Relaxing the limits on inflationary magnetogenesis

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Facts & Open Questions

Magnetic fields are everywhere

From the Earth all the way out to remote protogalaxies
and recently
in the intergalactic space (?)

Origin as yet unknown

Late-time or early time (?)

Many scenarios, but the issue is still open

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Early-time Magnetogenesis

Galactic dynamo requirements

Minimum coherence length

$(\lambda_B)_0 \sim 10$ Kpc

Strength-range

$10^{-22}\ \text{G} \lesssim B_0 \lesssim 10^{-12}\ \text{G}$

Post-inflationary B-Fields

Problem

Too small correlation lengths:
$(\lambda_B)_0 \ll 10$ Kpc

Solution

"Inverse cascade" (?)

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Typically too weak:
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Outside classical EM (?)

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Advantage

Very large correlation lengths: $(\lambda_B)_0 \gg 10$ Kpc

Disadvantage

Extremely weak today: $B_0 \lesssim 10^{-53}$ G

Reason

Adiabatic magnetic decay

$$B \propto a^{-2}$$

Throughout the lifetime of the universe

On all scales

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Inflationary Magnetic Fields

On spatially flat FRW backgrounds

$$\ddot{B}_a + 5H\dot{B}_a + 3(1-w)H^2B_a - \mathrm{D}^2B_a = \mathcal{J}_a\,,$$

to linear order. Setting $\mathcal{B}_a = a^2B_a$,

$$\mathcal{B}_a'' - a^2\mathrm{D}^2\mathcal{B}_a = a^2\mathcal{J}_a\,.$$

During inflation

The universe is a very poor conductor (i.e. $\mathcal{J}_a = 0$). Therefore.

$$\mathcal{B}_a'' - a^2\mathrm{D}^2\mathcal{B}_a = 0\,.$$

Then, at horizon crossing,

$$\mathcal{B}_{(k)} = a^2B_{(k)} = C_1\cos(k\eta) + C_2\sin(k\eta)\,.$$

Well outside the horizon

When $\lambda_B \gg \lambda_H \Leftrightarrow k\eta \ll 1$,

$$a^2B_{(k)} = \mathcal{C}_1 + \mathcal{C}_2k\eta\,, \qquad with \quad a = a(\eta)\,.$$

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Large-scale, Post-inflationary Magnetic Evolution (I)

On subhorizon scales

- Electric currents are formed
- The currents freeze the *B*-fields into the matter
- The magnetic flux remains conserved
- The *B*-fields decay adiabatically

On superhorizon scales

- There are no electric currents
- The magnetic fields are causally disconnected
- The magnetic freezing-in process is causal
- The *B*-fields will freeze-in once they have come into full causal contact
- The *B*-fields still retain the memory of their distant past

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Large-scale, Post-inflationary Magnetic Evolution (II)

Evolution during reheating & dust $(a \propto \eta^2)$

$$B = -\left(3B_* + \eta_* B'_*\right)\left(\frac{a_*}{a}\right)^2 + \left(4B_* + \eta_* B'_*\right)\left(\frac{a_*}{a}\right)^{3/2} .$$

Evolution during radiation $(a \propto \eta)$

$$B = -\left(B_* + \eta_* B'_*\right)\left(\frac{a_*}{a}\right)^2 + \left(2B_* + \eta_* B'_*\right)\left(\frac{a_*}{a}\right) .$$

Superadiabatic amplification when $\lambda_B \gg \lambda_H$

As long as $4B_* + \eta_* B'_* \neq 0$ and $2B_* + \eta_* B'_* \neq 0$,

$$B \propto a^{-3/2} \quad \text{and} \quad B \propto a^{-1} .$$

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The Role of the Initial Conditions

Scenario No 1 (standard)

Adiabatic decay during inflation & no “surface layers” ($[H]^{+}_{-} = 0$)

The slowly decaying modes do not survive ($B_0 \lesssim 10^{-53}$ G)

Scenario No 2 (non-conventional)

Non-adiabatic decay during inflation (e.g. $B \propto a^{-m}$, with $0 < m < 2$)

The slowly decaying modes survive ($B_0 \gg 10^{-53}$ G)

Scenario No 3 (conventional)

Adiabatic decay during inflation & finite “surface layers” ($[H]^{+}_{-} \neq 0$)

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Scenario No 2

Strong inflationary amplification ($B \propto a^{-1/2}$)

$B_0 \sim 10^{-2}$ G (for $M \sim 10^{17}$ GeV & $T_{RH} \sim 10^{10}$ GeV)

Mild inflationary amplification ($B \propto a^{-3/2}$)

$B_0 \sim 10^{-25}$ G (for $M \sim 10^{17}$ GeV & $T_{RH} \sim 10^{10}$ GeV)

Reverse engineering the galactic-dynamo constraints

Today the dynamo requires

10^{-22} G $\lesssim B_0 \lesssim 10^{-12}$ G

At the end of inflation,

λ_0 (Mpc)	T_{HC} (GeV)	B_{DS} (G)
10^{-2}	10^{-6}	$10^{22} \lesssim B_{DS} \lesssim 10^{32}$
1	10^{-8}	$10^{20} \lesssim B_{DS} \lesssim 10^{30}$
$10^{3/2}$	10^{-10}	$10^{18} \lesssim B_{DS} \lesssim 10^{28}$
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Scenario No 3

Residual magnetic field

$$(\lambda_B)_0 \sim 10 \text{ Kpc} \quad \mapsto \quad T_{HC} \sim 10^{-6} \text{ GeV}$$

$$B_0 \simeq 10^{-33} \left(\frac{M}{10^{17}}\right)^{2/3} \left(\frac{T_{RH}}{10^{10}}\right)^{1/3} \text{ G}\,.$$

Subsequent amplification

Spherically symmetric galactic collapse $\mapsto$ $B_0 \sim 10^{-29}$ G

Anisotropic galactic collapse $\mapsto$ $B_0 \sim 10^{-27}$ G

Additional amplification (?)

When $w = 1$ (stiff matter) $\Rightarrow$ $B =$ constant

Stiff-matter era (from, say, $T_{RH} \sim 10^{10}$ GeV to $T \sim 10^{4}$ GeV)

$B_0 \sim 10^{-21}$ G

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Summary

By appealing to causality

- The large-scale magnetic decay may slow down
- Strong inflationary amplification may not be necessary
- Conventional magnetogenesis might still work

Thanks

and

Merry Christmas