Relativistic effects in large-scale structure surveys

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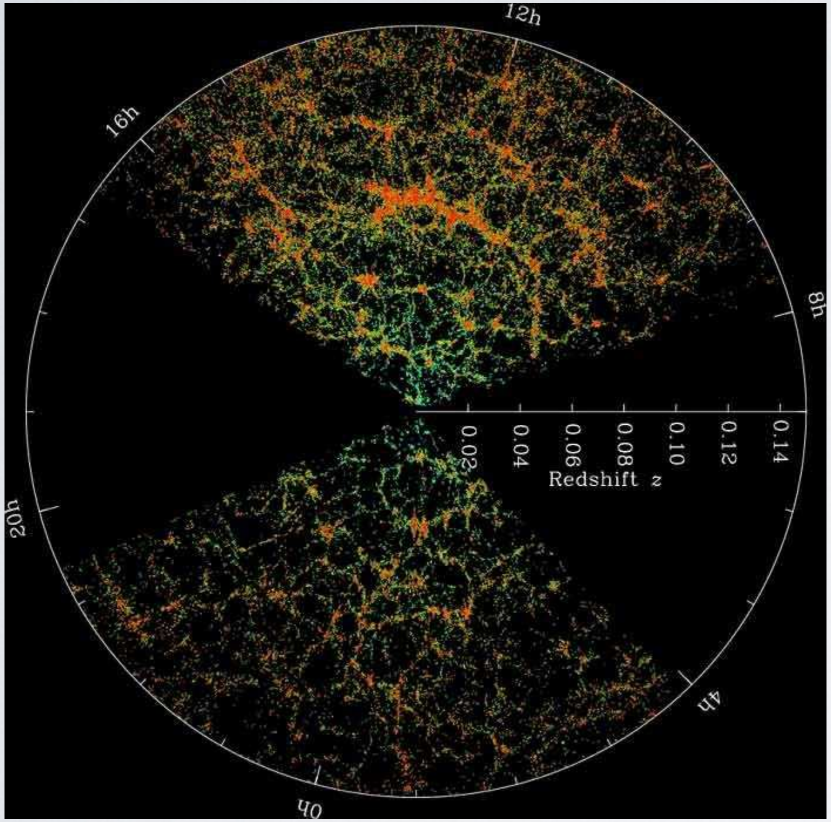
Galaxy survey

The **distribution** of galaxies is sensitive to:



- the initial conditions
- the theory of gravity
- the content of the universe

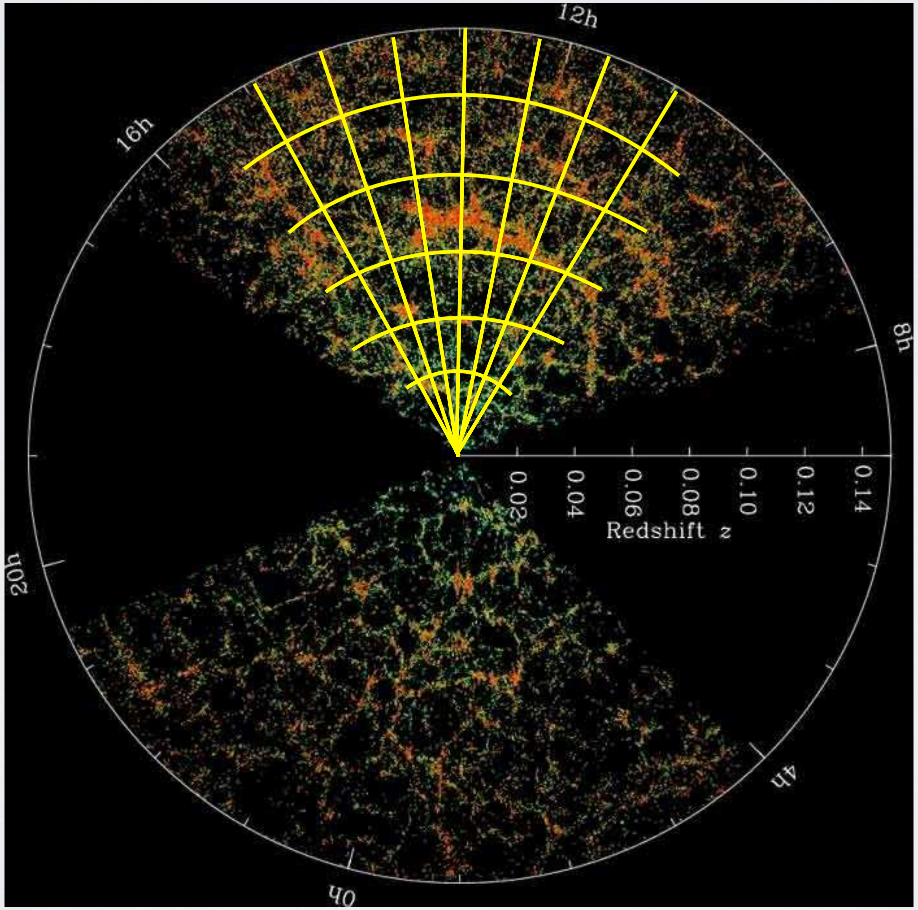
→ The large-scale structure contains valuable **information**

To interpret properly this information, we need to understand **what** we are **measuring**.

Galaxy survey

- We count the number of **galaxies** per **pixel**: $\Delta = \dfrac{N - \bar{N}}{\bar{N}}$
- How is Δ related to: the initial conditions, the theory of gravity and dark energy?

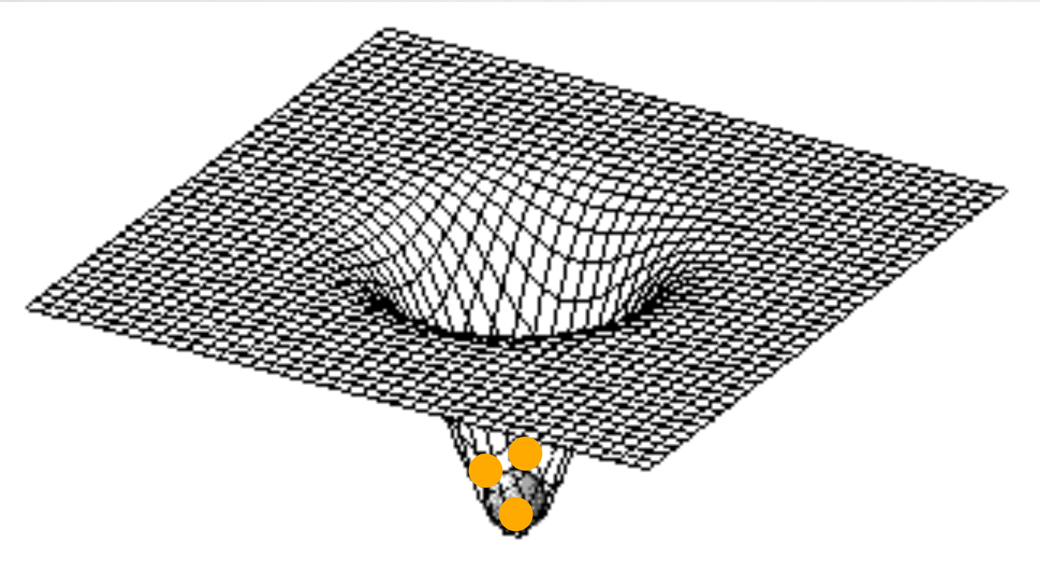
Credit: M. Blanton, SDSS



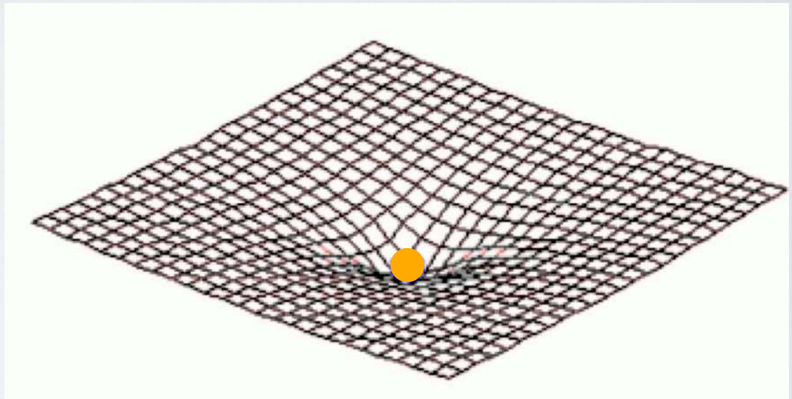
Galaxy distribution

Simple picture:
- **dark matter** is not homogeneously distributed
- it creates **gravitational potential** wells
- **baryons** fall into them and form galaxies

More dark matter



Less dark matter



$$\Delta = \frac{\delta\rho}{\rho} \equiv \delta$$

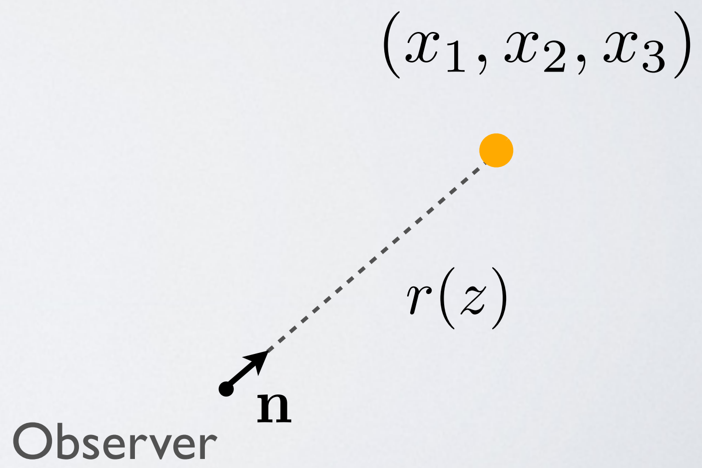
Complications

- **Bias**: the distribution of galaxies does not trace directly the distribution of dark matter $\Delta = b \cdot \delta$

- We never observe directly the **position** of galaxies, we observe the **redshift** z and the **direction** of incoming photons $\mathbf{n}$.

In a **homogeneous** universe:

- we calculate the distance $r(z)$
- light propagates on straight line

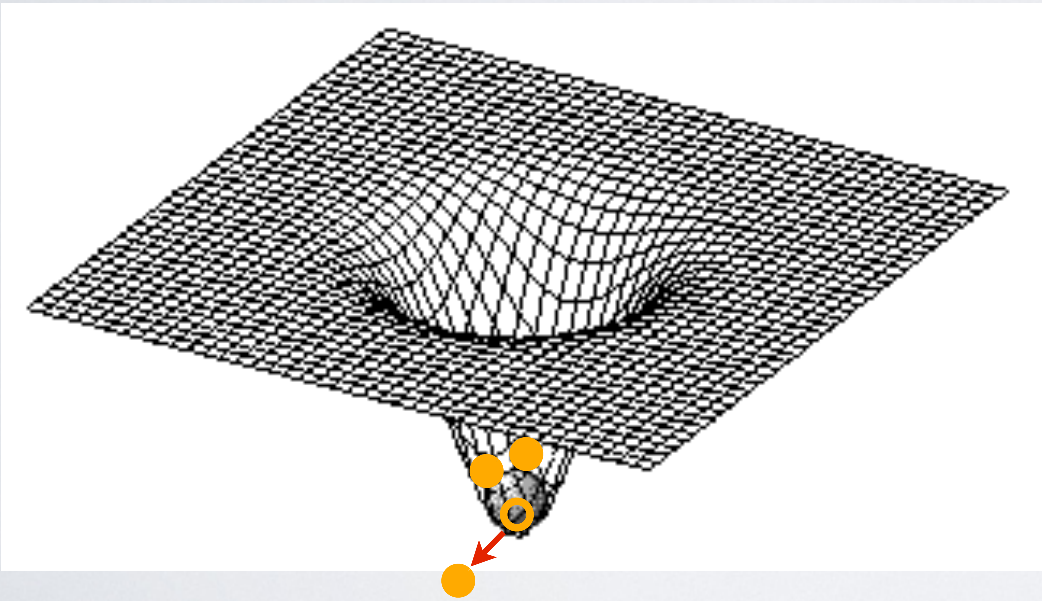


Redshift

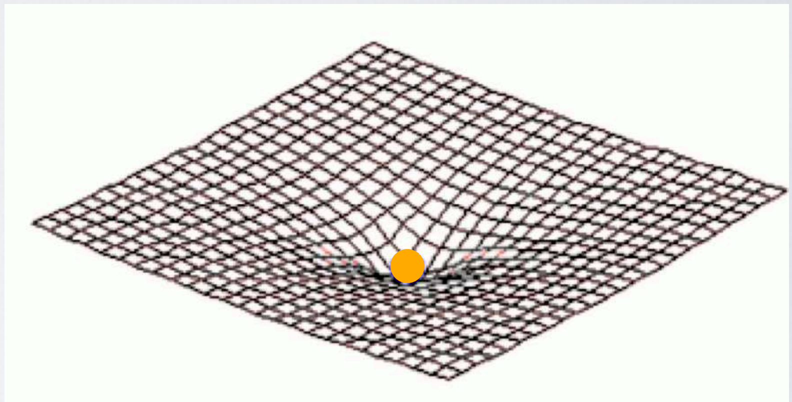
In an **inhomogeneous** universe: the redshift is affected by fluctuations, e.g. **Doppler** effect due to peculiar velocities.

→ **radial** shift in the galaxy position

More dark matter



Less dark matter



Redshift distortions

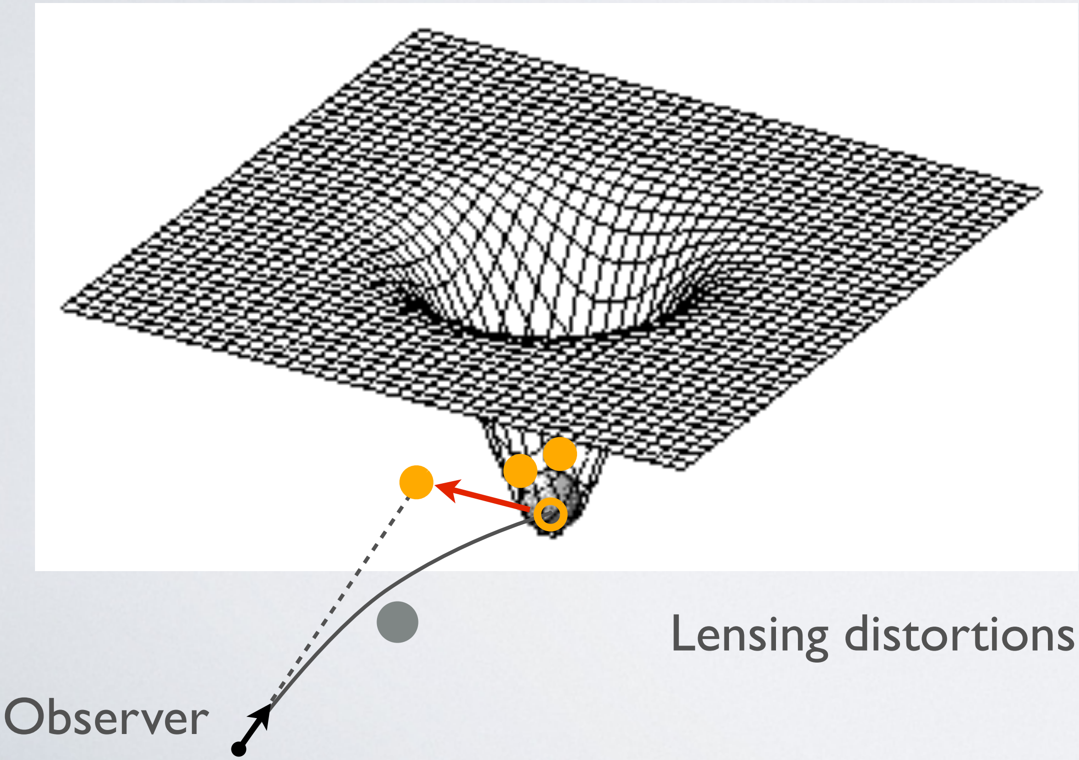
Observer

Lensing

In an **inhomogeneous** universe: light is **lensed** by matter between the galaxies and the observer

→ **transverse** shift in the galaxy position

More dark matter



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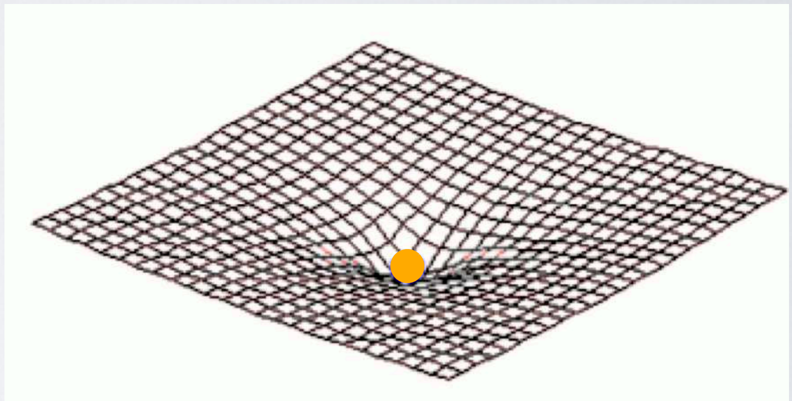
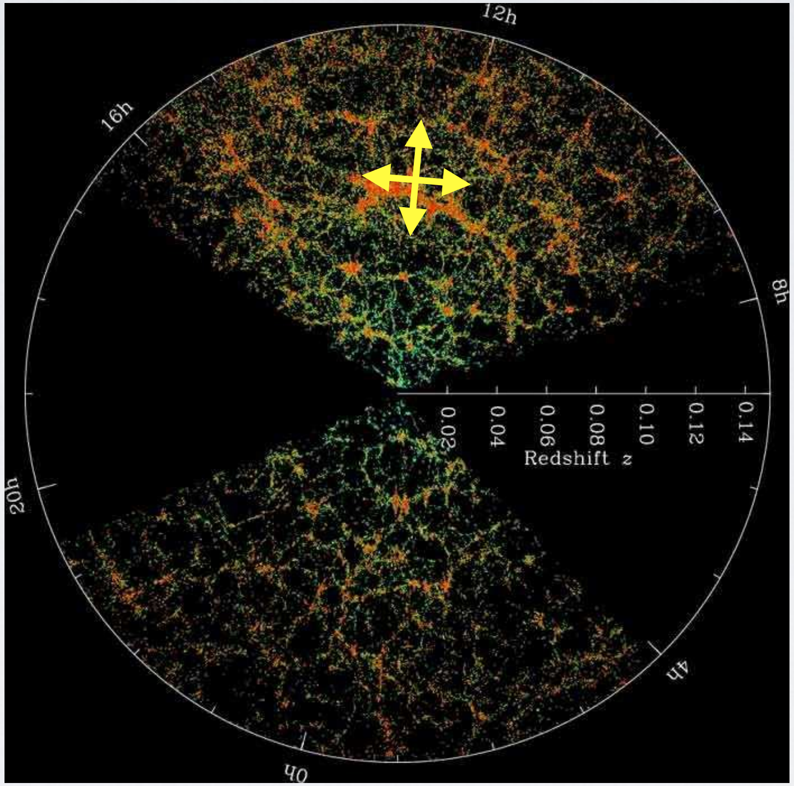


Galaxy distribution

The **structures** seen on a galaxy map do **not reflect** directly the underlying dark matter structures. The observed **position** of galaxies are **shifted** radially and transversally.

Credit: M. Blanton, SDSS



To extract **information** from a galaxy map, we need to understand exactly which **distortions** there are.

Outline

- Expression for Δ encoding all distortions at linear order

$$\Delta = \text{density} + \text{redshift distortions} + \text{lensing} + \text{relativistic effects}$$

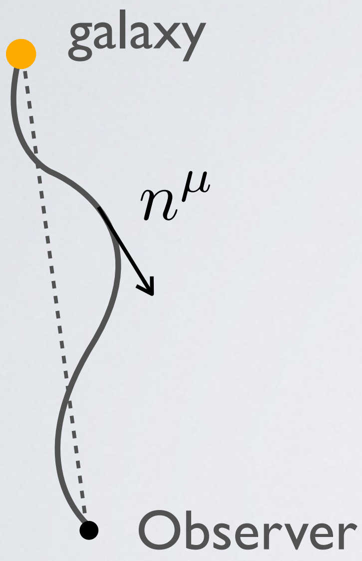
- Impact of the different terms on the **correlation** function

→ We can construct estimators to **separate** the contributions and use them to test **gravity**.

Calculating the distortions

Perturbed Friedmann universe:

$$ds^2 = -a^2\left[\left(1+2\Psi\right)d\eta^2 + \left(1-2\Phi\right)\delta_{ij}dx^i dx^j\right]$$



We calculate the **propagation** of **photons**, i.e. the null geodesics and infer:

- the changes in **energy**
- the changes in **direction**

The number count at linear order

Yoo et al (2010)
CB and Durrer (2011)
Challinor and Lewis (2011)

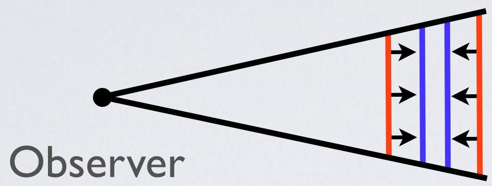
$$
\begin{aligned}
\Delta(z,\mathbf{n}) = b\cdot\delta - \frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n}) \\
- \int_0^r dr'\frac{r-r'}{rr'}\Delta_\Omega(\Phi+\Psi) \\
+ \left(1-\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}-\frac{2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\partial_r\Psi \\
+ \Psi - 2\Phi + \frac{1}{\mathcal{H}}\dot{\Phi} - 3\frac{\mathcal{H}}{k}V + \frac{2}{r}\int_0^r dr'(\Phi+\Psi) \\
+ \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{2}{r\mathcal{H}}\right)\left[\Psi + \int_0^r dr'(\dot{\Phi}+\dot{\Psi})\right]
\end{aligned}
$$

Distortions

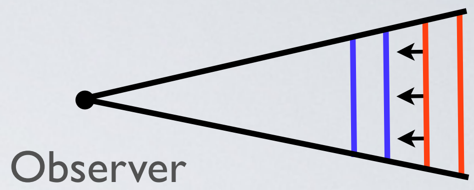
♦ Velocities

Kaiser (1987), Lilje & Efstathiou (1989), Hamilton (1992)

Change in the bin size:
Redshift distortions



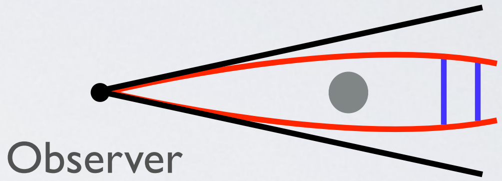
Change in the bin position:
Doppler effect



♦ Lensing

Gunn (1967), Schneider (1989), Broadhurst, Taylor & Peacock (1995)

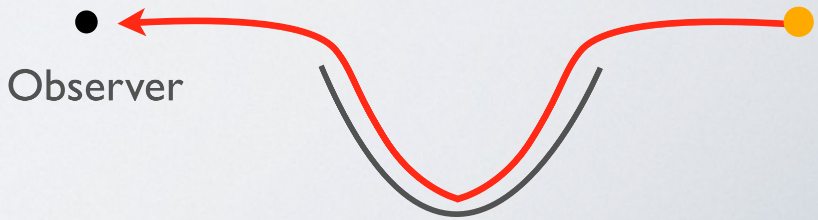
Change in the solid angle



♦ Potentials

Local terms:
e.g. gravitational redshift

Integrated terms: e.g. Shapiro time-delay and Integrated Sachs-Wolfe



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& + \left(1 - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - \frac{2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\partial_r\Psi \\
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\end{aligned}
$$

The number count at linear order

density — redshift distortions

Yoo et al (2010)
CB and Durrer (2011)
Challinor and Lewis (2011)

$$\Delta(z,\mathbf{n}) = b\cdot\delta - \frac{1}{\mathcal{H}}\partial_r \mathbf{V}\cdot\mathbf{n}$$

$$-\int_0^r dr' \frac{r-r'}{rr'}\Delta_\Omega(\Phi+\Psi)$$

gravitational lensing

$$+\left(1-\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}-\frac{2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n}+\frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n}+\frac{1}{\mathcal{H}}\partial_r\Psi$$

gravitational redshift

$$+\Psi-2\Phi+\frac{1}{\mathcal{H}}\dot{\Phi}-3\frac{\mathcal{H}}{k}V+\frac{2}{r}\int_0^r dr'(\Phi+\Psi)$$

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$$-\int_0^r dr'\frac{r-r'}{rr'}\Delta_\Omega(\Phi+\Psi)$$ gravitational lensing

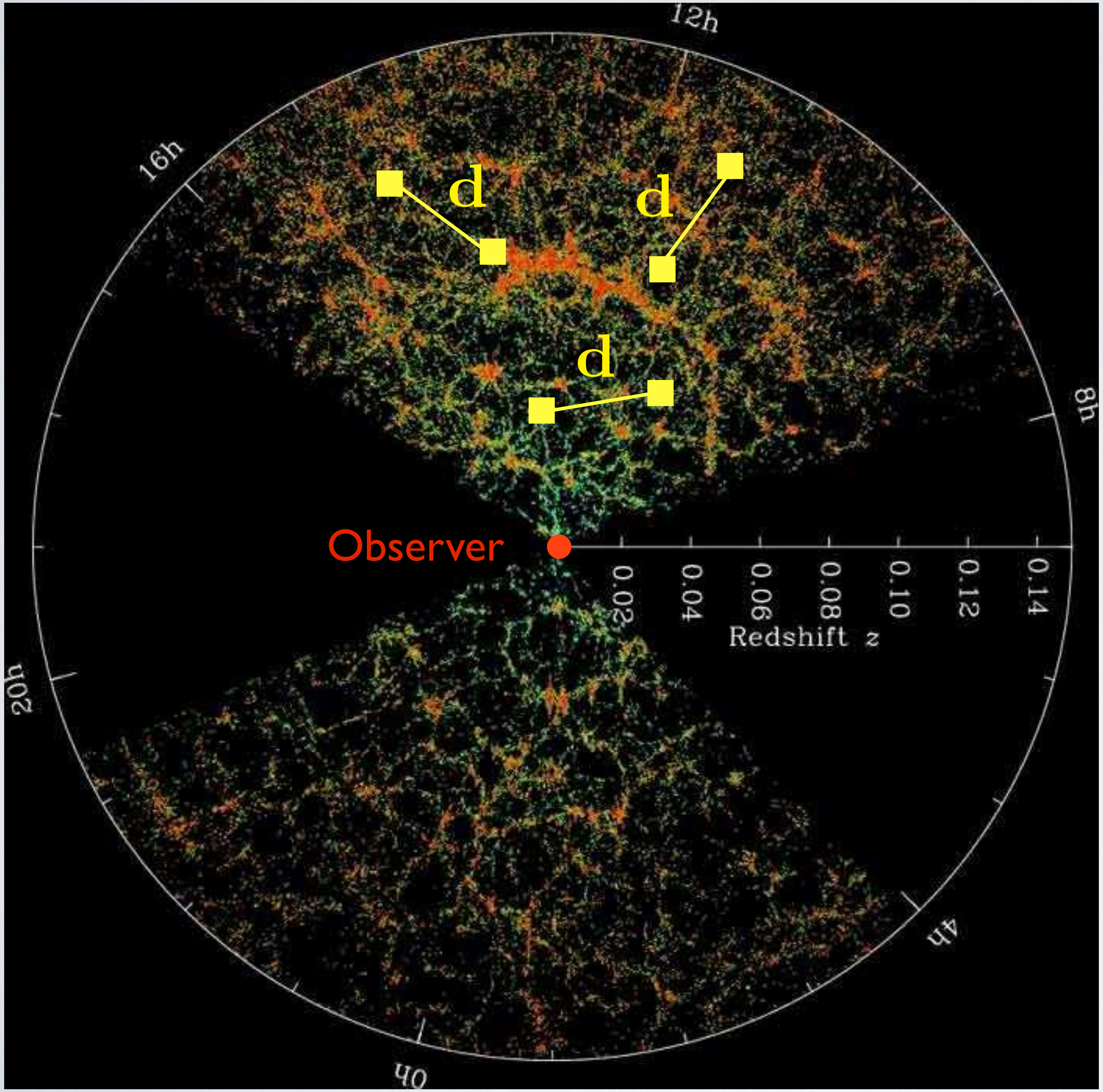
$$+\left(1-\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}-\frac{2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n}+\frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n}+\frac{1}{\mathcal{H}}\partial_r\Psi$$ gravitational redshift

δ	$\longleftrightarrow$	Φ	Poisson equation
Ψ	$\longleftrightarrow$	Φ	Anisotropic stress
V	$\longleftrightarrow$	Ψ	Euler equation

Correlation function

The various terms affect the two-point function differently.

Credit: M. Blanton, SDSS



$$\xi = \langle \Delta(\mathbf{x}) \Delta(\mathbf{x}') \rangle$$

The dark matter fluctuations generate **isotropic** correlations

$$\Delta = b \cdot \delta$$

$$\xi(d) = C_0(d)$$

Correlation function

Kaiser (1987), Lilje & Efstathiou (1989) Hamilton (1992)

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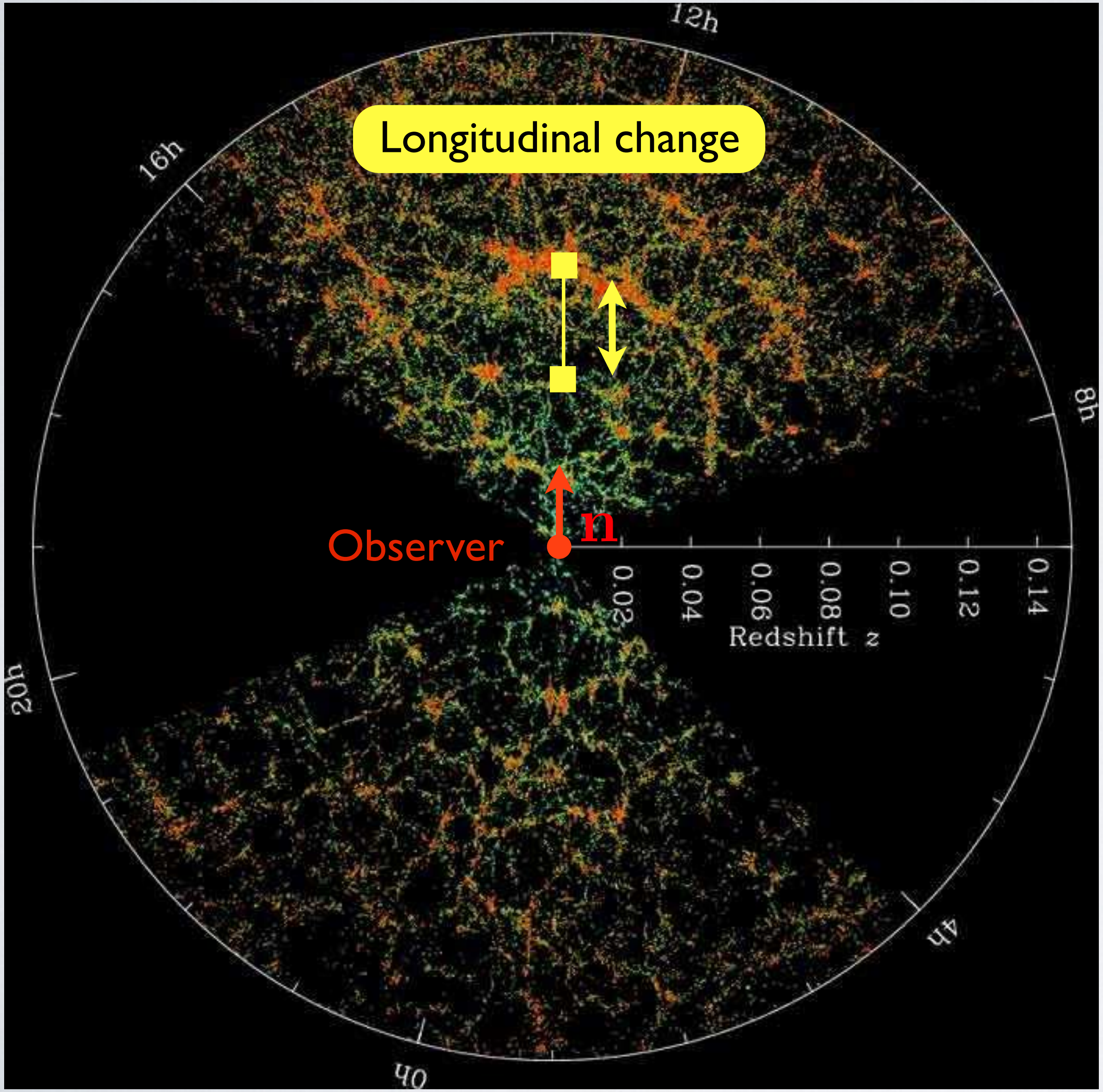

$$\xi = \langle \Delta(\mathbf{x})\Delta(\mathbf{x}')\rangle$$

Redshift distortions **break** the **isotropy**

$$\Delta = -\frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n})$$

$$\xi = C_0(d) + C_2(d)P_2(\cos\beta) + C_4(d)P_4(\cos\beta)$$

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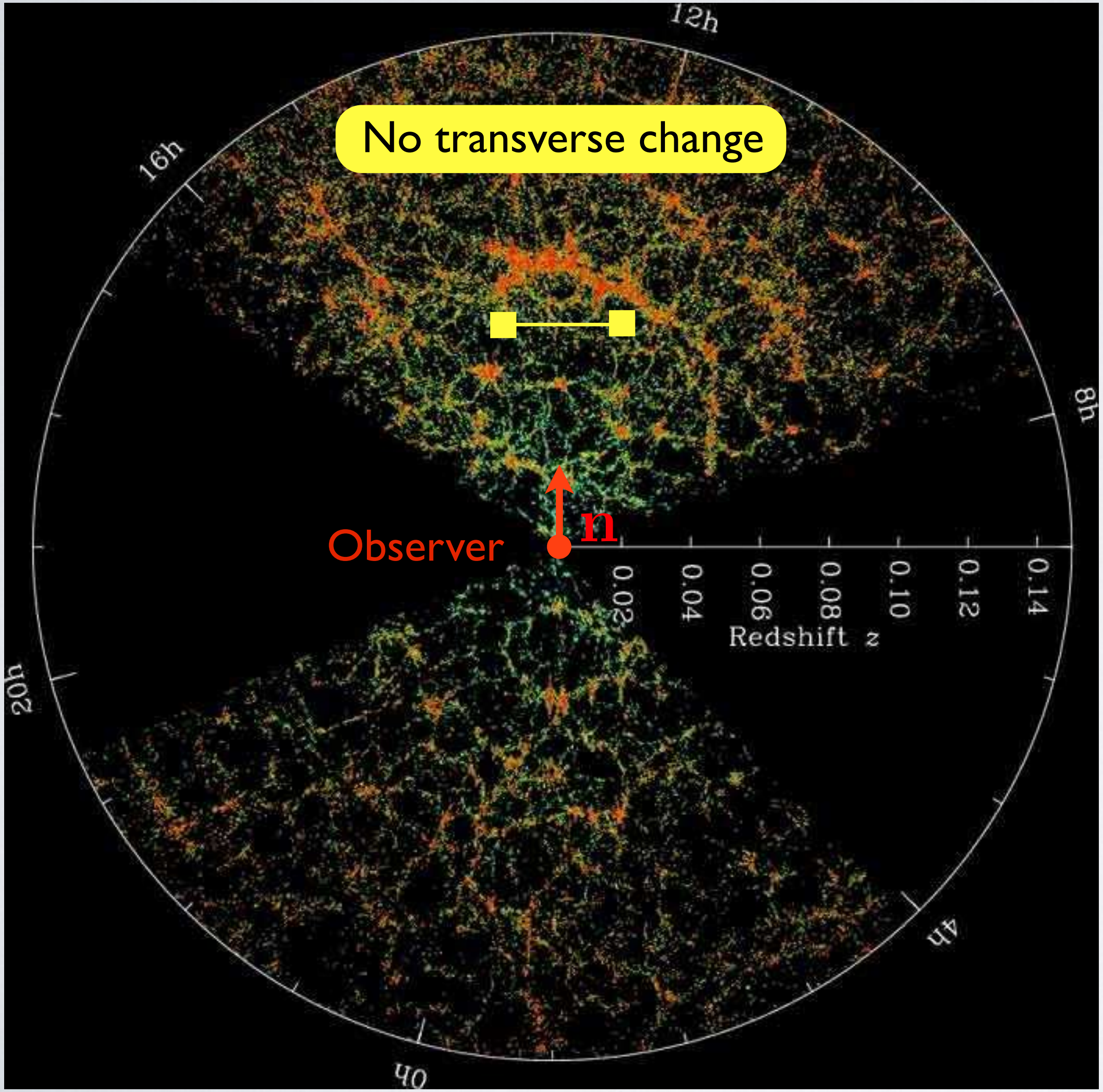

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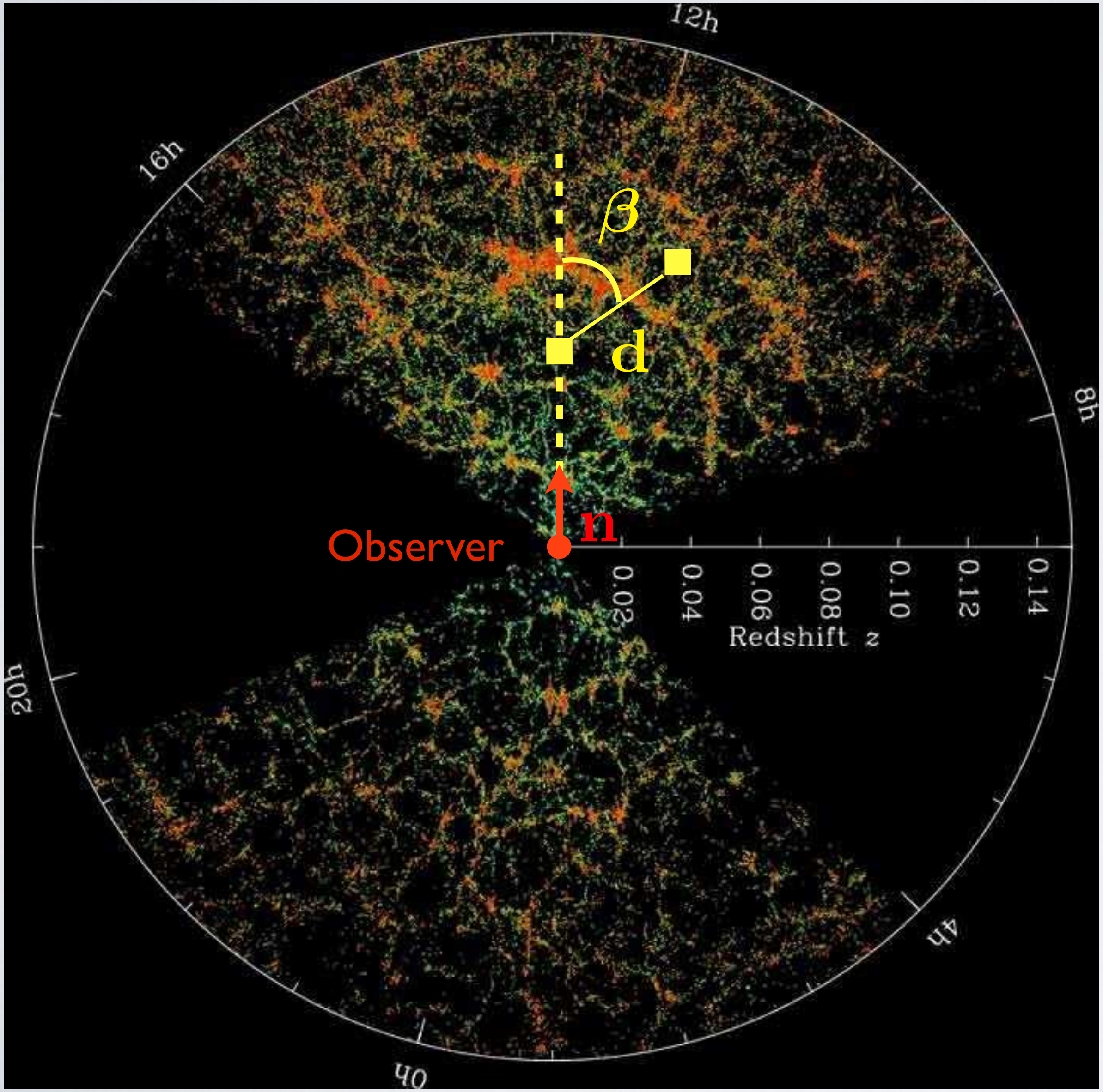

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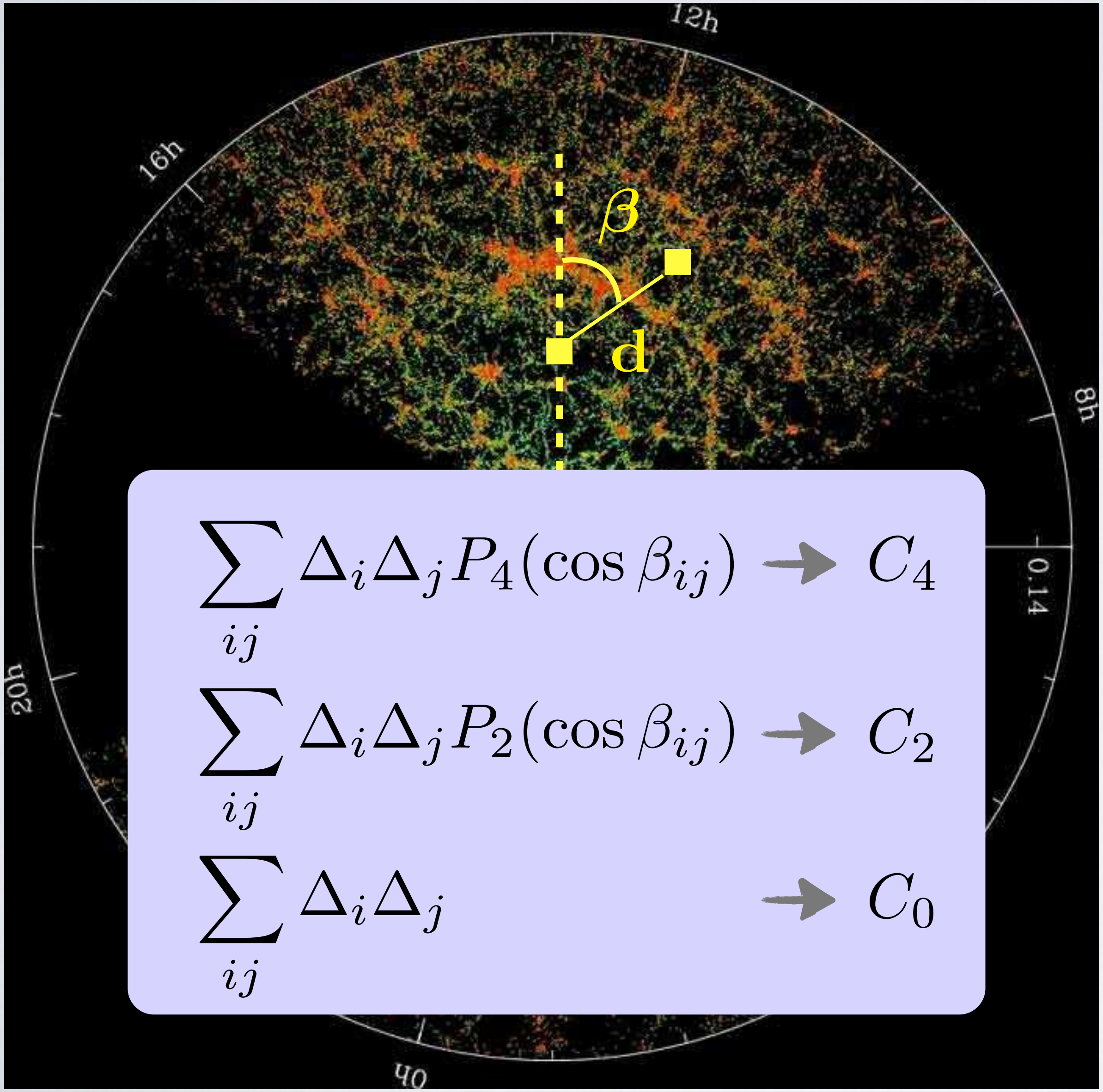

$$\sum_{ij}\Delta_i\Delta_j P_4(\cos\beta_{ij}) \rightarrow C_4$$

$$\sum_{ij}\Delta_i\Delta_j P_2(\cos\beta_{ij}) \rightarrow C_2$$

$$\sum_{ij}\Delta_i\Delta_j \rightarrow C_0$$

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Correlation function

McDonald (2009); Yoo et al (2012) Croft (2013); CB, Hui & Gaztanaga (2014); Raccanelli et al (2014)

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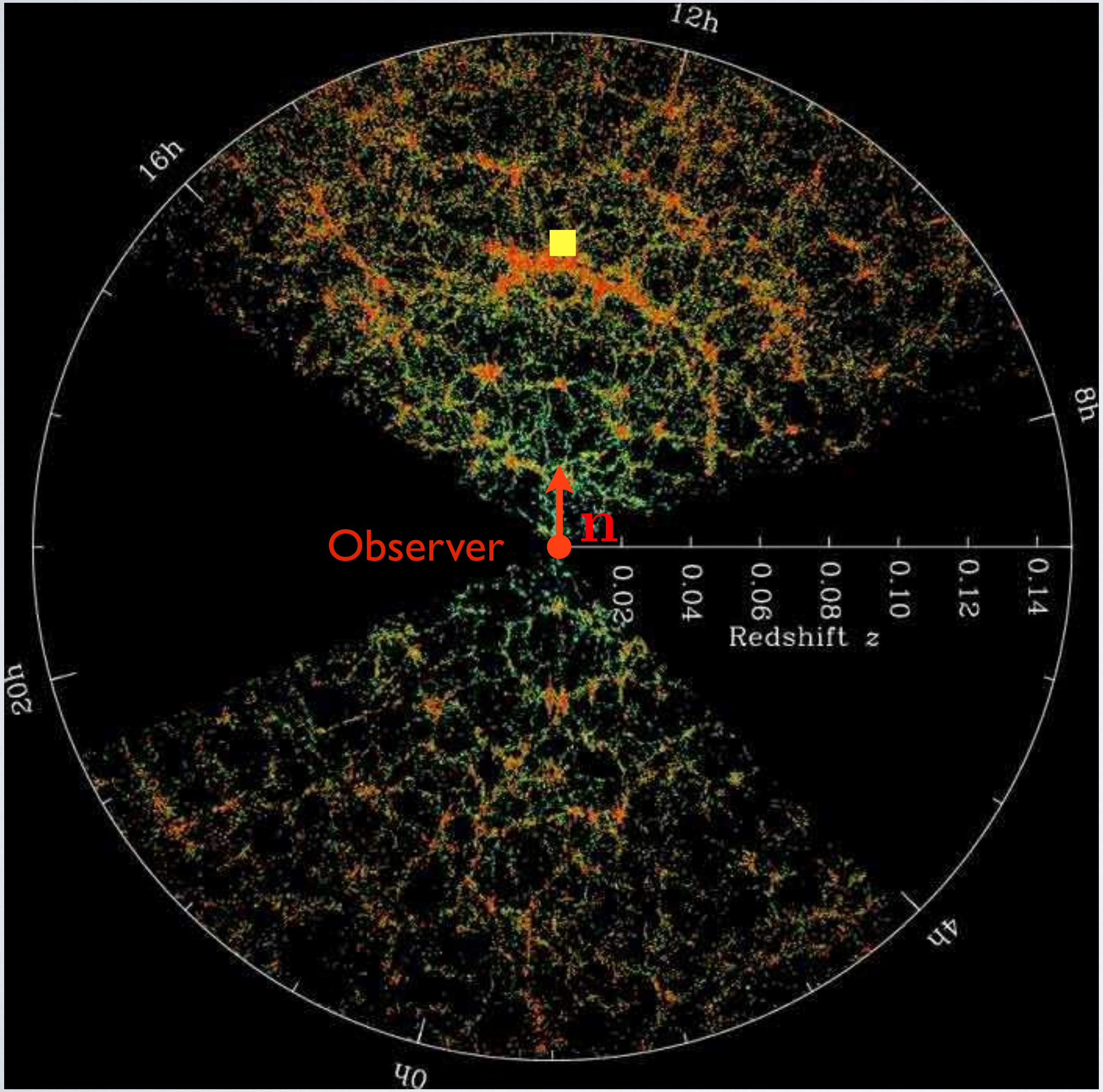

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Gravitational redshift breaks the **symmetry back-front**

$$\Delta = \frac{1}{\mathcal{H}} \partial_r \Psi$$

We need **two** galaxies' **populations** to measure this asymmetry

Correlation function

McDonald (2009); Yoo et al (2012) Croft (2013); CB, Hui & Gaztanaga (2014); Raccanelli et al (2014)

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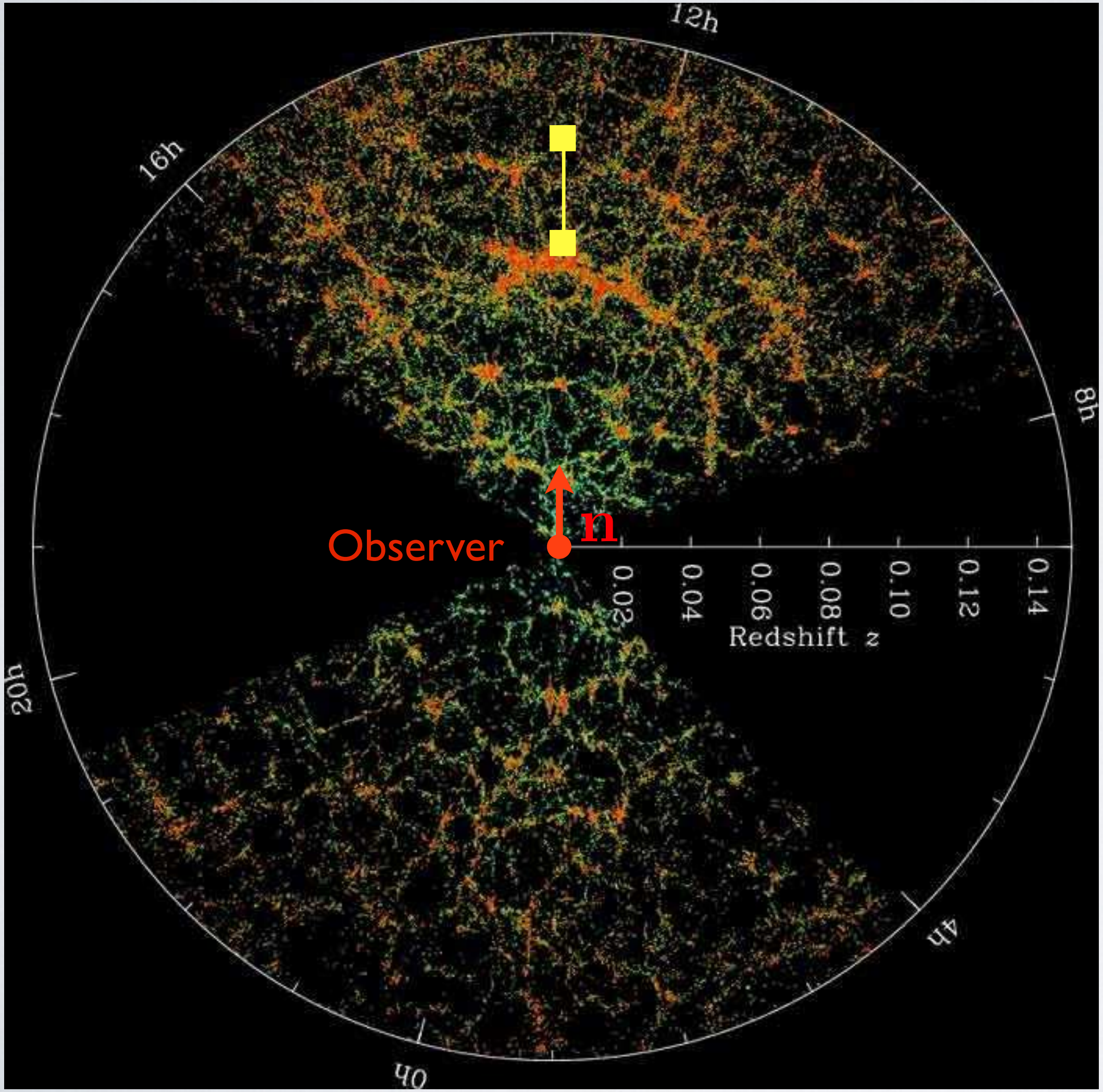

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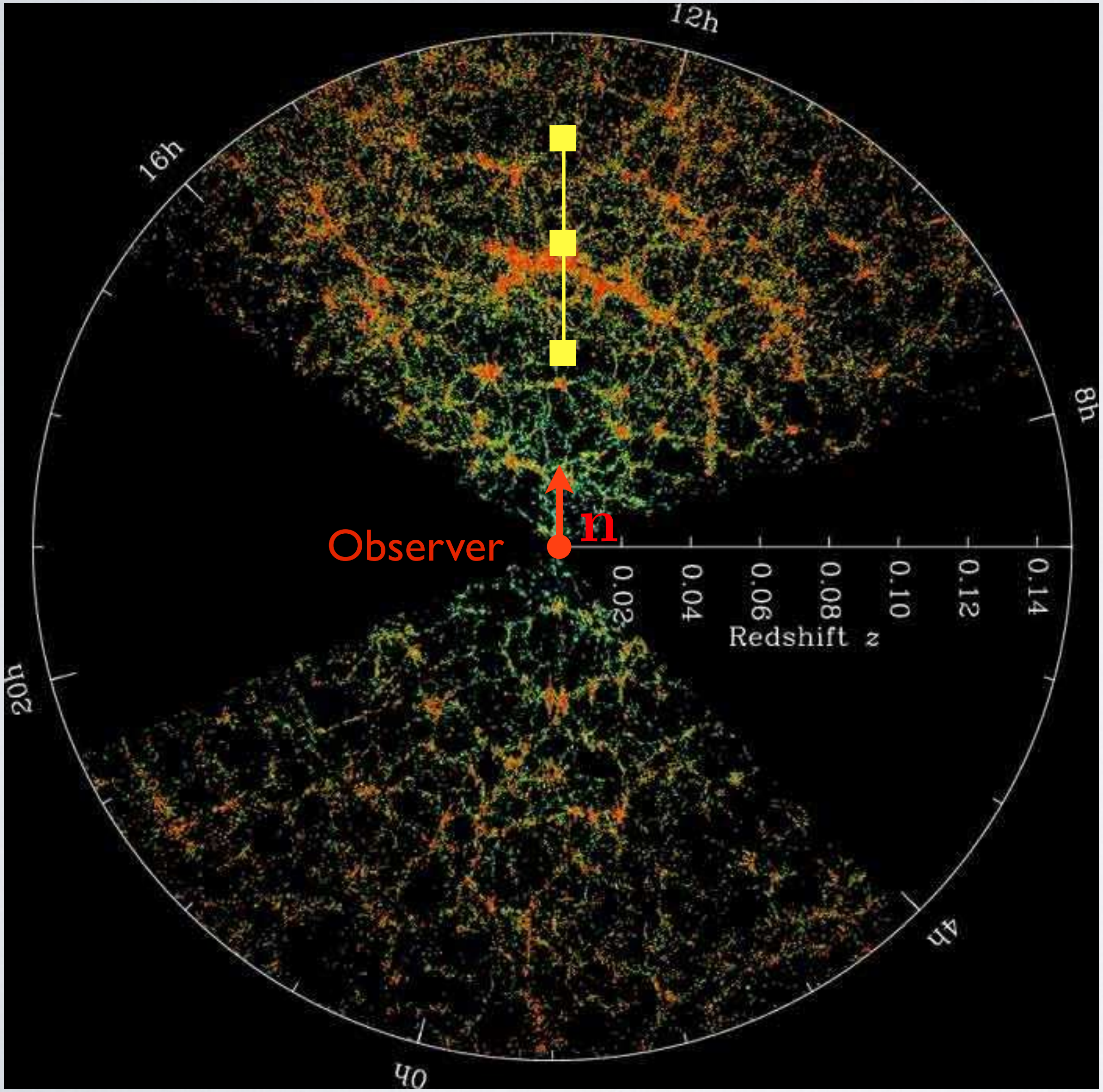

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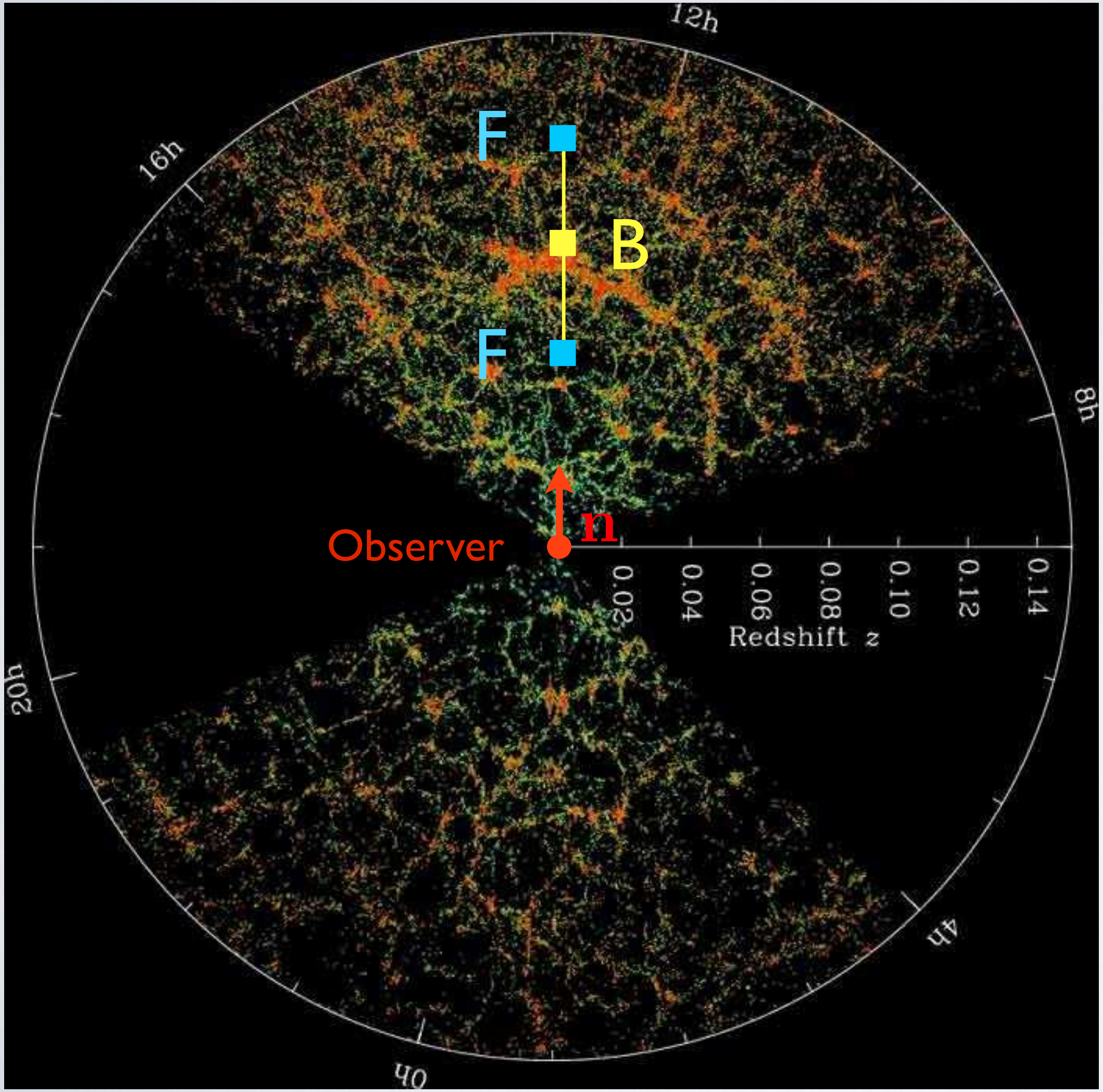

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CB, Hui & Gaztanaga (2014)

Breaking of symmetry from gravitational redshift

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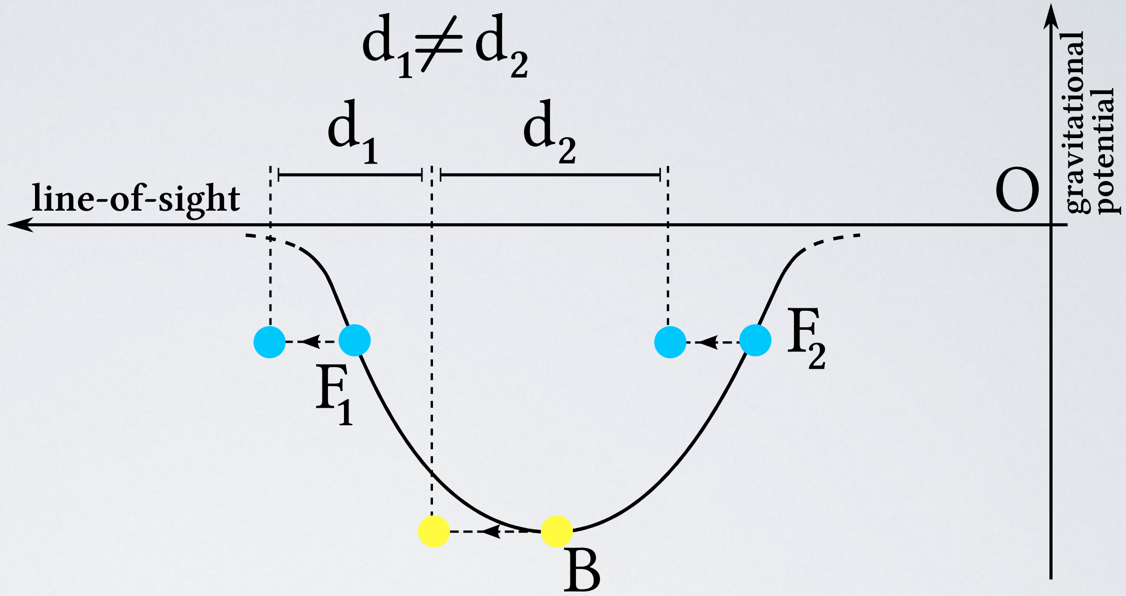

Which kind of estimator do we need to **isolate** those terms?

$$\xi = (b_{\rm B} - b_{\rm F}) C_1(d) \cos\beta \quad \rightarrow \quad \sum_{ij} \Delta_i \Delta_j \cos\beta_{ij}$$

The number count at linear order

Yoo et al (2010)
CB and Durrer (2011)
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$$\Delta(z,\mathbf{n}) = b\cdot\delta - \frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n})$$

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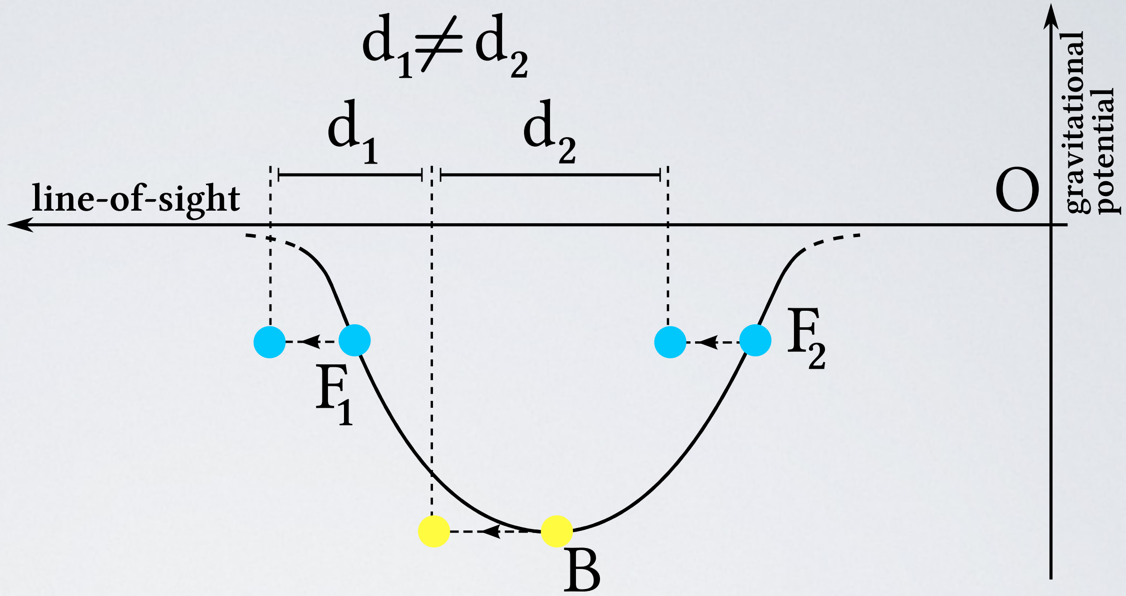

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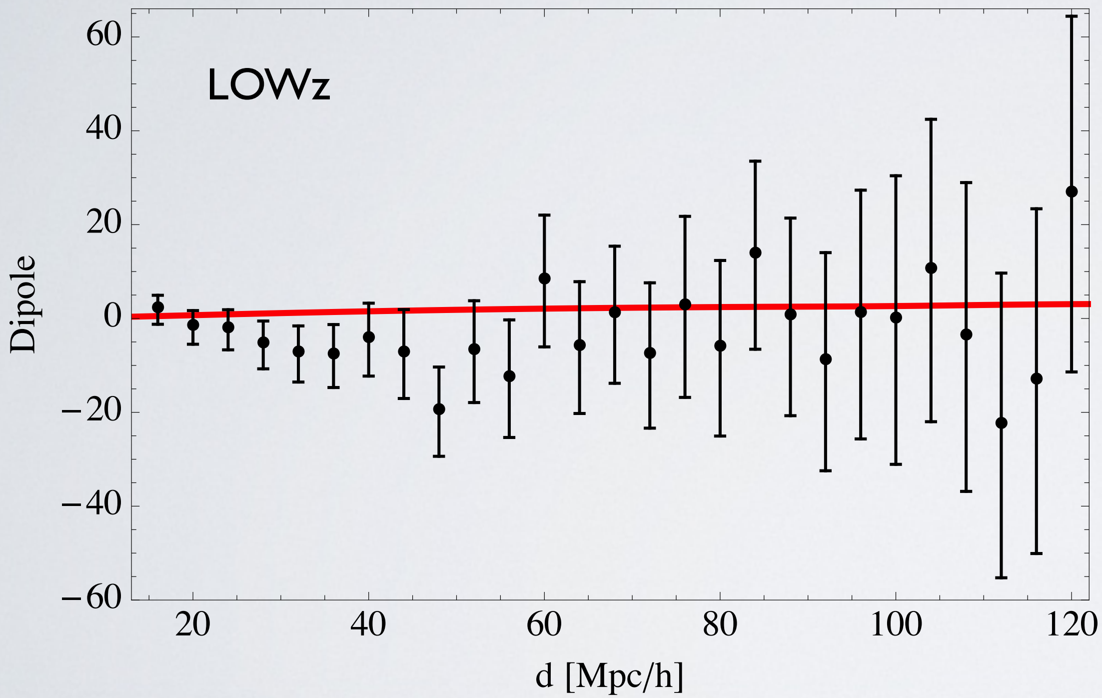
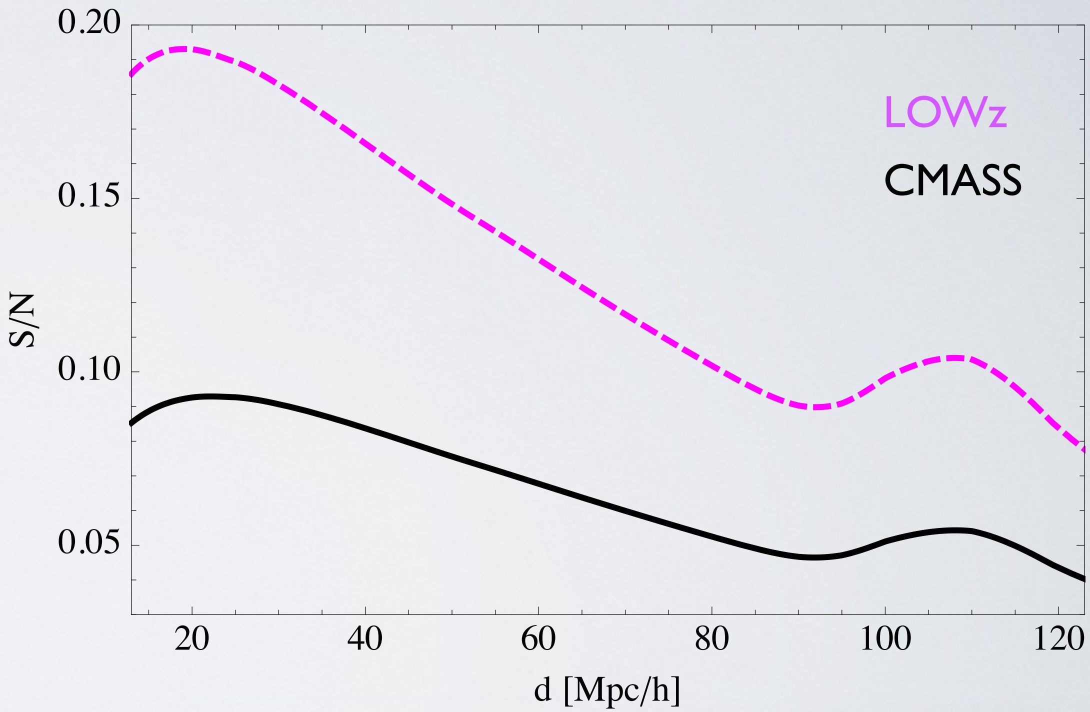
Measuring the dipole in BOSS

We split the LOWz and CMASS samples into 2 populations and measure the dipole.





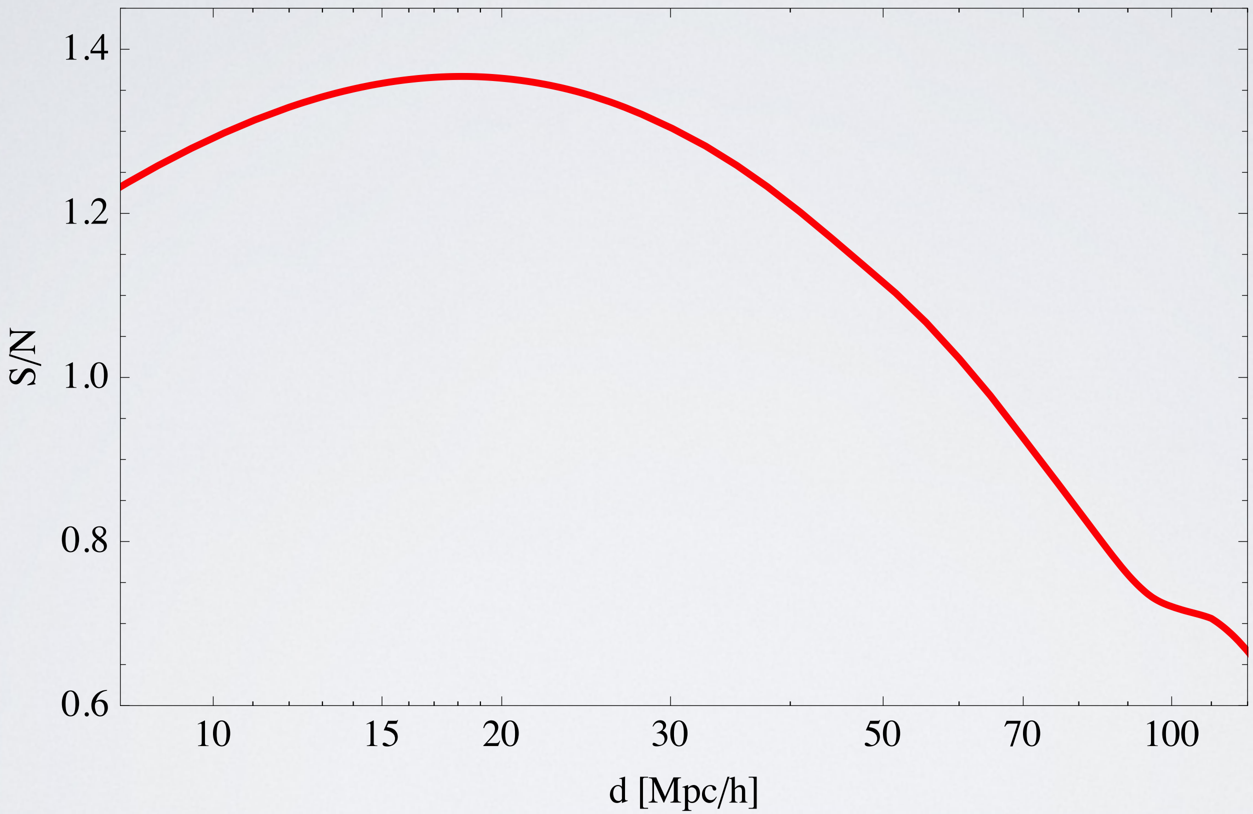
Improvements

- Measure the dipole at **lower** redshift to increase the signal.
- Use a sample with **diverse** populations to increase the bias difference.
- Divide the sample into **more** than 2 **populations** to gain in statistics.
- Use an **optimal** estimator: weight each pair by the bias difference.

Forecasts

CB, Hui & Gaztanaga, (2015)

Main sample of SDSS: 465'000 galaxies in total,
6 populations with bias from 0.96 to 2.16 Percival et al (2007)

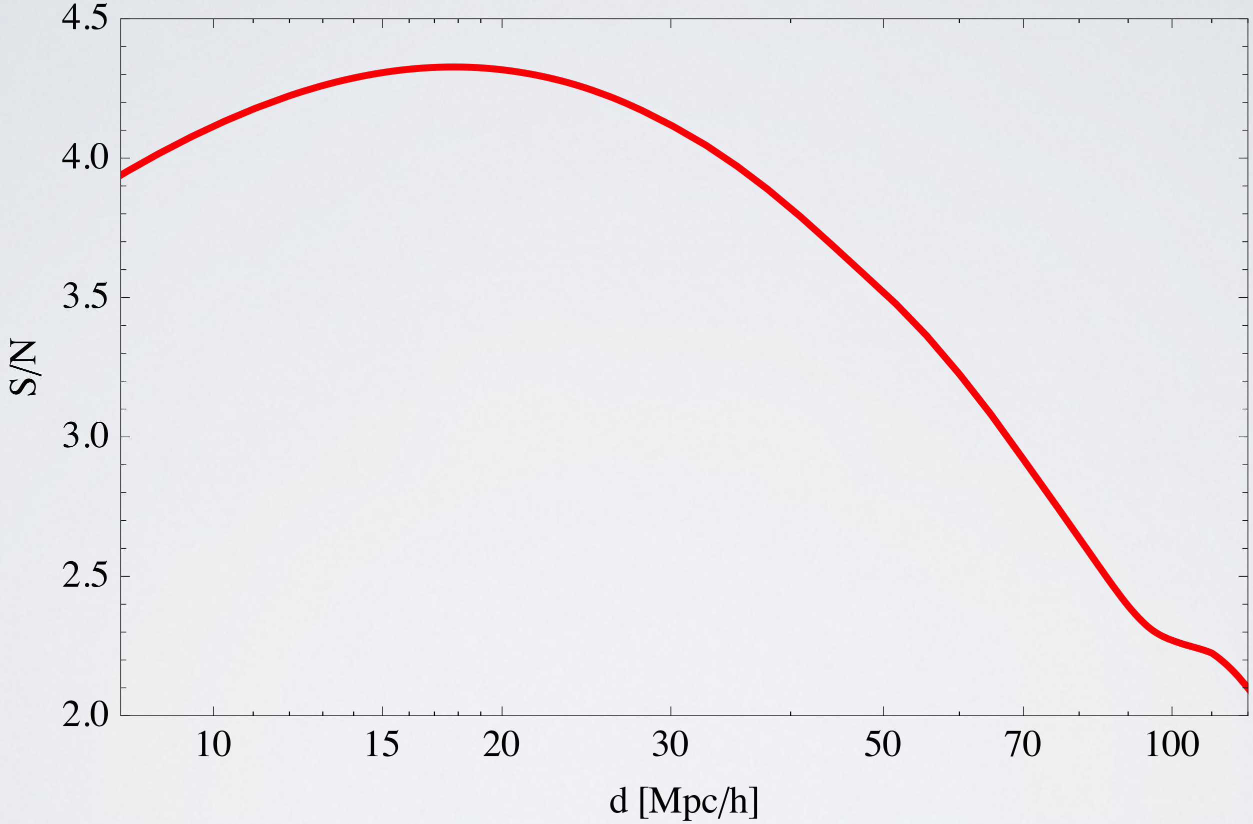


Cumulative signal-to-noise of 2.4

Forecasts

CB, Hui & Gaztanaga, (2015)

DESI Bright Sample: 10 million galaxies,
6 populations with bias from 0.96 to 2.16



Cumulative signal-to-noise of 7.4

Conclusion

- The **fluctuations** in the number of **galaxies** is affected by many effects besides the matter density fluctuations.

- These effects have a different **signature** in the **correlation** function:
    - density → monopole
    - redshift distortions → quadrupole and hexadecapole
    - relativistic effects → dipole

- By measuring the multipoles separately we can **test** the **relations** between the density, velocity and gravitational potential.

- The dipole should be detectable in the near future (DESI).

Interest

The dipole is sensitive to the **gravitational potential**.

$$\Delta_{\mathrm{rel}} = \frac{1}{\mathcal{H}}\partial_r\Psi + \left(1 - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - \frac{2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n}$$

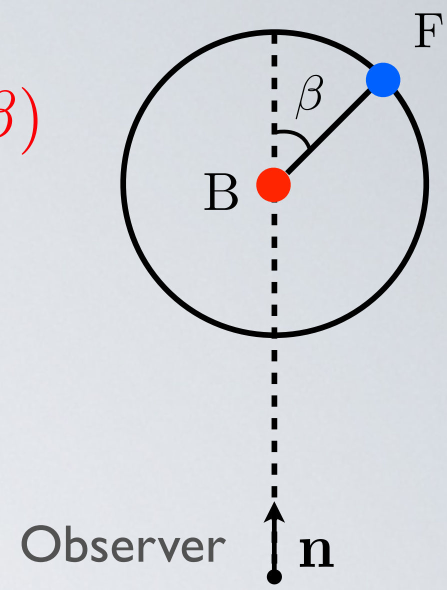
In general relativity, Euler equation: $\dot{\mathbf{V}}\cdot\mathbf{n} + \mathcal{H}\mathbf{V}\cdot\mathbf{n} + \partial_r\Psi = 0$

$$\Delta_{\mathrm{rel}} = -\left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n}$$

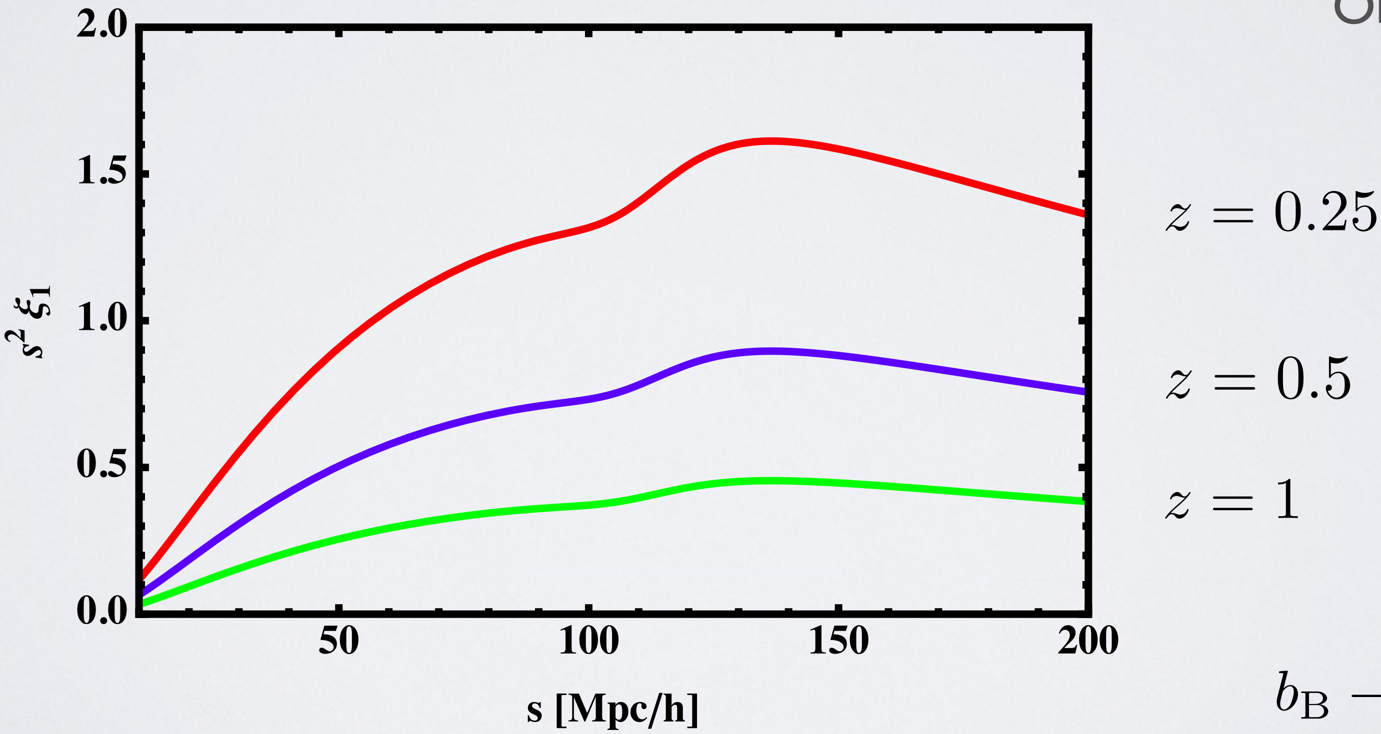
Combining the dipole with the quadrupole, we can test **Euler equation**.

Dipole in the correlation function

$$\xi(d,\beta) = D_1^2\, f\, \frac{\mathcal{H}}{\mathcal{H}_0}\left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2}{r\mathcal{H}}\right)(b_{\rm B} - b_{\rm F})\nu_1(d)\cdot \cos(\beta)$$



$$\nu_1(d) = \frac{A}{2\pi^2}\int \frac{dk}{k}\left(\frac{k}{H_0}\right)^{n_s-1} T_\delta(k) T_\Psi(k)\, j_1(k\cdot d)$$



$b_{\rm B} - b_{\rm F} \simeq 0.5$

CB, Hui and Gaztanaga (2013)

Redshift

$$ds^2 = -a^2\left[\left(1+2\Psi\right)d\eta^2 + \left(1-2\Phi\right)\delta_{ij}dx^i dx^j\right]$$

Effect of inhomogeneities on the redshift: $1+z = \frac{\nu_S}{\nu_O} = \frac{E_S}{E_O}$

Photons travel on **null geodesics**.

$$1+z = \frac{a_O}{a_S}\left[1 + \mathbf{V}_S\cdot\mathbf{n} - \mathbf{V}_O\cdot\mathbf{n} + \Psi_O - \Psi_S - \int_0^{r_S} dr(\dot{\Phi}+\dot{\Psi})\right]$$

Doppler — Gravitational redshift — Integrated Sachs-Wolfe

Gravitational redshift:

Multipoles

- Monopole $$C_0 = D_1^2\, b^2 \mu_0(d)$$

- Quadrupole $$C_2 = -D_1^2 \left(\frac{4fb}{3} + \frac{4f^2}{7}\right) \mu_2(d) P_2(\cos\beta)$$

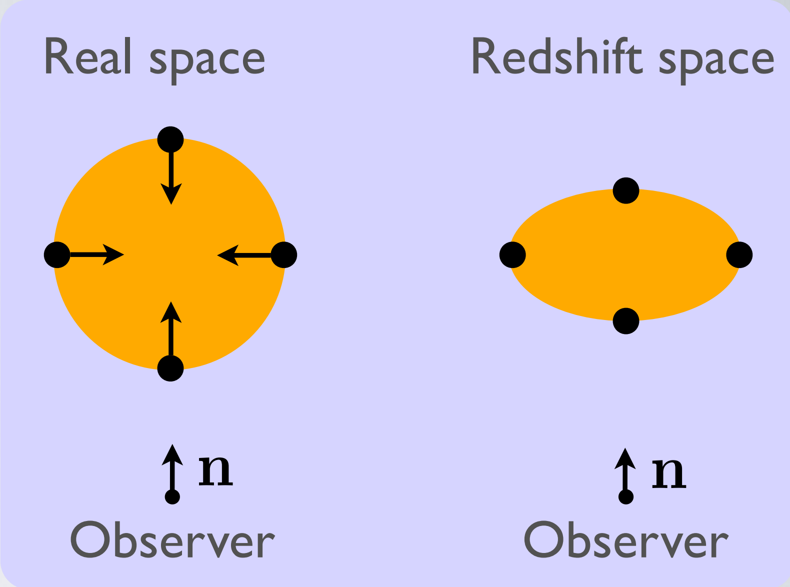
- Hexadecapole $$C_4 = D_1^2\, \frac{8f^2}{35} \mu_4(d) P_4(\cos\beta)$$

$$\mu_\ell(d) = \frac{A}{2\pi^2} \int \frac{dk}{k} \left(\frac{k}{H_0}\right)^{n_s - 1} T_\delta^2(k)\, j_\ell(k \cdot d)$$

Redshift distortions

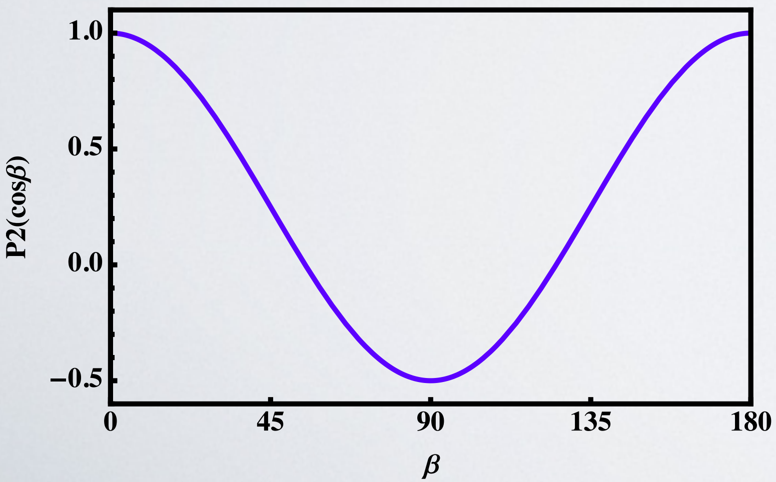
Redshift distortions **break** the **isotropy** of the correlation function.

$$\Delta = b\cdot\delta - \frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n})$$



Quadrupole

$$P_2(\cos\beta) = \frac{3}{2}\cos^2\beta - \frac{1}{2}$$



Hexadecapole

$$P_4(\cos\beta) = \frac{1}{8}\left[35\cos^4\beta - 30\cos^2\beta + 3\right]$$