Relaxing the limits on inflationary magnetogenesis II

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Abstract

The typical residual strengths of inflation-produced primordial magnetic fields are considered too weak to sustain the galactic dynamo. The main reason is the so-called adiabatic decay of these fields, which is believed to apply at all times and on all scales. Non-conventional mechanisms of inflationary magnetogenesis can slow down the adiabatic magnetic depletion by breaking away from classical electromagnetic theory during the de Sitter phase. Maxwellian electromagnetism is usually restored after inflation and the fields are allowed to decay adiabatically ever since and on all scales. So, if the magnetic "amplification" during inflation is sufficiently strong, the residual fields are capable of seeding the dynamo today. However, a drastic magnetic enhancement during inflation could lead to unwanted effects and cause problems. We argue here that such a strong magnetic amplification is not necessary, if the decay rate of the field slows down after inflation. To demonstrate that this is possible, we employ causality arguments and claim that, contrary to the common belief, superhorizon-sized magnetic fields are not a priori frozen into the matter after inflation. As a result, on very large scales, the post-inflationary magnetic evolution becomes an issue of the initial conditions at the start of reheating and the adiabatic magnetic decay can slow down considerably. This makes it much easier for non-conventional magnetogenesis to produce primordial fields of astrophysical interest today. In fact, even conventional scenarios of inflationary magnetogenesis, namely those operating within classical electromagnetism, may be able to work.

Although magnetic (B) fields seem to be everywhere in the universe [1], their origin is still an open issue and the subject of ongoing debate [2]. Large-scale magnetic fields are produced "naturally" during inflation, but to seed the galactic dynamo the current strength of these fields should lie between 10^{-22} and 10^{-12} Gauss [3]. To achieve such magnitudes, most scenarios of primordial magnetogenesis break away from classical electromagnetism during inflation [2]. This way it is possible to strengthen the B-fields considerably all along the de Sitter phase of the expansion and thus produce seeds that fulfill the galactic-dynamo requirements. Maxwellian electromagnetism is typically restored once inflation is over and the B-field is assumed to decay adiabatically thereafter (i.e. $B \propto a^{-2}$, with a representing the cosmological scale factor). The same adiabatic decay is the reason the conventional mechanisms of inflationary magnetogenesis are considered unable to produce astrophysically relevant B-fields.

Assuming magnetic-flux freezing (namely that $B \propto a^{-2}$) on all scales after inflation, means applying the ideal magnetohydrodynamic (MHD) limit both inside and outside the Hubble horizon. However, the ideal-MHD approximation is the result of causal microphysical processes operating inside the horizon only. In particular, it is the electric currents that freeze the magnetic field into the highly conductive matter. These currents are produced after inflation and for this reason their scale is always smaller than the causal horizon. Inflationary magnetic fields, on the other hand, have already achieved superhorizon correlations. On these very large scales there are no electric currents, no causal physics and therefore no magnetic-flux freezing. Claiming the opposite would be like arguing that causal physical processes operating inside the Hubble radius can affect superhorizon perturbations, which clearly violates causality [4]. Instead of freezing into the matter, the aforementioned large-scale B-fields feel only the background expansion and retain the memory of the inflationary past, for as long as they remain outside the horizon. All these mean that, even after the de Sitter phase is over, on super-Hubble scales we are still dealing with the free magnetic fields left there from inflation. As a result, the adiabatic decay law $B \propto a^{-2}$ is not a priori guaranteed on all scales after inflation.

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The issue of causality was pointed out in [5], but an extensive discussion of the argument and an investigation of its implications for primordial magnetogenesis, conventional as well as non-conventional, was given in [6, 7]. It was demonstrated there that, on scales larger than the causal horizon, post-inflationary B-fields do not a priori decay adiabatically, but their depletion rates can slow down considerably. Put another way, large-scale magnetic fields could be superadiabatically amplified after inflation.[1] Then, both conventional and non-conventional magnetic fields can reach residual strengths much larger than those typically quoted in the literature. As a result, the standard limits on inflationary magnetogenesis (conventional or not) can relax considerably. What determines whether this happens or not is the field's evolution in the de Sitter phase and the specifics of the transition from inflation to reheating and subsequently to the radiation and the dust epochs. Recently, the same causality claim was adopted in the work of [8], with analogous conclusions and results. In what follows we will revise the standard de Sitter limits set on inflationary magnetic fields, starting from the galactic-dynamo requirements today and assuming that these B-fields have been superadiabatically amplified after inflation, for as long as they remained outside the causal horizon.

Let us begin by recalling that the typical magnetic strengths required at present, in order for the galactic dynamo to operate successfully, lie within the range

$$10^{-22}\ \mathrm{G} \lesssim B_0 \lesssim 10^{-12}\ \mathrm{G}\,, \tag{1}$$

where B_0 is the magnetic field today [3]. In addition, the B-seeds must have a minimum size of roughly 10 Kpc. Note that the strength is calculated at the time of completed galaxy formation, while the scale before the collapse of the protogalaxy. If magnetic fields were to decay adiabatically on all scales after inflation and given that $a \propto T^{-1}$ always (T is the temperature of the universe), we have $B_{RH} = B_0(T_{RH}/T_0)^2$ at the end of reheating. Then, recalling that $\rho \propto a^{-3}$ during reheating (ρ is the matter density), $\rho_{RH} \sim T_{RH}^4$ and $\rho_{DS} \sim M^4$ (M is the energy scale of inflation), gives

$$B_{DS} \sim B_0\, \frac{M^{8/3}}{T_0^2 T_{RH}^{2/3}}\,, \tag{2}$$

at the end of the de Sitter regime (with M, T_0 and T_{RH} in GeV). Combined, expressions (1) and (2) lead to the constraints

$$10^{-22}\frac{M^{8/3}}{T_0^2 T_{RH}^{2/3}} \lesssim B_{DS} \lesssim 10^{-12}\frac{M^{8/3}}{T_0^2 T_{RH}^{2/3}}\,. \tag{3}$$

If magnetic fields were to decay adiabatically all along their post-inflationary life and still seed the galactic dynamo today, their required strength (in Gauss) at the end of inflation must lie within the above limits. Setting $T_0 \sim 10^{-13}$ GeV, $T_{RH} \sim 10^{10}$ GeV and $M \sim 10^{17}$ GeV, we obtain

$$10^{43}\ \mathrm{G} \lesssim B_{DS} \lesssim 10^{53}\ \mathrm{G}\,. \tag{4}$$

Magnetic strengths within the above range at the end of inflation are impossible to achieve within the framework of conventional electromagnetic theory [2]. Even outside Maxwellian electromagnetism, it is generally very hard to satisfy condition (4), at least not without some unwanted side effects.

The standard picture, namely constraint (4), can drastically relax when our aforementioned causality argument is accounted for. As stated previously, causality demands that there are no electric currents and no magnetic-flux freezing on super-Hubble scales after inflation. Let us consider the implications of this claim for the post-inflationary magnetic evolution, referring the reader to [6, 7] for further discussion and details. We begin by recalling that, in the absence of electric currents and on spatially flat Friedmann-Robertson-Walker (FRW) backgrounds, the magnetic component (B_a) of the Maxwell field obeys the linear wave-like formula [2]

$$\mathcal{B}_a'' - a^2\mathrm{D}^2\mathcal{B}_a = 0\,. \tag{5}$$

[1] Inside the horizon causal physics takes over, the electric currents freeze the magnetic fields into the matter and the ideal-MHD limit applies. Therefore, on subhorizon scales, B-fields decay adiabatically (i.e. $B \propto a^{-2}$) at all times.

Note that $\mathcal{B}_a = a^2 B_a$ by definition, primes are conformal-time derivatives and $\mathrm{D}^2 = \mathrm{D}^a\mathrm{D}_a$ is the 3-D covariant Laplacian. Equation (5) accepts the solution

$$\mathcal{B}_{(n)} = \mathcal{C}_1 \cos(n\eta) + \mathcal{C}_2 \sin(n\eta)\,, \tag{6}$$

where n (with $n \geq 0$) is the comoving wavenumber of the magnetic mode. On superhorizon lengths, where $n\eta \ll 1$, the above reduces to the power law

$$\mathcal{B} = a^2 B = \mathcal{C}_1 + \mathcal{C}_2 n\eta\,. \tag{7}$$

Despite the fact that $n\eta \ll 1$, the second magnetic mode is not negligible. In fact, this can turn out to be the key mode that will decide the large-scale magnetic evolution after inflation. Indeed, when $\mathcal{C}_2 \gg \mathcal{C}_1$, the second mode on the right-hand side of solution (7) can dominate. We therefore need to evaluate the integration constants. These depend on the initial conditions, which at the beginning of reheating are determined by the magnetic evolution during inflation and by the specifics of the transition from the de Sitter regime to reheating. Similarly, one can set the initial conditions at the start of the radiation and the dust eras. In what follows, we will consider three characteristic and complementary scenarios of initial conditions.

Scenario A: Suppose that the magnetic field has been decaying adiabatically throughout inflation, as it happens in conventional primordial magnetogenesis. Then we have $B'^{\,-}_* = -2a^-_* H^-_* B^-_*$ at the end of the de Sitter expansion.[2] After inflation, throughout reheating (as well as later during the dust era) the scale factor and the conformal time are related by $a \propto \eta^2$. In these epochs, solution (7) reads

$$B = \left[B^+_* - \eta^+_*\left(2a^+_* H^+_* B^+_* + B'^{\,+}_*\right)\right]\left(\frac{a^+_*}{a}\right)^2 + \eta^+_*\left(2a^+_* H^+_* B^+_* + B'^{\,+}_*\right)\left(\frac{a^+_*}{a}\right)^{3/2}, \tag{8}$$

with $a \geq a^+_*$. Let us also assume that the Hubble parameter experiences no discontinuity during the transition from the de Sitter phase to reheating, namely that $H^+_* = H^-_*$. In line with Israel's "junction conditions" [9], this ensures that there are no surface layers on the transit surface (Σ), which is the hypersurface of constant energy density [10]. Then, given that $a^+_* = a^-_*$ always and setting $B^+_* = B^-_0$ and $B'^{\,+}_* = B'^{\,-}_*$ on either side of Σ, we obtain $B'^{\,+}_* = -2a^+_* H^+_* B^+_*$ at the start of reheating. Employing these initial conditions, solution (8) reduces to

$$B = B^+_*\left(\frac{a^+_*}{a}\right)^2, \tag{9}$$

to guarantee the adiabatic magnetic decay throughout the reheating era. A straightforward calculation shows that the adiabatic decay law persists into the radiation and the dust epochs, as long as the Hubble parameter remains continuous on the transition hypersurfaces.

Scenario B: Let us now assume that B-fields are superadiabatically amplified during inflation. More specifically, suppose that $B \propto a^{-m}$, with $0 < m < 2$, throughout the de Sitter phase. This occurs in non-conventional inflationary magnetogenesis (e.g. see [11]), giving $B'^{\,-}_* = -ma^-_* H^-_* B^-_*$ at the end of inflation proper. When there is no discontinuity in the Hubble parameter on Σ, the above becomes $B'^{\,+}_* = -ma^+_* H^+_* B^+_*$ at the start of reheating and solution (8) reads

$$B = -(3-2m)B^+_*\left(\frac{a^+_*}{a}\right)^2 + 2(2-m)B^+_*\left(\frac{a^+_*}{a}\right)^{3/2}. \tag{10}$$

Given that $m \neq 2$, the second mode on the right-hand side survives, leading to the superadiabatic amplification of the B-field throughout reheating and also during the subsequent eras of radiation and dust. In particular, as long as the field remains outside the Hubble horizon, we find $B \propto a^{-3/2}$ along reheating and dust (see solution (10) above) and $B \propto a^{-1}$ in the radiation epoch [6, 7].

[2] The $*$-suffix will always indicate the transition from one cosmological epoch to the next, while the $-$ and $+$ superscripts will respectively denote the moments just prior and immediately after the aforementioned transition.

Scenario C: Finally, suppose that magnetic fields decay adiabatically during inflation, but allow for a discontinuity in the value of the Hubble parameter on Σ (i.e. set $H_*^+ \neq H_*^-$). This implies the presence of surface layers, with a non-vanishing energy-momentum tensor, on the transition surface [9]. Then, Σ is no-longer the hypersurface of constant energy density, but that of constant conformal time [12]. Adiabatic magnetic decay during inflation means $B_*'^- = -2a_*^- H_*^- B_*^-$ at the end of that period (see scenario A above). Given that $H = -1/a\eta$ throughout the de Sitter expansion, the latter recasts into $\eta_*^- B_*'^- = 2B_*^-$ and then into $\eta_*^+ B_*'^+ = -2B_*^+$, since $\eta_*^+ = -\eta_*^-$ (recall that $\eta < 0$ during inflation and $\eta > 0$ afterwards). Substituting these initial conditions into the right-hand side of (8), while keeping in mind that $H = 2/a\eta$ all along reheating, gives

$$B = -B_*^+ \left(\frac{a_*^+}{a}\right)^2 + 2B_*^+ \left(\frac{a_*^+}{a}\right)^{3/2}, \tag{11}$$

where $a \geq a_*^+$. This result guarantees the superadiabatic amplification of the B-field throughout reheating, as well as during the subsequent eras of radiation and dust. As in scenario B before, we find that $B \propto a^{-1}$ during the radiation epoch and $B \propto a^{-3/2}$ during dust [6, 7].

The first of the above scenarios has essentially reproduced the standard model of conventional inflationary magnetogenesis, which leads to astrophysically irrelevant magnetic fields today. Scenario B affects the non-conventional mechanisms of primordial magnetic generation, which typically enhance their B-fields during inflation and can lead to even stronger residual B-fields than those typically expected. This is “good news” for mechanisms that achieve relatively mild magnetic enhancement during the de Sitter phase. Indeed, since the amplification of the field persists after inflation as well, the final magnetic strength can come within the galactic dynamo requirements. On the other hand, scenario B is “bad news” for mechanisms achieving strong inflationary amplification, because the subsequent enhancement of the field can lead to excessively strong magnetic seeds that are at odds with the observational constraints. Finally, scenario C affects conventional magnetogenesis. Typically, the residual magnetic strengths achieved in this scenario are not within the standard dynamo requirements. Nevertheless, the final magnitudes are close enough to suggest that conventional magnetogenesis may still be able to provide the seeds for the galactic dynamo. In what follows we will take a closer look at the last two scenarios.

The magnetic strengths achieved through scenarios B and C depend on the time the B-field reenters the Hubble horizon, which marks the transition from superadiabatic amplification (outside the Hubble radius) to adiabatic decay (inside the Hubble length). The latter is decided by the coherence scale of the field, with the larger scales crossing inside the horizon later. In general, the longer a magnetic mode stays outside the horizon the stronger its superadiabatic amplification, in which case the standard de Sitter conditions (see expressions (3) and (4)) can relax considerably. Next we will re-evaluate these constraints for magnetic fields reentering the Hubble radius in the late radiation epoch and during the dust era respectively.

(i) Suppose that a magnetic mode reenters the Hubble radius during the radiation epoch. After that the B-field decays adiabatically (i.e. $B \propto a^{-2}$). Then if B_0 is the current strength of the field, we have $B_{HC} = B_0(T_{HC}/T_0)^2$ at horizon crossing. Earlier in the radiation era, as well as throughout reheating, the mode was outside the Hubble scale and decayed as $B \propto a^{-1}$ and $B \propto a^{-3/2}$ respectively. Then, a straightforward calculation gives

$$B_{DS} \sim B_0 \, \frac{T_{HC} M^2}{T_0^2 T_{RH}}, \tag{12}$$

at the end of the de Sitter regime (recall that $\rho_{DS} \sim M^4$ and $\rho_{RH} \sim T_{RH}^4$ in natural units). Solving for B_0 and substituting into (1) we arrive at

$$10^{-22} \frac{T_{HC} M^2}{T_0^2 T_{RH}} \lesssim B_{DS} \lesssim 10^{-12} \frac{T_{HC} M^2}{T_0^2 T_{RH}}. \tag{13}$$

Table 1: The strength-range of inflationary magnetic fields, measured at the end of the de Sitter phase, capable of seeding the galactic dynamo today (compare to the standard – 'adiabatic' – range given in Eq. (4)). The B-fields of the first row have the minimum required (comoving) coherence scale. In the third and the forth row the associated magnetic modes have re-entered the Hubble horizon at recombination and today respectively. Note that $M \sim 10^{17}$ GeV and $T_{RH} \sim 10^{10}$ GeV in all cases [7].

λ_0 (Mpc)	T_{HC} (GeV)	B_{DS} (G)
10^{-2}	10^{-6}	$10^{22} \lesssim B_{DS} \lesssim 10^{32}$
1	10^{-8}	$10^{20} \lesssim B_{DS} \lesssim 10^{30}$
$10^{3/2}$	10^{-10}	$10^{18} \lesssim B_{DS} \lesssim 10^{28}$
10^{3}	10^{-13}	$10^{17} \lesssim B_{DS} \lesssim 10^{27}$

Magnetic fields with strengths (in Gauss) within the above range at the end of inflation will be able to seed the galactic dynamo today. Let us consider a particular case, namely B-fields with current scales around 10 Kpc (the minimum required by the dynamo). Given that $\lambda \propto a$ at all times and $\lambda_H \propto a^{3/2}$ in the dust epoch, we obtain $(\lambda_H/\lambda)_{EQ} = (\lambda_H/\lambda)_0(T_0/T_{EQ})^{1/2} \sim 10^3$ at the time of matter-radiation equality. Setting $T_{EQ} \sim 10^{-9}$ GeV, $T_0 \sim 10^{-13}$ GeV and $(\lambda_H)_0 \sim 10^3$ Mpc, a mode of approximately 10 Kpc at present crossed inside the horizon before equipartition, which means that $(\lambda_H/\lambda)_{HC} = (\lambda_H/\lambda)_{EQ}(T_{EQ}/T_{HC})$. The latter ensures that $T_{HC} \sim 10^{-6}$ GeV. Then, setting $T_{RH} \sim 10^{10}$ GeV and adopting GUT-scale inflation with $M \sim 10^{17}$ GeV, we obtain

$$10^{22}\ \mathrm{G} \lesssim B_{DS} \lesssim 10^{32}\ \mathrm{G}\,. \tag{14}$$

Consequently, inflationary B-fields close to 10 Kpc today can seed the galactic dynamo provided their strength lies within the above given range. These constraints are weaker than their standard "adiabatic" counterparts by many orders of magnitude (compare the above to condition (4)).

(ii) Let us now consider magnetic fields crossing the Hubble radius after matter-radiation equality. If B_0 is the magnetic strength today, then $B_{HC} = B_0(T_{HC}/T_0)^2$ at horizon entry. Given that $B \propto a^{-3/2}$ on super-Hubble scales throughout the dust and the reheating eras, while $B \propto a^{-1}$ during the radiation epoch, an exactly analogous calculation gives

$$B_{DS} \sim B_0 \frac{T_{HC}^{1/2} T_{EQ}^{1/2} M^2}{T_0^2 T_{RH}}\,, \tag{15}$$

at the end of inflation. This field will lie within the dynamo strength-requirements today if

$$10^{-22} \frac{T_{HC}^{1/2} T_{EQ}^{1/2} M^2}{T_0^2 T_{RH}} \lesssim B_{DS} \lesssim 10^{-12} \frac{T_{HC}^{1/2} T_{EQ}^{1/2} M^2}{T_0^2 T_{RH}}\,, \tag{16}$$

by the end of the de Sitter expansion. For example, a magnetic field entering the horizon today (i.e. with $T_{HC} \sim 10^{-13}$ GeV) will seed the galactic dynamo provided its strength satisfied the constraint

$$10^{17}\ \mathrm{G} \lesssim B_{DS} \lesssim 10^{27}\ \mathrm{G}\,, \tag{17}$$

at the end of inflation. The above limits are more relaxed than those of (14) and far more than their adiabatic counterparts of (4). Overall, once the current scale of a magnetic mode is given, we can calculate the time of horizon entry and then find the required strength range at the end of the de Sitter phase, if these B-fields were to seed the galactic dynamo today (see TABLE 1).

The relaxed de Sitter limits given in (14) and (17) are relatively straightforward to satisfy by non-conventional scenarios of inflationary magnetogenesis, without facing the so-called backreaction problems [13]. For instance, mechanisms producing magnetic fields around 10^{22} G by the end of the de Sitter phase, with current close to 1 Mpc, have been discussed in [14]. Although such fields cannot satisfy the standard (adiabatic) conditions (1), they lie within the new relaxed limits discussed here (see second row in TABLE 1). Thus, causality and the absence of superhorizon-sized electric currents make it much easier for (non-conventional) inflationary magnetogenesis to produce the seeds required by the galactic dynamo today. There is no longer the need for a strong magnetic amplification during the de Sitter regime. Conventional inflationary magnetic fields do not satisfy the new relaxed conditions, but they can come fairly close. For instance, a conventionally produced B-field, with current size close 10 Kpc, had strength around 10^{12} G at the end of the de Sitter regime, which lies below the lower limits given in (14) and (17). Nevertheless, these limits can relax further if we allow for a very brief period of stiff-matter domination between the reheating and the radiation epochs [15]. During such a phase, $a \propto \eta^{1/2}$ and the dominant mode of (7) remains constant [6], while the matter energy density depletes as $\rho \propto a^{-6}$. All these mean that a stiff-matter era between $T_{RH} \sim 10^{10}$ GeV and $T_{SM} \sim 10^{7}$ GeV will weaken condition (17) to 10^{10} G $\lesssim B_{DS} \lesssim 10^{20}$ G at the end of inflation. The latter is readily satisfied by typical conventionally produced inflationary magnetic fields.

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