Self-similar magnetic, turbulent and thermal energy evolution in massive galaxy clusters

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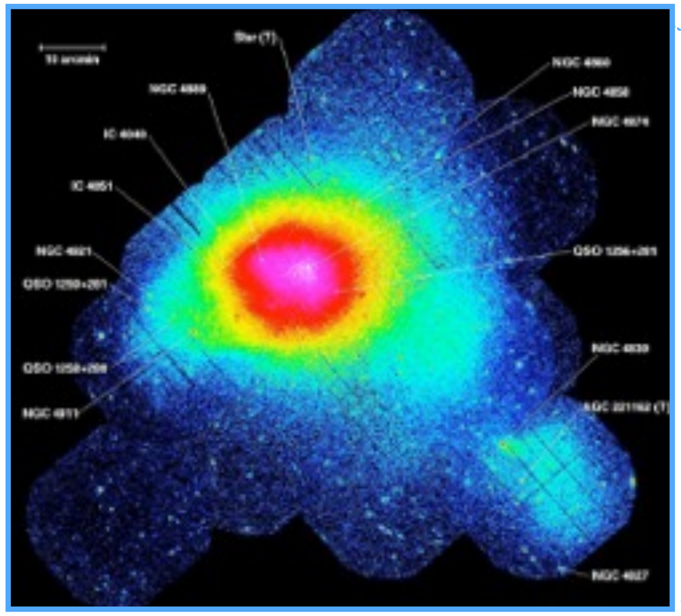
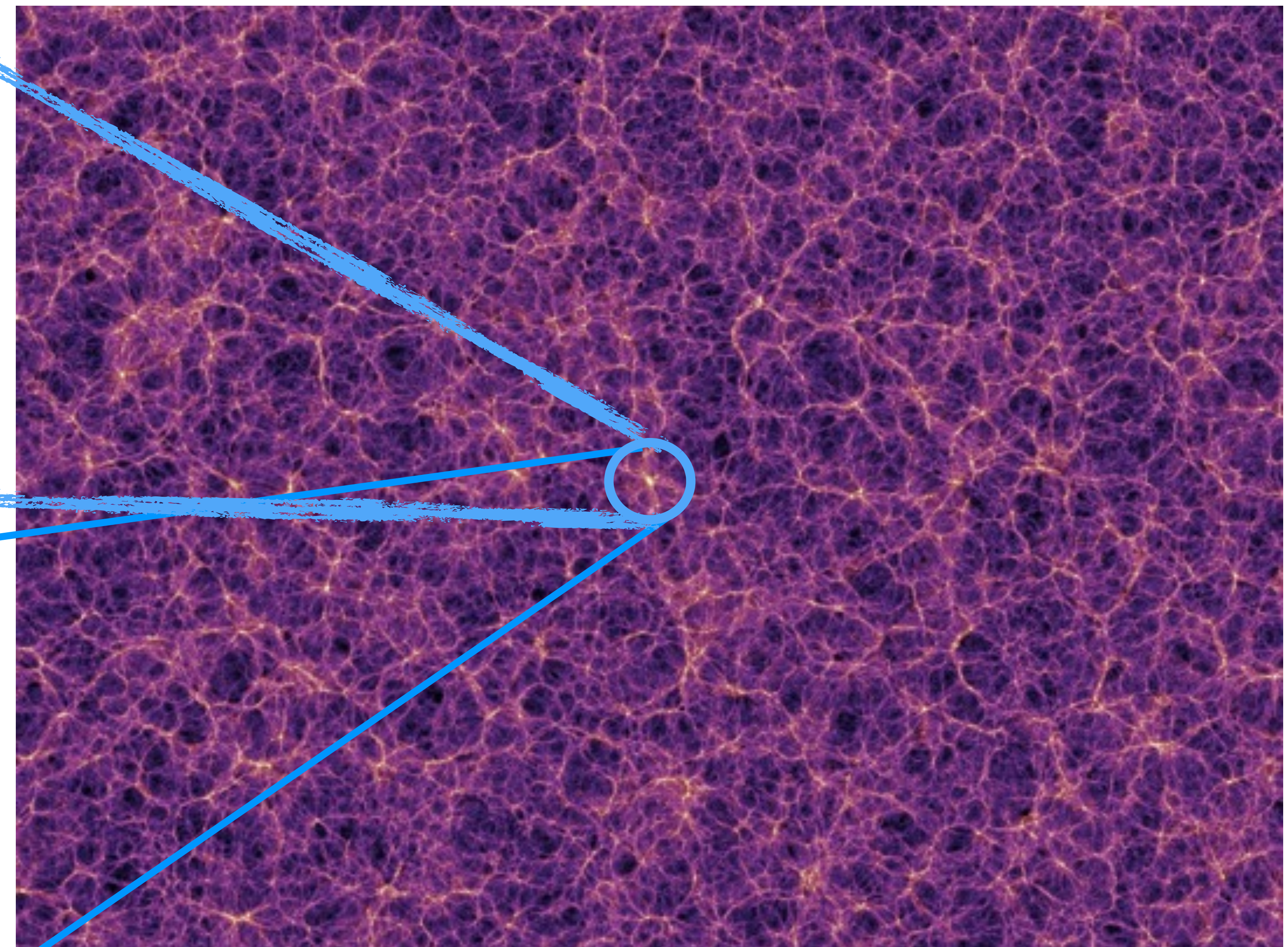
Self-similarity of Cosmic Structure

Moore+ 1999

- Matter Power Spectrum (Harrison 1970, Zel'dovich 1972)
- Cluster Scaling Relations (Kaiser 1986)
- Halo Density Profile (Navarro-Frenk-White 1996, 1997)
- Dark Matter Substructure (Moore et al. 1999)

Galaxy Custers





$$N_{gal} \approx 30 - \text{few} \times 10^3$$

$$\tau_{cr} \approx 10^9\,\text{yr}\left(\frac{R_{GC}}{\text{Mpc}}\right)\left(\frac{\sigma_{gal}}{10^3\,\text{km s}^{-1}}\right) \ll \tau_{\text{H}}$$

$$M_{vir} \approx \sigma_{gal}^2 \frac{R_{GC}}{G} \approx 10^{14} - 10^{15} h^{-1} M_\odot \gg M_{gal}$$

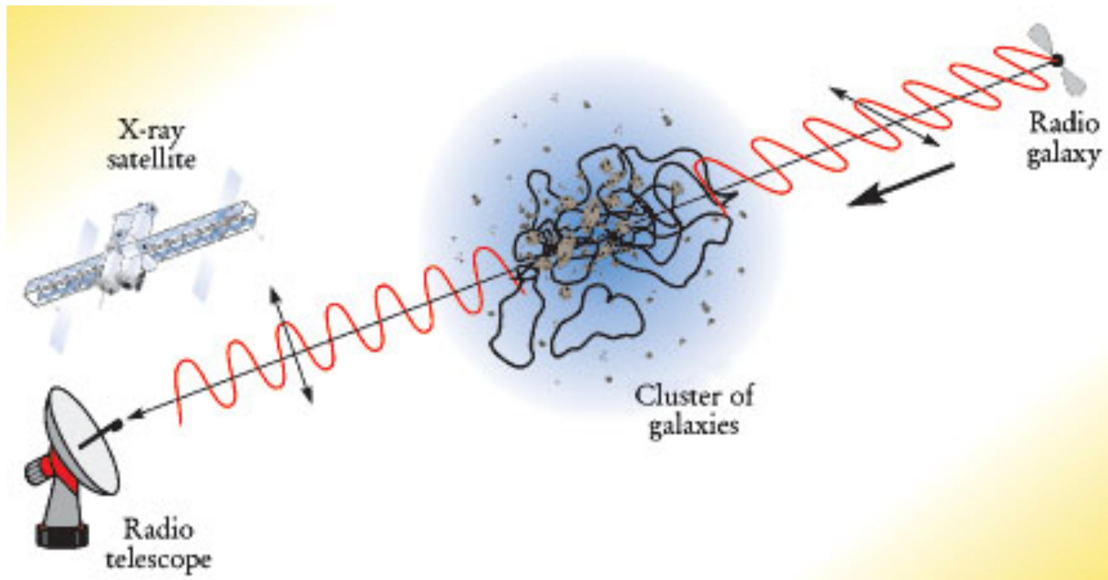
$$\left\langle \frac{M}{L_B} \right\rangle \approx 300h \frac{M_\odot}{L_\odot}$$

$$n_{gas} \approx 10^{-2} - 10^{-3}\ \text{cm}^{-3}$$

$$T_x \approx 10^7 - 10^8\ \text{K}$$

$$L_x \approx 10^{44} - 10^{45}\ \text{erg s}^{-1}$$

ICM Magnetic Field Measurements



Faraday Rotation Effect

$$\chi = \chi_0 + \mathrm{RM}\,\lambda^2$$

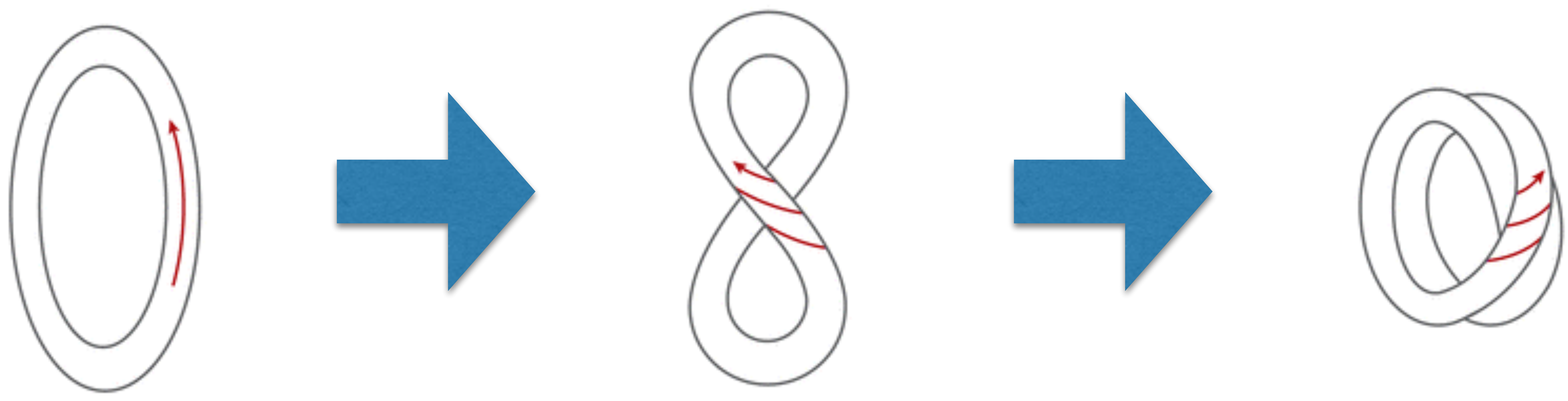
$$\mathrm{RM}=0.8\int n_e \vec{B}\cdot d\vec{l}$$

- ICM magnetic fields very challenging. The emerging picture, based on Faraday Rotation Effect, is essentially that (Clarke+ 2001, Guidetti 2008, Govoni+2010, Bonafede+ 2010, Kuchar+2011)

    1. B~few-several μG
    2. beta within 1 Mpc ~ 40-50
    3. magnetic field coherence length ~ tens of kpc
    4. the power spectrum of E_B is steep, i.e. Kolmogorov-like
    5. the magnetic field decreases towards the cluster outskirts

- trend with, e.g., cluster temperature/mass etc. being sought after

Stretch, twist and fold dynamo mechanism



In the kinematic regime the magnetic energy growth rate depends on Reynolds number like:

$$\gamma \approx \frac{\mathrm{Re}^{1/2}}{30\ \tau_L}$$

Beresnyak 2012;
Haugen, Brandenburg, Dobler (2004)
Schekochihin et al. (2004)

Pseudo-Linear Growth and Saturation

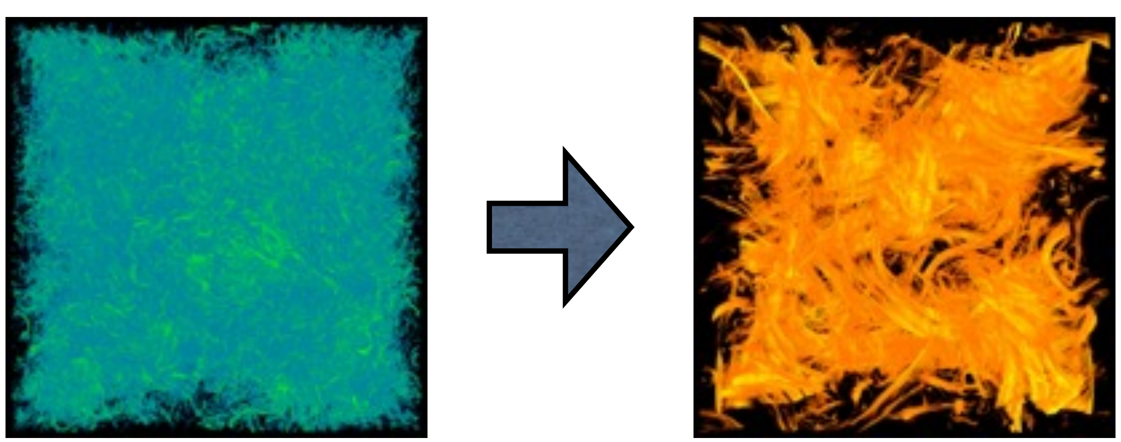
(solenoidal case)

- if ℓ_s is the scales where stretching is most efficient so that roughly $\delta u_{\ell_s}^2 \sim \langle B^2 \rangle$ (Kulsrud & Anderson 1992, Cho & Vishniac 2000, Schekochihin & Cowley 2007, Jones et al. 2011)

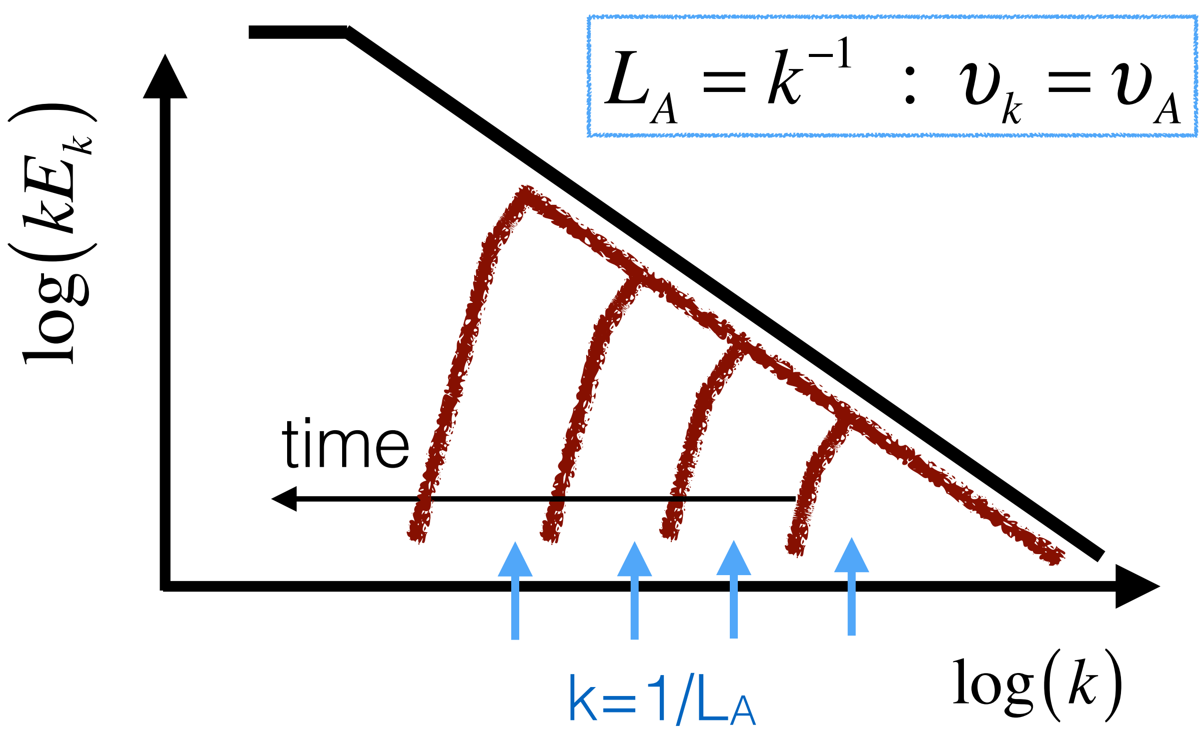
$$\frac{d}{dt}\langle B^2 \rangle \approx \frac{\delta u_{\ell_s}}{\ell_s}\langle B^2 \rangle \approx \frac{\delta u_{\ell_s}^3}{\ell_s} \sim \varepsilon_{turb} \quad \Rightarrow \quad \langle B^2 \rangle(t) \approx C_E \int^t \varepsilon_{turb}(\tau)\, d\tau$$

- C_E~4-5 % according to recent numerical simulations (Beresnyak 2012, Beresnyak and *FM* 2015)

- Finally, E_B~E_K and E_B growth saturates



Jones et al. (2011)



Numerical Models of ICM Turbulent Dynamo

- cosmological numerical models of MHD dynamo in the ICM typically achieve only modest magnetic field amplification, by factors of order ~ 10^3 (Miniati et al. 2001, Dolag et al. 2001, Dubois and Teyssier 2008, Xu et al. 2012)

$$Re \approx 100 \;\Rightarrow\; \gamma \approx \frac{Re^{1/2}}{30\;\tau_L} \leq \frac{1}{\tau_L} \approx \frac{1}{Gyr}$$

- can't measure magnetic field structure

Turbulence in GCs

- Turbulence Drivers - Gravity of course through:
  i) asymmetric smooth accretion induced by tidal fields
  ii) mergers/substructure

- Scales:
  i) Injection scales: $L_{inj} \approx R_{vir} \approx 3\ \mathrm{Mpc} \left(\frac{M}{10^{15} M_\odot} \right)^{1/3}$
  iii) Velocity scale: $u_L \approx u_{\mathrm{vir}} = \left(\frac{G\mathrm{M}_{\mathrm{vir}}}{\mathrm{R}_{\mathrm{vir}}} \right)^{1/2}$
  iv) Mean free path:
    a. collisional $\lambda \approx \mathrm{kpc} \left(\frac{\mathrm{n}_{\mathrm{ICM}}}{3\mathrm{x}10^{-3}\mathrm{cm}^{-3}} \right)^{-1} \left(\frac{\mathrm{T}}{10\mathrm{keV}} \right)^{3/2}$
    b. Larmor radius $\lambda \approx 10^{-4}\ \mathrm{a.u.} \left(\frac{E}{\mathrm{keV}} \right) \left(\frac{B}{\mu G} \right)^{-1}$
    c. micro-instabilities (e.g. Schecochikhin et al. 2005)

$$\mathrm{Re} \gg 10^3$$

- Timescale for turbulence cascade:

$$\tau \simeq \frac{L}{u} \approx \frac{R_{\mathrm{vir}}}{u_{\mathrm{vir}}} = \Delta_c^{-1/2} \tau_H \ll \tau_H$$

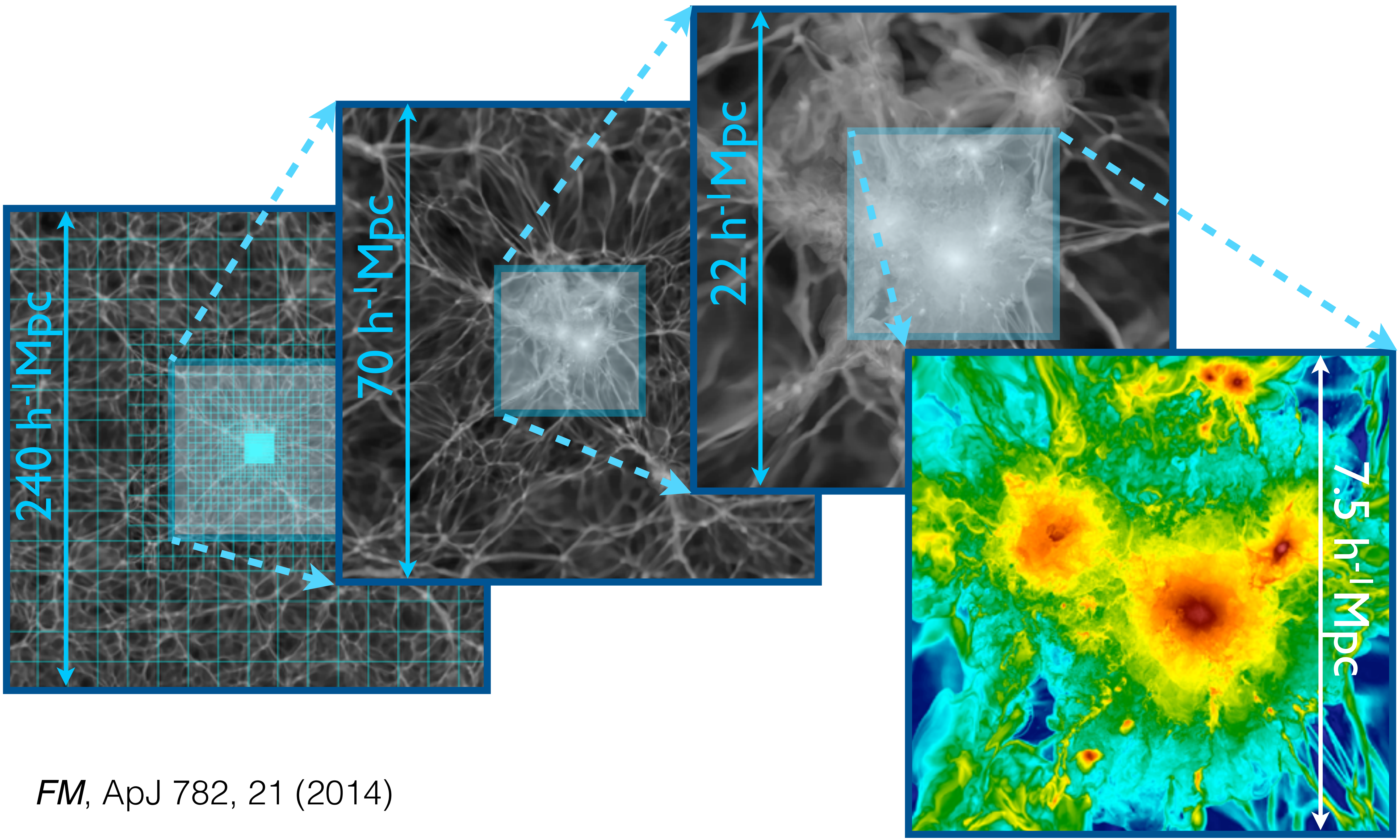
FM, ApJ 782, 21 (2014)

Numerics

- Box size: L_{box}= 100 Mpc to give proper tidal effects
- Resolution: $\Delta x \leq 10^{-3} L_{inj}$ to have enough dynamic range of scales
- For PPM (3rd order in space) numerical dissipation affects turbulence cascade up to 32 res. elements (Porter & Woodward 1994)
- ⇒ dynamic range ~10^5 — 10^6
- N_{re}~ $10^{9\text{-}10}$, N_{step}~ 10^4, => N_{re} x N_{step} ~ $10^{14\text{-}15}$ i.e. Tera-Peta flop scale

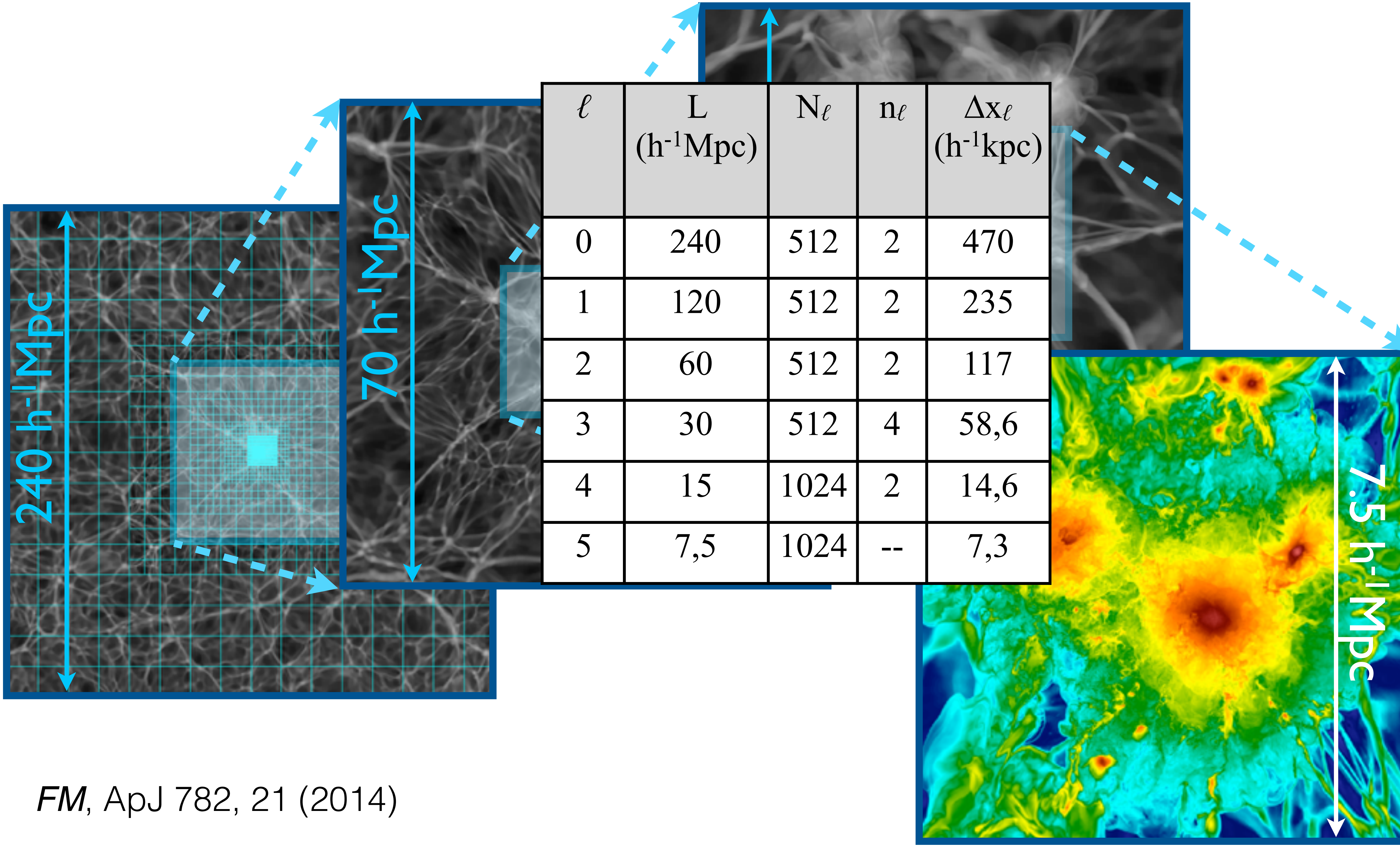
- AMR resolution criterion:
  - i) based on mass threshold (lagrangian)
  - ii) vorticity (Iapichino & Niemeyer 2008, Paul et al 2011), and velocity threshold (Vazza et al 2011)
- high surface-to-volume => potential issues:
  - iii) highly structured grids -- inefficiencies
  - iv) accuracy drops @ fine/coarse boundary
- high order schemes vs plenty of shocks

Eulerian Refinement Strategy: Zoom-in + Matryoshka of grids



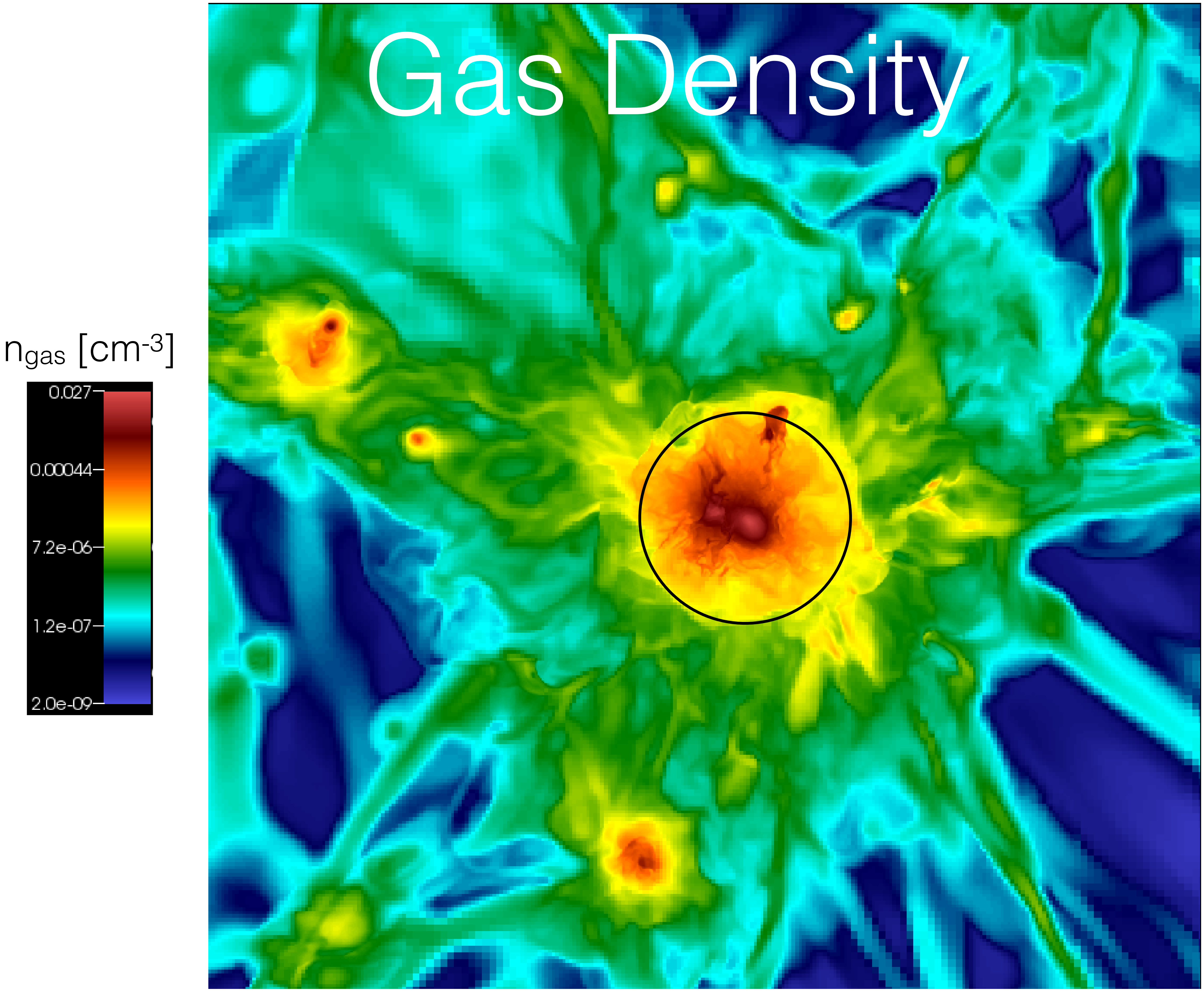
FM, ApJ 782, 21 (2014)

Eulerian Refinement Strategy: Zoom-in + Matryoshka of grids

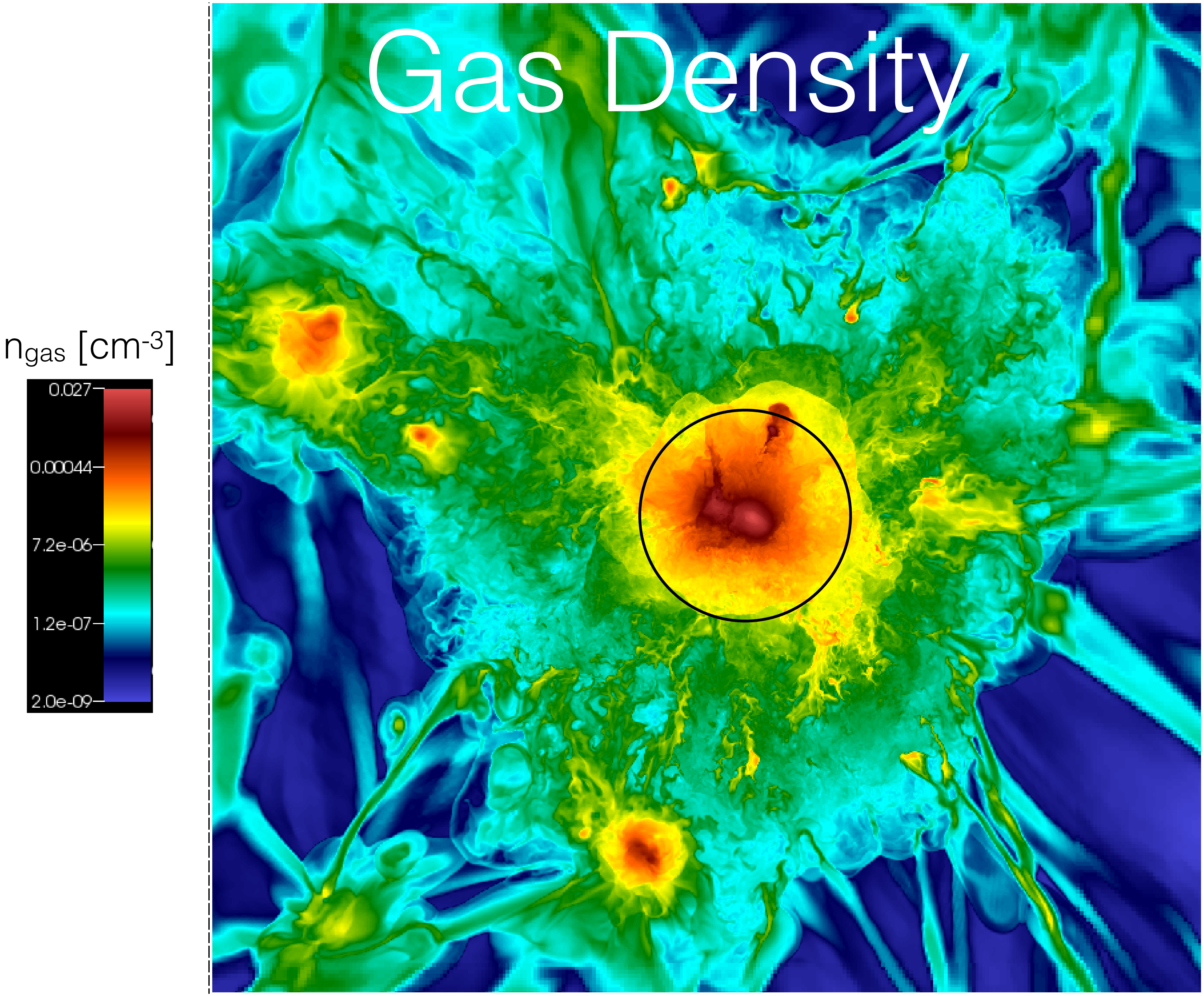


ℓ	L (h^{-1}Mpc)	N_ℓ	n_ℓ	Δx_ℓ (h^{-1}kpc)
0	240	512	2	470
1	120	512	2	235
2	60	512	2	117
3	30	512	4	58,6
4	15	1024	2	14,6
5	7,5	1024	--	7,3

FM, ApJ 782, 21 (2014)



Gas Density
n_gas [cm^-3]
0.027
0.00044
7.2e-06
1.2e-07
2.0e-09



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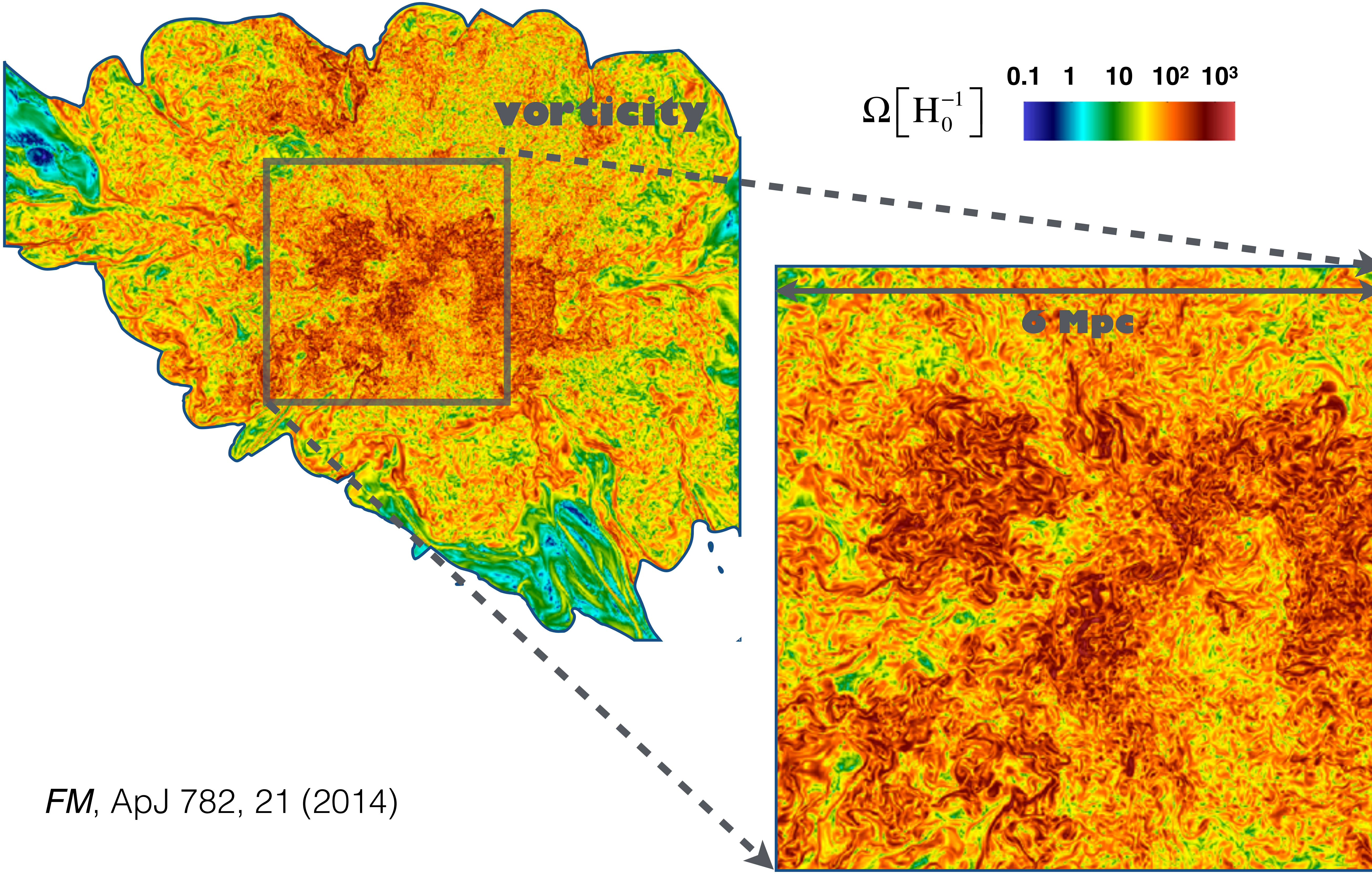
Vorticity



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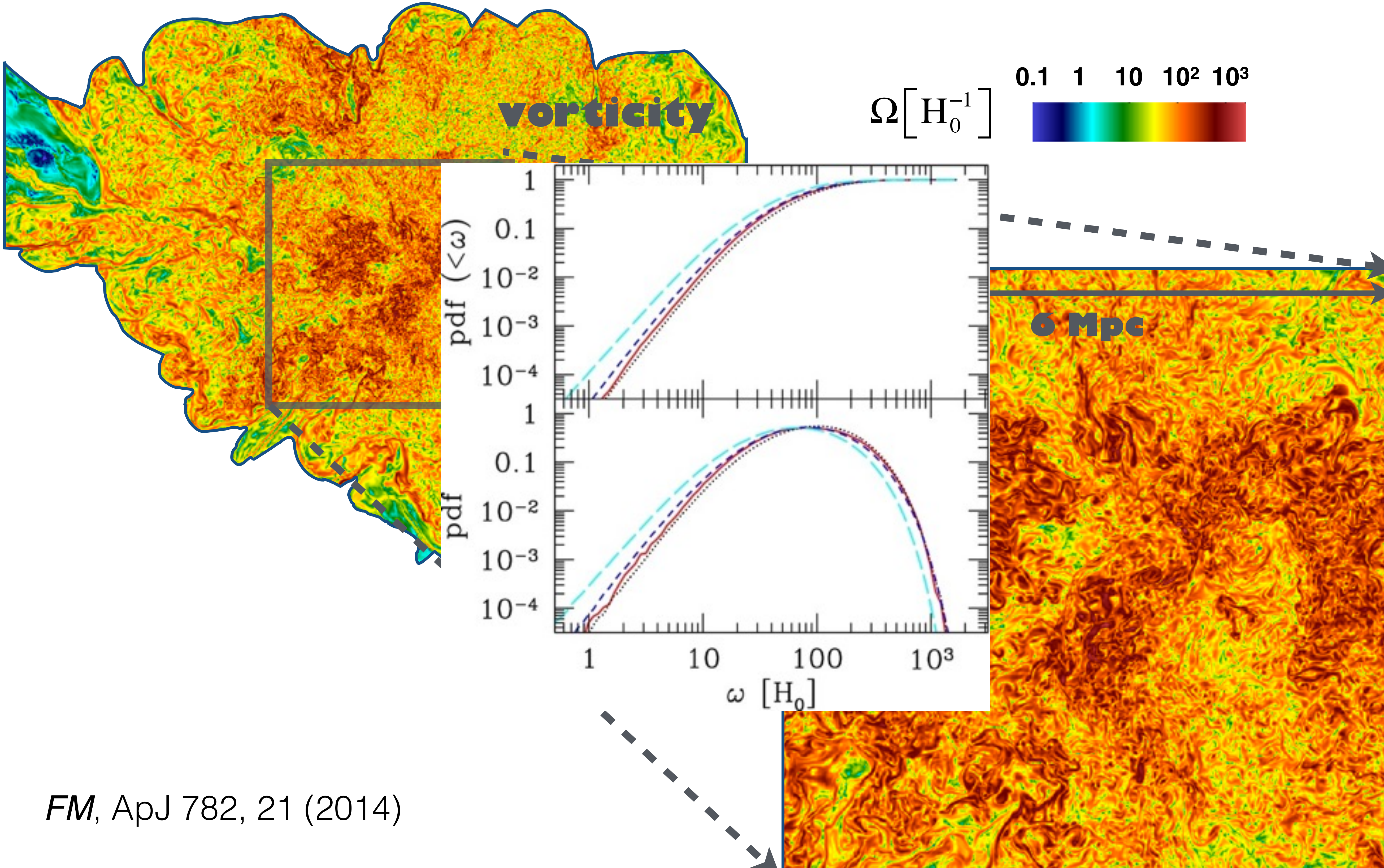


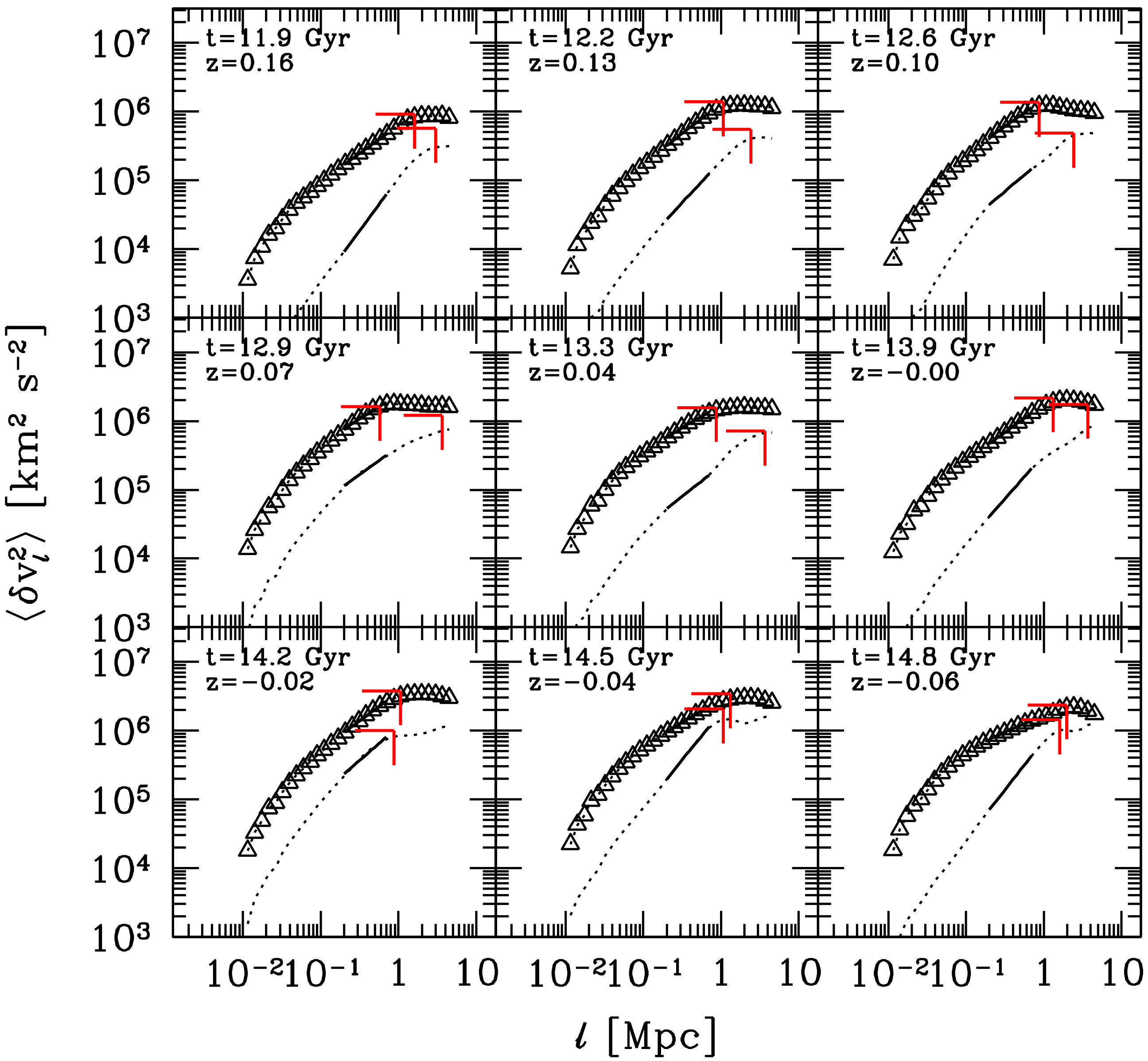
FM, ApJ 782, 21 (2014)

Statistics

Hodge-Helmholtz decomposition

$$\vec{v}_s = -\nabla\phi, \quad \vec{v}_c = \nabla\times\vec{A}$$

$$\phi = \frac{1}{4\pi}\int\frac{\nabla\cdot\vec{v}}{r}d^3r, \quad \vec{A} = \frac{1}{4\pi}\int\frac{\nabla\times\vec{v}}{r}d^3r$$



FM, ApJ 800, 60 (2015)

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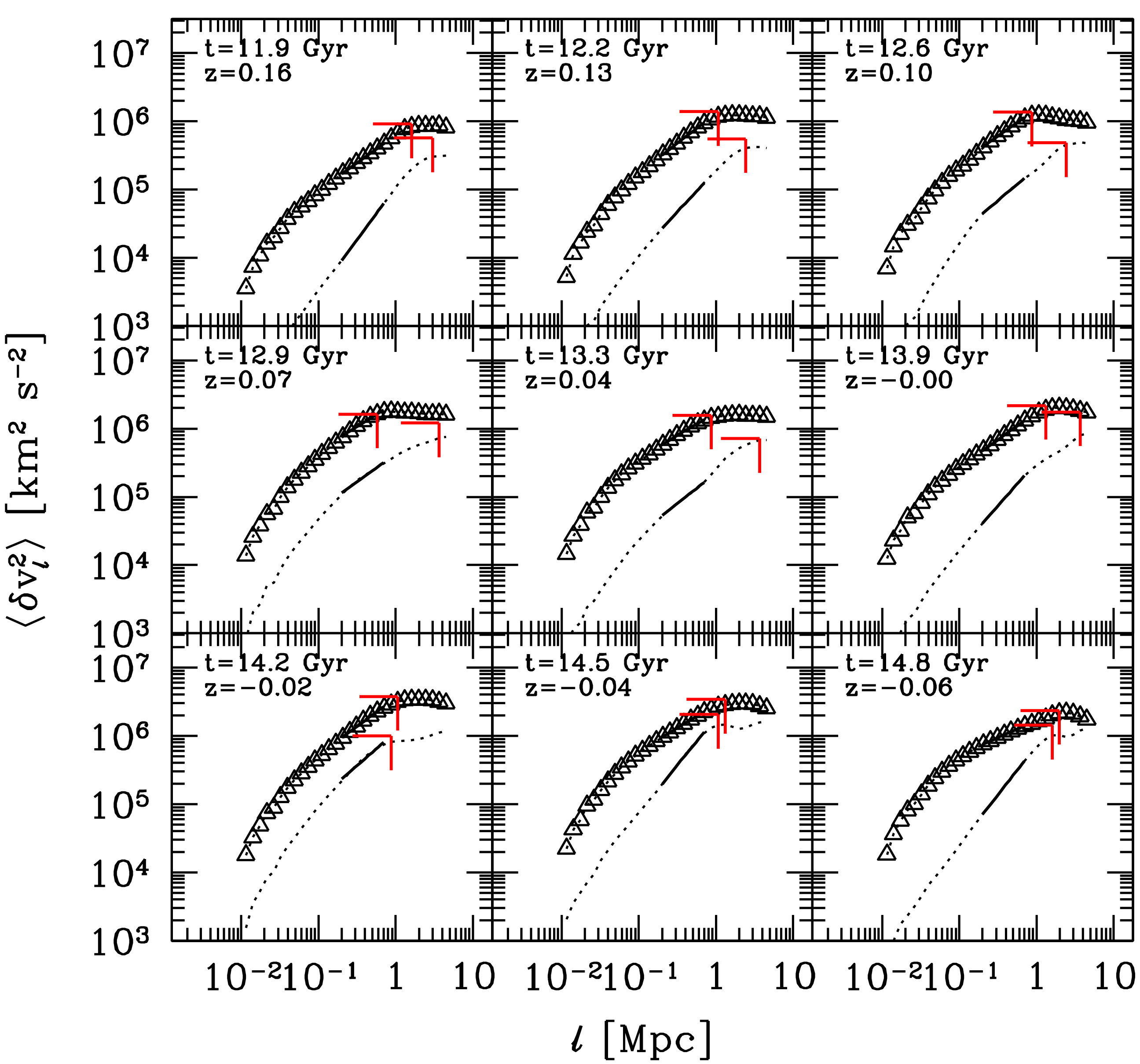
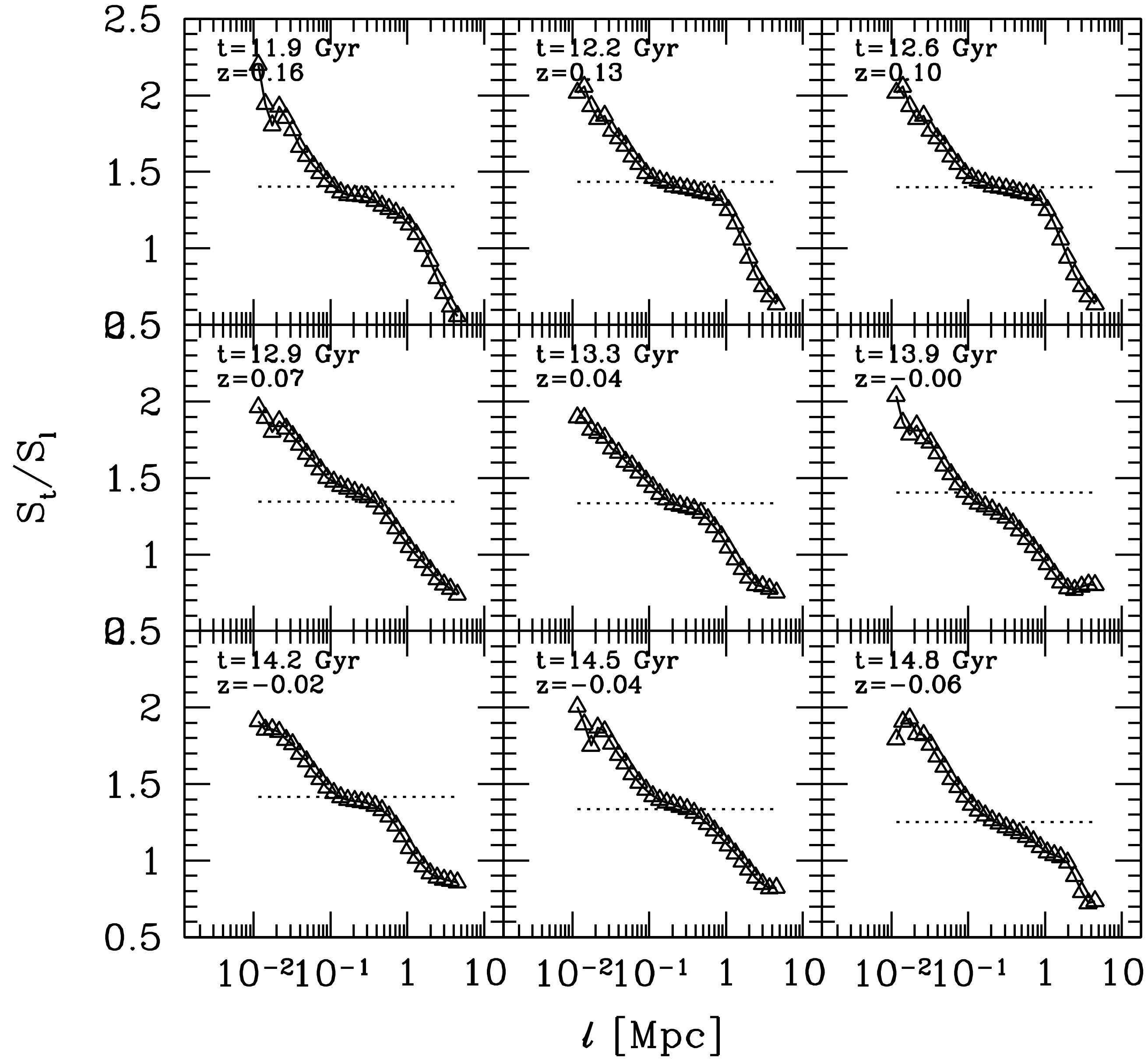
$$S_{t,s}^{(2)} = \frac{2+\zeta_2}{2}S_{l,s}^{(2)}$$

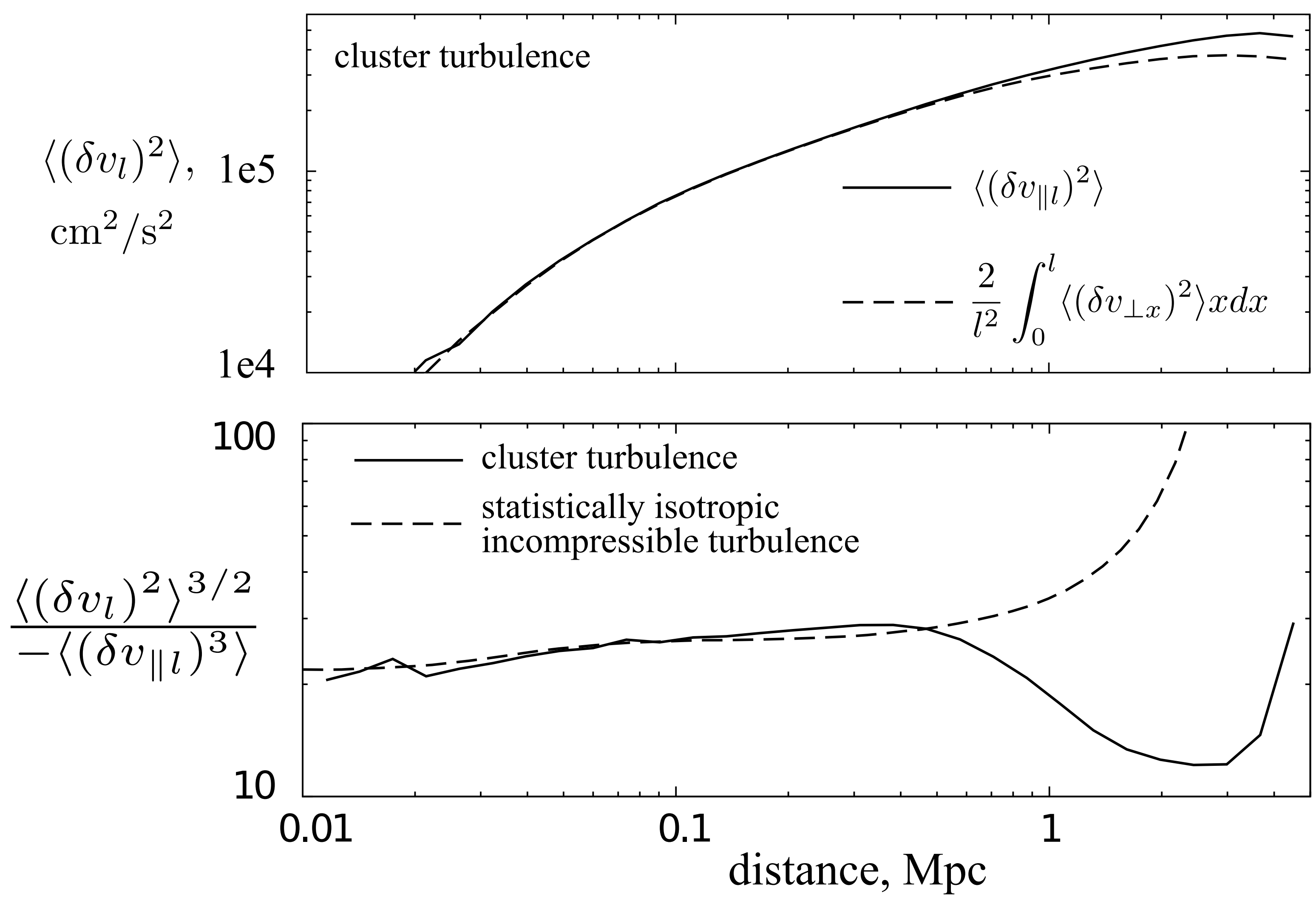
(von Karman & Howarth, 1938)





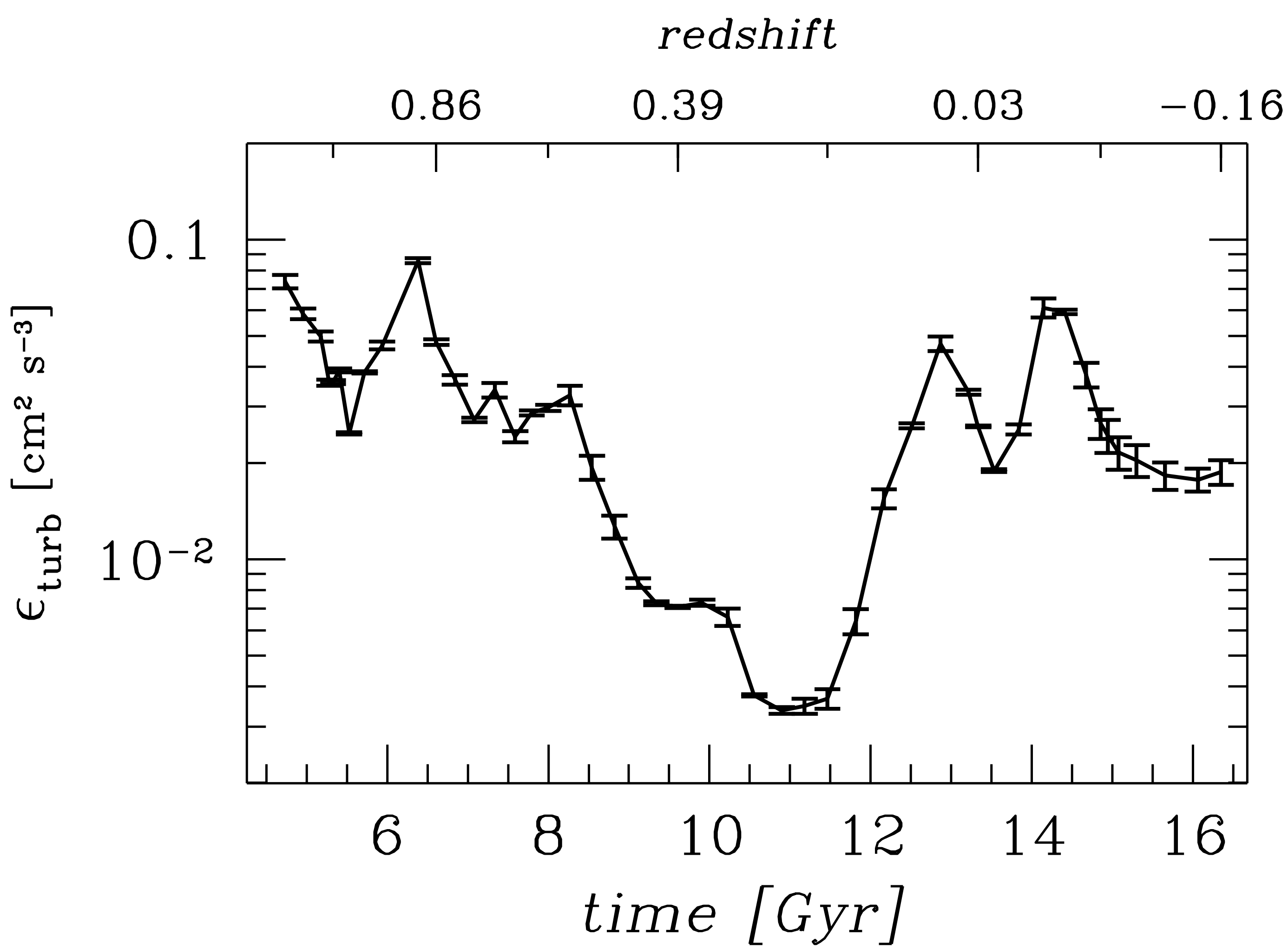
FM, ApJ 800, 60 (2015)

Comparison with DNS



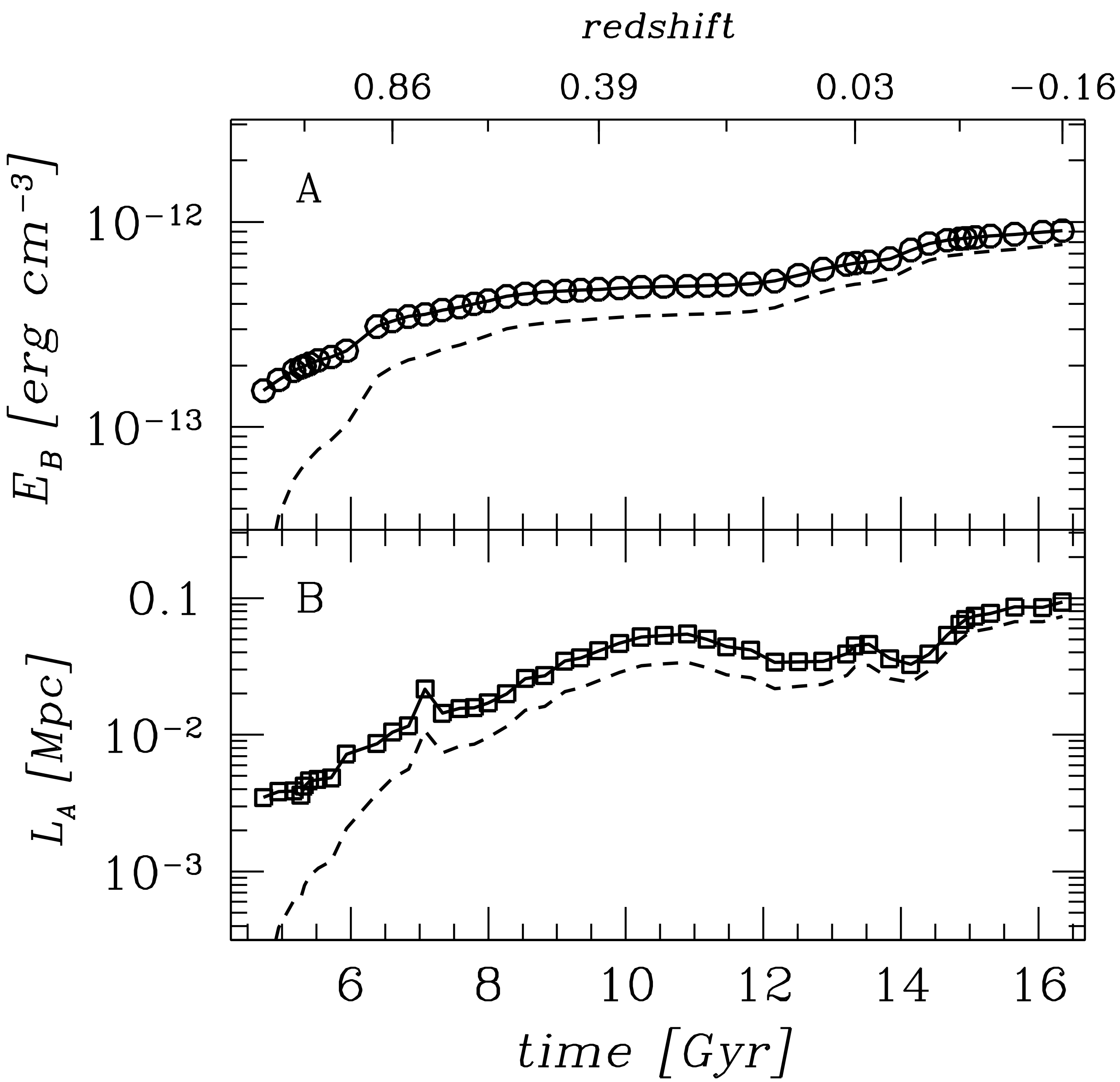
Beresnyak & *FM* (2015)

Turbulent Dissipation Rate



Beresnyak & ***FM*** (2015)
FM & Beresnyak (2015)

Magnetic Energy and Alfvèn Scale

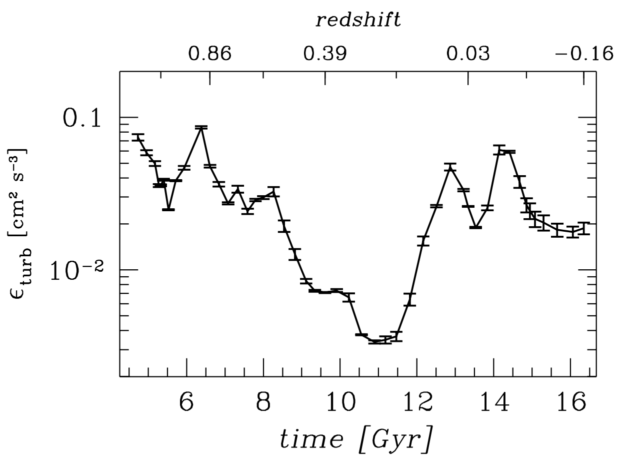


$$E_B(t) = C_E \int^t d\tau \; \rho \varepsilon_{turb}(\tau)$$

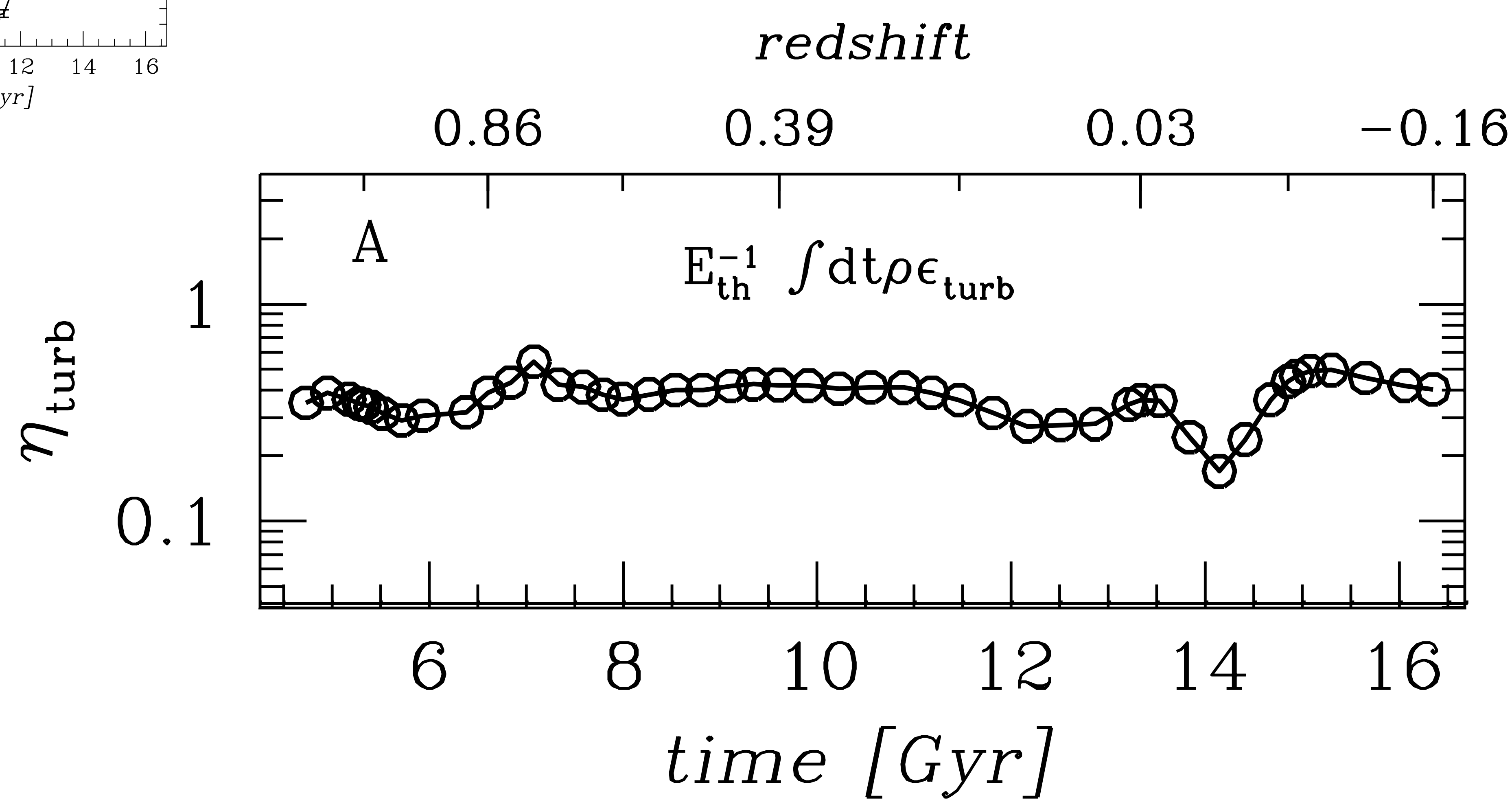
$$L_A(t) = \frac{V_A^3}{C^{3/2} \langle \varepsilon_{turb} \rangle_{\tau_{eddy}}}$$

Beresnyak & ***FM*** (2015)

Turbulent Dissipation Efficiency

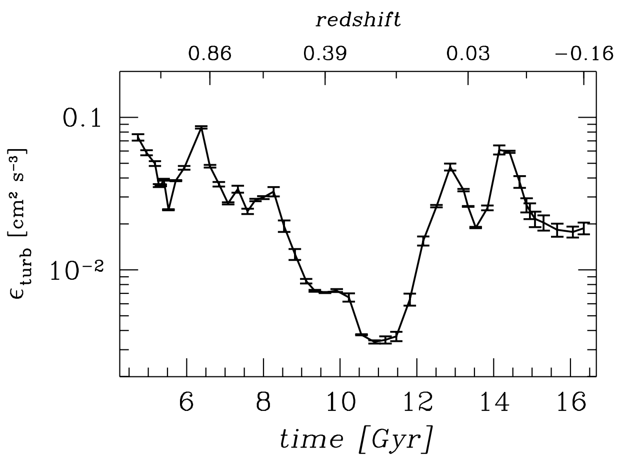


$$\eta_{turb}(t) = \frac{1}{E_{th}(t)} \int^{t} d\tau \, \rho \varepsilon_{turb}(\tau)$$



FM & Beresnyak (Nature, 523, 59, 2015)

ICM plasma-beta



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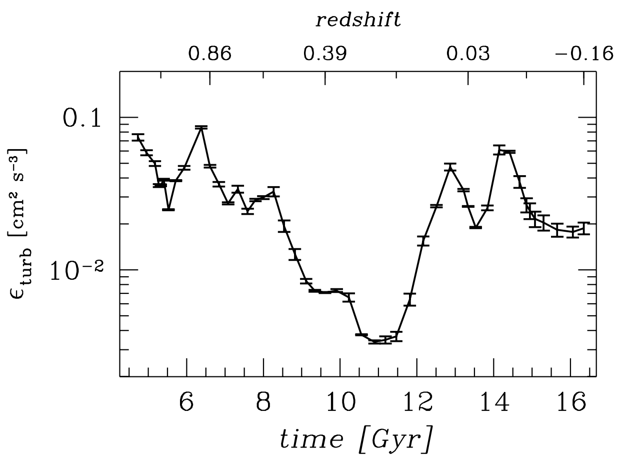
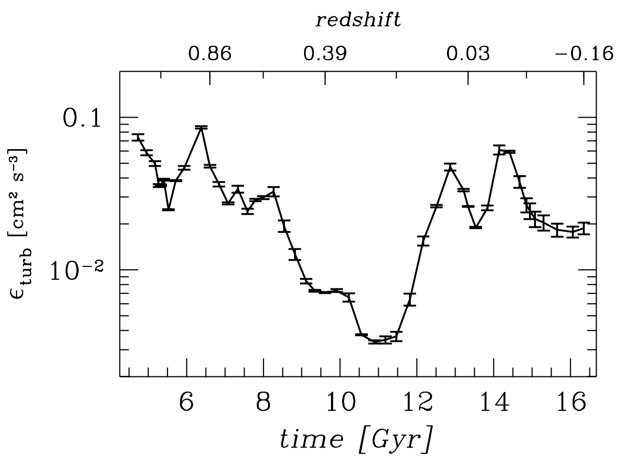
ICM plasma-beta



$$\beta_{plasma}(t) \equiv \frac{P_{gas}}{E_B} = \frac{(\gamma - 1)}{\eta_{turb}} \frac{\int^t \rho \varepsilon_{turb}\, d\tau}{E_B} = 40 \left(\frac{\eta_{turb}}{1/3} \right)^{-1} \left(\frac{C_E}{0.05} \right)^{-1}$$

FM & Beresnyak (Nature, 523, 59, 2015)

ICM plasma-beta



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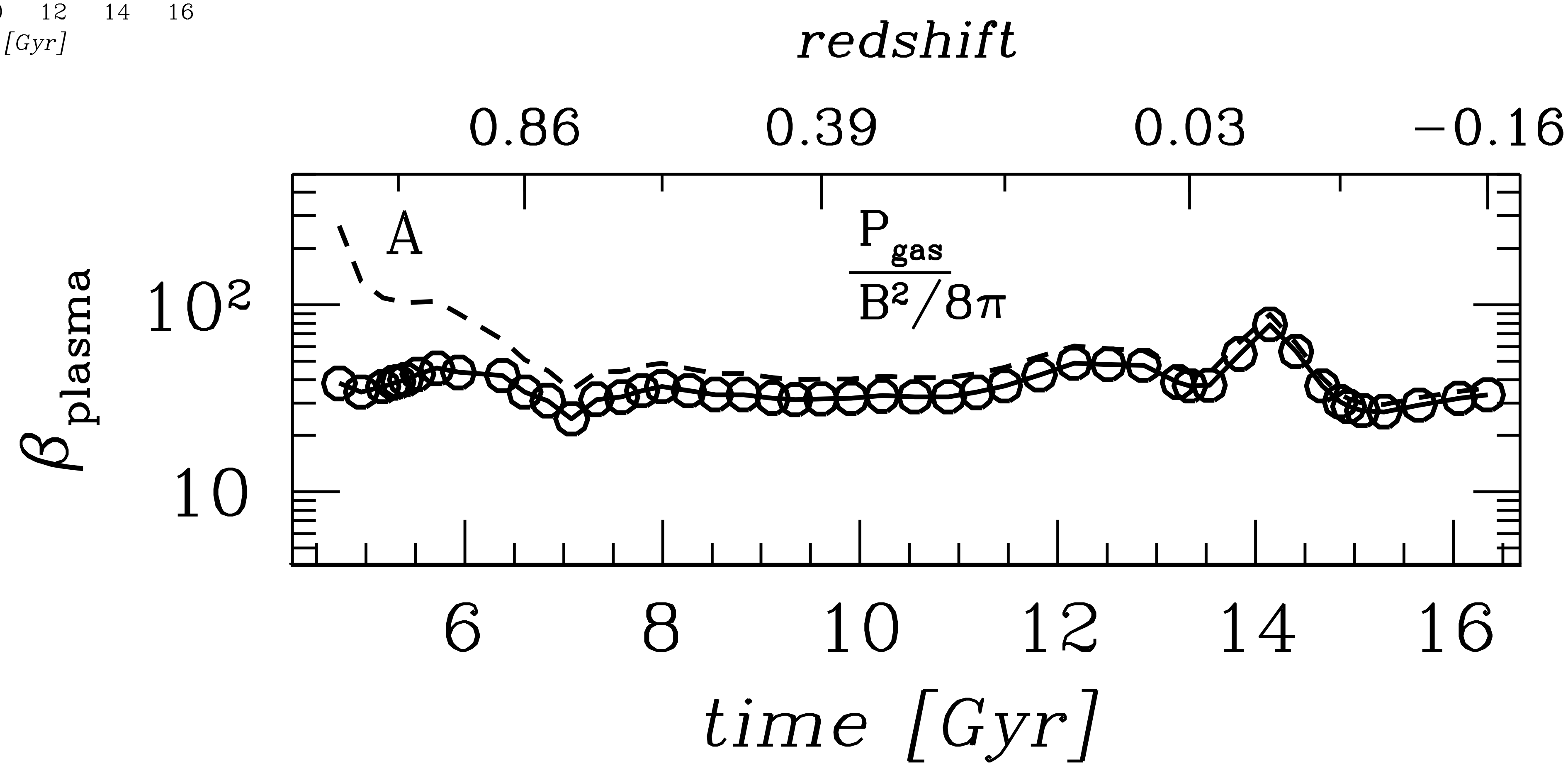
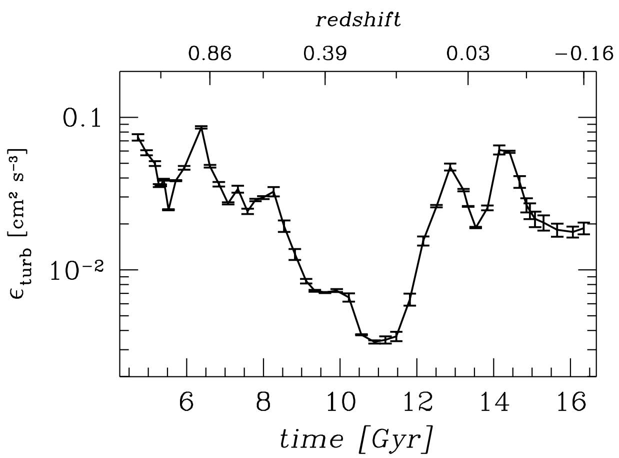
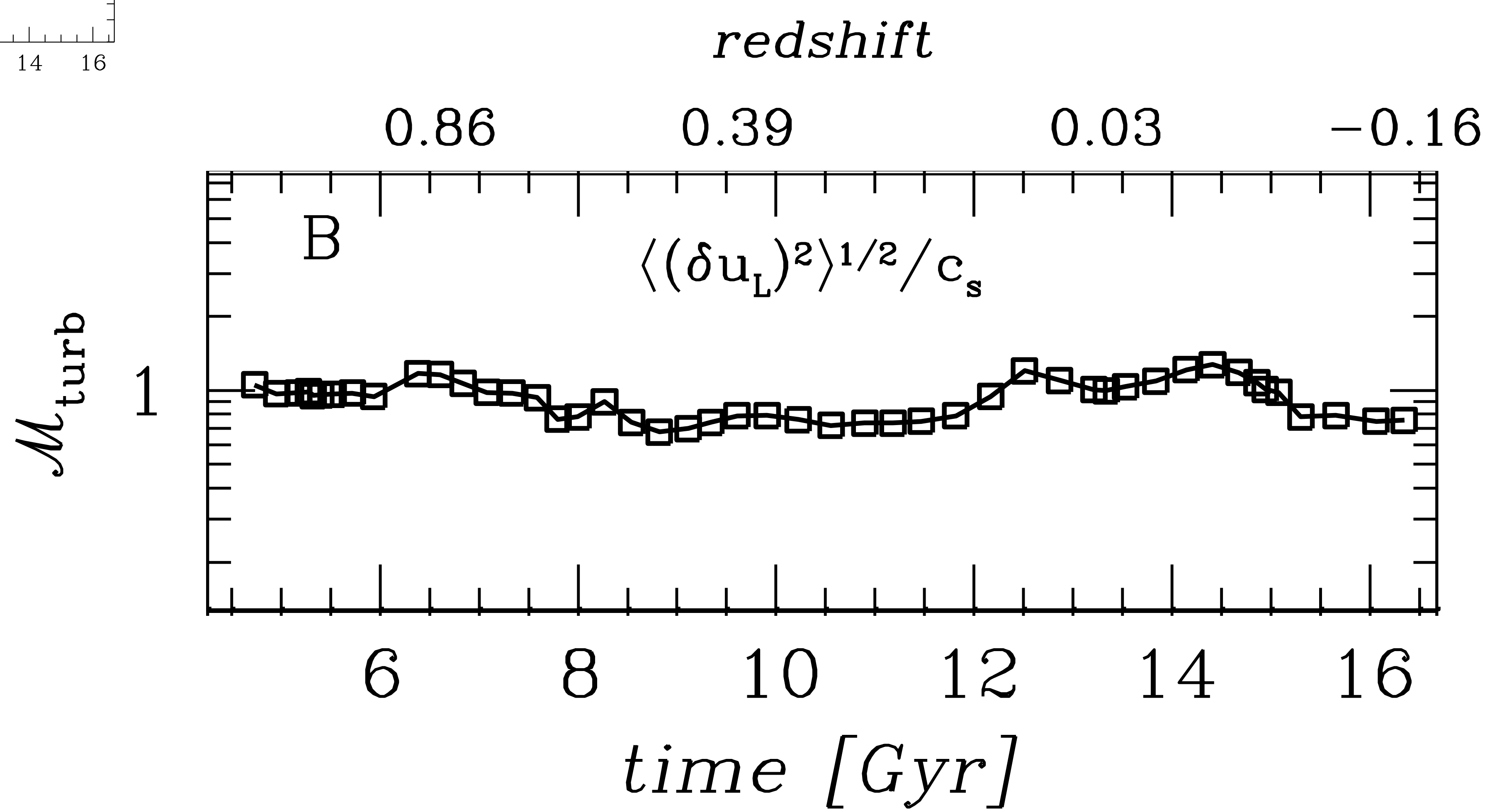


FM & Beresnyak (Nature, 523, 59, 2015)

Turbulent Mach Number

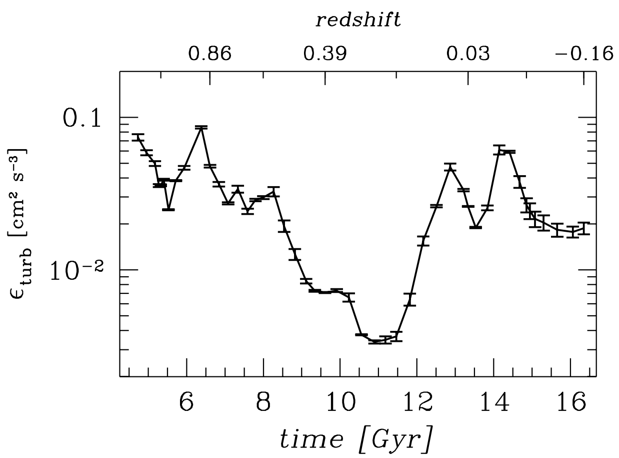


$$M_{turb}(t) = \frac{\left\langle (\delta u_L)^2 \right\rangle^{1/2}}{c_s}$$

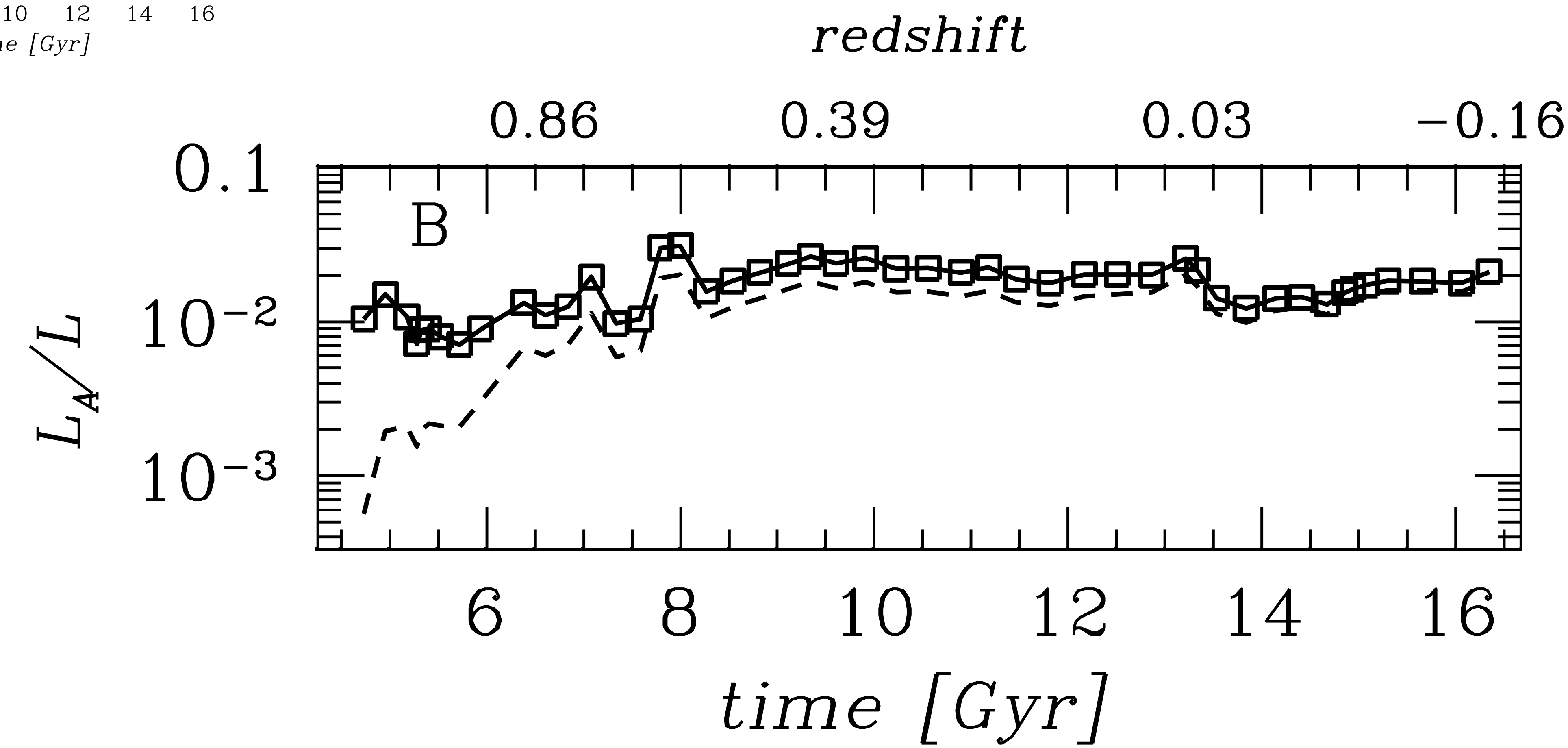


FM & Beresnyak (Nature, 523, 59, 2015)

Alfvèn Scale

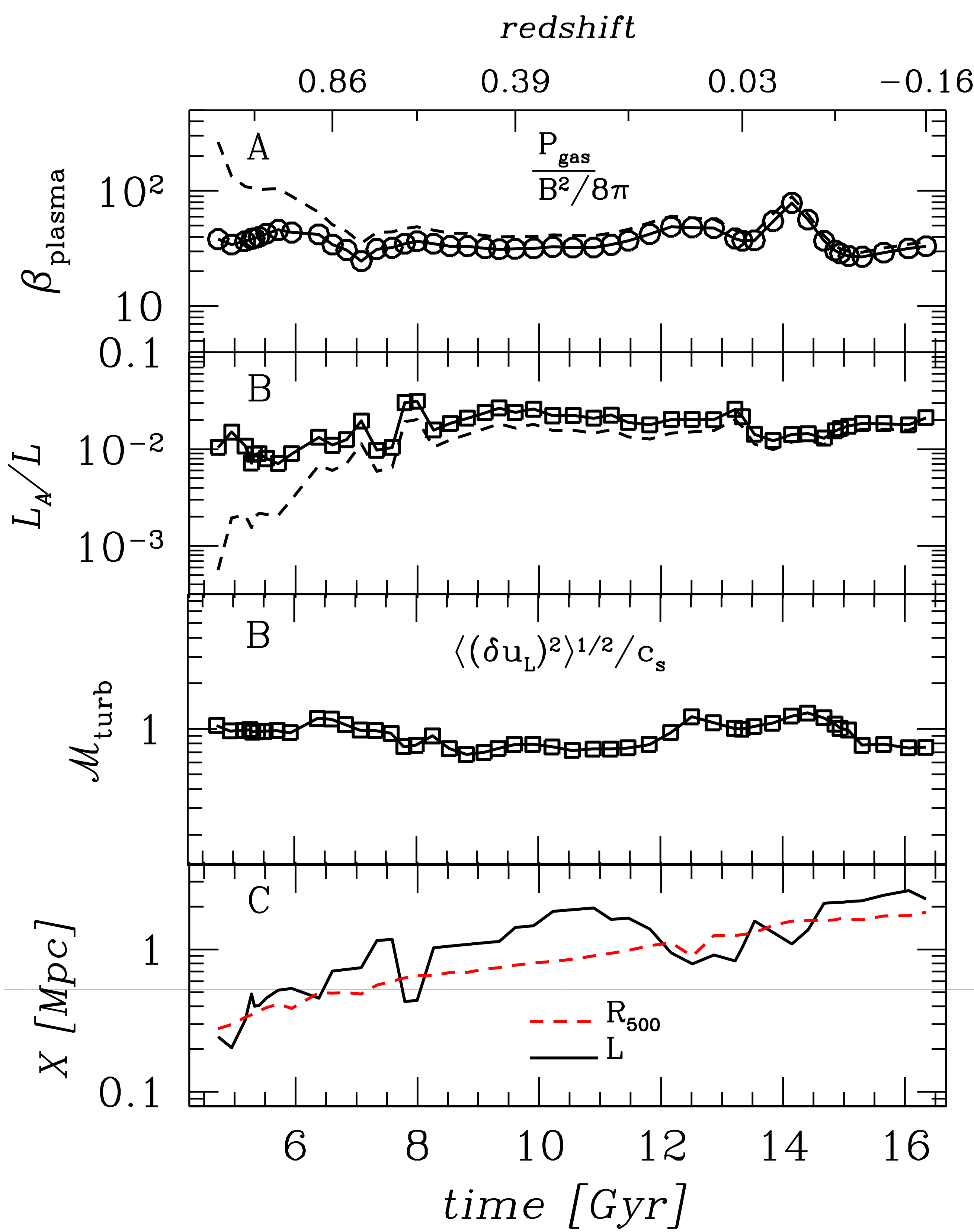


$$\frac{L_A}{L}(t) = \frac{V_A^3}{C^{3/2}\varepsilon_{turb}} = \frac{1}{100}\left(\frac{\beta_{plasma}}{40}\right)^{-\frac{3}{2}}\left(\frac{M_{turb}}{1}\right)^{-3}$$



FM & Beresnyak (Nature, 523, 59, 2015)

Self-similarity in the ICM



$$\beta_{plasma}(t) = \frac{P_{gas}}{E_B} = 40\left(\frac{\eta_{turb}}{1/3}\right)^{-1}\left(\frac{C_E}{0.05}\right)^{-1}$$

$$\frac{L_A}{L}(t) = \frac{V_A^3}{C^{3/2}\varepsilon_{turb}} = \frac{1}{100}\left(\frac{\beta_{plasma}}{40}\right)^{-\frac{3}{2}}\left(\frac{M_{turb}}{1}\right)^{-3}$$

$$M_{turb}(t) = \frac{\left\langle(\delta u_L)^2\right\rangle^{1/2}}{c_s} \approx \left(\frac{\alpha}{\sqrt{3}}\right)^{-\frac{1}{2}}\left(\frac{\eta_{turb}}{1/3}\right)^{\frac{1}{2}}$$

$$E_{th} : E_{turb} : E_B = 1 : \eta_{turb} : C_E\eta_{turb}$$

FM & Beresnyak (Nature, 523, 59, 2015)

Conclusions

- I have presented results from a recent numerical model of structure formation that resolves the ICM turbulent cascade for the first time
- Coupled with numerical studies of MHD turbulence our model reproduces remarkably well the observed properties of ICM magnetic field without any free parameter and independent of initial conditions!
- This calculation also shows that the evolution of ICM thermal, turbulent and magnetic field strength and structure are *self-similar*, with the turbulent dynamo far away from saturation as always
- The dimensionless numbers characterising the ratio of thermal to magnetic field (β_{plasma}), turbulent to thermal (M_{turb}) and magnetic to turbulent (L_A/L) reflect the values of the coefficients describing the efficiency of turbulent heating and of dynamo action

CHARM AMR-MHD-PIC Code

(FM & Colella 2007b)

$$\frac{\partial U}{\partial t} + \nabla \cdot F(U) = \Sigma(U)$$

$$U = (\rho, \rho\vec{u}, E, \vec{B})^{\mathrm{T}},\ \Sigma = (0, \vec{\nabla}\varphi, \dot{E}, 0)$$

Hyperbolic Solver for Baryonic Gas
8 Variables
~ 8000 flops/cell

- Eulerian representation
- Use un-split PPM (Colella 1990), Constrained-Transport MHD (FM & Martin 2011), Stiff Sources (FM&Colella, 2007a), Cosmic-Ray (FM 2007,2001)

$$\frac{d\vec{x}}{dt} = \vec{u}$$

$$\frac{d\vec{u}}{dt} = -\mathrm{H}\vec{u} - \vec{\nabla}\varphi$$

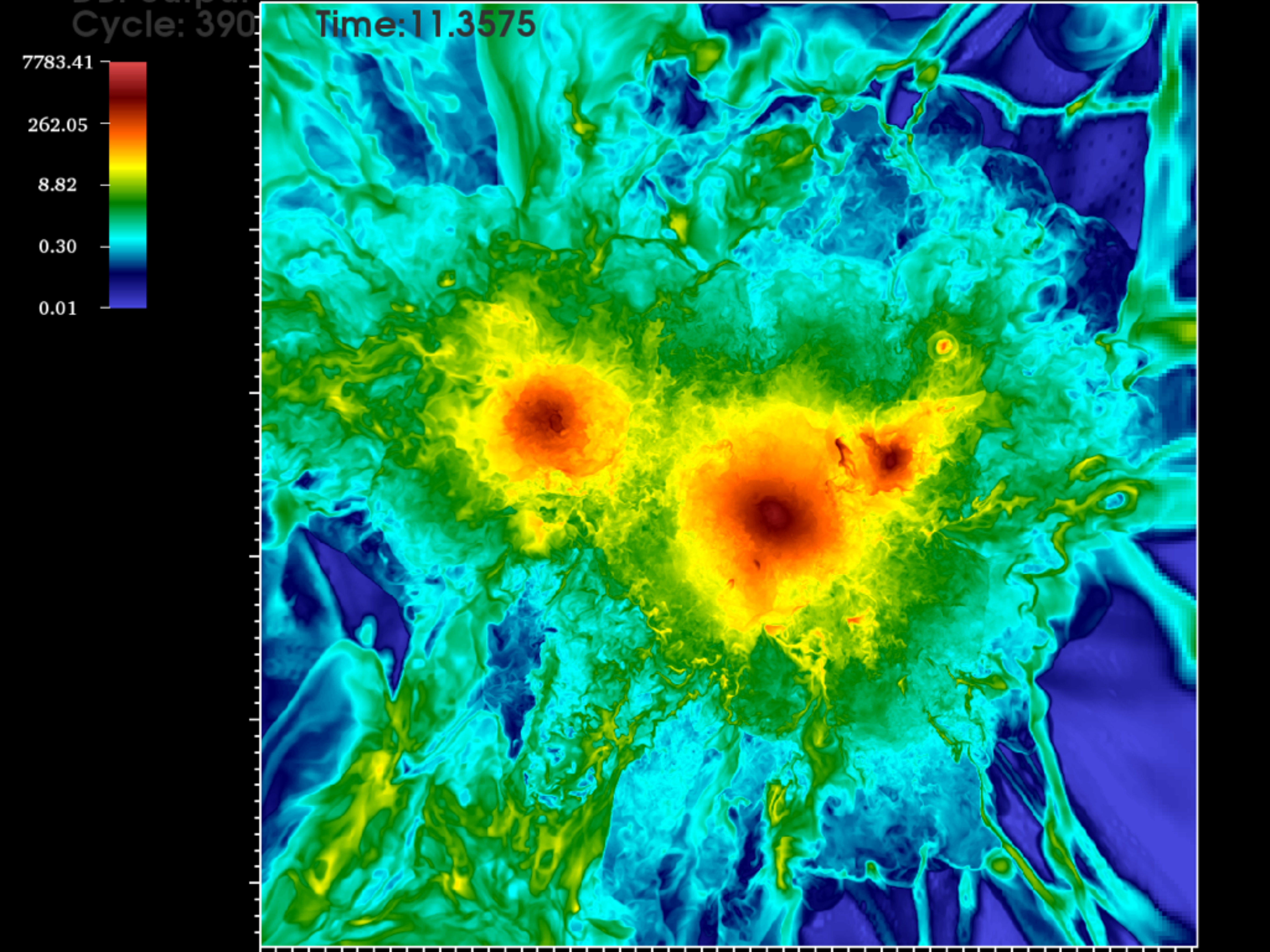
Vlasov-Poisson for collisionless Dark Mater
6 Variables
~ 500 flops/cell

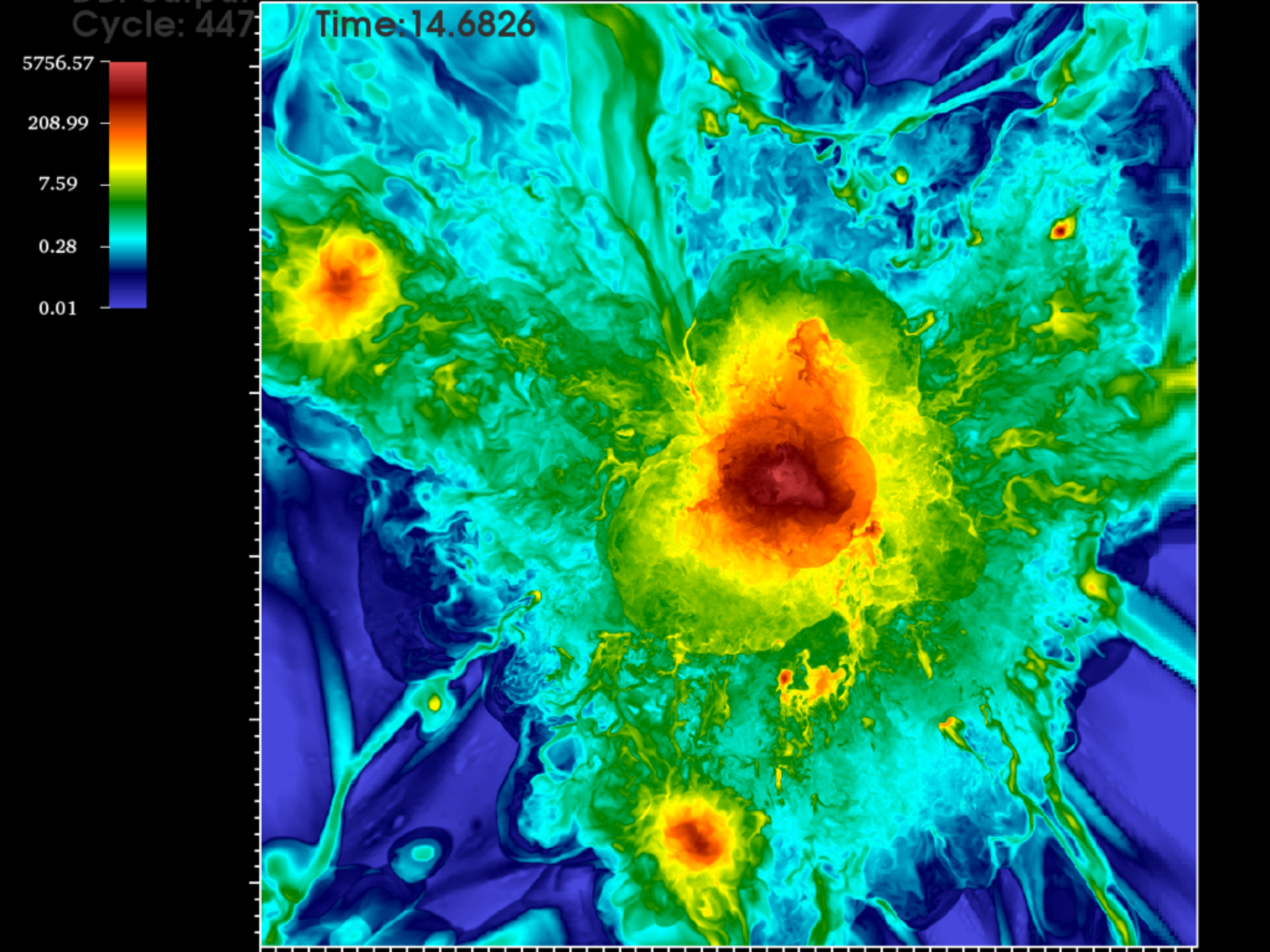
- Lagrangian representation
- Solve Vlasov-Poisson with Particle-Mesh method, time centered, modified symplectic scheme (Kick-Drift-Kick, Drift-Kick-Drift)

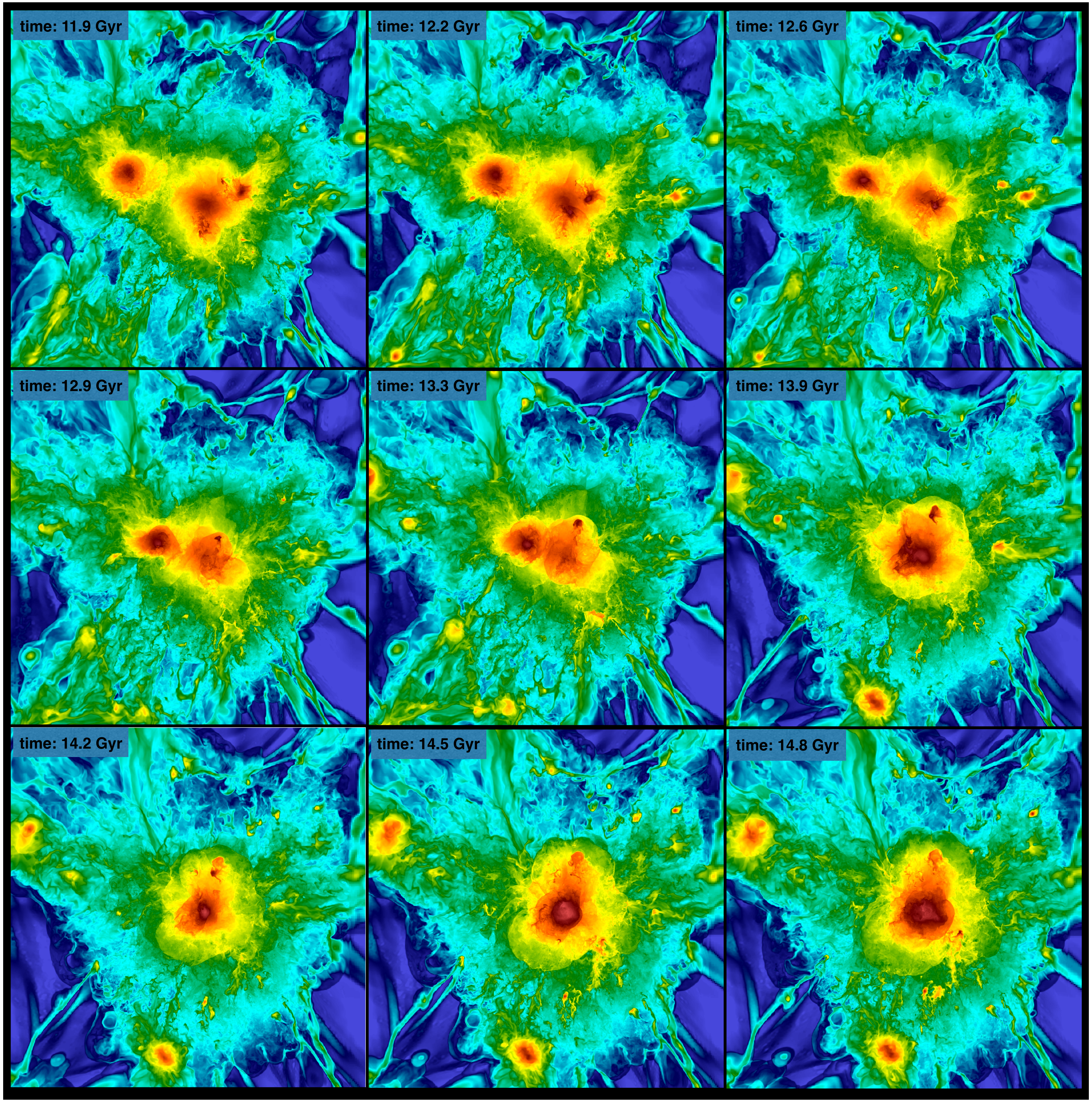
$$\Delta\varphi = 4\pi G\left(\rho - \langle\rho\rangle\right),\ \rho = \rho_{dm} + \rho_{gas}$$

- describes coupling between baryons and dark matter

Elliptic Solver
1 Variable
~ 1700 flops/cell

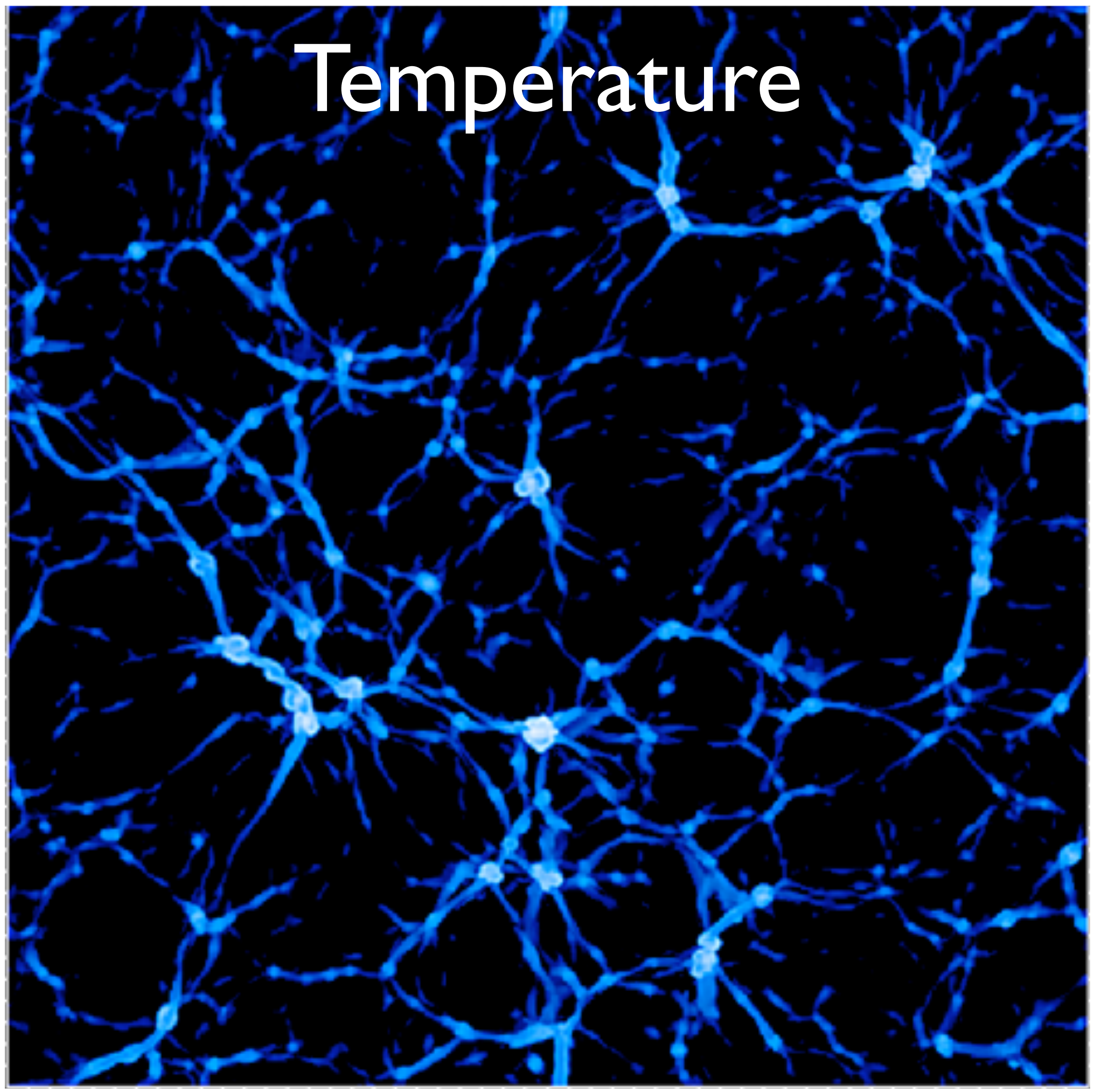

Cycle: 390
Time:11.3575
7783.41
262.05
8.82
0.30
0.01


Cycle: 447
Time:14.6826
5756.57
208.99
7.59
0.28
0.01



time: 11.9 Gyr
time: 12.2 Gyr
time: 12.6 Gyr
time: 12.9 Gyr
time: 13.3 Gyr
time: 13.9 Gyr
time: 14.2 Gyr
time: 14.5 Gyr
time: 14.8 Gyr

Accretion Flows



$$T_{IGM} \approx 10^3 - 10^4\,\mathrm{K}$$

Accretion onto filaments

$$M \sim 10\text{-}30,\ u \sim 100\ \mathrm{km/s}$$

Accretion onto clusters

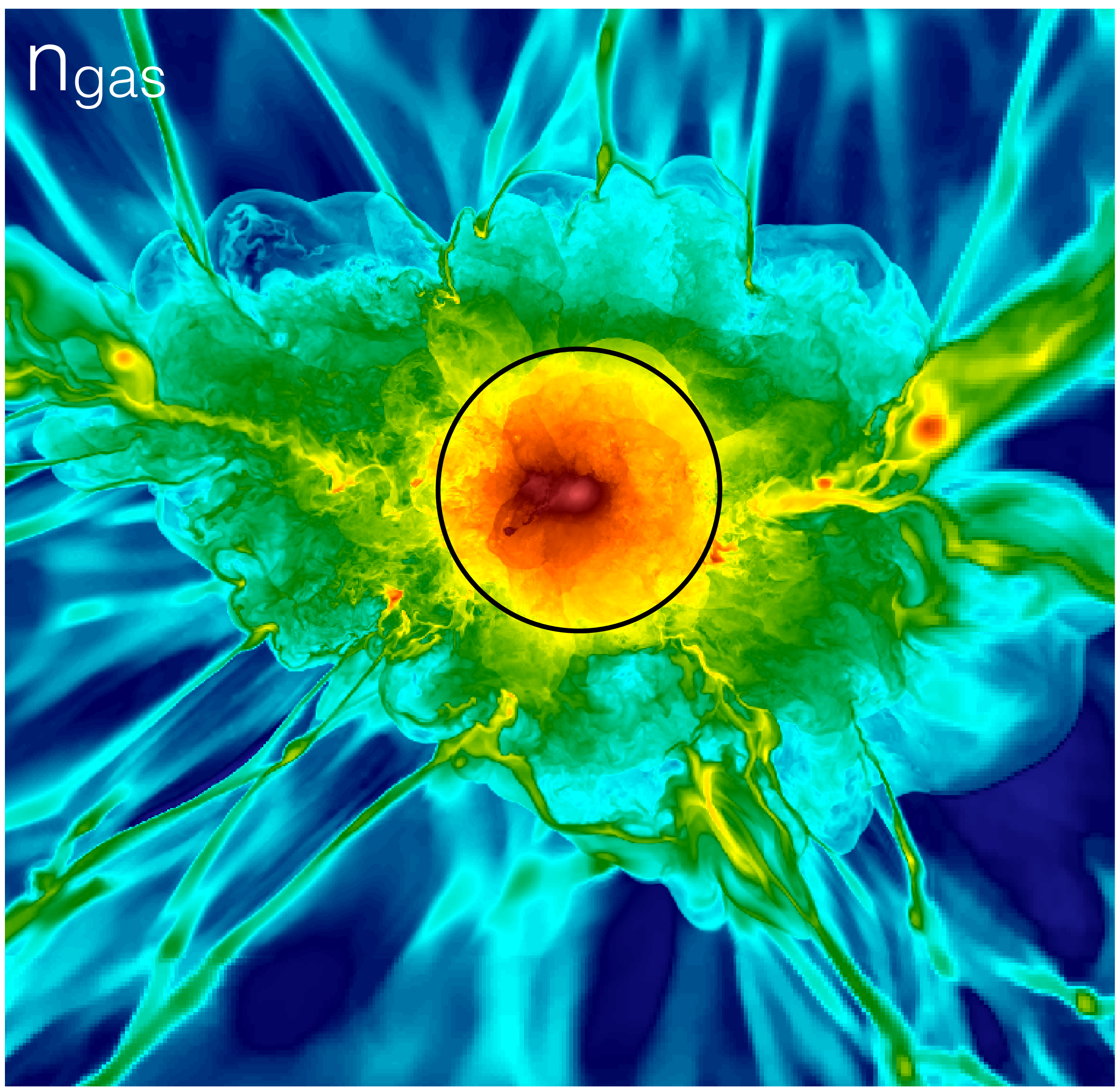
$$M \sim 100\text{-}300,\ u \sim 1000\ \mathrm{km/s}$$

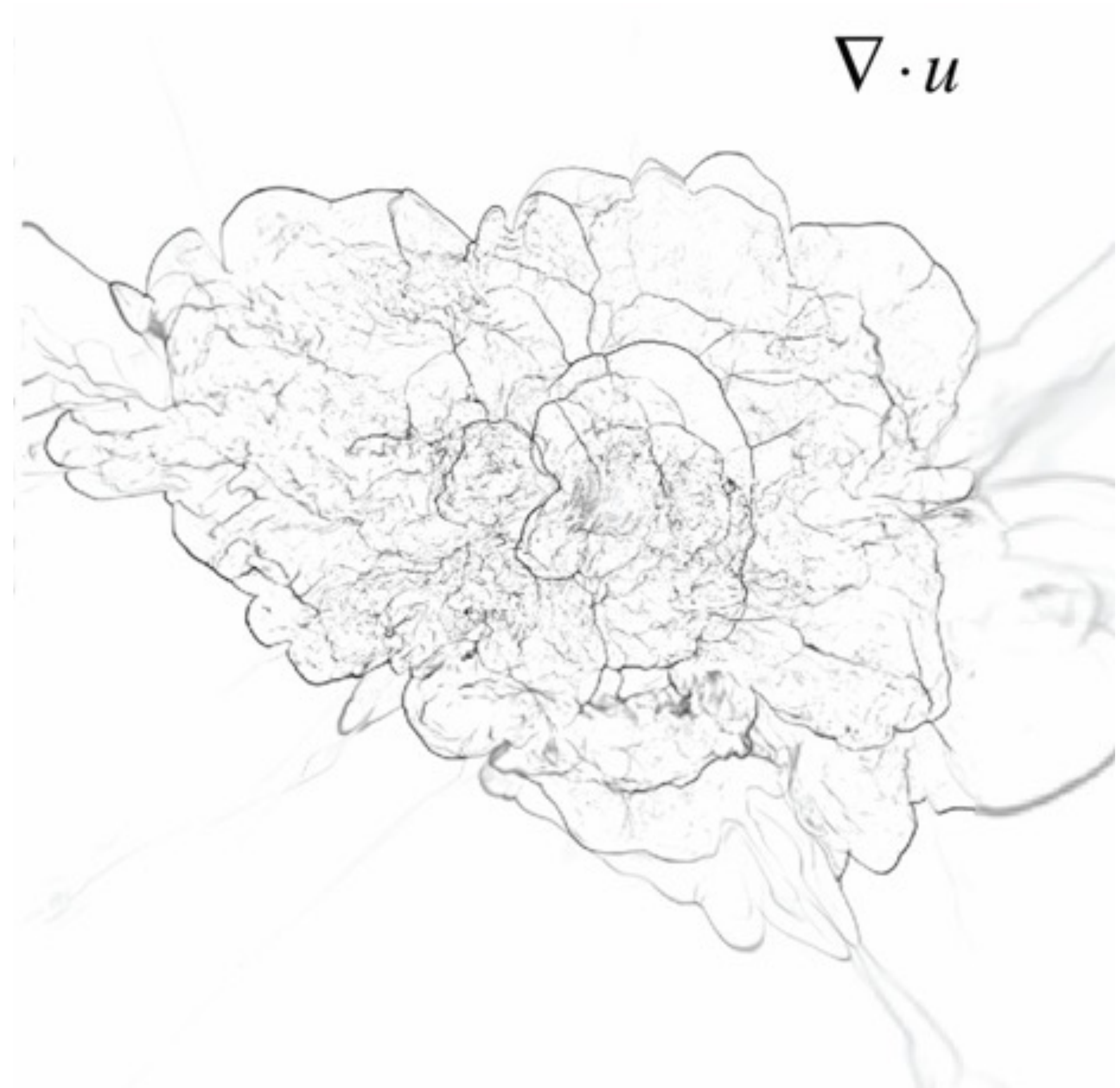
potential sites for occurrence of non-thermal processes: acceleration of cosmic-ray electrons and protons

Shocks

- *External* shocks have Mach numbers M>>10
- *Internal* shocks have Mach numbers ~ a few

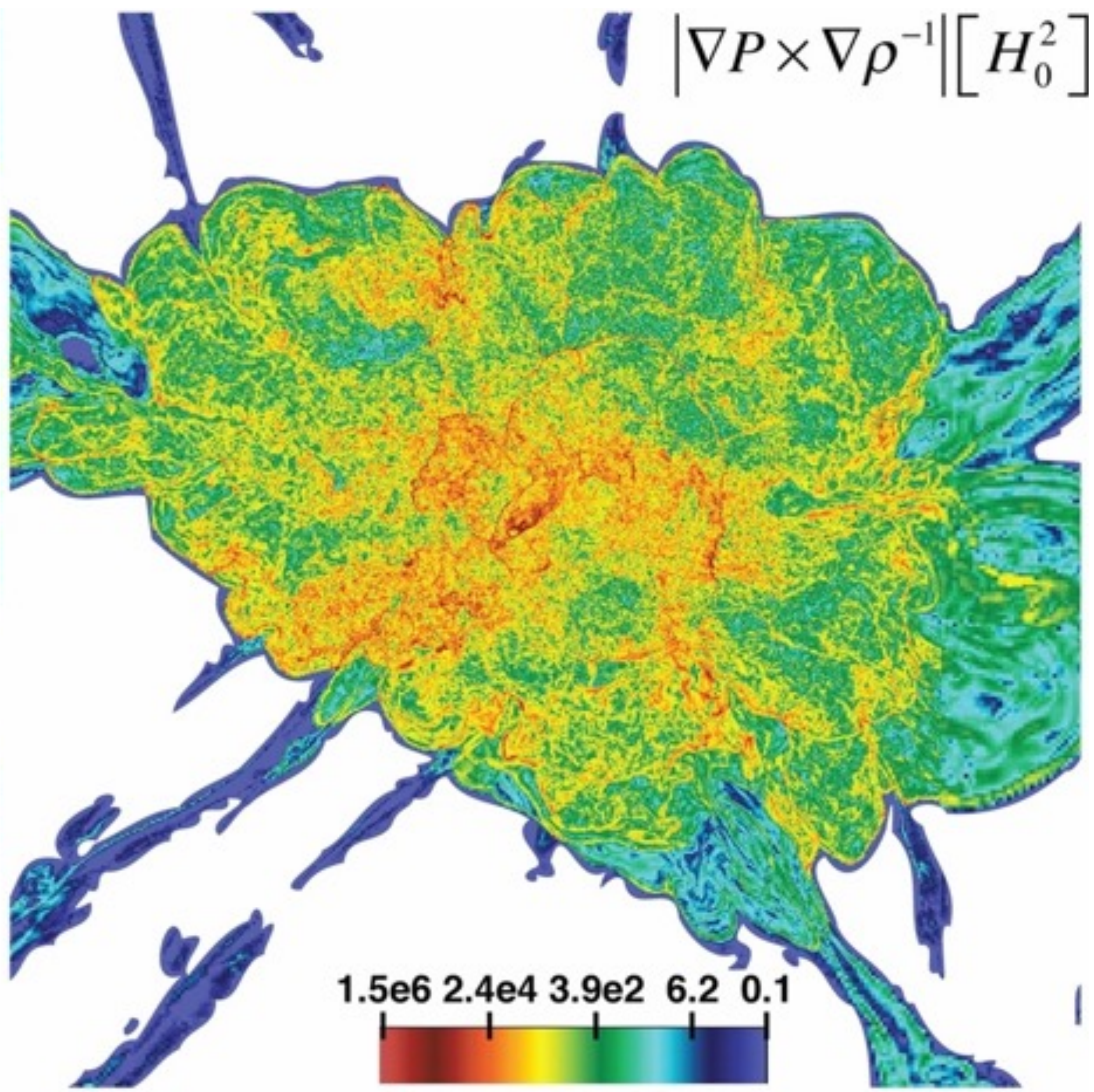
(Miniati+ 2000)

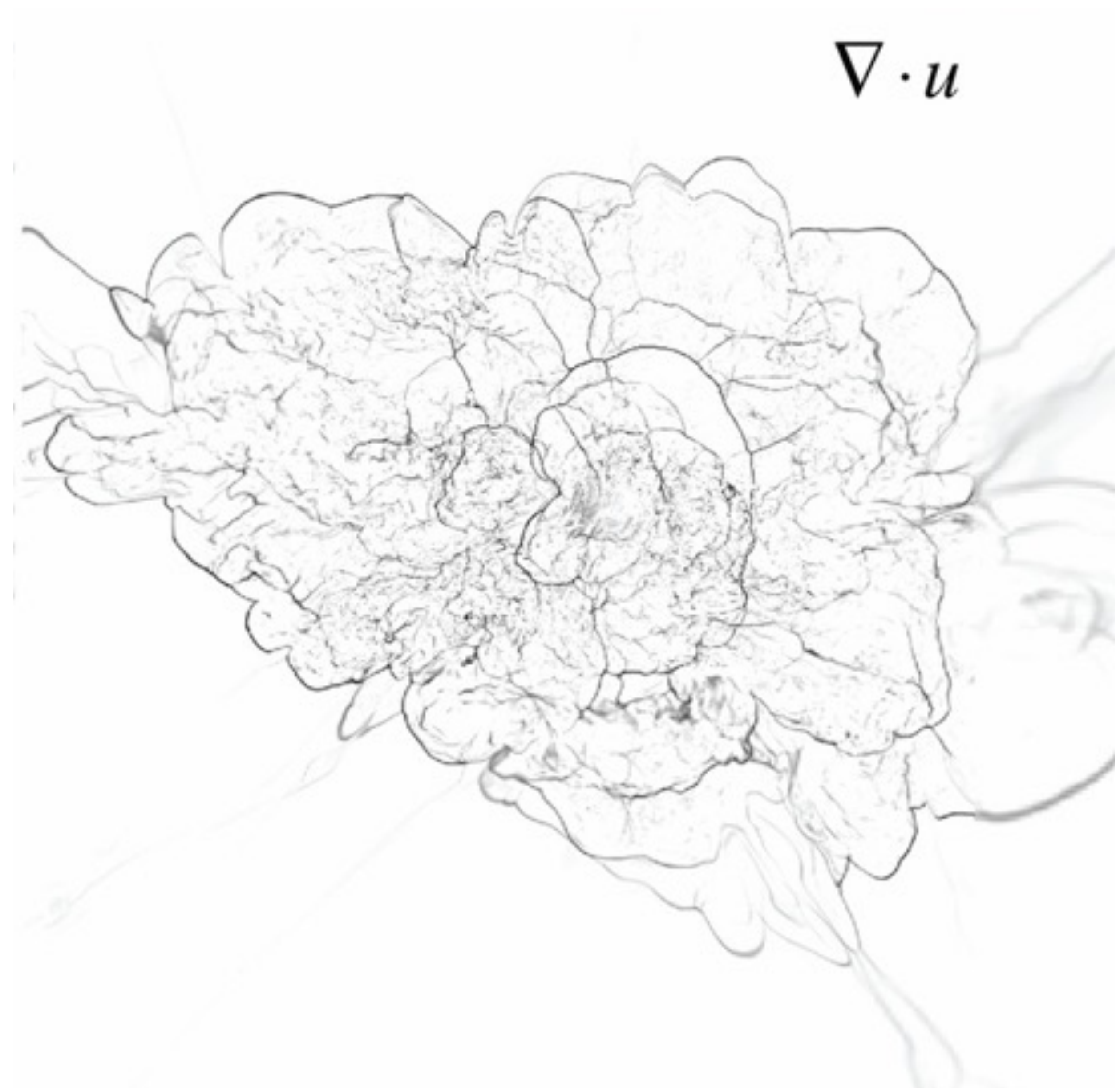




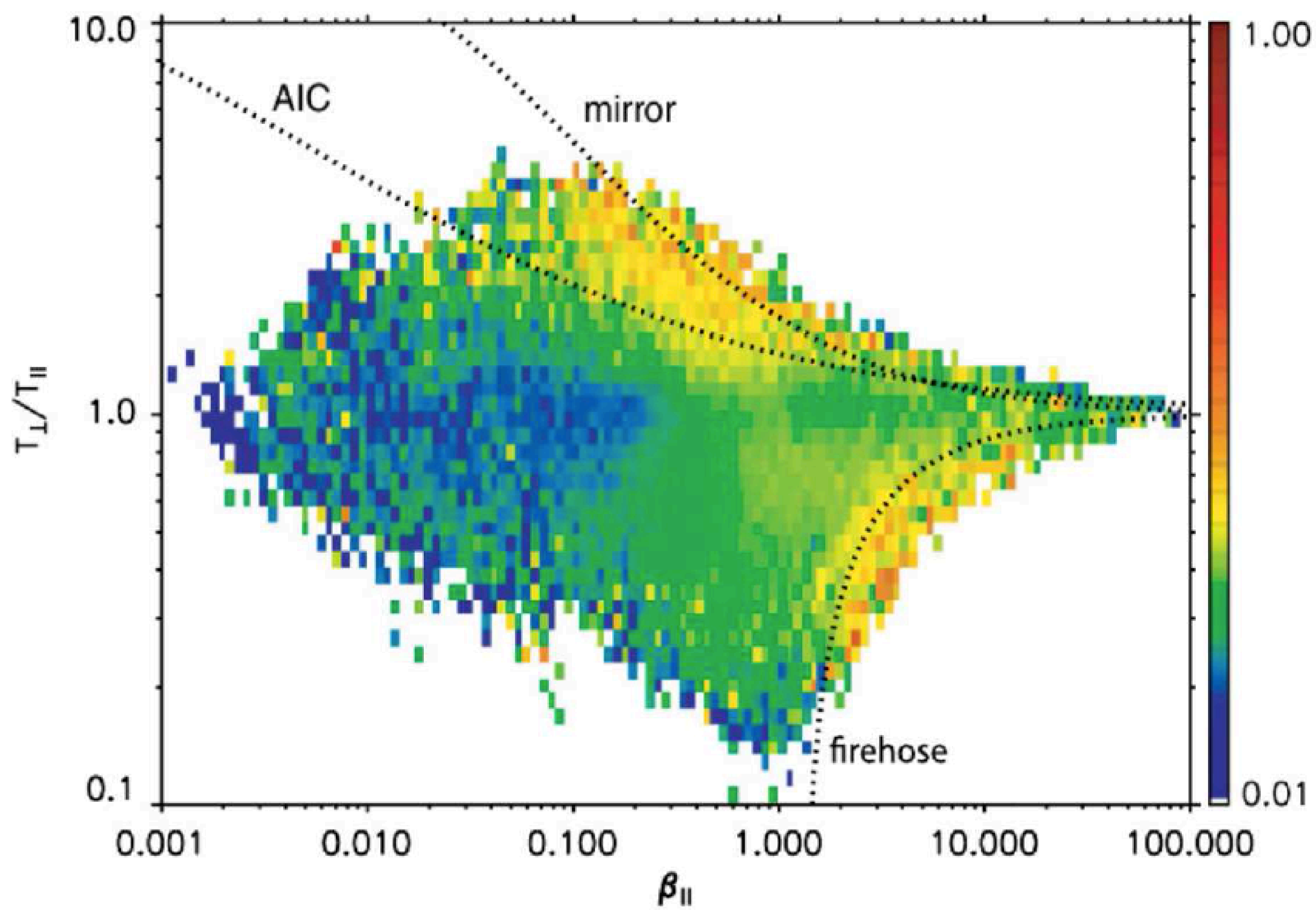
Baroclinicity

$$\frac{\partial \vec{\omega}}{\partial t} = \nabla \times (\vec{u} \times \vec{\omega}) + \nabla P \times \nabla \rho^{-1}$$





Anisotropy bounds from Mirror, Firehose Instabilities



Solar Wind Data from Bale et al. (PRL 103, 21101, 2009)