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Acoustically generated gravitational waves at a first order phase transition

PRL 112, 041301 (2014) [arXiv:1304.2433], arXiv:1504.03291 + work in progress

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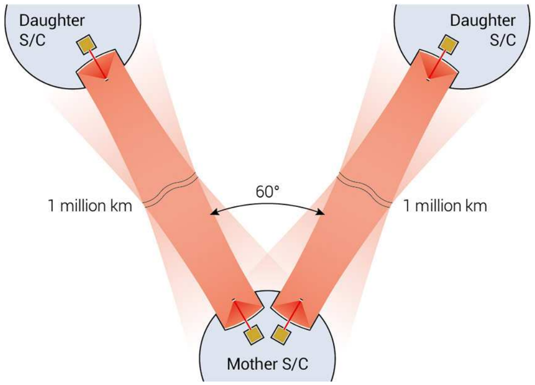
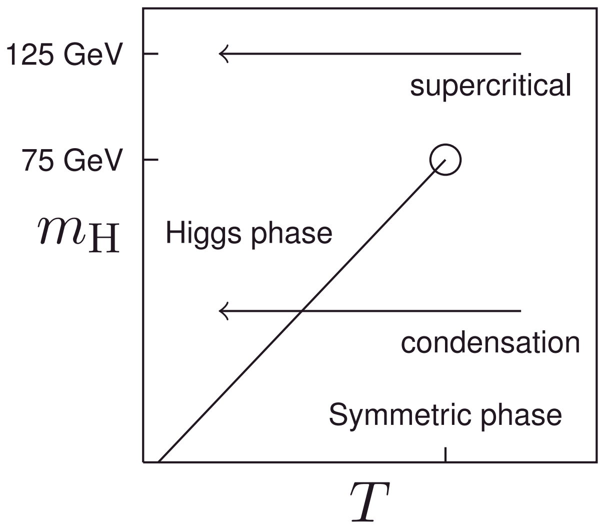
Motivation and context

- GWs are a unique and promising test of high energy physics (advanced LIGO and VIRGO; eLISA scheduled for 2034)
- First order PTs involve bubbles nucleating and growing: bubble collisions produce gravitational waves
- Standard Model EW PT is a crossover, but first order common in extensions (singlet, 2HDM, . . .)
  Andersen, Laine *et al.*, Kozaczuk *et al.*, Kamada and Yamada, Carena *et al.*, Bödeker *et al.*, Damgaard *et al.*
- First order PT around the EW scale *could* give right conditions for baryogenesis (but would then not give a good signal for GWs)
- What physics can we extract from the GW power spectrum at EW scales?



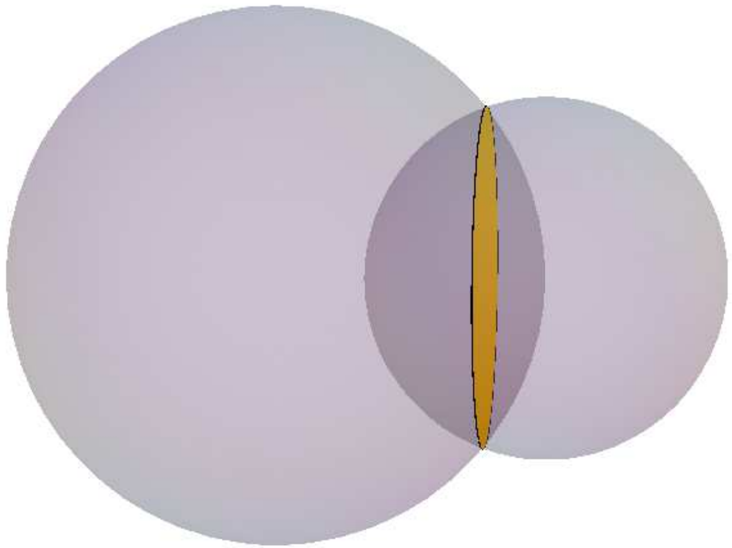


Envelope approximation

Kosowsky, Turner and Watkins; Kamionkowski, Kamionkowsky and Turner

- Thin-walled bubbles, no fluid
- Bubbles expand with velocity v_{w}
- Stress-energy tensor $\propto R^3$ on wall
- Overlapping bubbles $\rightarrow$ GWs
- Keep track of solid angle
- Collided portions of bubbles source gravitational waves
- Resulting power spectrum is simple
  - One scale (R_*)
  - Two power laws (k^3, k^{-1})
  - Amplitude

  $\Rightarrow$ 4 numbers define spectral form

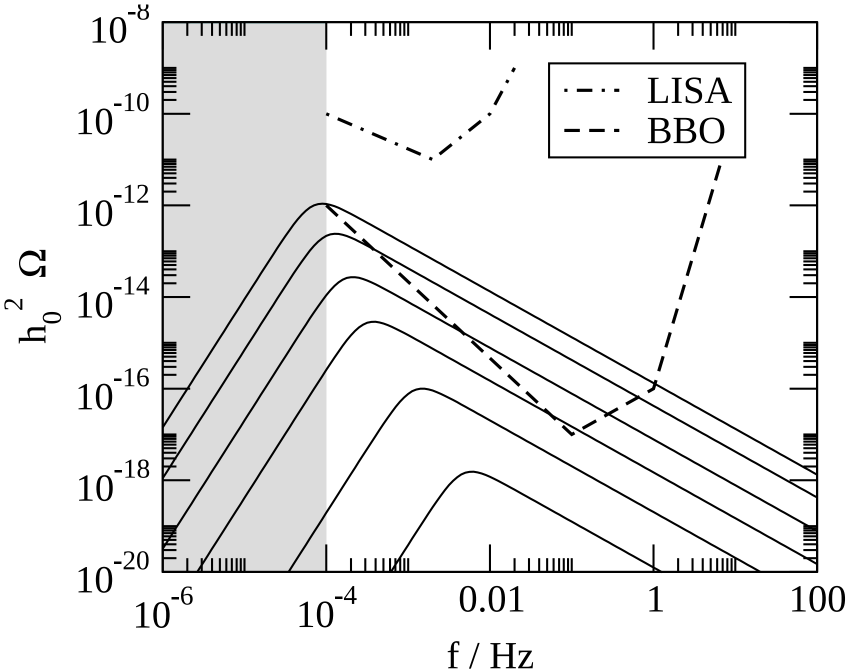


The envelope approximation makes predictions

Espinosa, Konstandin, No and Servant; Huber and Konstandin

4-5 numbers parametrise the transition:

- α, vacuum energy fraction
- v_{w}, bubble wall speed
- κ, conversion efficiency to fluid KE
- Transition rate:
  - H_*, Hubble rate at transition
  - β, bubble nucleation rate



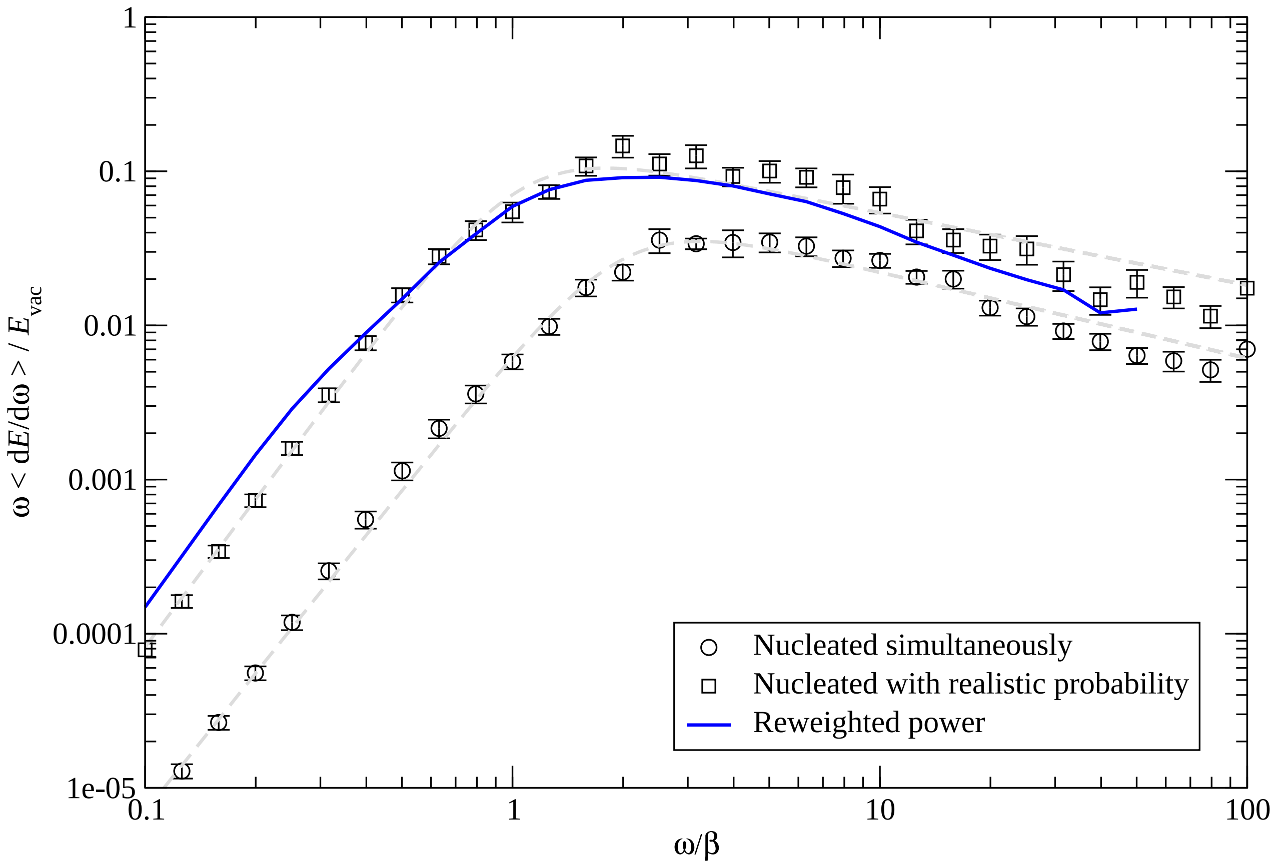
From Konstandin and Huber

Energy in GWs ($\Omega_{\mathrm{GW}} = \rho_{\mathrm{GW}}/\rho_{\mathrm{Tot}}$):

$$\Omega_{\mathrm{GW}}^{\text{envelope}} \approx \frac{0.11 v_{\mathrm{w}}^3}{0.42 + v_{\mathrm{w}}^2} \left(\frac{H_*}{\beta}\right)^2 \frac{\kappa^2 \alpha^2}{(\alpha+1)^2}$$

Envelope approximation power laws do not depend on nucleation

Work in progress!



- Re-implemented the method of Huber and Konstandin
- Bubbles nucleated at the same time have same power laws as bubbles nucleated 'properly'
- Can re-weight from equal time nucleation case to unequal time

Our approach: field+fluid system

- Scalar ϕ + ideal fluid u^μ (treated using standard SR hydro Wilson and Matthews)
  - Split stress-energy tensor $T^{\mu\nu}$ into field and fluid bits
    Ignatius, Kajantie, Kurki-Suonio and Laine

$$\partial_\mu T^{\mu\nu} = \partial_\mu (T^{\mu\nu}_{\text{field}} + T^{\mu\nu}_{\text{fluid}}) = 0$$

  - Parameter η sets the scale of friction due to plasma

$$\partial_\mu T^{\mu\nu}_{\text{field}} = \eta u^\mu \partial_\mu \phi \partial^\nu \phi \qquad \partial_\mu T^{\mu\nu}_{\text{fluid}} = -\eta u^\mu \partial_\mu \phi \partial^\nu \phi$$

- Effective potential $V(\phi, T)$ can be kept simple

$$V(\phi,T) = \tfrac{1}{2}\gamma(T^2 - T_0^2)\phi^2 - \tfrac{1}{3}AT\phi^3 + \tfrac{1}{4}\lambda\phi^4$$

  - γ, T_0, A, λ chosen to match scenario of interest

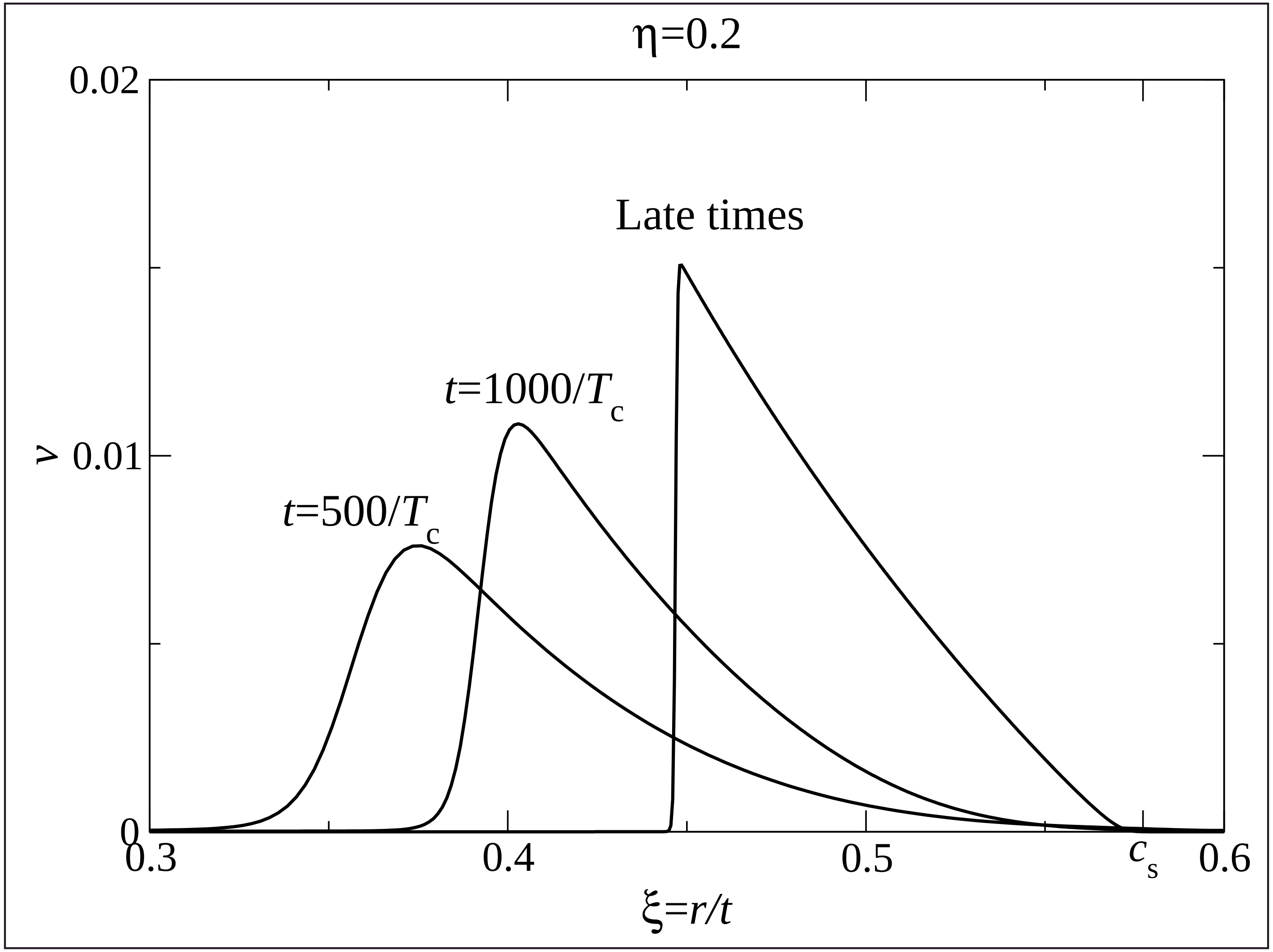
- Equations of motion (+ continuity equation)

$$\partial_\mu \partial^\mu \phi + \frac{\partial V(\phi,T)}{\partial \phi} = -\eta u^\mu \partial_\mu \phi$$

$$\partial_\mu \left\{ [\epsilon + p]\, u^\mu u^\nu - g^{\mu\nu}[p - V(\phi,T)] \right\} = \left(\eta u^\mu \partial_\mu \phi + \frac{\partial V(\phi,T)}{\partial \phi} \right) \partial^\nu \phi$$

Velocity profile development - deflagration [optional movie]

Here, $\eta = 0.2$ (deflagration)



- Metric perturbations evolve as

$$\ddot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G T_{ij}^{\mathrm{TT}}$$

equivalently Garcia-Bellido and Figueroa; Easther, Giblin and Lim

$$\ddot{u}_{ij} - \nabla^2 u_{ij} = 16\pi \tau_{ij}$$

and project $h_{ij}(k) = \Lambda_{ij,lm}(k) u_{ij}(k)$ later

- Consider only terms at leading order in the perturbation h_{ij}

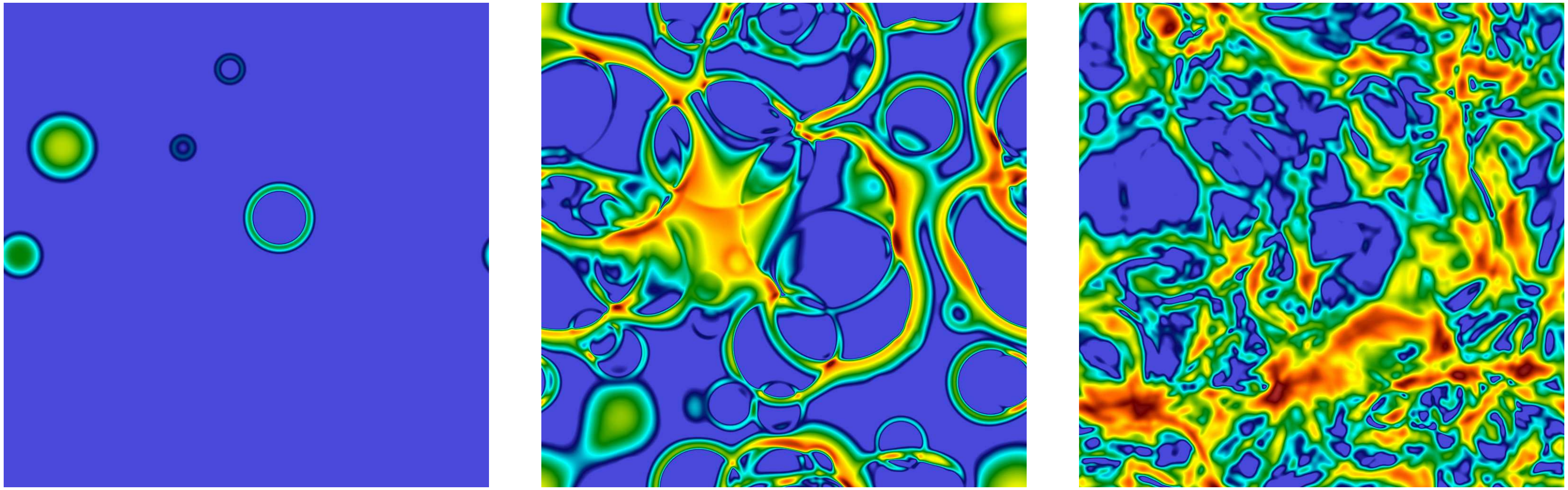
$$\tau_{ij}^{\mathrm{f}} = W^2(\epsilon + p) V_i V_j \qquad \tau_{ij}^{\phi} = \partial_i \phi \partial_j \phi$$

- Power $\rho_{\mathrm{GW}} = T_{00}^{\mathrm{grav}}$ per logarithmic interval,

$$\frac{d\rho_{\mathrm{GW}}}{d\ln k} = \frac{1}{32\pi G V} \frac{k^3}{(2\pi)^3} \int d\Omega \; \Lambda_{ij,lm}(\hat{\mathbf{k}}) \dot{u}_{ij}(t,\mathbf{k}) \dot{u}_{lm}^*(t,\mathbf{k})$$

Simulation slice example [optional movie]

Simulations at 1024^3, deflagration, fluid kinetic energy density, $\sim$250 bubbles

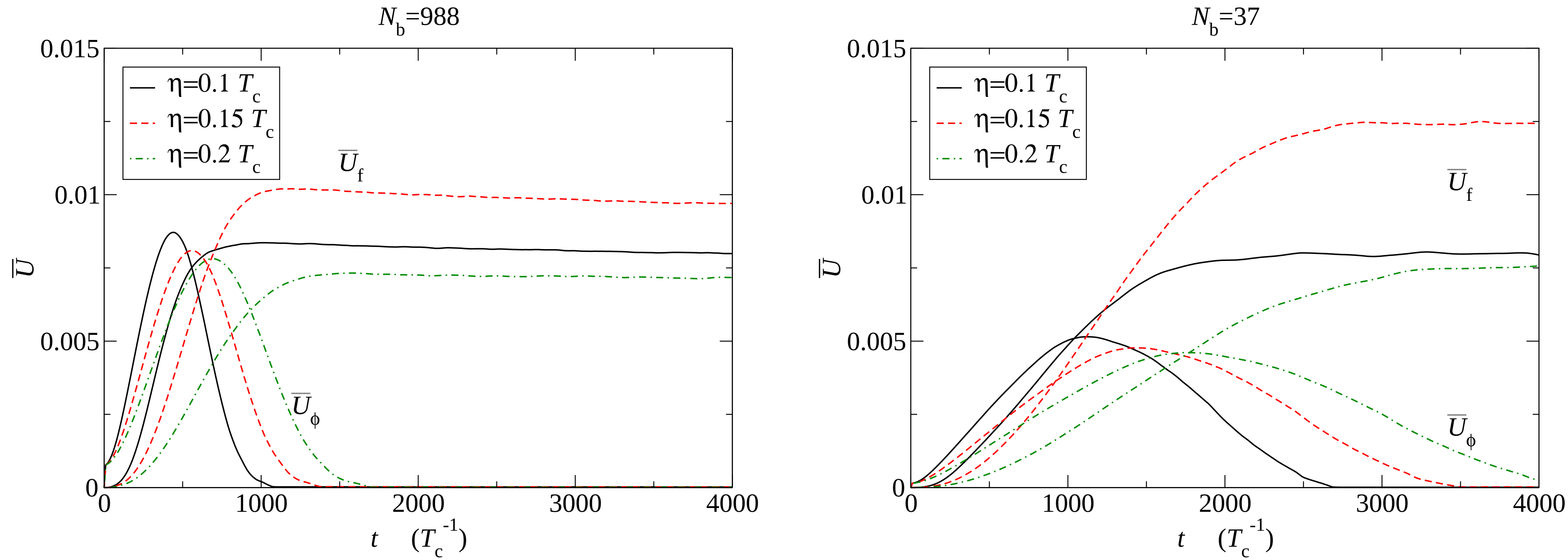


$t = 500\ T_c^{-1}$ $t = 750\ T_c^{-1}$ $t = 1000\ T_c^{-1}$

How the sources behave over time

- $\overline{U}_\mathrm{f}$ is the rms fluid velocity; $\overline{U}_\phi$ the analogous field quantity
- Constructed from τ^f_{ii} and τ^ϕ_{ii}, they indicate how strong each source is

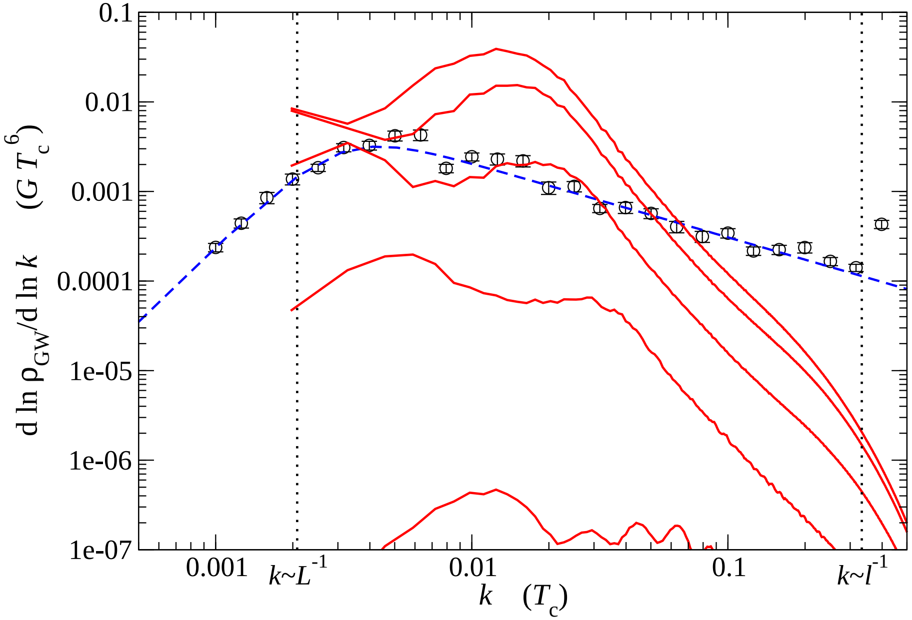
$$(\bar\epsilon + \bar p)\overline{U}_\mathrm{f}^2 = \frac{1}{V}\int d^3x\, \underbrace{W^2(\epsilon + p)}_{(\tau^\mathrm{f}_{ii})^2} \qquad (\bar\epsilon + \bar p)\overline{U}_\phi^2 = \frac{1}{V}\int d^3x\, \underbrace{(\partial_i\phi)^2}_{(\tau^\phi_{ii})^2}$$



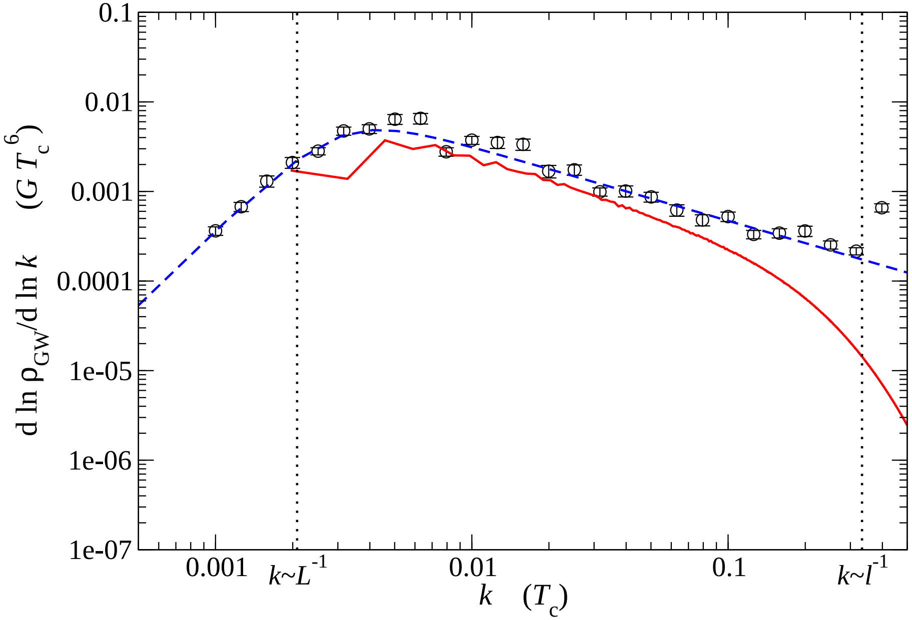
So does the envelope approximation really work?

- Compare field+fluid simulation with envelope approximation
- Nucleate 125 bubbles in same locations

Fluid source



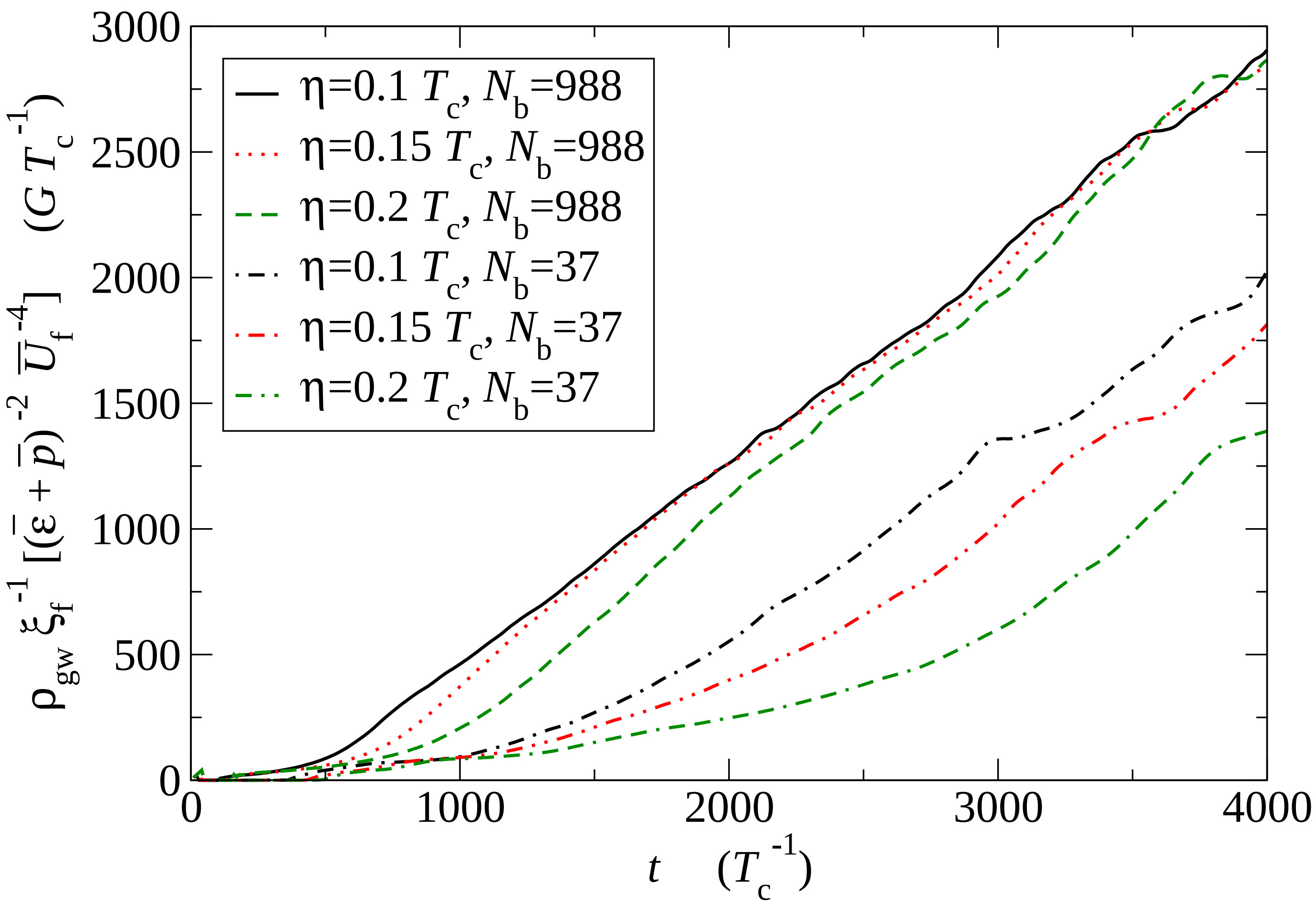
Field source



- Power laws for fluid source totally different
- Field source OK (overestimated), but will be subdominant anyway

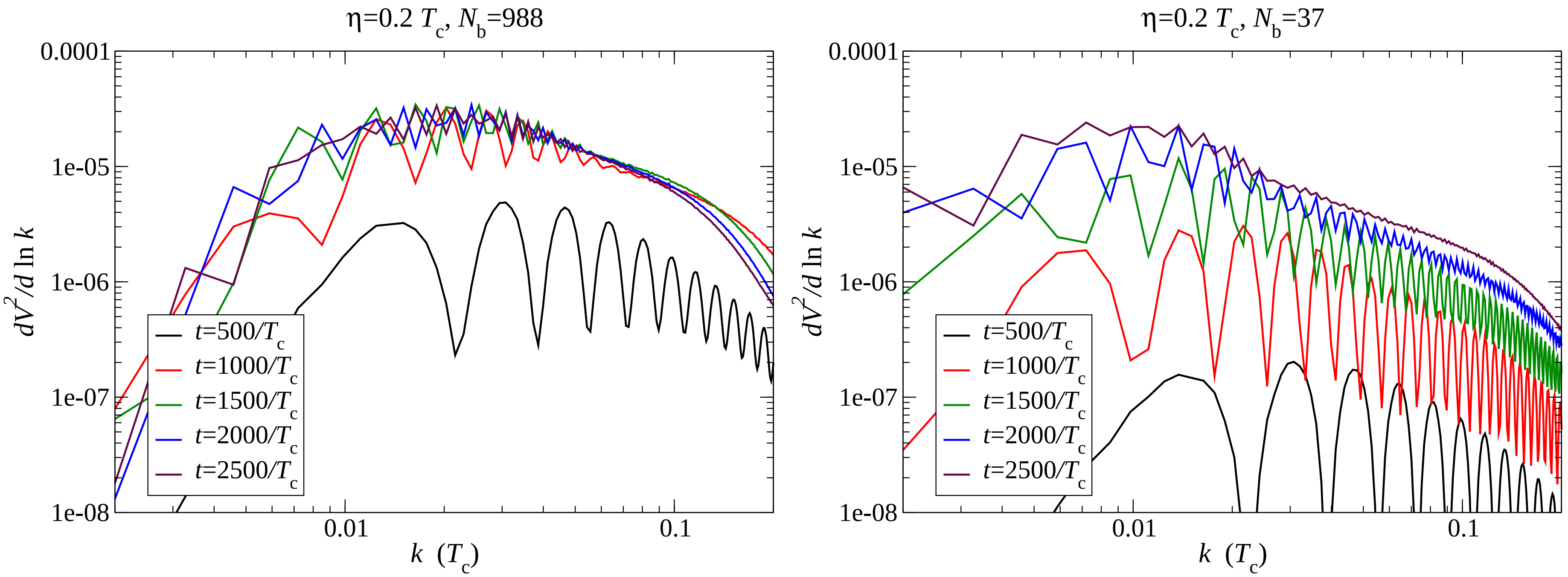
Acoustic waves source linear growth of gravitational waves

- Sourced by $T^{\rm f}_{ij}$ only (T^{ϕ}_{ij} source is small constant shift)



- Source generically scales as $\rho_{\rm GW} \propto t[G\xi_{\rm f}(\bar{\epsilon}+\bar{p})^2\overline{U}_f^4]$

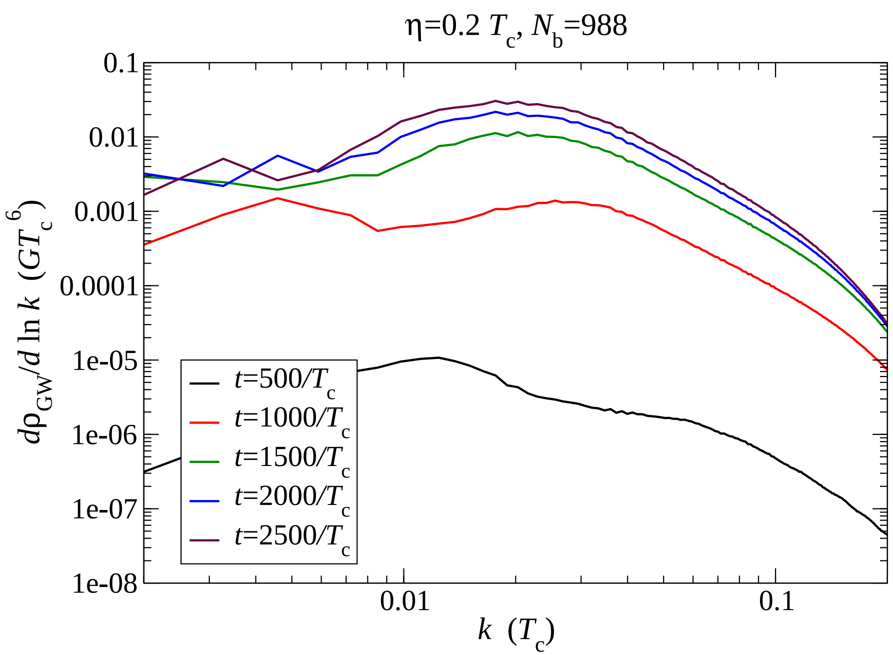
Velocity power spectra and power laws

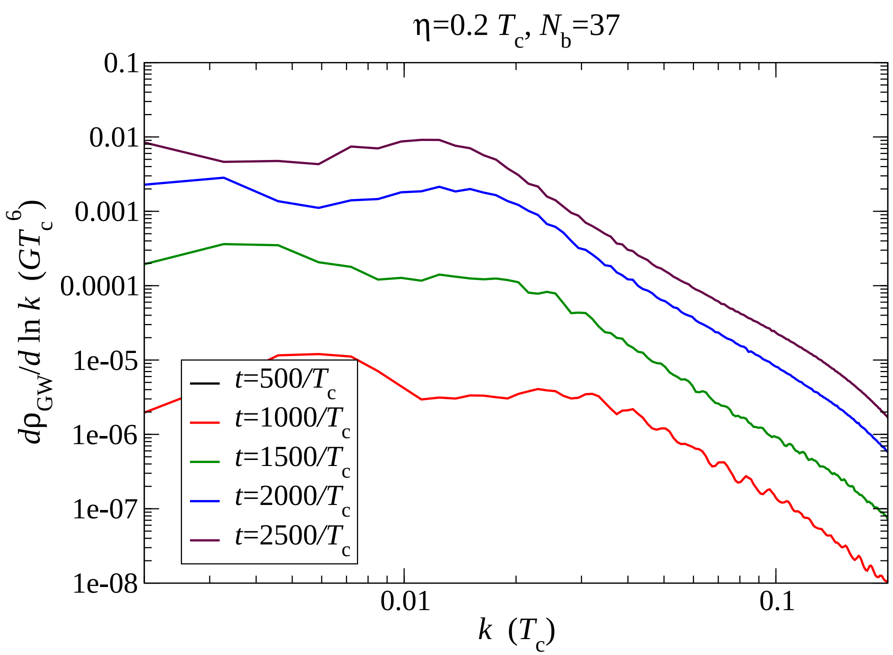


- Weak transition: $\alpha_{T_\mathrm{N}} = 0.01$, $v_\mathrm{w} = 0.44$
- Power law behaviour above peak (approximately k^{-1} here)
- “Ringing” due to simultaneous bubble nucleation, not physically important
- Power is in the longitudinal modes – acoustic waves, not turbulence
- If we know $\mathrm{d}V^2/\mathrm{d}\ln k$, can work out $\dot{\rho}_\mathrm{GW}/\mathrm{d}\ln k\ldots$?

GW power spectra and power laws

- Sourced by T^{f}_{ij} only





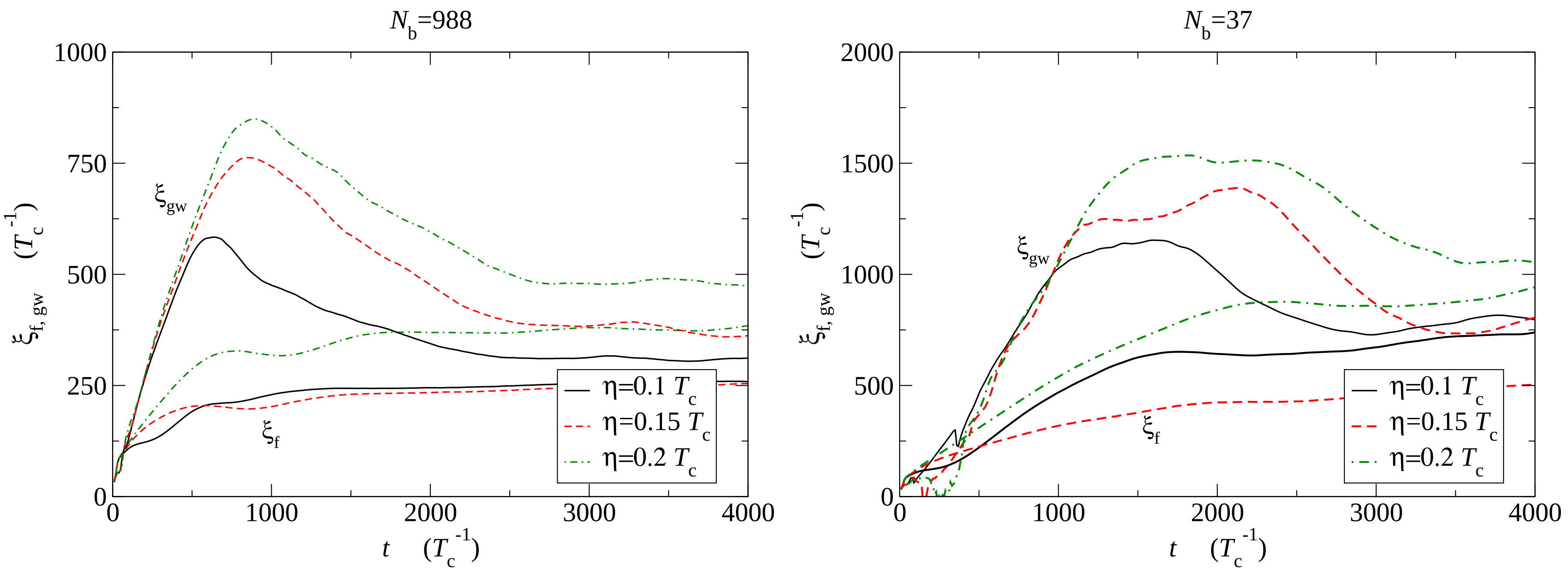
- Approximate k^{-3} power spectrum
- Finite size of box means that we choose not to probe behaviour below peak k

Fluid characteristic length scale is imprinted in GW power spectrum

Define the fluid integral scale

$$\xi_{\mathrm{f}} = \frac{1}{\langle V^2 \rangle} \int \frac{d^3k}{(2\pi)^3} |k|^{-1} P_V(k)$$

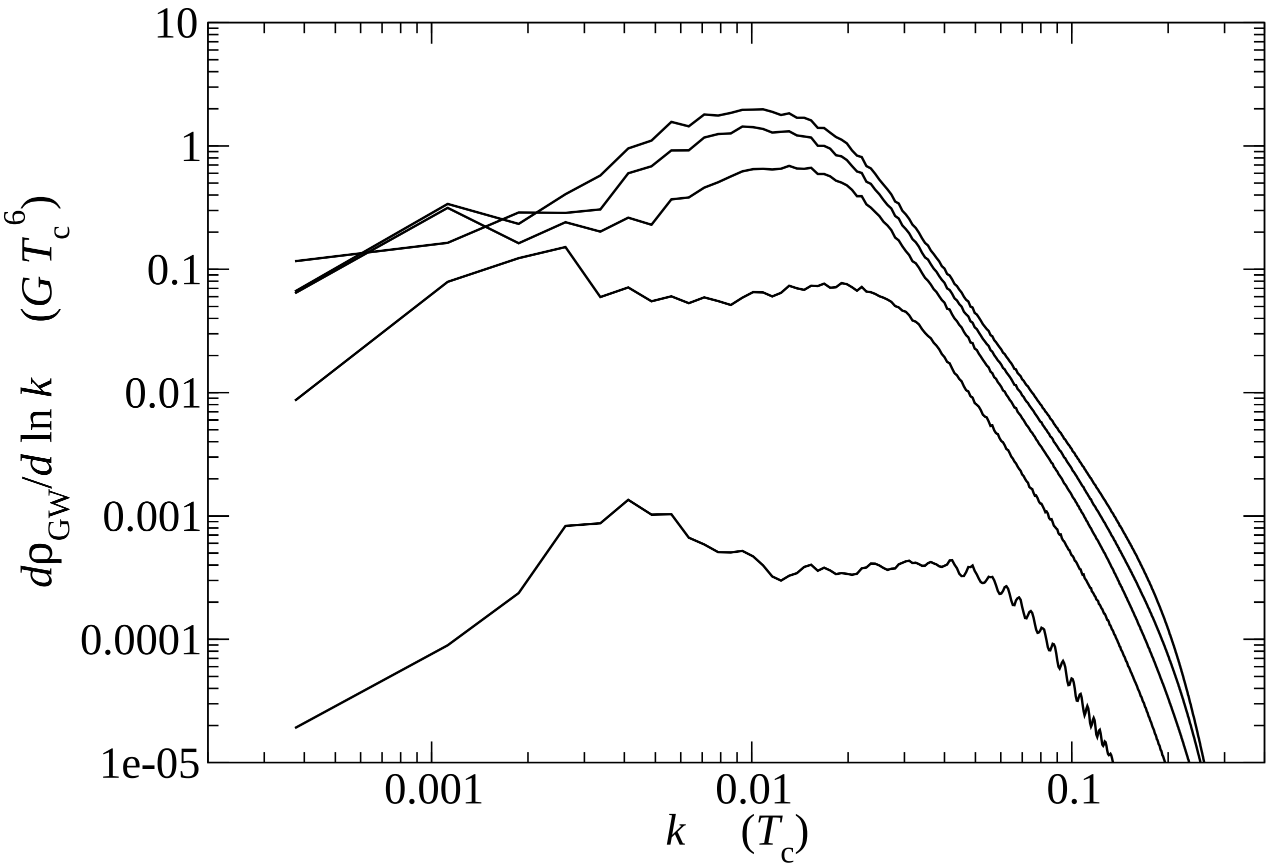
and the analogous quantity ξ_{GW} for the gravitational wave power spectrum.



This length scale is what sets the peak of the fluid power spectrum.

Latest results: 4200^3 using PRACE access

Some results from our latest runs (4200^3 lattice) – also work in progress!



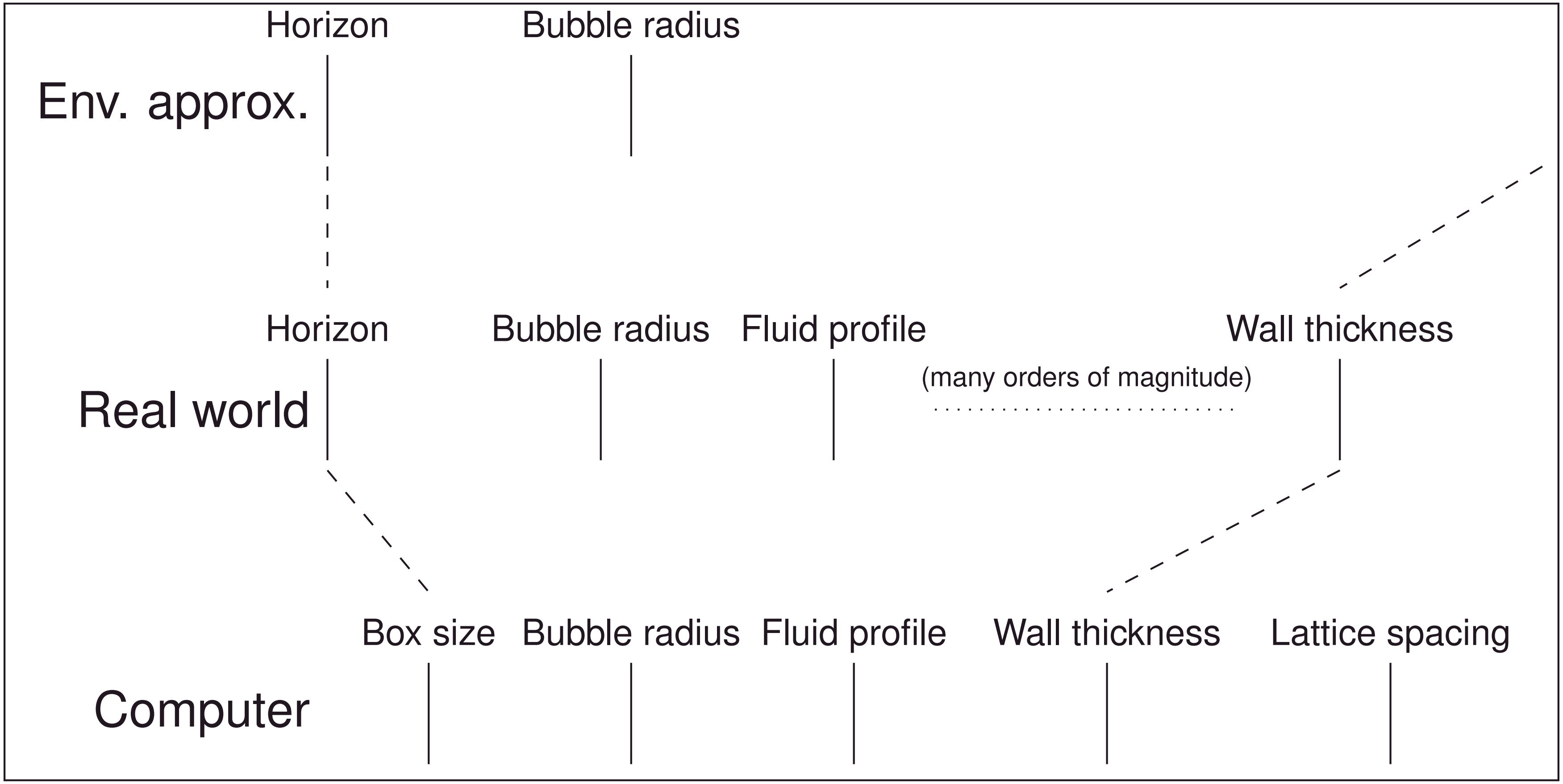
- Curves every $1000/T_\mathrm{c}$
- Here $v_\mathrm{w} = 0.68$ (detonation)
- Friction parameter now dimensionless, damping term $\eta \to \eta\phi^2/T$

Summary and outlook

- New source of GWs: sound waves from colliding bubble droplets
- Rate of GW energy production is **generically** $\rho_{\mathrm{GW}} \propto t[G\xi_{\mathrm{f}}(\bar{\epsilon}+\bar{p})^2\overline{U}_f^4]$
- Large enhancement over envelope approximation at EW scale
  $\rightarrow$ good news for models that do not produce strongly first-order PTs
- Power laws different from envelope approximation
- Functional form of power spectrum still a broken power law
- Currently trying to understand power laws with larger simulations
  – 18M CPU hours awarded by PRACE
- Building a science case for eLISA – Caprini *et al.*

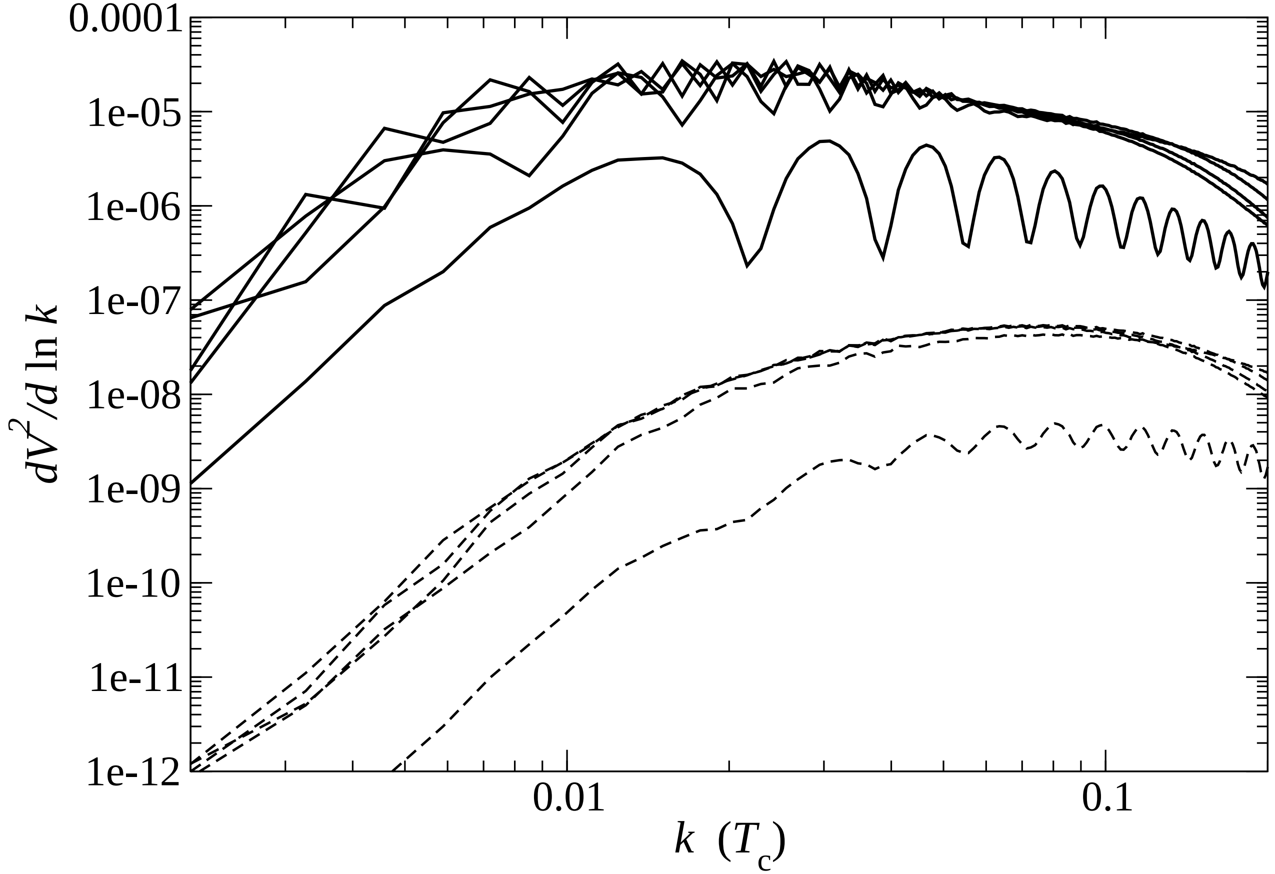
Dynamic range issues

- Most realtime lattice simulations in the early universe have a single [nontrivial] length scale
- Here, many length scales important



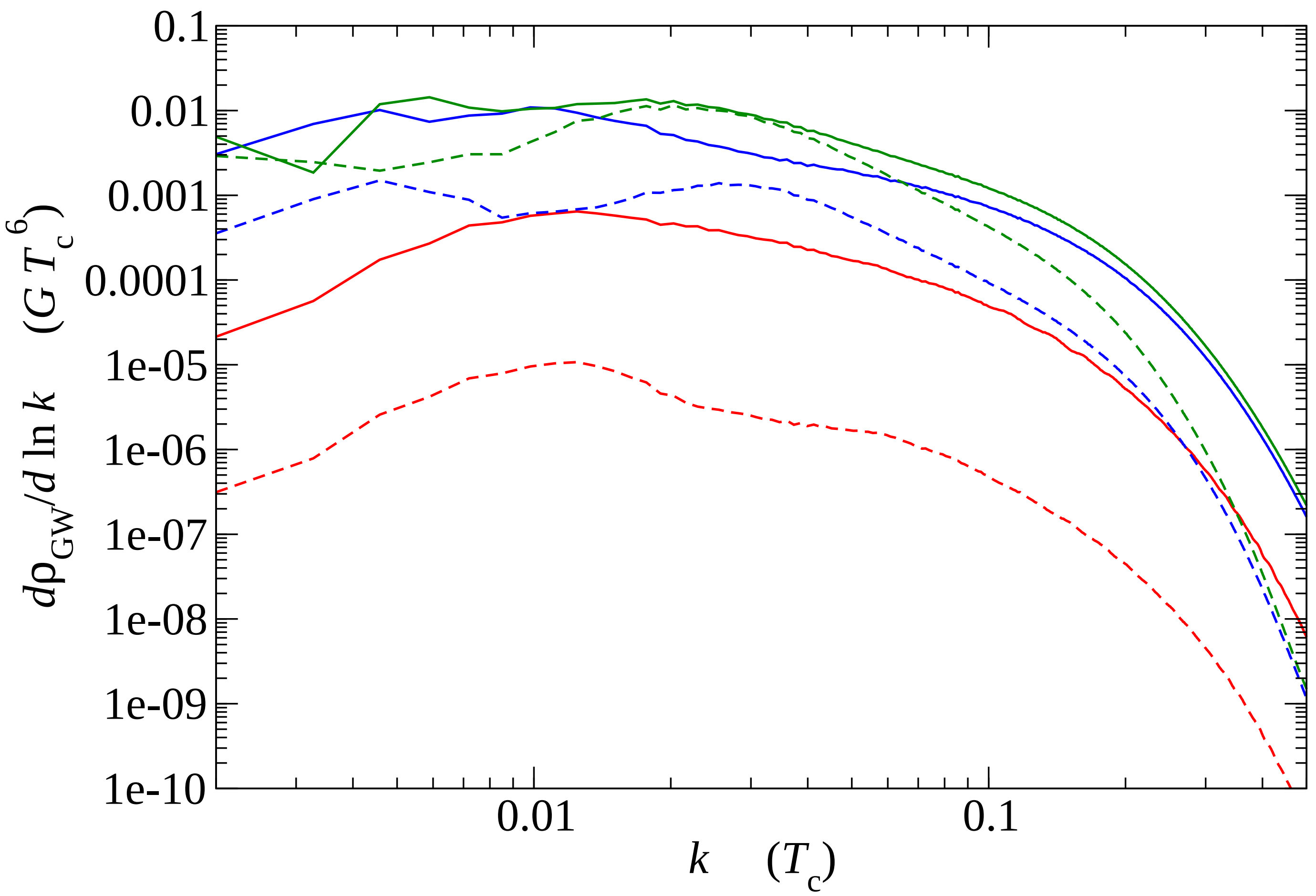
- Simulations in arXiv:1504.03291 are with 2400^3 lattice, $\delta x = 2/T_c$ $\rightarrow$ approx 200k CPU hours each ($\sim$ 3M total)

Transverse versus rotational modes – turbulence?



- Most power is in the longitudinal modes – acoustic waves, not turbulence
- System is quite linear. Reynolds number is ~ 100 due to discretisation.

GW power spectra – field and fluid sources



- By late times, fluid source dominates at all length scales
- $500/T_{\mathrm{c}}$, $1000/T_{\mathrm{c}}$, $1500/T_{\mathrm{c}}$ (‘before’, ‘during’, ‘after’ collision)
- Fluid source shown by dashed lines, total power solid lines

Lifetime of sound waves and increase in GW power

- Does the acoustic source matter?
  - Sound is damped by (bulk and) shear viscosity Arnold, Dogan and Moore; Arnold, Moore and Yaffe

$$\left(\frac{4}{3}\eta_{\rm s} + \zeta\right) \nabla^2 V^i_{\|} + \ldots \Rightarrow \tau_\eta(R) \sim \frac{R^2 \epsilon}{\eta_{\rm s}}$$

  - Compared to $\tau_{H_*} \sim H_*^{-1}$, on length scales

$$R^2 \gg \frac{1}{H_*}\frac{\eta_{\rm s}}{\epsilon} \sim 10^{-11} \frac{v_{\rm w}}{H_*}\left(\frac{T_{\rm c}}{100\,{\rm GeV}}\right)$$

the Hubble damping is faster than shear viscosity damping.

- Does the acoustic source enhance GWs?
  - Yes, we have

$$\Omega_{\rm GW} \approx \left(\frac{\kappa\alpha}{\alpha+1}\right)^2 (H_*\tau_{H_*})(H_*\xi_{\rm f}) \Rightarrow \frac{\Omega_{\rm GW}}{\Omega_{GW}^{\textsf{envelope}}} \gtrsim 60\frac{\beta}{H_*}.$$

extra slide