TSPT: Time-Sliced Perturbation Theory for Large Scale Sctructure

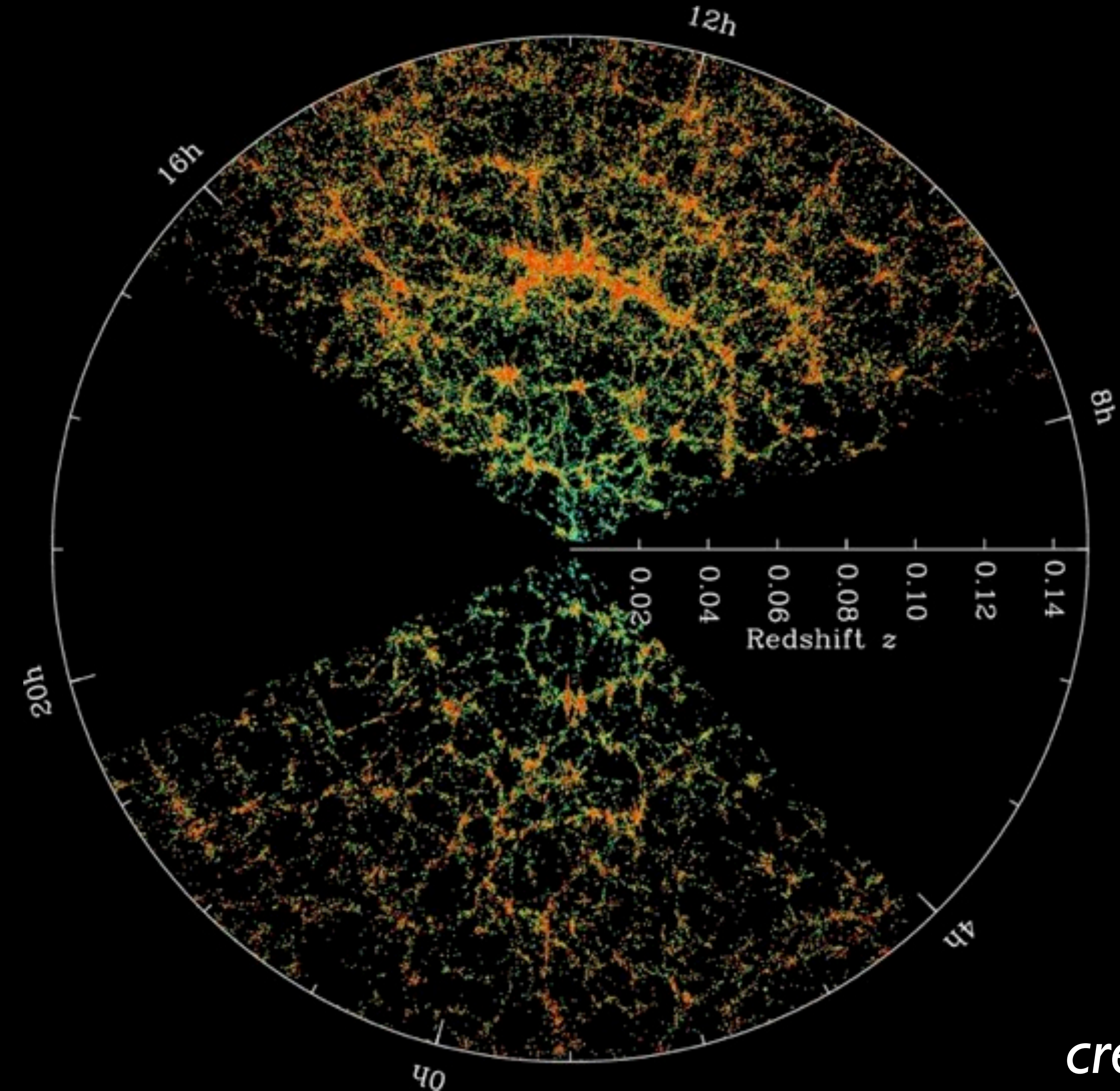
Sergey Sibiryakov

with D. Blas, M. Garny and M.M. Ivanov

arXiv: 1512.xxxxx

28th Texas Symposium, Geneva, 2015

The Universe has a beautiful structure



credit: SDSS

Physics with LSS

- evolution of perturbations

  ➡ properties of dark matter (e.g. fifth force, WDM) and dark energy (e.g. clustering)

- baryon acoustic oscillations = standard ruler in the Universe

  ➡ dark energy equation of state

- primordial non-gaussianity

  ➡ interactions in the inflationary sector

Challenges of non-linear dynamics

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

$$\nabla_\mu T^{\mu\nu} = 0$$

Newtonian approximation ($l \ll 10^4$ Mpc)

+ fluid description ($l \gg 10$ Mpc)

$$\frac{\partial \delta_\rho}{\partial \tau} + \nabla \left[(1 + \delta_\rho) \mathbf{u} \right] = 0$$

$$\frac{\partial \mathbf{u}}{\partial \tau} + \mathcal{H}(\tau) \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla \phi$$

$$\nabla^2 \phi = \frac{3}{2} \Omega_m(\tau) \mathcal{H}^2(\tau) \delta_\rho$$

Non-perturbative method: N-body simulations

- advantage: "exact", goes beyond fluid description
- drawback: too costly -- cannot be used to test many theories beyond the Standard Cosmological Model

Recall that fluid description appears valid up to $k \sim 0.5 \cdot h \cdot \mathrm{Mpc}^{-1}$

➡ use perturbation theory to solve the Euler - Poisson system

Standard perturbation theory (SPT)

$$\frac{\partial \delta_\rho(k)}{\partial \tau} + \theta(k) = -\int d^3q \; \alpha(q, k-q) \; \theta(q)\delta_\rho(k-q)$$

$$\frac{\partial \theta(k)}{\partial \tau} + \mathcal{H}\theta(k) + \frac{3}{2}\Omega_m \mathcal{H}^2 \delta_\rho(k) = -\int d^3q \; \beta(q, k-q) \; \theta(q)\theta(k-q)$$

Standard perturbation theory (SPT)

$$\frac{\partial \delta_\rho(k)}{\partial \tau} + \theta(k) = -\int d^3q\, \alpha(q, k-q)\, \theta(q)\delta_\rho(k-q)$$

$$\frac{\partial \theta(k)}{\partial \tau} + \mathcal{H}\theta(k) + \frac{3}{2}\Omega_m \mathcal{H}^2 \delta_\rho(k) = -\int d^3q\, \beta(q, k-q)\, \theta(q)\theta(k-q)$$

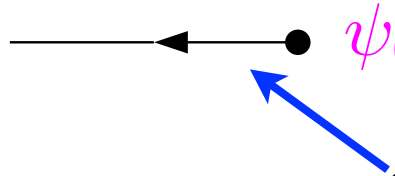
Solve for time evolution iteratively: $\psi = \psi^{(1)} + \psi^{(2)} + \psi^{(3)} + \ldots$

Standard perturbation theory (SPT)

$$\frac{\partial \delta_\rho(k)}{\partial \tau} + \theta(k) = -\int d^3q\, \alpha(q, k-q)\, \theta(q)\delta_\rho(k-q)$$

$$\frac{\partial \theta(k)}{\partial \tau} + \mathcal{H}\theta(k) + \frac{3}{2}\Omega_m \mathcal{H}^2 \delta_\rho(k) = -\int d^3q\, \beta(q, k-q)\, \theta(q)\theta(k-q)$$

Solve for time evolution iteratively: $\psi = \psi^{(1)} + \psi^{(2)} + \psi^{(3)} + \ldots$

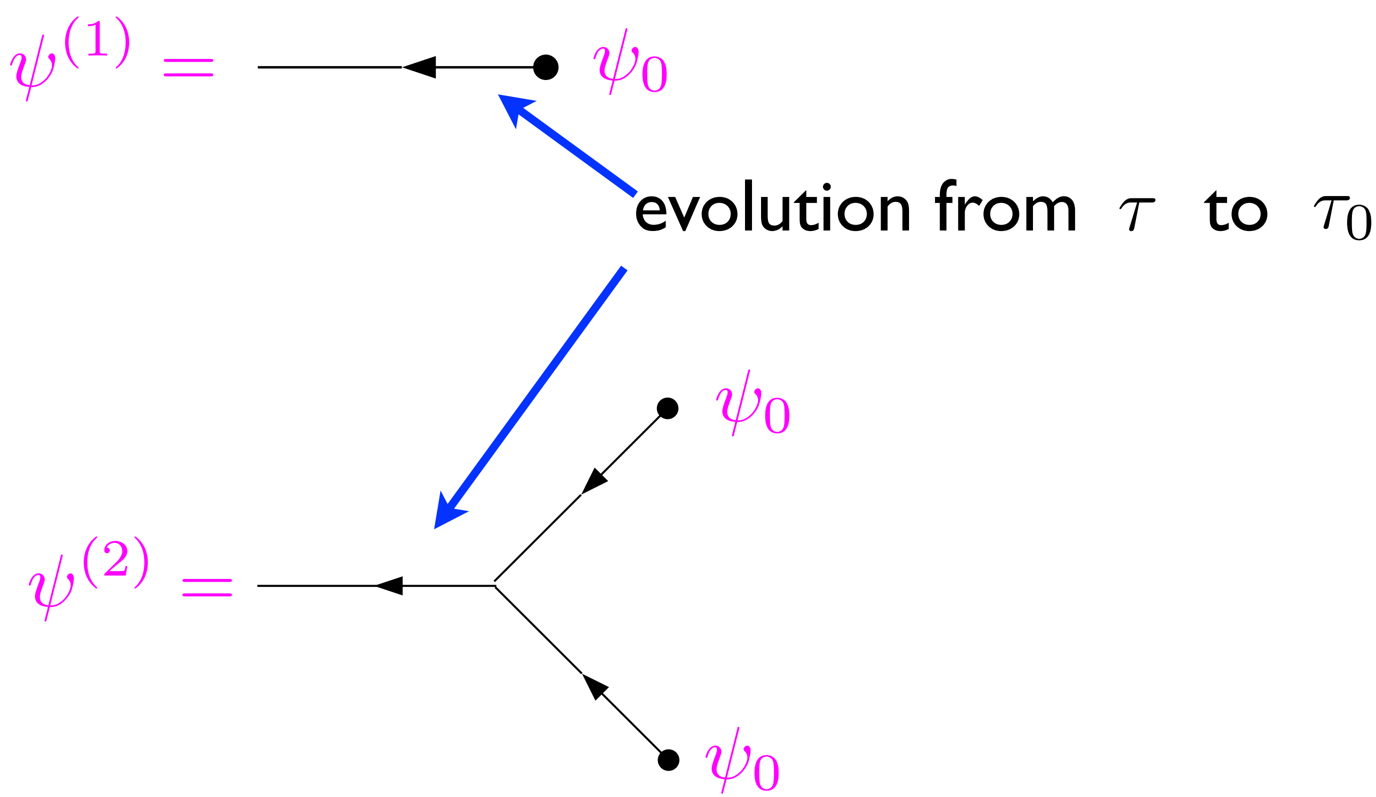
$\psi^{(1)} =$  ψ_0

Standard perturbation theory (SPT)

$$\frac{\partial \delta_\rho(k)}{\partial \tau} + \theta(k) = -\int d^3q\, \alpha(q, k-q)\, \theta(q)\delta_\rho(k-q)$$

$$\frac{\partial \theta(k)}{\partial \tau} + \mathcal{H}\theta(k) + \frac{3}{2}\Omega_m \mathcal{H}^2 \delta_\rho(k) = -\int d^3q\, \beta(q, k-q)\, \theta(q)\theta(k-q)$$

Solve for time evolution iteratively: $\psi = \psi^{(1)} + \psi^{(2)} + \psi^{(3)} + \dots$

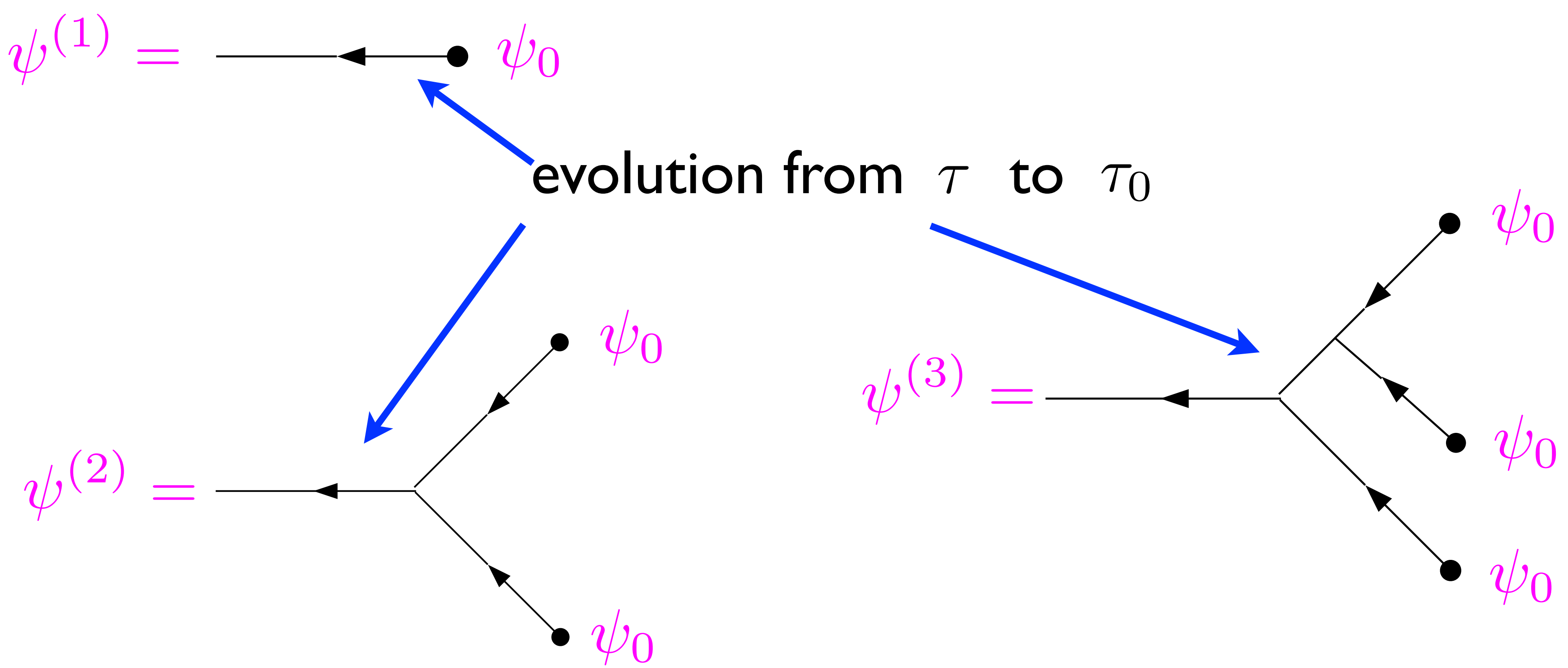


Standard perturbation theory (SPT)

$$\frac{\partial \delta_\rho(k)}{\partial \tau} + \theta(k) = -\int d^3q \; \alpha(q, k-q) \; \theta(q)\delta_\rho(k-q)$$

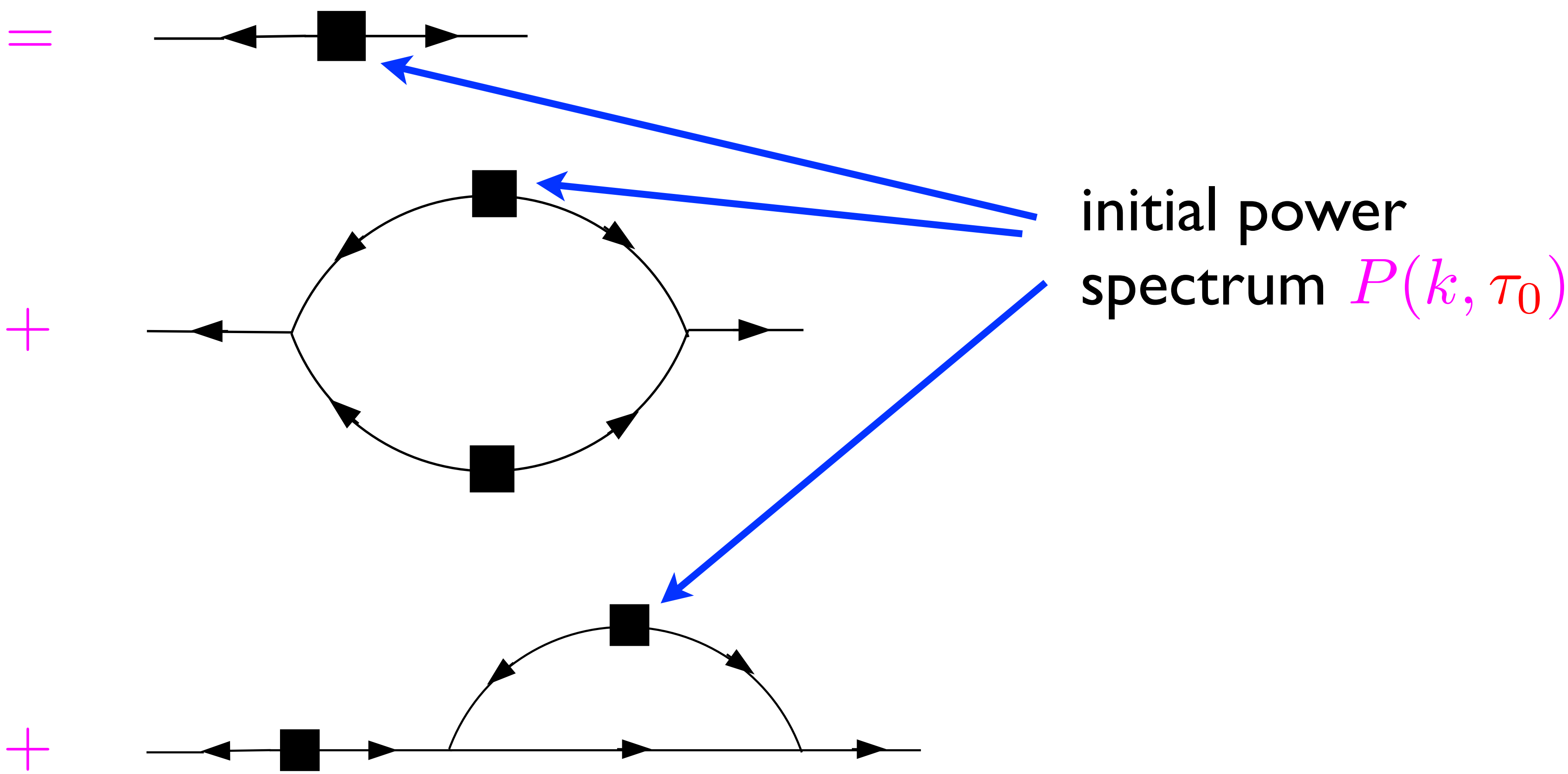
$$\frac{\partial \theta(k)}{\partial \tau} + \mathcal{H}\theta(k) + \frac{3}{2}\Omega_m \mathcal{H}^2 \delta_\rho(k) = -\int d^3q \; \beta(q, k-q) \; \theta(q)\theta(k-q)$$

Solve for time evolution iteratively: $\psi = \psi^{(1)} + \psi^{(2)} + \psi^{(3)} + \ldots$



Average over the ensemble of initial conditions:

$$\langle \psi(k_1, \tau) \psi(k_2, \tau) \rangle = \langle \psi^{(1)} \psi^{(1)} \rangle + \langle \psi^{(2)} \psi^{(2)} \rangle + 2 \langle \psi^{(1)} \psi^{(3)} \rangle + \ldots =$$



Problems of SPT

"Ultraviolet" Loop integrals run over all momenta including short modes where the fluid description is not applicable.
We must introduce a UV cutoff

➡ EFT of LSS

Carrasco, Hertzberg, Senatore (2012)

Add counterterms into the equations of motion to account for deviations from fluid description

Problems of SPT

"Ultraviolet" Loop integrals run over all momenta including short modes where the fluid description is not applicable.
We must introduce a UV cutoff

→ EFT of LSS

Carrasco, Hertzberg, Senatore (2012)

Add counterterms into the equations of motion to account for deviations from fluid description

Complication: counterterms must have non-local time-dependence to consistently renormalize the loop integrals

Abolhasani, Mirbabayi, Pajer (2015)

Problems of SPT

"Infrared" Kernels α, β in the e.o.m.'s behave as $1/q$

➡ individual loop diagrams diverge at small momenta

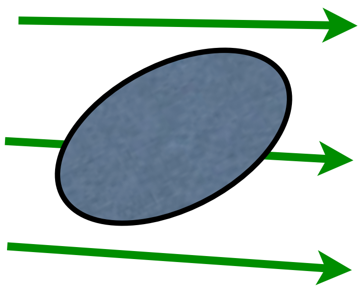
When summed, the divergences cancel in equal-time correlators

Problems of SPT

"Infrared" Kernels α, β in the e.o.m.'s behave as $1/q$

individual loop diagrams diverge at small momenta

When summed, the divergences cancel in equal-time correlators



overdensity is moved by an almost homogeneous flow

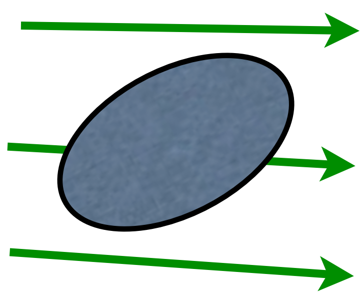
accumulation of the effect with time

Problems of SPT

"Infrared" Kernels α, β in the e.o.m.'s behave as $1/q$

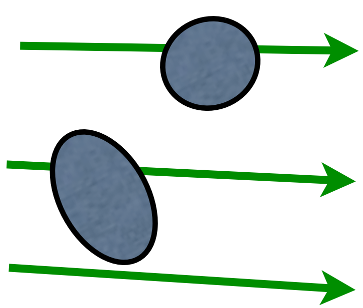
individual loop diagrams diverge at small momenta

When summed, the divergences cancel in equal-time correlators



overdensity is moved by an almost homogeneous flow

accumulation of the effect with time



two overdensities will move identically

cancellation in equal-time correlators; subleading effect: flow gradients

TSPT: time-sliced perturbation theory

Main idea: Evolve the whole statistical ensemble in time

TSPT: time-sliced perturbation theory

Main idea: Evolve the whole statistical ensemble in time

Example: Consider a single variable with random initial conditions

$$\dot{\psi} = \Omega\psi + \sum_{m=2} \frac{A_m}{m!}\psi^m \quad \Rightarrow \quad \psi(\tau;\psi_0)$$

TSPT: time-sliced perturbation theory

Main idea: Evolve the whole statistical ensemble in time

Example: Consider a single variable with random initial conditions

$$\dot{\psi} = \Omega\psi + \sum_{m=2} \frac{A_m}{m!}\psi^m \quad \Rightarrow \quad \psi(\tau;\psi_0)$$

SPT: $\int d\psi_0 \, e^{-\Gamma_0[\psi_0]}\psi(\tau;\psi_0)^2$ $\qquad \Gamma_0[\psi_0] = \frac{\psi_0^2}{2P}$

TSPT: time-sliced perturbation theory

Main idea: Evolve the whole statistical ensemble in time

Example: Consider a single variable with random initial conditions

$$\dot{\psi} = \Omega\psi + \sum_{m=2} \frac{A_m}{m!}\psi^m \quad \Longrightarrow \quad \psi(\tau;\psi_0)$$

SPT: $$\int d\psi_0\, e^{-\Gamma_0[\psi_0]}\psi(\tau;\psi_0)^2 \qquad \Gamma_0[\psi_0] = \frac{\psi_0^2}{2P}$$

TSPT: $$\int d\psi\, e^{-\Gamma[\psi;\tau]}\psi^2 \qquad \Gamma[\psi;\tau] = \sum_n \frac{\Gamma_n(\tau)}{n!}\,\psi^n$$

Two integrals must coincide

→ equation for the "vertices"

$$\frac{d}{d\tau}\left(d\psi e^{-\Gamma[\psi;\tau]}\right) = 0$$

→ $$\dot{\Gamma}_n = -n\Omega\Gamma_n - \underbrace{\sum_{m=2}^{n} C_n^m A_m \Gamma_{n-m+1}}_{} + A_{n+1}$$

contains only $\Gamma_{n'}$ with $n' < n$

Two integrals must coincide

➡ equation for the "vertices"

$$\frac{d}{d\tau}\left(d\psi e^{-\Gamma[\psi;\tau]}\right) = 0$$

➡ $$\dot{\Gamma}_n = -n\Omega\Gamma_n - \underbrace{\sum_{m=2}^{n} C_n^m A_m \Gamma_{n-m+1}}_{} + A_{n+1}$$

contains only $\Gamma_{n'}$ with $n' < n$

The same logic for fields in space with the substitution:
integral $\Longrightarrow$ path integral

Generating functional for cosmological correlators

$$Z[J, J_\delta; \tau] = \int [\mathcal{D}\theta] \exp\left\{ -\Gamma[\theta;\tau] + \int \theta J + \int \delta_\rho[\theta;\tau] J_\delta \right\}$$

$$\Gamma = \frac{1}{2}\int \frac{\theta^2}{\hat{P}(k)} + \sum_{n=3}^{\infty} \frac{1}{n!} \int \Gamma_n(\tau)\theta^n$$

$$\delta_\rho = \sum_{n=1}^{\infty} \frac{1}{n!} \int K_n(\tau)\theta^n$$

TSPT - 3d Euclidean QFT vocabulary:

- Γ --- 1PI effective action
- δ_ρ --- composite source
- τ --- external parameter

Advantages

- All Γ_n are explicitly IR finite, and so are the individual loop diagrams

Advantages

- All Γ_n are explicitly IR finite, and so are the individual loop diagrams

- For gaussian initial conditions the time dependence factorize

$$\Gamma_n = \frac{1}{g^2(\tau)} \hat{\Gamma}_n$$

effective coupling constant

Advantages

- All Γ_n are explicitly IR finite, and so are the individual loop diagrams

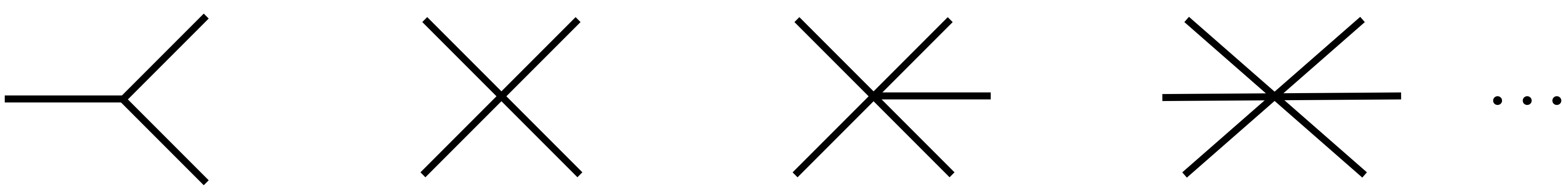
- For gaussian initial conditions the time dependence factorize

$$\Gamma_n = \frac{1}{g^2(\tau)} \hat{\Gamma}_n$$

effective coupling constant

- Simplified diagrammatic technique

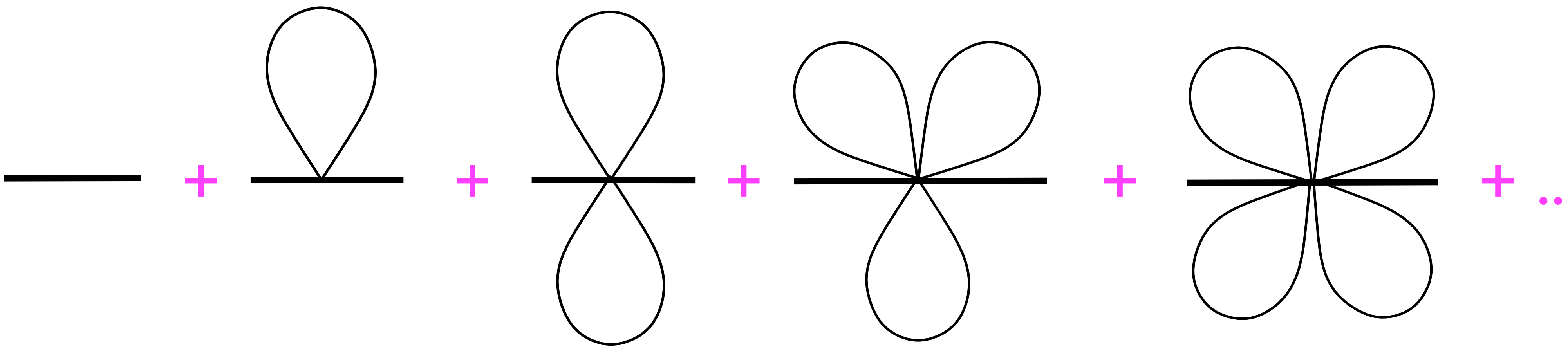
 $= \hat{P}(k)$



IR resummation

- Large IR contributions can be resummed. Important for the correct calculation of BAO

IR contributions are dominated by daisy diagrams

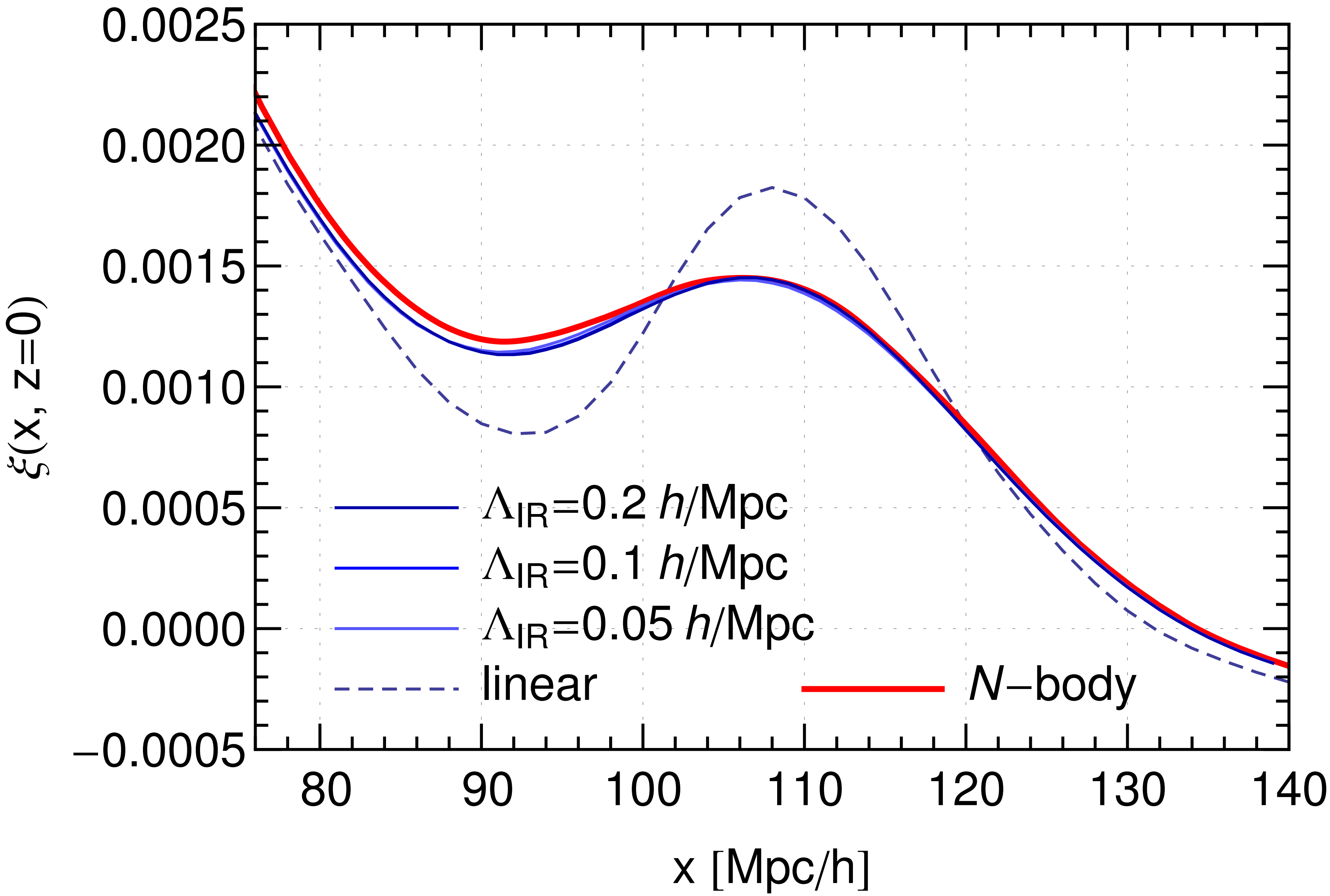


$$= P_{BAO}(k)\exp\left[-k^2\sigma(\Lambda_{IR},\tau)\right]$$

BAO wavelength

$$\sigma = -\frac{4\pi}{3}\int_0^{\Lambda_{IR}} dq\, P(q)\big(1 - j_0(qr_s) + 2j_2(qr_s)\big)$$

IR resummed BAO 1-loop, z=0



See talk of Mikhail Ivanov on Thursday

EFT of LSS reformulated

Introduce a hard cutoff:

$$\hat{P}(k) \mapsto \hat{P}^{\Lambda}(k) = \begin{cases} \hat{P}(k), & k < \Lambda \\ 0, & k > \Lambda \end{cases}$$

$$\Gamma_n \mapsto \Gamma_n^{\Lambda}$$

Wilsonian RG:

$$\frac{d\Gamma_n^{\Lambda}}{d\Lambda} = \mathcal{F}_n[\hat{P}^{\Lambda}, \Gamma^{\Lambda}]$$

Boundary conditions = counterterms of EFT

NB. Counterterms depend on time time (parameter) τ only locally

Summary and Outlook

- time-sliced perturbation theory (TSPT) is a new approach to LSS in the mildly non-linear regime
  20 Mpc < l < 100 Mpc
- TSPT is free from IR divergences, diagrammatic resummation of IR-enhanced contributions
- TSPT provides a suitable framework for UV renormalization à la Wilsonian RG

Summary and Outlook

- time-sliced perturbation theory (TSPT) is a new approach to LSS in the mildly non-linear regime
  20 Mpc < l < 100 Mpc
- TSPT is free from IR divergences, diagrammatic resummation of IR-enhanced contributions
- TSPT provides a suitable framework for UV renormalization à la Wilsonian RG
- inclusion of baryons
- applications: primordial non-gaussianity
- applications: non-standard dark matter (WDM, fifth force, etc.)