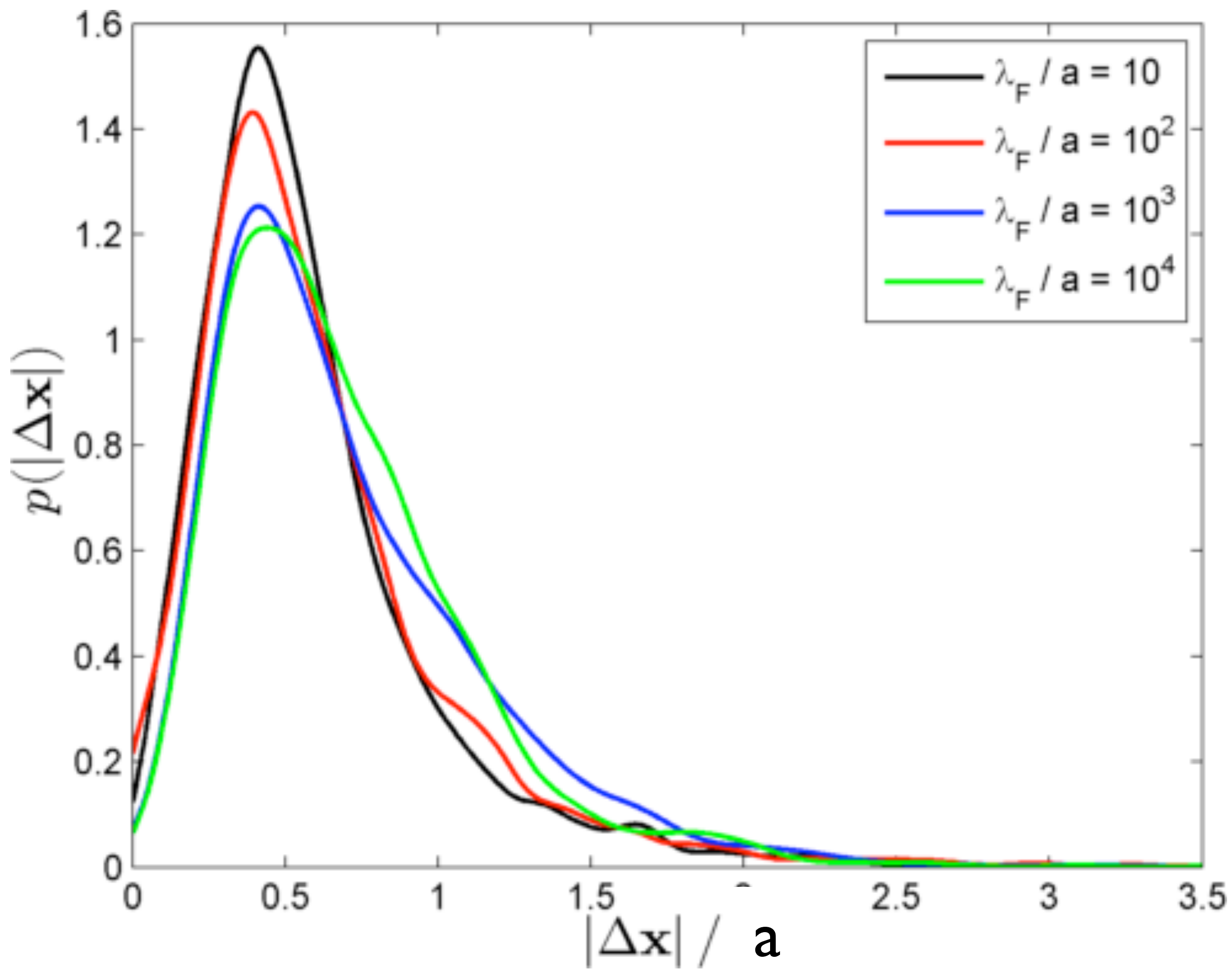




$$\mathcal{R} \approx 0.3$$

10^4 vortices

N-body results



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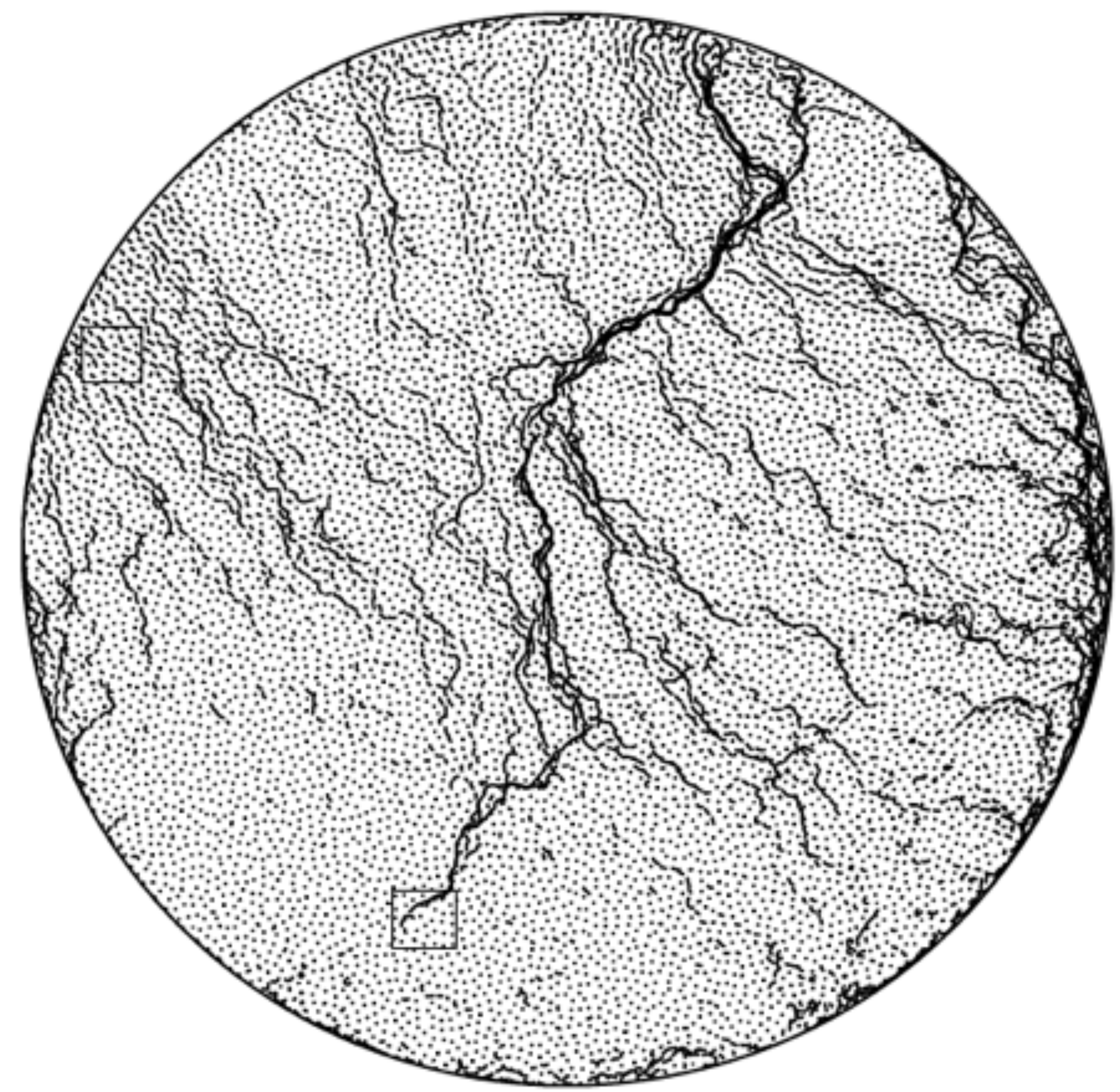


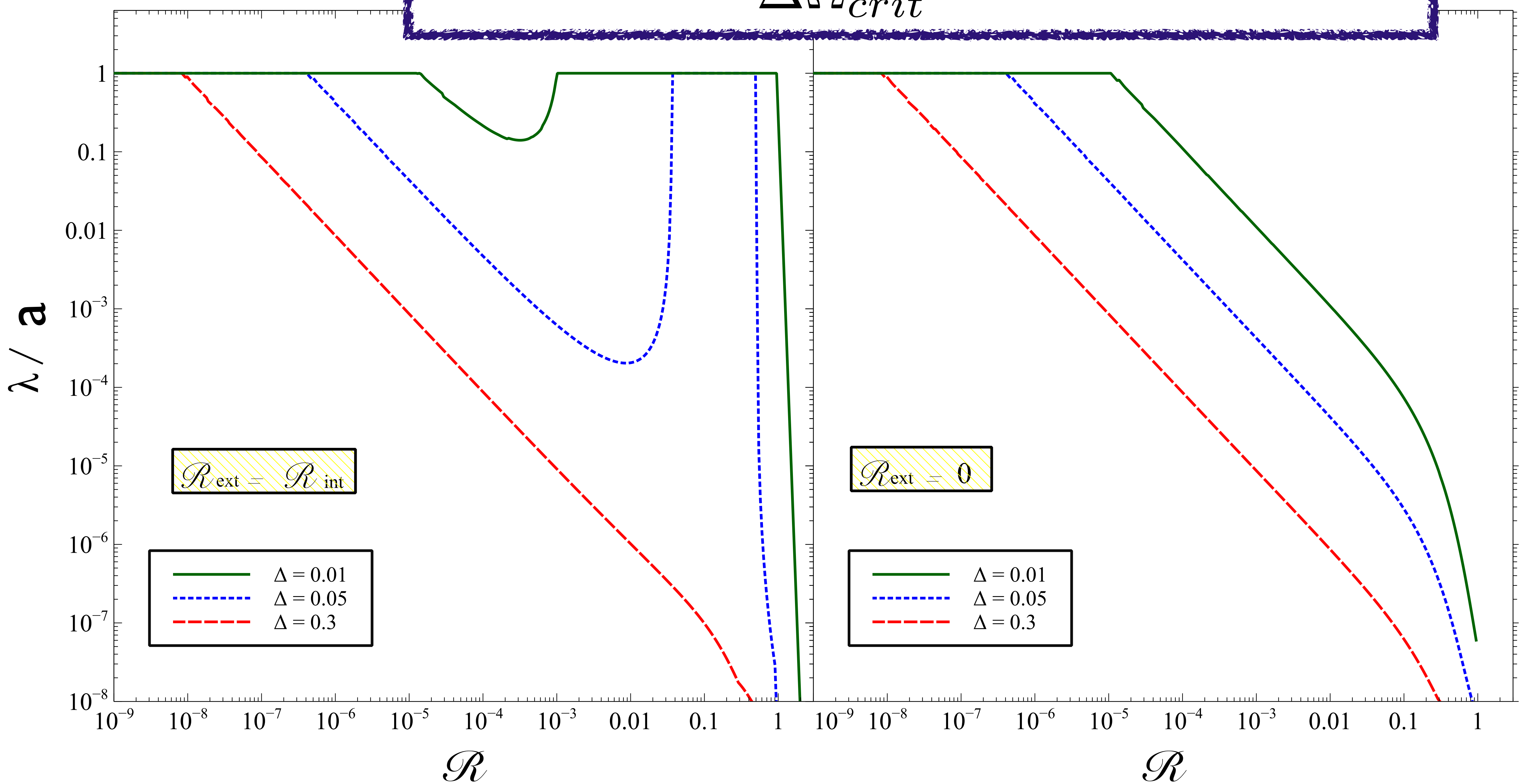
10^4 vortices

(Douglass, Melatos & BH in preparation)

Analytic Results:

(BH & Melatos, 2015)

$$\Delta = \frac{\Delta\Omega_{crit} - \Delta\Omega}{\Delta\Omega_{crit}} \approx 0.001 - 0.1$$



Equations of motion

$$\dot{\Omega}_{\rm n}(\tilde{r}) = \kappa n_v \frac{\mathcal{B}(\Omega_{\rm p}-\Omega_{\rm n})}{(1-\varepsilon_{\rm n}-\varepsilon_{\rm p})} - f(\varepsilon_{\rm p})\mathcal{A}\Omega_{\rm p}^3$$

$$\dot{\Omega}_{\rm p}(\tilde{r}) = -\kappa n_v \frac{\rho_{\rm n}}{\rho_{\rm p}} \frac{\mathcal{B}(\Omega_{\rm p}-\Omega_{\rm n})}{(1-\varepsilon_{\rm n}-\varepsilon_{\rm p})} - \mathcal{A}\Omega_{\rm p}^3$$

$$\kappa n_v = f(T, \Omega_p - \Omega_n)$$

$$\mathcal{B} = \frac{\mathcal{R}}{1+\mathcal{R}^2}$$

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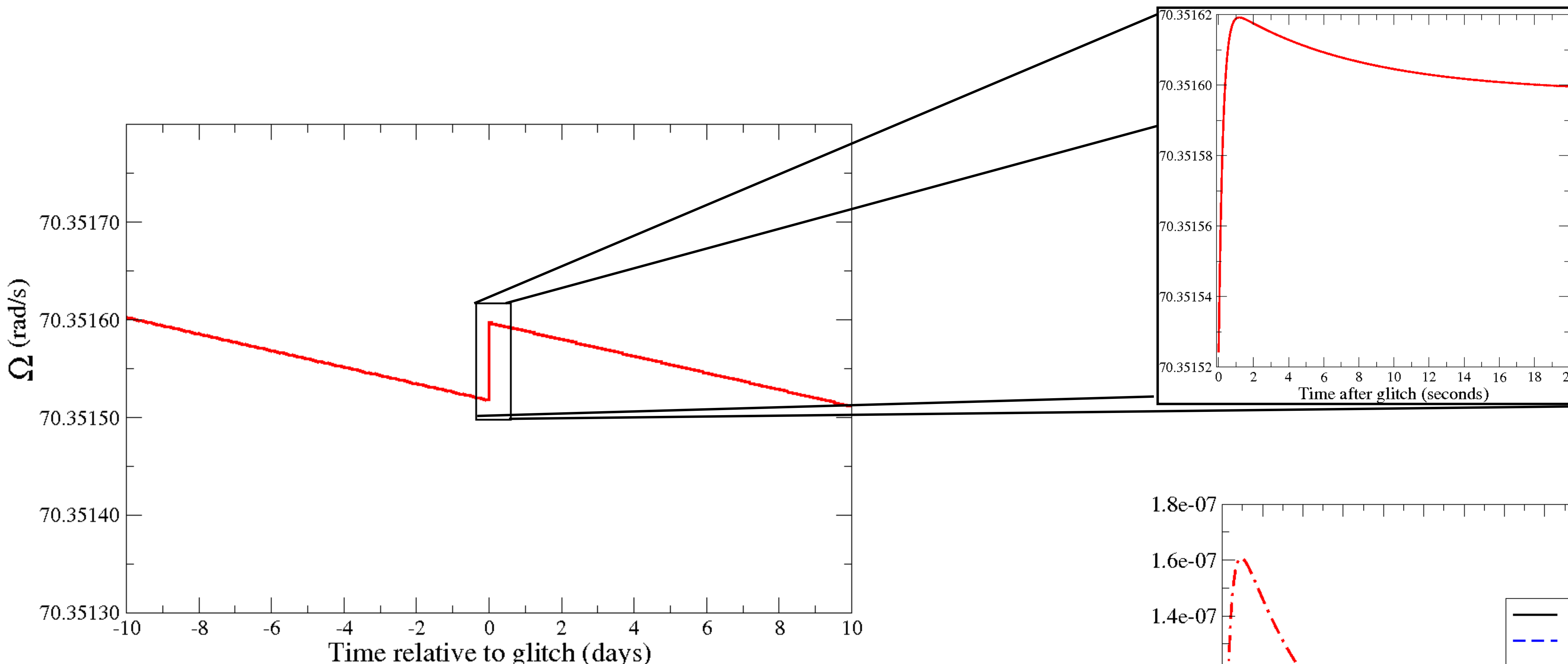
$$\kappa n_v = f(T, \Omega_p - \Omega_n)$$

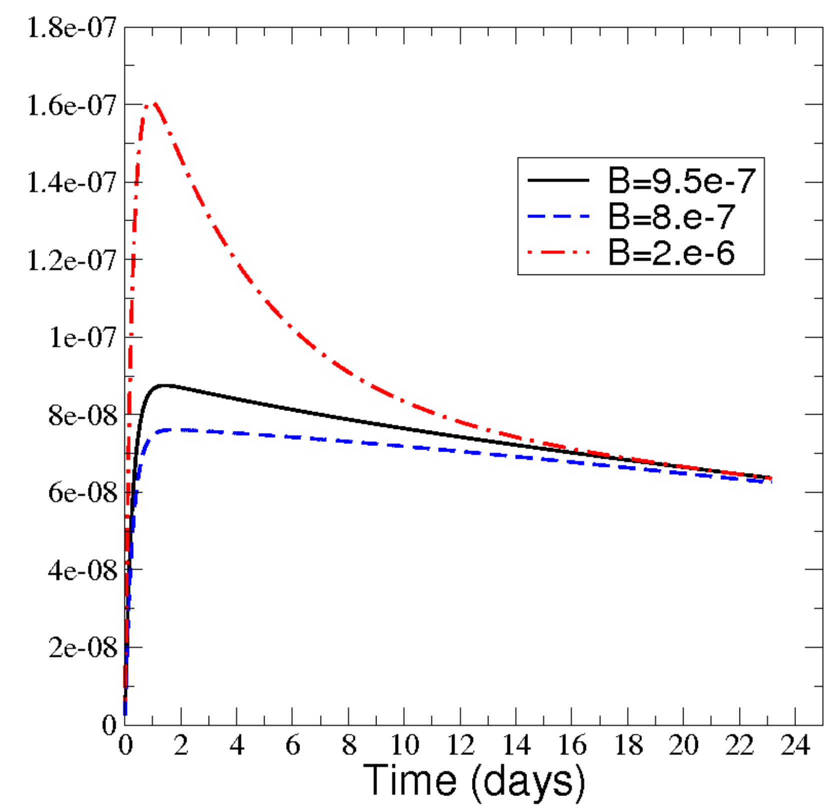
$$\mathcal{B} = \frac{\mathcal{R}}{1 + \mathcal{R}^2}$$

Important in the crust!

(Chamel 2012, Andersson et al. 2012)
(Newton, Berger & BH 2015)

Hydrodynamical Response:





(BH et al. 2012, BH & Antonopoulou 2014)

Hydrodynamical Response:

- Assume realistic profile for pinning force
- draw number of unpinned vortices and size of unpinning region from a power-law distribution

$$\gamma \kappa n_v \mathcal{B}$$

$$I_u \approx I \frac{\gamma - 1}{\gamma_{MAX}}$$

- draw waiting time between unpinning events from an exponential distribution

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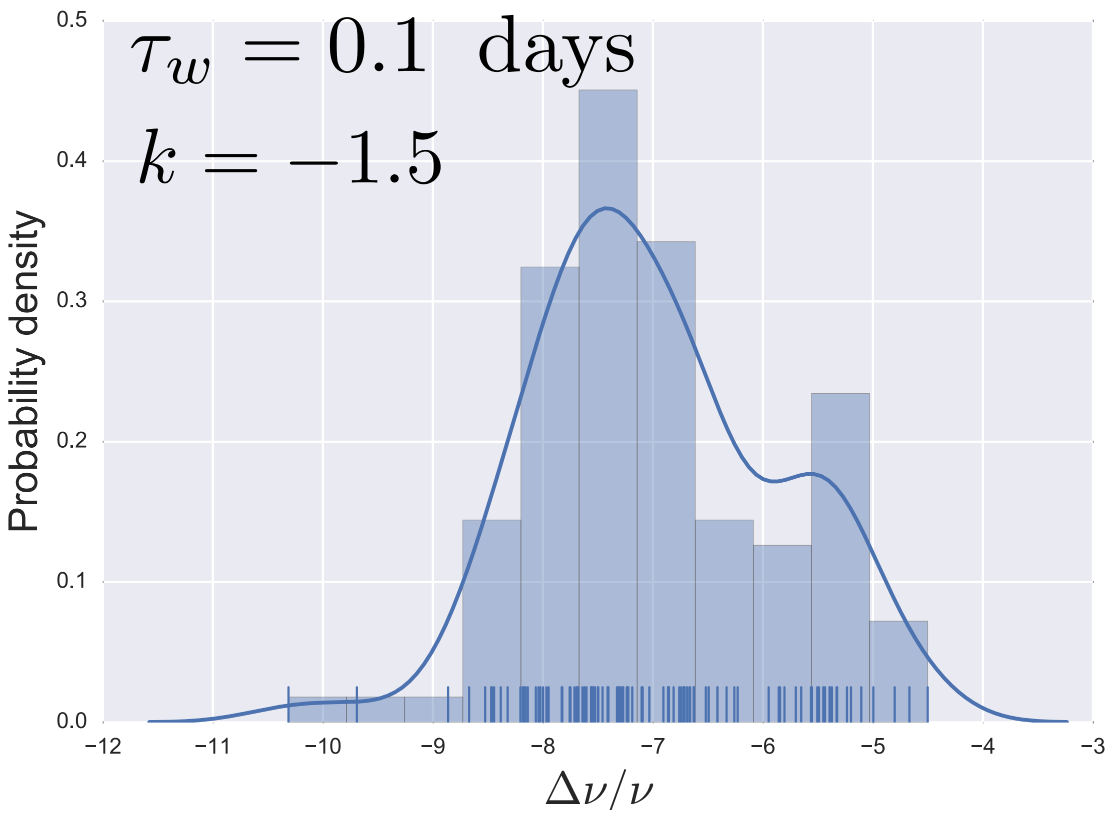
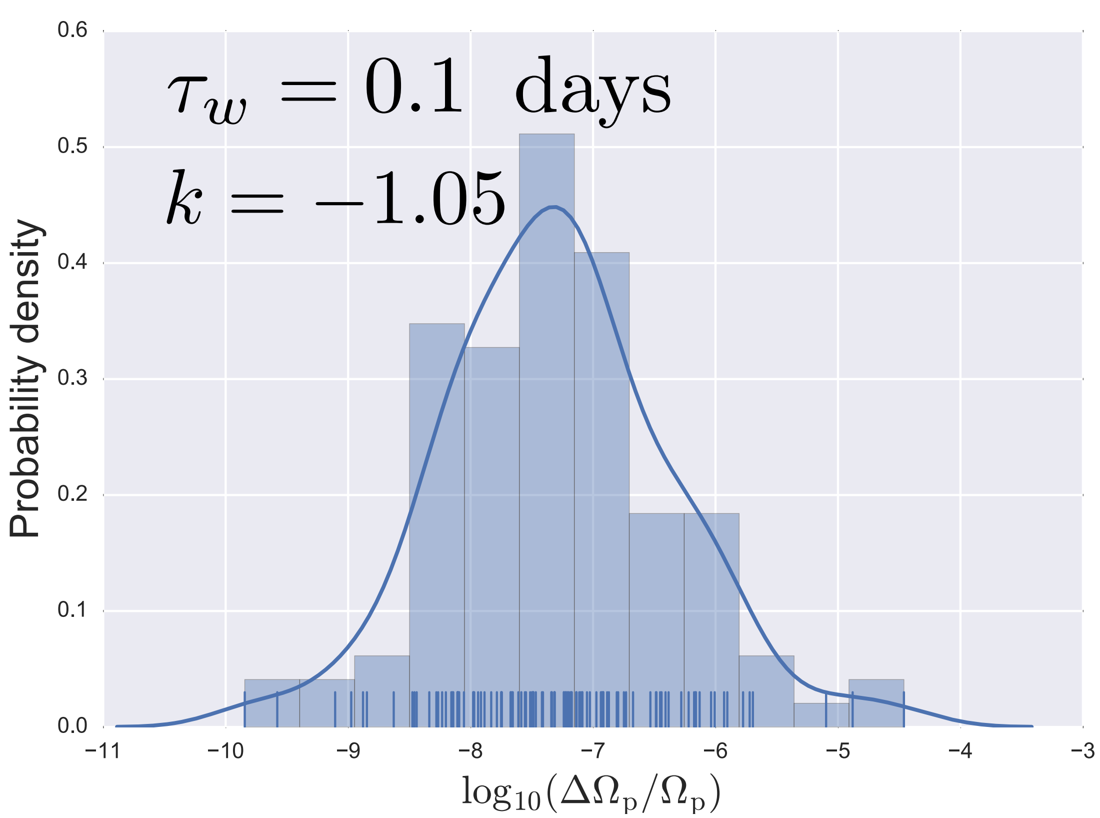
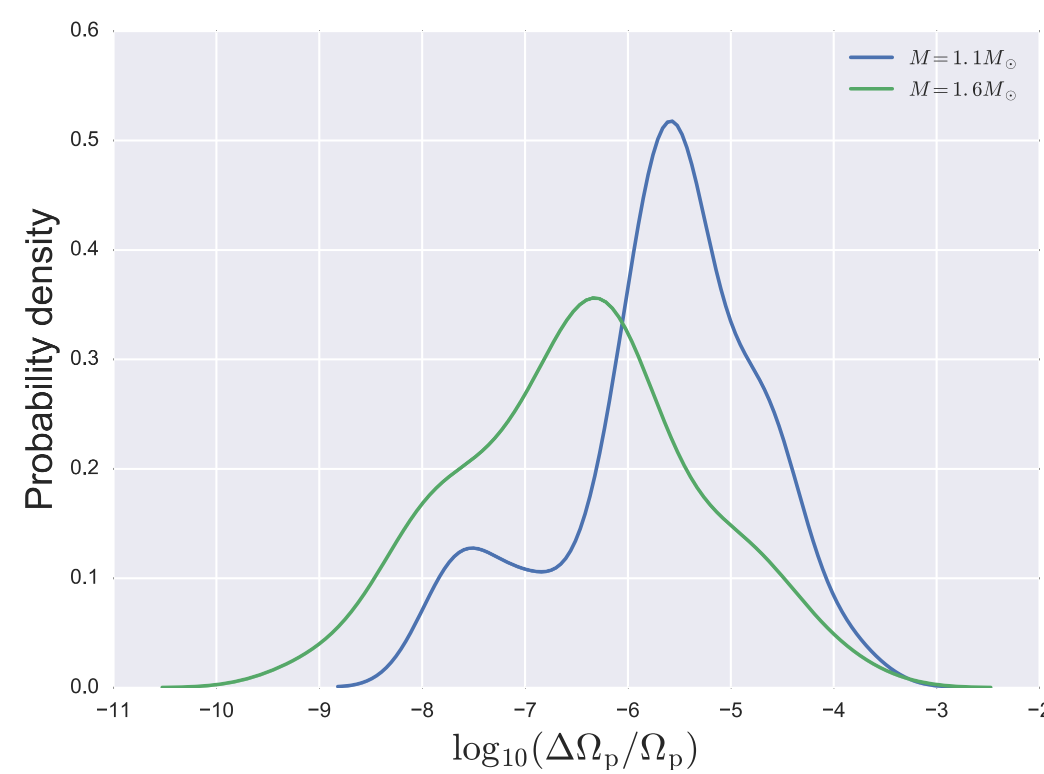
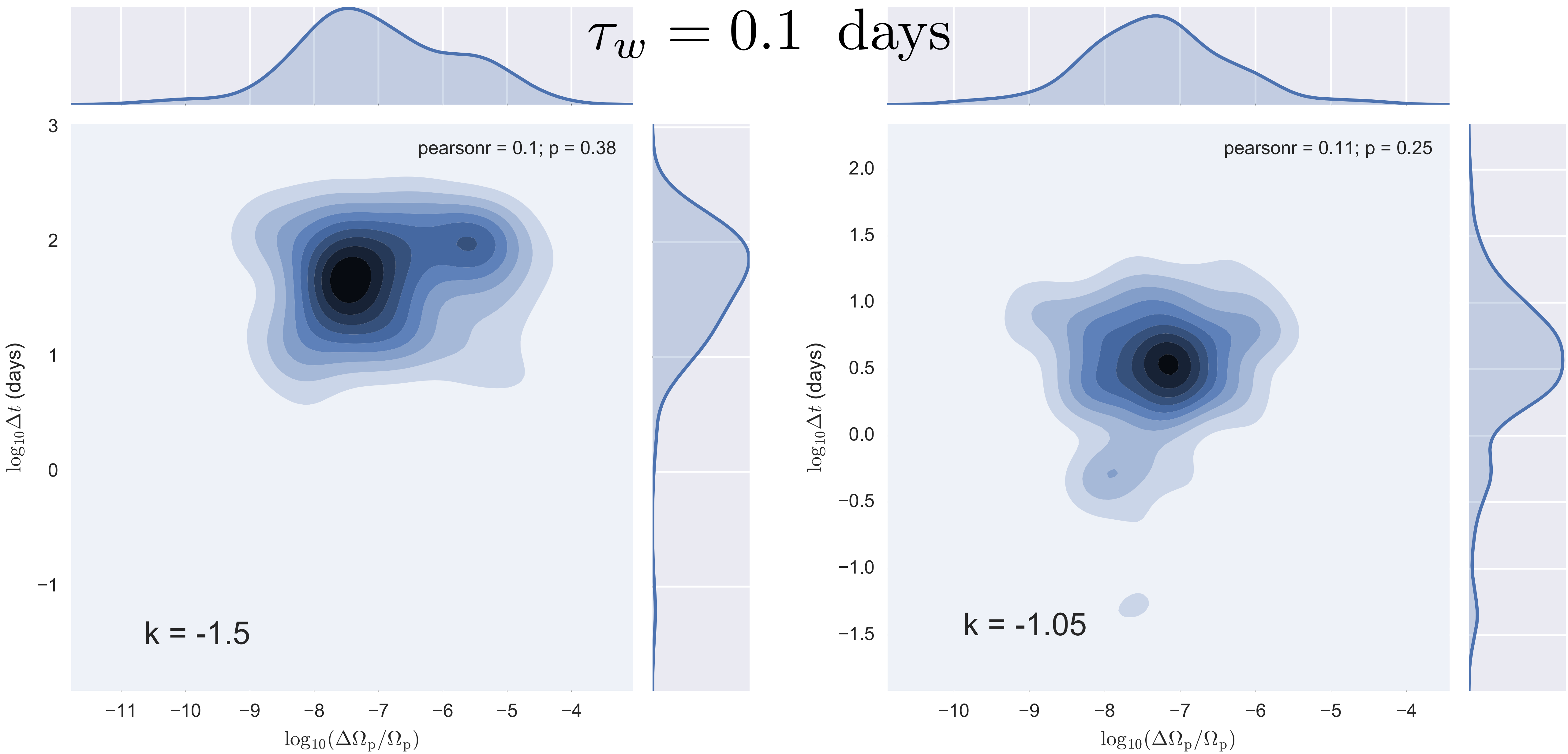


- Size distributions deviate from power-laws for low sizes, consistent with distribution of Crab glitches (Espinoza et al. 2014)
- Steeper microscopic power-law indices lead to larger glitches
- Higher mass stars have more small glitches





$$\tau_w = 0.1 \ \ \text{days}$$



(BH, in preparation)

Conclusions

- Vortex avalanches **can** propagate in NS interiors

(need better constraints on superfluid drag and the role of tension)

- Coupling of the fluid to vortex motion is crucial

(size distributions deviate from power-laws)

Region	1	2	3	4
$\rho/(10^{14}\mathrm{g/cm^3})$	0.015	0.096	0.34	0 .78
$\Delta = 0.3$				
λ/a	–	1.7×10^{-7}	–	1.0×10^{-5}
$\Delta = 0.05$				
λ/a	–	8.8×10^{-6}	–	5.0×10^{-4}
$\Delta = 0.01$				
λ/a	–	2.3×10^{-4}	–	1.3×10^{-2}
$\Delta = 0.001$				
λ/a	7.5×10^{-9}	2.3×10^{-2}	5.4×10^{-9}	1.3

$\mathcal{R}_{ext} = 0$

[BH & Melatos, in preparation]

$\mathcal{R}_{ext} = \mathcal{R}_{in}$

Region	1	2	3	4
$\rho/(10^{14}\mathrm{g/cm^3})$	0.015	0.096	0.34	0 .78
$\Delta = 0.3$				
λ/a	–	3.4×10^{-7}	–	1.1×10^{-5}
$\Delta = 0.05$				
λ/a	–	> 1	–	7.0×10^{-4}
$\Delta = 0.01$				
λ/a	–	> 1	–	0.42
$\Delta = 0.001$				
λ/a	2.3×10^{-8}	> 1	3.2×10^{-8}	> 1