Rapid particle acceleration at perpendicular shocks

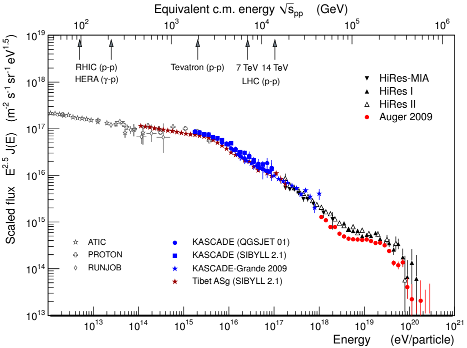
John Kirk[1], Makoto Takamoto[2]

[1]Max-Planck-Institut für Kernphysik
Heidelberg
[2]Department of Earth and Planetary Science
University of Tokyo

28th Texas Symposium on Relativistic Astrophysics, Geneva, 14th December 2015

Galactic Cosmic Rays

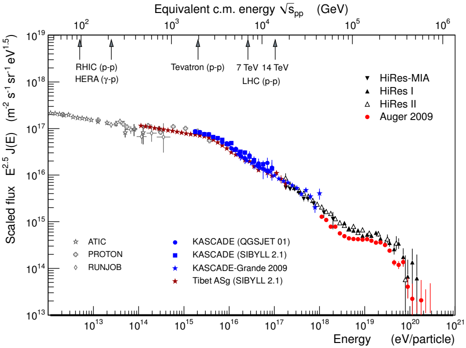
- $r_g <$ galactic disk
  $\Rightarrow \quad E_p < 10^{18}$ eV
- Single mechanism
- SNR shocks?



d'Enterria et al, Astroparticle Phys. (2011)

Galactic Cosmic Rays

- $r_g <$ galactic disk
  $\Rightarrow \quad E_p < 10^{18}$ eV
- Single mechanism
- SNR shocks?
- DSA:
  $E_p <$ few $\times\ 10^{15}$ eV



d'Enterria et al, Astroparticle Phys. (2011)

An old story

- For test particles, DSA too slow: $\epsilon = u_{\rm s}/v_{\rm cr} \ll 1$, a first order Fermi process, $(\Delta p/p \sim \epsilon)$ but $t_{\rm cycle}^{-1} \sim u_{\rm s} v_{\rm cr}/\kappa \sim \epsilon\omega_{\rm g}$, $\Rightarrow t_{\rm acc}^{-1} \sim \epsilon^2\omega_{\rm g}$

An old story

- For test particles, DSA too slow: $\epsilon = u_{\rm s}/v_{\rm cr} \ll 1$, a first order Fermi process, $(\Delta p/p \sim \epsilon)$ but $t_{\rm cycle}^{-1} \sim u_{\rm s} v_{\rm cr}/\kappa \sim \epsilon\omega_{\rm g}$,
  $\Rightarrow t_{\rm acc}^{-1} \sim \epsilon^2\omega_{\rm g}$
- The problem applies to parallel shocks, where $\kappa = \kappa_{\|} \approx \eta\omega_{\rm g}$ and $\eta \gtrsim 1$. *Particles spend a long time wandering up and down field lines, without crossing the shock*

An old story

- For test particles, DSA too slow: $\epsilon = u_{\rm s}/v_{\rm cr} \ll 1$, a first order Fermi process, $(\Delta p/p \sim \epsilon)$ but $t_{\rm cycle}^{-1} \sim u_{\rm s} v_{\rm cr}/\kappa \sim \epsilon\omega_{\rm g}$, $\Rightarrow t_{\rm acc}^{-1} \sim \epsilon^2\omega_{\rm g}$
- The problem applies to parallel shocks, where $\kappa = \kappa_{\|} \approx \eta\omega_{\rm g}$ and $\eta \gtrsim 1$. *Particles spend a long time wandering up and down field lines, without crossing the shock*
- *Not so at perpendicular shocks* (Jokipii 1987): $\kappa = \kappa_{\perp} \approx \omega_{\rm g}/\eta$, so that $t_{\rm acc}^{-1} \sim \eta\epsilon^2\omega_{\rm g}$, which is rapid for $\eta \gg 1$.

An old story

- For test particles, DSA too slow: $\epsilon = u_s/v_{cr} \ll 1$, a first order Fermi process, ($\Delta p/p \sim \epsilon$) but $t_{cycle}^{-1} \sim u_s v_{cr}/\kappa \sim \epsilon\omega_g$, $\Rightarrow t_{acc}^{-1} \sim \epsilon^2\omega_g$
- The problem applies to parallel shocks, where $\kappa = \kappa_\parallel \approx \eta\omega_g$ and $\eta \gtrsim 1$. *Particles spend a long time wandering up and down field lines, without crossing the shock*
- *Not so at perpendicular shocks* (Jokipii 1987): $\kappa = \kappa_\perp \approx \omega_g/\eta$, so that $t_{acc}^{-1} \sim \eta\epsilon^2\omega_g$, which is rapid for $\eta \gg 1$.
- Fundamental objection: diffusion approximation valid only for small anistropy. At a parallel shock, anistropy $\sim \epsilon$. But, for $\eta \gg 1$, a perpendicular shock should drive a strong anisotropy.

Transport model

Diffusion in direction of motion:

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \vec{\nabla} f + \omega_{\mathrm{g}} \frac{\partial f}{\partial \phi} = \frac{\nu_{\mathrm{coll}}}{2} \left[\frac{1}{\sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{\sin^2\theta} \frac{\partial^2 f}{\partial \phi^2} \right]$$

Valid up/downstream. No deflection by the shock itself.

Transport model

Diffusion in direction of motion:

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \vec{\nabla} f + \omega_{\mathrm{g}} \frac{\partial f}{\partial \phi} = \frac{\nu_{\mathrm{coll}}}{2} \left[\frac{1}{\sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{\sin^2\theta} \frac{\partial^2 f}{\partial \phi^2} \right]$$

Valid up/downstream. No deflection by the shock itself.

Previous work:

Transport model

Diffusion in direction of motion:

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \vec{\nabla} f + \omega_{\rm g} \frac{\partial f}{\partial \phi} = \frac{\nu_{\rm coll}}{2} \left[\frac{1}{\sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{\sin^2\theta} \frac{\partial^2 f}{\partial \phi^2} \right]$$

Valid up/downstream. No deflection by the shock itself.

Previous work:

- MC simulation. Many papers. Recent ‘full’ simulations: Ellison & Double (2004), Summerlin & Baring (2012). Nonrelativistic case expensive, only stationary solutions.

Transport model

Diffusion in direction of motion:

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \vec{\nabla} f + \omega_{\mathrm{g}} \frac{\partial f}{\partial \phi} = \frac{\nu_{\mathrm{coll}}}{2} \left[\frac{1}{\sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{\sin^2\theta} \frac{\partial^2 f}{\partial \phi^2} \right]$$

Valid up/downstream. No deflection by the shock itself.

Previous work:

- MC simulation. Many papers. Recent 'full' simulations: Ellison & Double (2004), Summerlin & Baring (2012). Nonrelativistic case expensive, only stationary solutions.
- Expansion in spherical harmonics and finite difference solution: Bell, Schure & Reville (2011), only stationary solutions.

Transport model

Diffusion in direction of motion:

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \vec{\nabla} f + \omega_{\rm g} \frac{\partial f}{\partial \phi} = \frac{\nu_{\rm coll}}{2} \left[\frac{1}{\sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{\sin^2\theta} \frac{\partial^2 f}{\partial \phi^2} \right]$$

Valid up/downstream. No deflection by the shock itself.

Previous work:

- MC simulation. Many papers. Recent 'full' simulations: Ellison & Double (2004), Summerlin & Baring (2012). Nonrelativistic case expensive, only stationary solutions.
- Expansion in spherical harmonics and finite difference solution: Bell, Schure & Reville (2011), only stationary solutions.

This work, *(details in ApJ (2015) and arXiv 1506.04354)*:

- Analytic approach using eigenfunctions (stationary case).
- SDE solution ($\approx$ MC), stationary and time-dependent cases.

Stationary solution

Separation of variables:

$$f(z,\vec{p}) = p^{-s}\sum_i c_i \mathrm{e}^{\Lambda_i z\omega_\mathrm{g}/v} Q_i(\mu,\phi)$$

$$\Lambda_i\left(\hat{v}_z - u\right) Q_i = \left\{-\frac{\partial}{\partial\phi} + \frac{1}{2\eta}\left[\frac{\partial}{\partial\mu}\left(1-\mu^2\right)\frac{\partial}{\partial\mu} + \frac{1}{1-\mu^2}\frac{\partial^2}{\partial\phi^2}\right]\right\} Q_i$$

$(\hat{v}_z = \sqrt{1-\mu^2}\sin\phi,\ \eta = \omega_\mathrm{g}/\nu_\mathrm{coll}.)$

Stationary solution

Separation of variables:

$$f(z,\vec{p}) = p^{-s} \sum_i c_i \mathrm{e}^{\Lambda_i z \omega_\mathrm{g}/v} Q_i(\mu,\phi)$$

$$\Lambda_i \left(\hat{v}_z - u\right) Q_i = \left\{ -\frac{\partial}{\partial\phi} + \frac{1}{2\eta}\left[\frac{\partial}{\partial\mu}\left(1-\mu^2\right)\frac{\partial}{\partial\mu} + \frac{1}{1-\mu^2}\frac{\partial^2}{\partial\phi^2}\right]\right\} Q_i$$

$(\hat{v}_z = \sqrt{1-\mu^2}\sin\phi,\ \eta = \omega_\mathrm{g}/\nu_\mathrm{coll}.)$

- Similar to method used for relativistic shocks (ApJ 2000).

Stationary solution

Separation of variables:

$$f(z,\vec{p}) = p^{-s}\sum_i c_i \mathrm{e}^{\Lambda_i z\omega_\mathrm{g}/v} Q_i(\mu,\phi)$$

$$\Lambda_i\left(\hat{v}_z - u\right) Q_i = \left\{-\frac{\partial}{\partial\phi} + \frac{1}{2\eta}\left[\frac{\partial}{\partial\mu}\left(1-\mu^2\right)\frac{\partial}{\partial\mu} + \frac{1}{1-\mu^2}\frac{\partial^2}{\partial\phi^2}\right]\right\} Q_i$$

$(\hat{v}_z = \sqrt{1-\mu^2}\sin\phi,\ \eta = \omega_\mathrm{g}/\nu_\mathrm{coll}.)$

- Similar to method used for relativistic shocks (ApJ 2000).
- But two-parameter (η, u) problem in two-dimensions (μ,ϕ).

Stationary solution

Separation of variables:

$$f(z,\vec{p}) = p^{-s} \sum_i c_i \mathrm{e}^{\Lambda_i z \omega_{\mathrm{g}}/v} Q_i(\mu,\phi)$$

$$\Lambda_i \left(\hat{v}_z - u\right) Q_i = \left\{ -\frac{\partial}{\partial\phi} + \frac{1}{2\eta}\left[\frac{\partial}{\partial\mu}\left(1-\mu^2\right)\frac{\partial}{\partial\mu} + \frac{1}{1-\mu^2}\frac{\partial^2}{\partial\phi^2}\right]\right\} Q_i$$

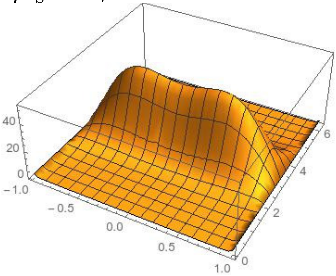
$(\hat{v}_z = \sqrt{1-\mu^2}\sin\phi,\ \eta = \omega_{\mathrm{g}}/\nu_{\mathrm{coll}}.)$

- Similar to method used for relativistic shocks (ApJ 2000).
- But two-parameter (η, u) problem in two-dimensions (μ, ϕ).
- Approximate by retaining only the 'leading' upstream eigenfunction.

Approximate analytic solution for $u_\mathrm{s} \sim 1/\eta \sim \epsilon \ll 1$

- $Q = \mathrm{e}^{\Lambda v \sqrt{1-\mu^2}\cos\phi} \mathrm{Ps}_0^0\left(\mu, -\Lambda^2/2\right)$
  Ps_n^m: angular, oblate, spheroidal wave function.

Leading eigenfunction, $\eta u_\mathrm{s} = 2$,

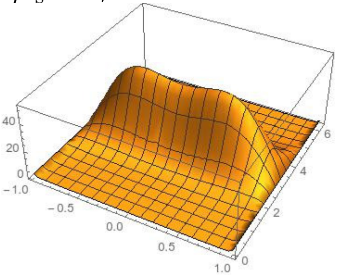


Anisotropic at order ϵ^0, as suggested by Schatzman (1963).

Approximate analytic solution for $u_\mathrm{s} \sim 1/\eta \sim \epsilon \ll 1$

- $Q = \mathrm{e}^{\Lambda v\sqrt{1-\mu^2}\cos\phi}\,\mathrm{Ps}_0^0\left(\mu, -\Lambda^2/2\right)$
  Ps_n^m: angular, oblate, spheroidal wave function.
- $\lambda_0^0\left(-\Lambda^2/2\right) = \Lambda\left(\Lambda + 2\eta u\right)$
  $\lambda_n^m(-\gamma^2)$: spheroidal eigenvalue.

Leading eigenfunction, $\eta u_\mathrm{s} = 2$,

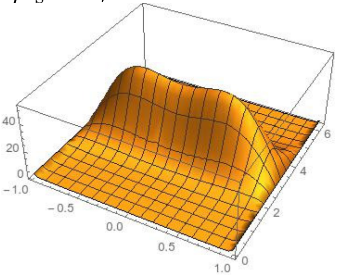


Anisotropic at order ϵ^0, as suggested by Schatzman (1963).

Approximate analytic solution for $u_{\rm s} \sim 1/\eta \sim \epsilon \ll 1$

- $Q = \mathrm{e}^{\Lambda v \sqrt{1-\mu^2}\cos\phi} \mathrm{Ps}_0^0\left(\mu, -\Lambda^2/2\right)$
  Ps_n^m: angular, oblate, spheroidal wave function.
- $\lambda_0^0\left(-\Lambda^2/2\right) = \Lambda\left(\Lambda + 2\eta u\right)$
  $\lambda_n^m(-\gamma^2)$: spheroidal eigenvalue.
- Diffusive transport: $\Lambda \sim \epsilon$ (Fisch & Kruskal 1980).
  Here, $\Lambda \sim \epsilon^0$.

Leading eigenfunction, $\eta u_{\rm s} = 2$,

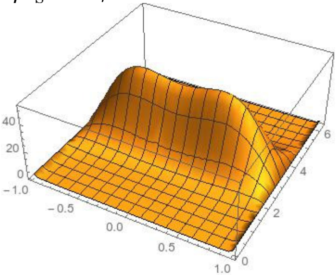


Anisotropic at order ϵ^0, as suggested by Schatzman (1963).

Approximate analytic solution for $u_{\mathrm{s}} \sim 1/\eta \sim \epsilon \ll 1$

- $Q = \mathrm{e}^{\Lambda v \sqrt{1-\mu^2}\cos\phi} \mathrm{Ps}_0^0\left(\mu, -\Lambda^2/2\right)$
  Ps_n^m: angular, oblate, spheroidal wave function.
- $\lambda_0^0\left(-\Lambda^2/2\right) = \Lambda\left(\Lambda + 2\eta u\right)$
  $\lambda_n^m(-\gamma^2)$: spheroidal eigenvalue.
- Diffusive transport: $\Lambda \sim \epsilon$
  (Fisch & Kruskal 1980).
  Here, $\Lambda \sim \epsilon^0$.
- Power-law index fixed by b.c.'s

Leading eigenfunction,
$\eta u_{\mathrm{s}} = 2$,



Anisotropic at order ϵ^0,
as suggested by
Schatzman (1963).

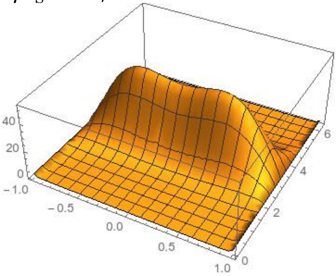
Approximate analytic solution for $u_{\mathrm{s}} \sim 1/\eta \sim \epsilon \ll 1$

- $Q = \mathrm{e}^{\Lambda v \sqrt{1-\mu^2}\cos\phi} \mathrm{Ps}_0^0\left(\mu, -\Lambda^2/2\right)$
  Ps_n^m: angular, oblate, spheroidal wave function.
- $\lambda_0^0\left(-\Lambda^2/2\right) = \Lambda\left(\Lambda + 2\eta u\right)$
  $\lambda_n^m(-\gamma^2)$: spheroidal eigenvalue.
- Diffusive transport: $\Lambda \sim \epsilon$
  (Fisch & Kruskal 1980).
  Here, $\Lambda \sim \epsilon^0$.
- Power-law index fixed by b.c.'s
- Series in ηu:

$$s = \frac{3r}{r-1} + \frac{9\left(r+1\right)}{20r(r-1)}\eta^2 u_{\mathrm{s}}^2 + \mathrm{O}\left(\eta^4 u_{\mathrm{s}}^4\right)$$

(r = compression ratio)

Leading eigenfunction, $\eta u_{\mathrm{s}} = 2$,

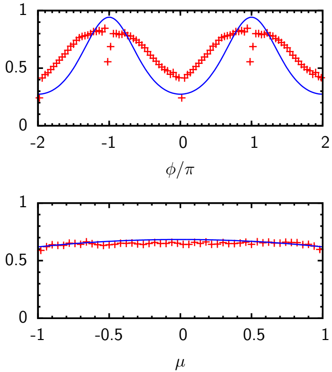


Anisotropic at order ϵ^0, as suggested by Schatzman (1963).

Numerical solution of SDE's

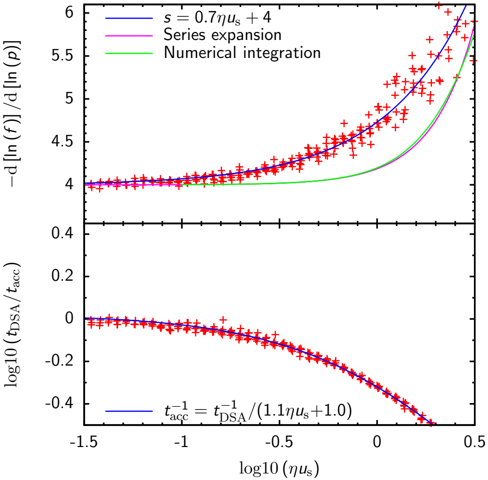
Angular distribution

- $\eta = 22,\ u_{\rm s} = 0.012.$
- Upper: $\int_{-1}^{1} {\rm d}\mu f\,(\mu, \phi)$
- Lower: $\int_{0}^{2\pi} {\rm d}\phi f\,(\mu, \phi)$



Numerical solution of SDE's

- $\eta = 1\text{–}100$ (10 values) $u_{\rm s} = 0.01\text{–}0.2$, (30 values)
- Top: spectral index
- Bottom: acceleration rate/ DSA (Jokipii) prediction



Conclusions

- DSA valid only for strong "collisionality" ($\eta u_{\rm s} \ll 1$)

Conclusions

- DSA valid only for strong “collisionality” ($\eta u_{\rm s} \ll 1$)
- For weak collisionality: fan beam, opening angle $\approx (\eta u_{\rm s})^{-1}$

Conclusions

- DSA valid only for strong "collisionality" ($\eta u_\mathrm{s} \ll 1$)
- For weak collisionality: fan beam, opening angle $\approx (\eta u_\mathrm{s})^{-1}$
- For $\eta u_\mathrm{s} \sim 1$, s softens by ~ 1, acceleration rate slows by factor ~ 2

Conclusions

- DSA valid only for strong "collisionality" ($\eta u_{\rm s} \ll 1$)
- For weak collisionality: fan beam, opening angle $\approx (\eta u_{\rm s})^{-1}$
- For $\eta u_{\rm s} \sim 1$, s softens by ~ 1, acceleration rate slows by factor ~ 2
- Maximum CR energy for SN in WR-star wind (DSA):

$$\begin{aligned} E_{\rm max} &= \frac{3}{8}\eta u_{\rm s}\left(\frac{R_*\Omega}{v_{\rm w}}\right) eB_* R_* \\ &= 1.7 \times 10^{16} \eta u_{\rm s} \left(\frac{R_*\Omega}{v_{\rm w}}\right) \left(\frac{B_*}{50\,{\rm G}}\right) \left(\frac{R_*}{3.10^{12}\,{\rm cm}}\right) {\rm eV} \end{aligned}$$

not changed much provided $\eta u_{\rm s} \sim 1$.