One-dimensional models of cosmological perturbations: direct integration in the Fourier space

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Introduction

Power spectrum of cosmological inhomogeneity is one of basic characteristics of large-scale structure of the Universe. Extensive efforts are made to achieve a sufficient accuracy of numerical methods to obtain this spectrum on 0.01÷0.1 Mpc scale. The well known approaches use the N-body simulations and/or modifications of a perturbation theory in the Fourier space. Here we propose an approach based on a direct integration of the Fourier transformed hydrodynamic equations for an ensemble of random initial data with subsequent calculation of ensemble averages. To test the method we consider some one-dimensional version of cosmological hydrodynamics. We have obtained solutions of one-dimensional equations in linear and significantly non-linear regime up to scales ~0.3 Mpc. We point out a significant growth of dispersion of the power spectrum in the non-linear region. The results show that it is possible to extend the method to scales ~0.1 Mpc in the 3-dimensional case within realistic computational time.

Hydrodynamical equations in the Fourier space

$$\frac{\partial \rho}{\partial t} + div_r(\rho \boldsymbol{U}) = 0$$
$$\frac{\partial \boldsymbol{U}}{\partial t} + (\boldsymbol{U}\nabla_r)\boldsymbol{U} = -\frac{\nabla_r p}{\rho} - \nabla_r \Phi$$
$$\Delta_r \Phi = 4\pi G\rho$$

$$\boldsymbol{U} = \boldsymbol{v} + \boldsymbol{U_h}(\boldsymbol{x})$$
$$\boldsymbol{\rho} = \boldsymbol{\rho_h}(\boldsymbol{1} + \boldsymbol{\delta})$$
$$\boldsymbol{\Phi} = \boldsymbol{\Phi_h} + \boldsymbol{\varphi}$$
$$\boldsymbol{\rho_h} = \boldsymbol{\rho_{h0}}\left(\frac{a_0}{a}\right)^3;\ \boldsymbol{U_h} = \left(\frac{\dot{a}}{a}\right)\boldsymbol{r}$$
$$\boldsymbol{r} = a\boldsymbol{x};\ t \to \tau: d\tau = \frac{dt}{a(t)}$$
$$\theta = div(\boldsymbol{v})$$

$$\frac{\partial \theta}{\partial \tau} + \frac{2}{\tau}\theta + \Delta\left(\frac{\boldsymbol{v}^2}{2}\right) = -\frac{6}{\tau^2}\delta$$
$$\frac{\partial \delta}{\partial \tau} + \theta + div(\delta \boldsymbol{v}) = 0$$

Discrete Fourier Transform

$$\frac{\partial a_k}{\partial \tau} + \frac{2}{\tau}a_k + \frac{6}{\tau^2}b_k + \frac{k^2}{2}\sum_{p\neq 0;k-p\neq 0}\frac{\boldsymbol{p}\cdot(\boldsymbol{k}-\boldsymbol{p})}{\boldsymbol{p}^2|\boldsymbol{k}-\boldsymbol{p}|^2}a_p a_{k-p} = 0$$
$$\frac{\partial b_k}{\partial \tau} + a_k + \sum_{p\neq 0}\frac{\boldsymbol{p}\cdot\boldsymbol{k}}{\boldsymbol{p}^2}a_p b_{k-p} = 0$$
$$\boldsymbol{p} = \frac{2\pi}{L}(m_1, m_2, m_3) \neq 0, m_i = 0, \pm 1, \pm 2, \ldots$$

Scoccimarro & Frieman 1996; Pietroni 2008; Scoccimarro 1997

Three dimensional case with one-dimensional initial data

$$\boldsymbol{k} = \frac{2\pi}{L}(n,0,0), n = 0, \pm 1, \pm 2, \ldots$$

$$\frac{\partial a_n}{\partial \tau} + \frac{2}{\tau} a_n + \frac{6}{\tau^2} b_n + n \sum_{p \neq 0} \frac{a_p a_{n-p}}{p} = 0$$

$$\frac{\partial b_n}{\partial \tau} + a_n + \sum_{p \neq 0} \frac{a_p a_{n-p}}{p} = 0$$

$$p = \pm 1, \pm 2, \ldots$$

Parameter variations

$$a_n = -2\tau {\varphi^1}_n + 3\tau^{-4} {\varphi^2}_n$$
$$b_n = \tau^2 {\varphi^1}_n + \tau^{-3} {\varphi^2}_n$$

$$\frac{\partial {\varphi^a}_n}{\partial \tau} = n \sum_{b,c=1,2} A^{abc} \sum_{p \neq 0} p^{-1} {\varphi^b}_p {\varphi^c}_{n-p} = 0$$

$$A^{111} = -2\tau, A^{112} = 0, A^{121} = 3\tau^{-4}, A^{122} = 0$$
$$A^{211} = 0\,, A^{212} = 2\tau, A^{221} = 0, A^{222} = -3\tau^{-4}$$

One-dimensional solution of hydrodynamical equations in Fourier space

$$\frac{\partial \tilde{\theta}\,(\boldsymbol{k})}{\partial \tau} + \frac{2}{\tau}\tilde{\theta}(\boldsymbol{k}) + \frac{6}{\tau^2}\tilde{\delta}(\boldsymbol{k}) = A(\tilde{\theta})$$

$$\frac{\partial \tilde{\delta}\,(\boldsymbol{k})}{\partial \tau} + \tilde{\theta}(\boldsymbol{k}) = B(\tilde{\delta}, \tilde{\theta})$$

$$A(\tilde{\theta}) = -\frac{k^2}{2}\int_{-L}^{L} dp \int_{-L}^{L} dq\; \delta_D(k-p-q)\frac{pq}{(p^2+1)(q^2+1)}\tilde{\theta}(p)\tilde{\theta}(q)$$

$$B(\tilde{\delta}, \tilde{\theta}) = -\int_{-L}^{L} dp \int_{-L}^{L} dq\; \delta_D(k-p-q)\frac{kq}{(q^2+1)}\tilde{\delta}(p)\tilde{\theta}(q)$$

Scoccimarro & Frieman 1996; Pietroni 2008; Scoccimarro 1997; Sliusar & Zhdanov 2013

Method description

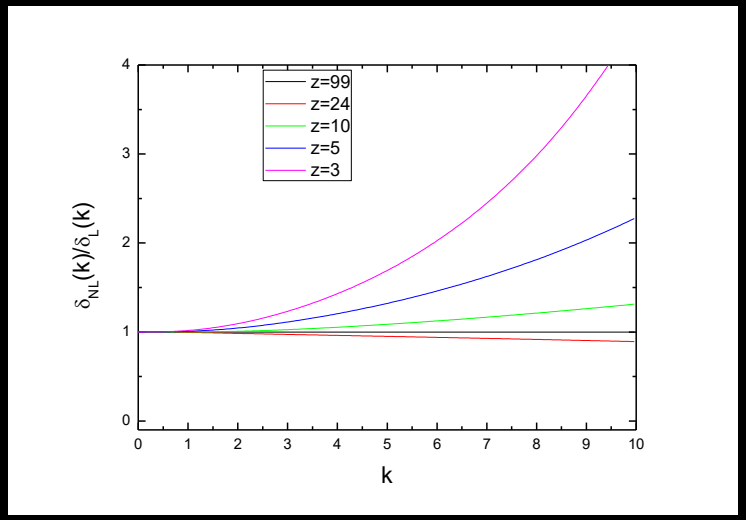
The idea of this method is not to study the dynamics of particles (as it is performed in N-Body simulations), but to integrate with some level of accuracy the hydrodynamical equations in Fourier space. This approach is effective mostly for the weakly nonlinear regime, but it is also possible to consider the strongly nonlinear regime. For the initial moment of time we set values of $\delta_{in}(k)$ $\theta_{in}(k)$ *according to the* Gaussian distribution with power-law power spectrum ($n = -2$).

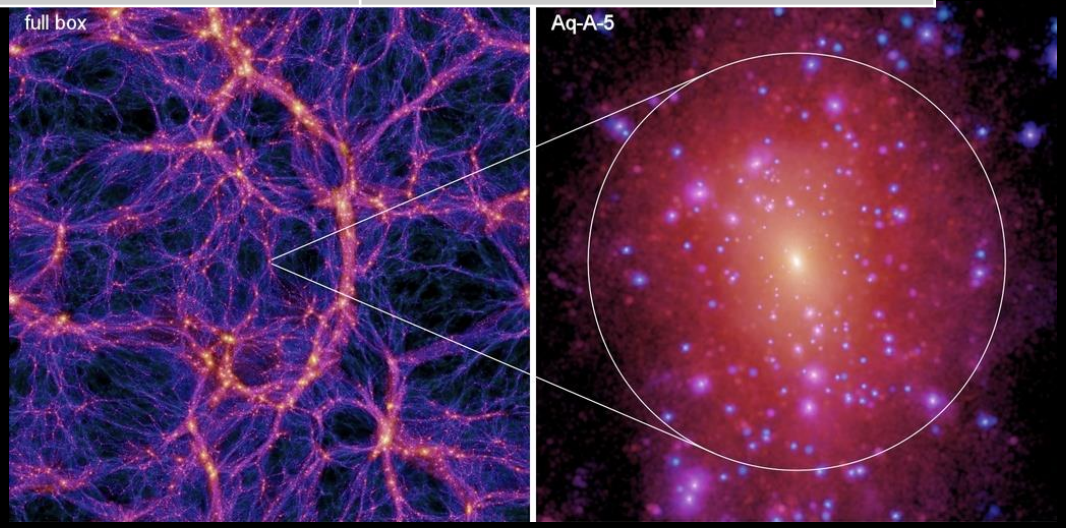
The main advantage of this method is it's CPU and memory consumption. We can easily perform parallel calculation of different realizations on multiple cores or cluster nodes, we also can perform the calculation of a single realization simultaneously in parallel environment because the summation can also be divided among different cores. So it was also interesting to use the GPGPU platform. The OpenCL framework was chosen to have the possibility to run the calculations on either CPUs or GPUs.

The results of trial one dimensional and three dimensional run with one dimensional initial conditions show that it is possible to work with totally three dimensional equations.

Comparison with N-Body method

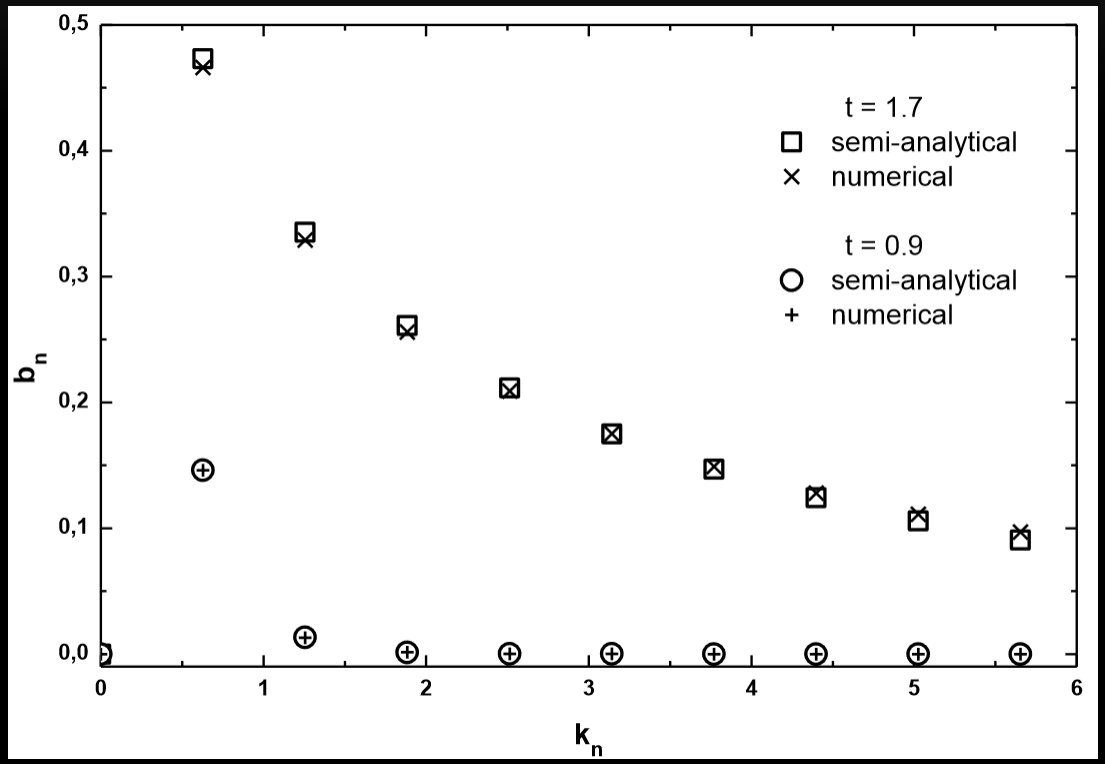
	Direct power spectrum calculations	N-Body simulations
Method	• Direct integration of hydrodynamic equations in Fourier space. The solution is considered for numerous realizations of initial conditions.	• Calculation of the dynamical particle evolution by consideration of the impact of each particle on each other
Advantages	• High resolution • Wide z-range • Low CPU consumption • Low memory consumption	• Simple initial conditions generation • Nice result pictures :)
Disadvantages	• Medium CPU and memory consumption	• Huge CPU consumption • Huge memory consumption





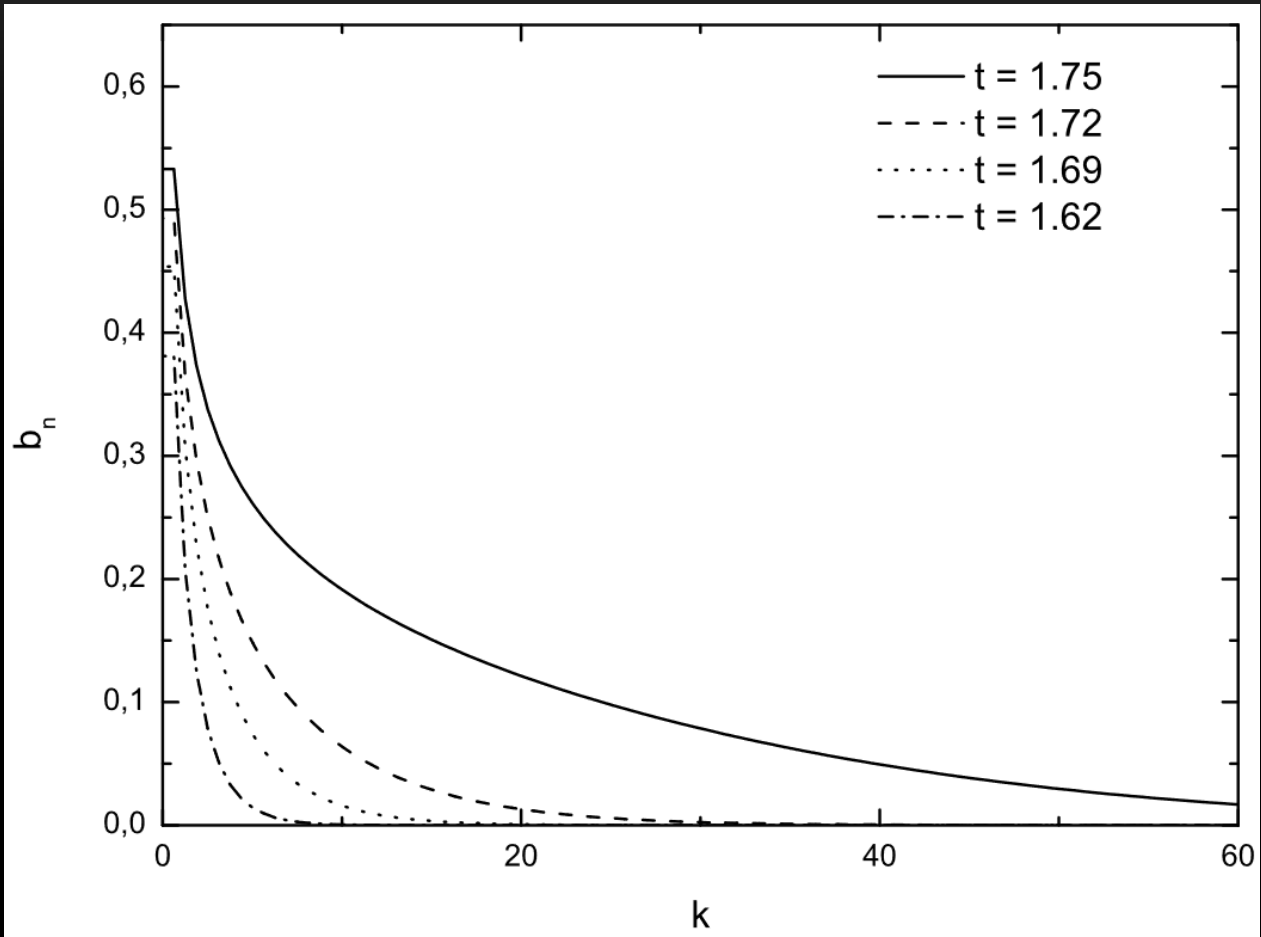
Aquarius simulation; Springel et al. (2008)

One-dimensional case: numerical and semi-analytical results



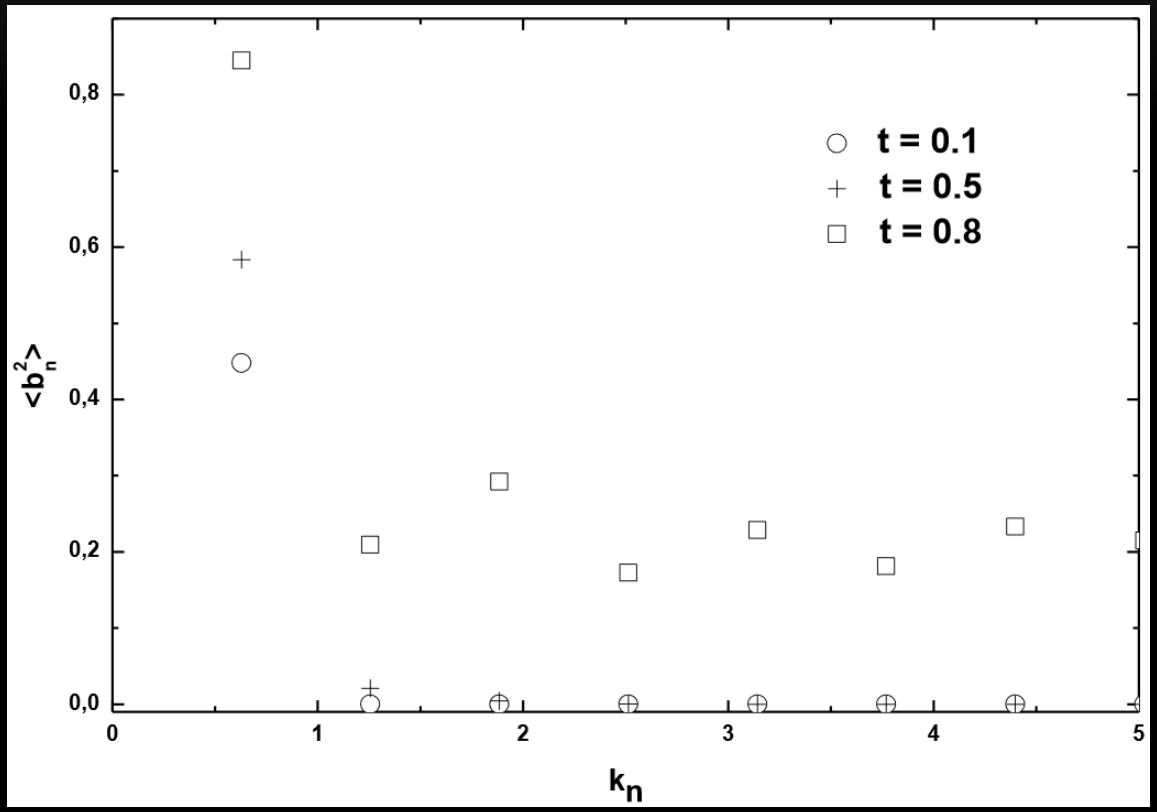
Coefficients $b_n(t)$ for t = 0.9 (bottom) and t = 1.7 (top) determined by semi-analytical and numerical methods with the only nonzero initial conditions value $b_{\pm1}(0) = 0.1$

One-dimensional case: evolution of perturbations



Coefficients $b_n(t)$ determined by numerical method with the only nonzero initial conditions value $b_{\pm 1}(0) = 0.1$

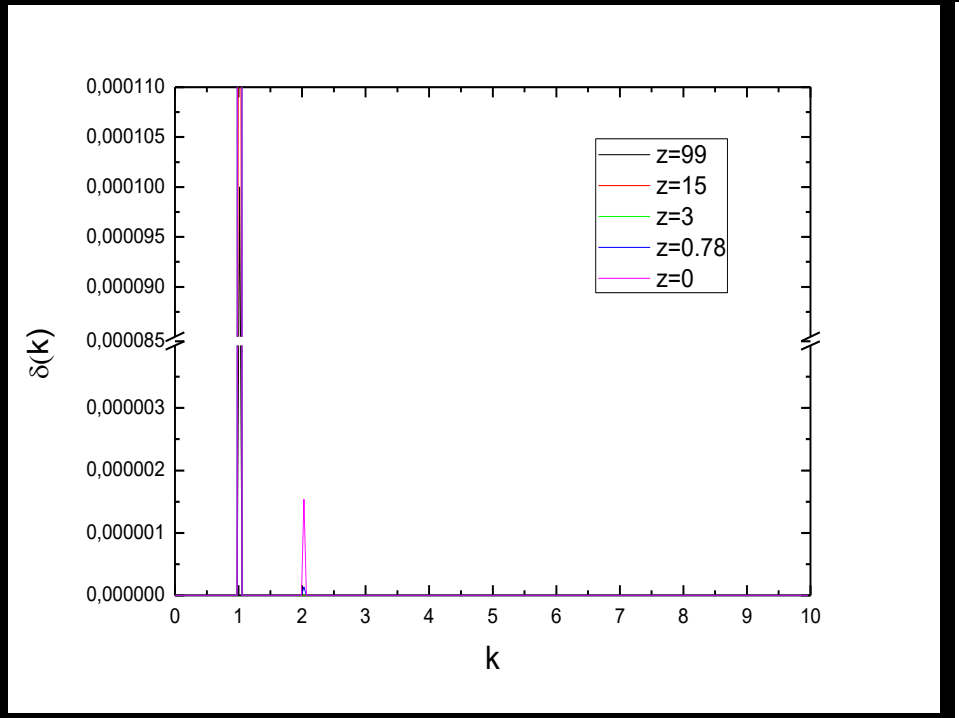
One-dimensional case: normally distributed initial conditions



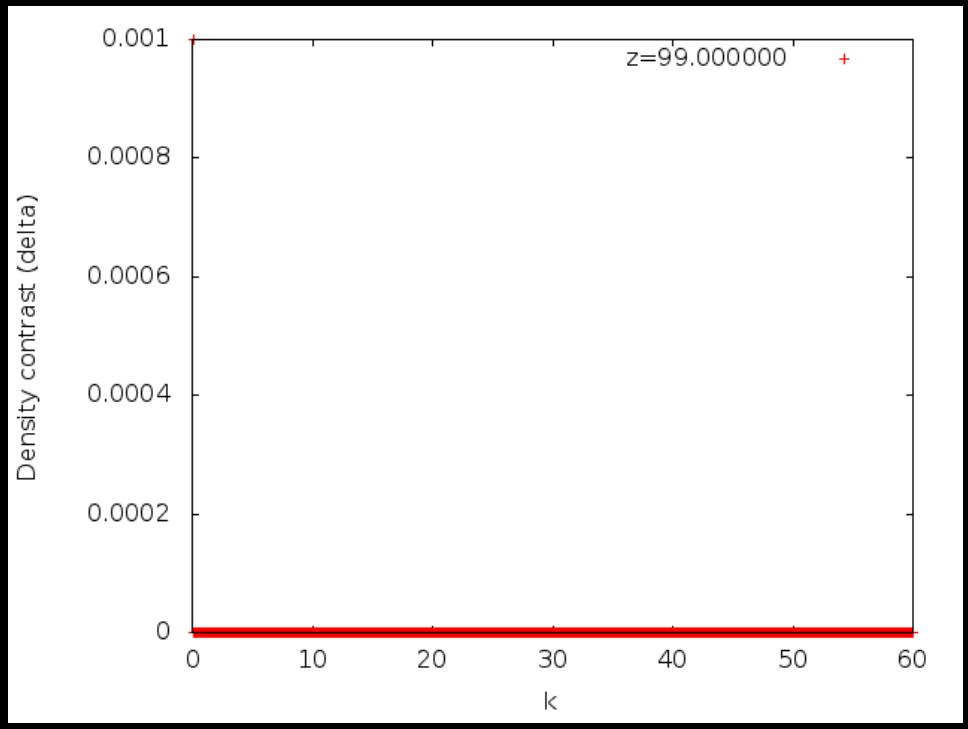
Power spectrum $\langle b_n^2 \rangle$ calculated for 100 realizations of randomly generated initial conditions: uniform distribution of $b_{\pm 1}(0)$, $\langle b_{\pm 1}^2(0) \rangle = 0.5$; the other initial coefficients equal to zero. We see that for $t = 0.8$ the perturbation becomes nonzero for larger k_n due to the non-linear interaction.

Inter wave number interaction

One of the most basic tests that we performed to test the method was where we set the initial conditions in such a way that the density contrast δ had non-zero value for only one value of k. This allows us to see how the perturbations appear on other k-scales.



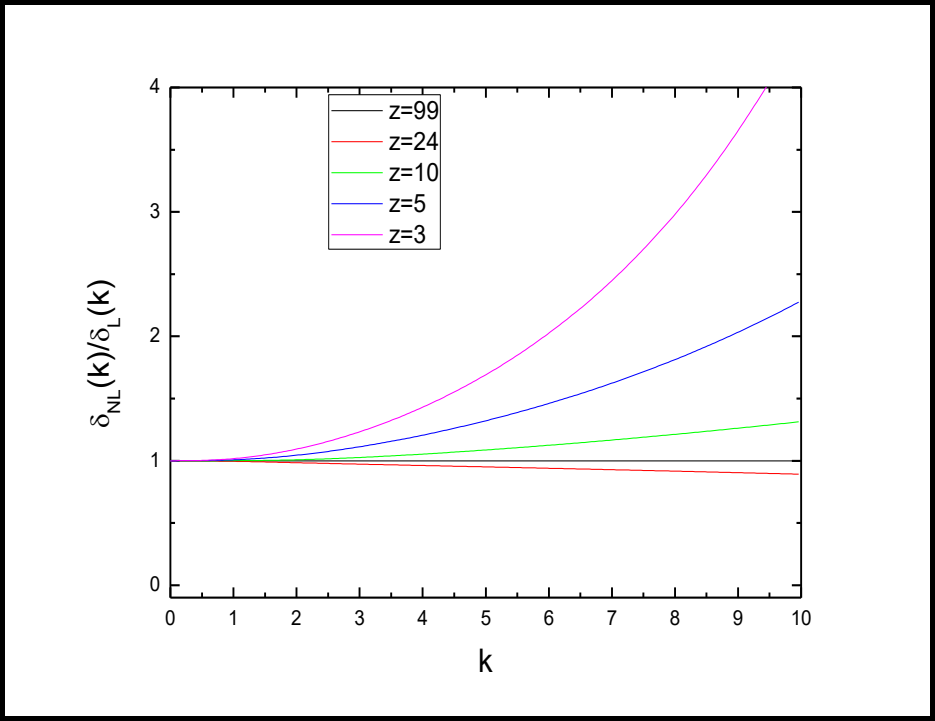
1d case



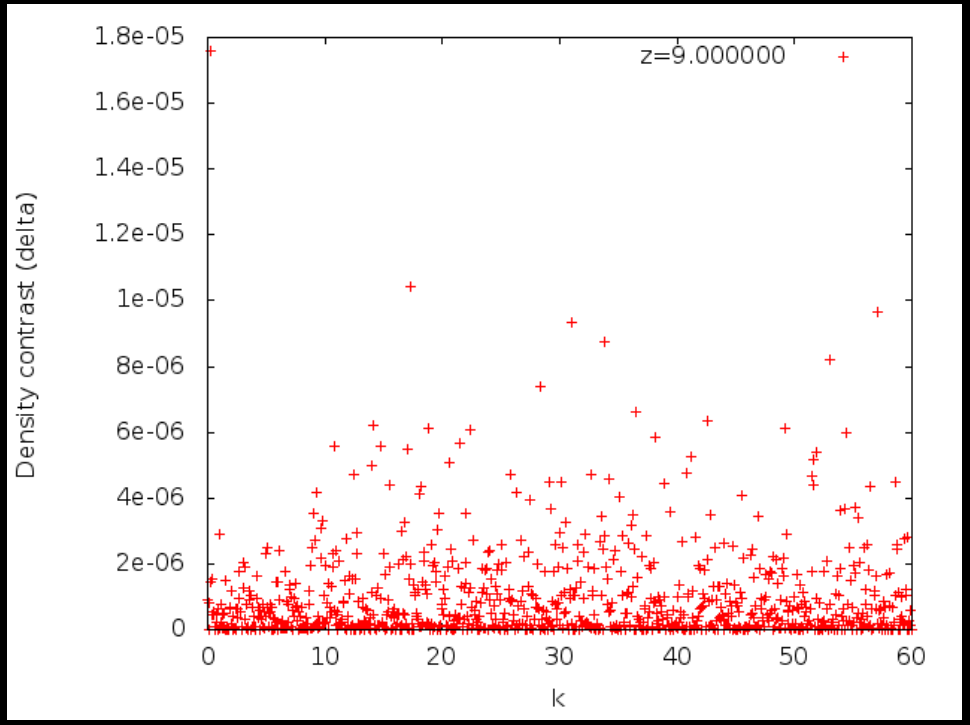
3d+1d case

Normally distributed initial conditions

It is very complicated or even impossible to solve the previously shown equations analytically, so it was interesting to test the solutions in different cases. On this slide you can see the density contrast rations in case where all the initial conditions of δ were set according to the Gaussian distribution with constant dispersion.



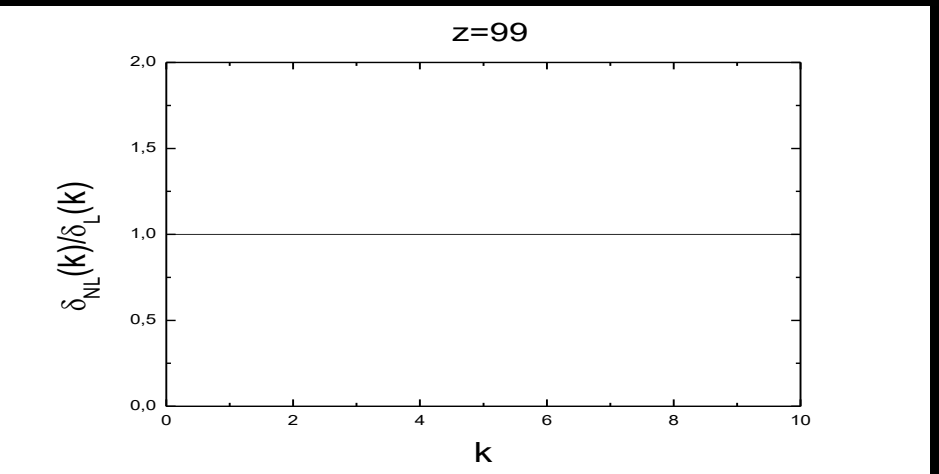
1d case

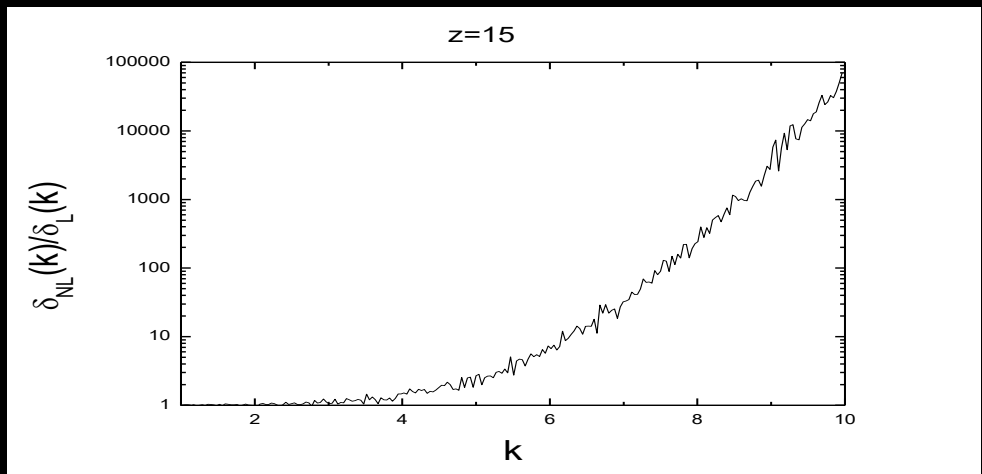


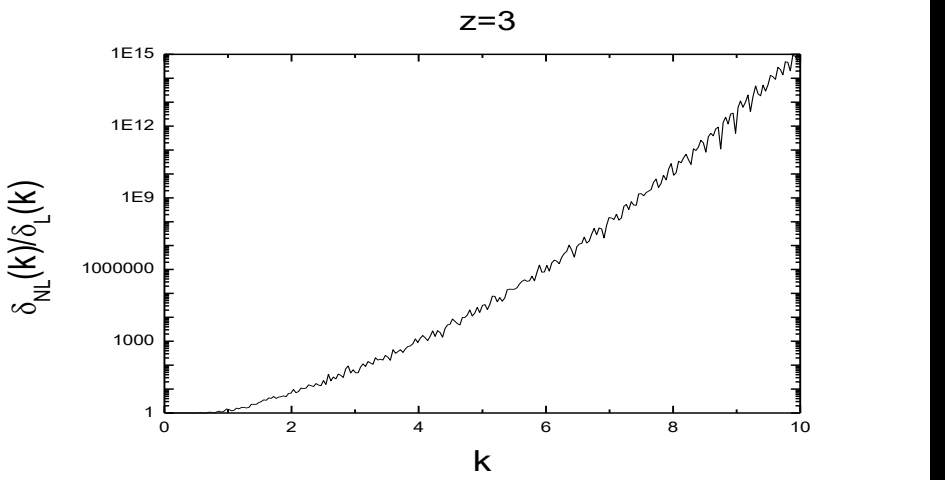
3d+1d case

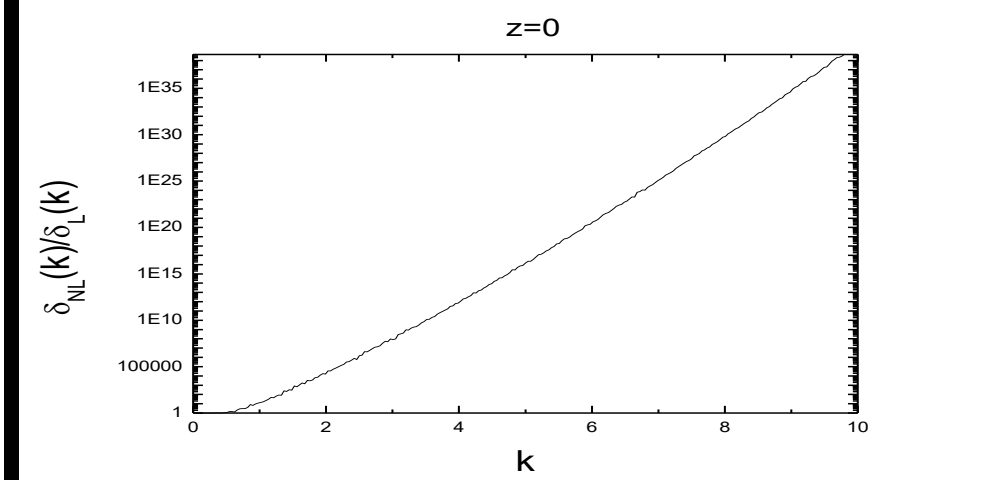
Results

We integrate the equations while $k \in [0;10], z \in [99;0]$. Initial conditions are specified for *z = 99*. Totally we calculated 100 random realizations of initial conditions that are ensemble averaged to get the resulting curves of density contrast ratios in nonlinear and linear regime. Time spent for evolution of one realization is 5 seconds.









Conclusions

- The method was tested in one dimensional case and in three dimensional case with one dimensional initial conditions and it works.
- At the moment with the help of OpenCL software we can reach the 0.1 Mpc/h scale and at the moment there are no limits to reach 0.01 Mpc/h scale and even smaller.

Thank you
for your attention