28TH TEXAS SYMPOSIUM ON RELATIVISTIC ASTROPHYSICS
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MODELLING INHOMOGENEOUS COSMOLOGIES WITH NUMERICAL RELATIVITY

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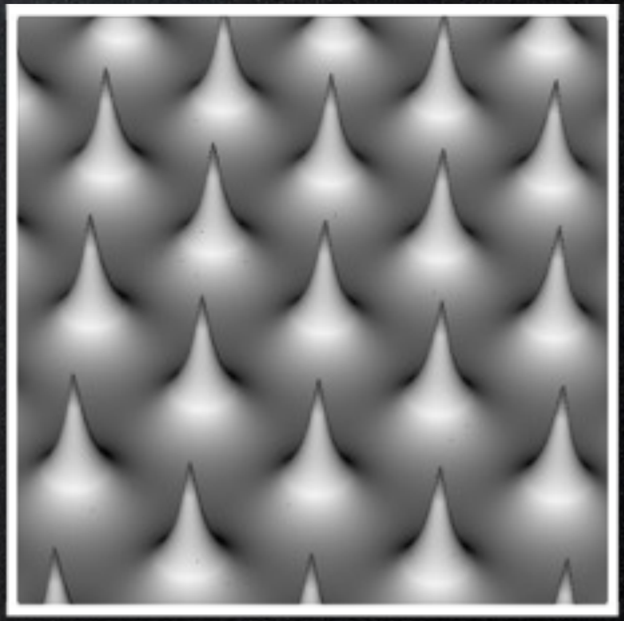
Overview

- **Modelling the Universe**
  - ΛCDM and beyond
  - Effects of Inhomogeneities & observations
- **Numerical Relativity for Cosmology**
  - Contracting black-hole lattices
  - Expanding black-hole lattices
  - Dust cosmologies
- **Requirements and feasibility**



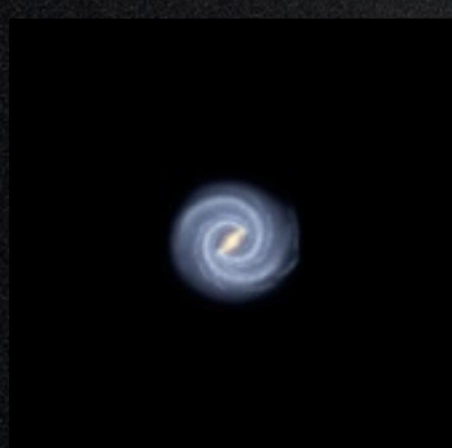
Inhomogeneous cosmologies

ΛCDM and Beyond

Standard model ("ΛCDM") based on three ingredients:

1. Class of exact solutions: homogeneous and isotropic, Friedmann-Lemaitre-Robertson-Walker (FLRW);
2. Cosmological perturbation theory around FLRW: large-scale fluctuations;
3. Newtonian methods (such as N-body codes): small scales fluctuations, non-perturbative collapse and structure formation.

Non-Relativistic
Non-Linear



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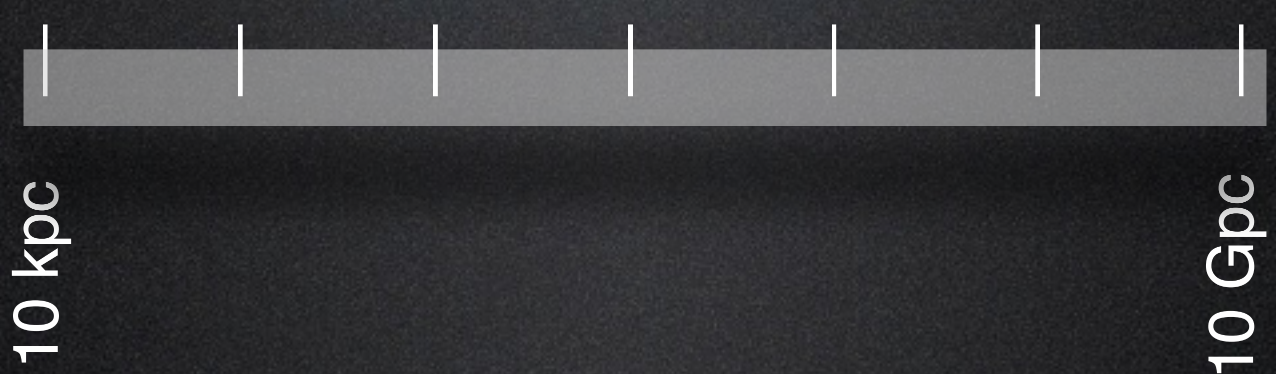

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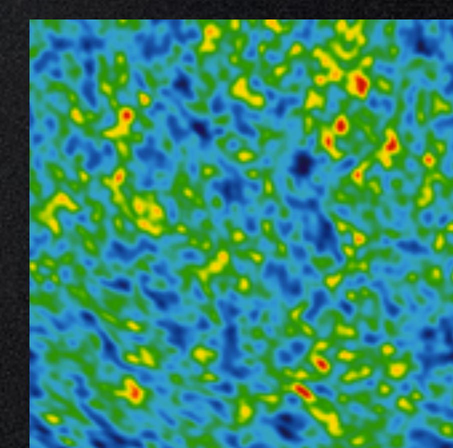

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Inhomogeneous cosmologies

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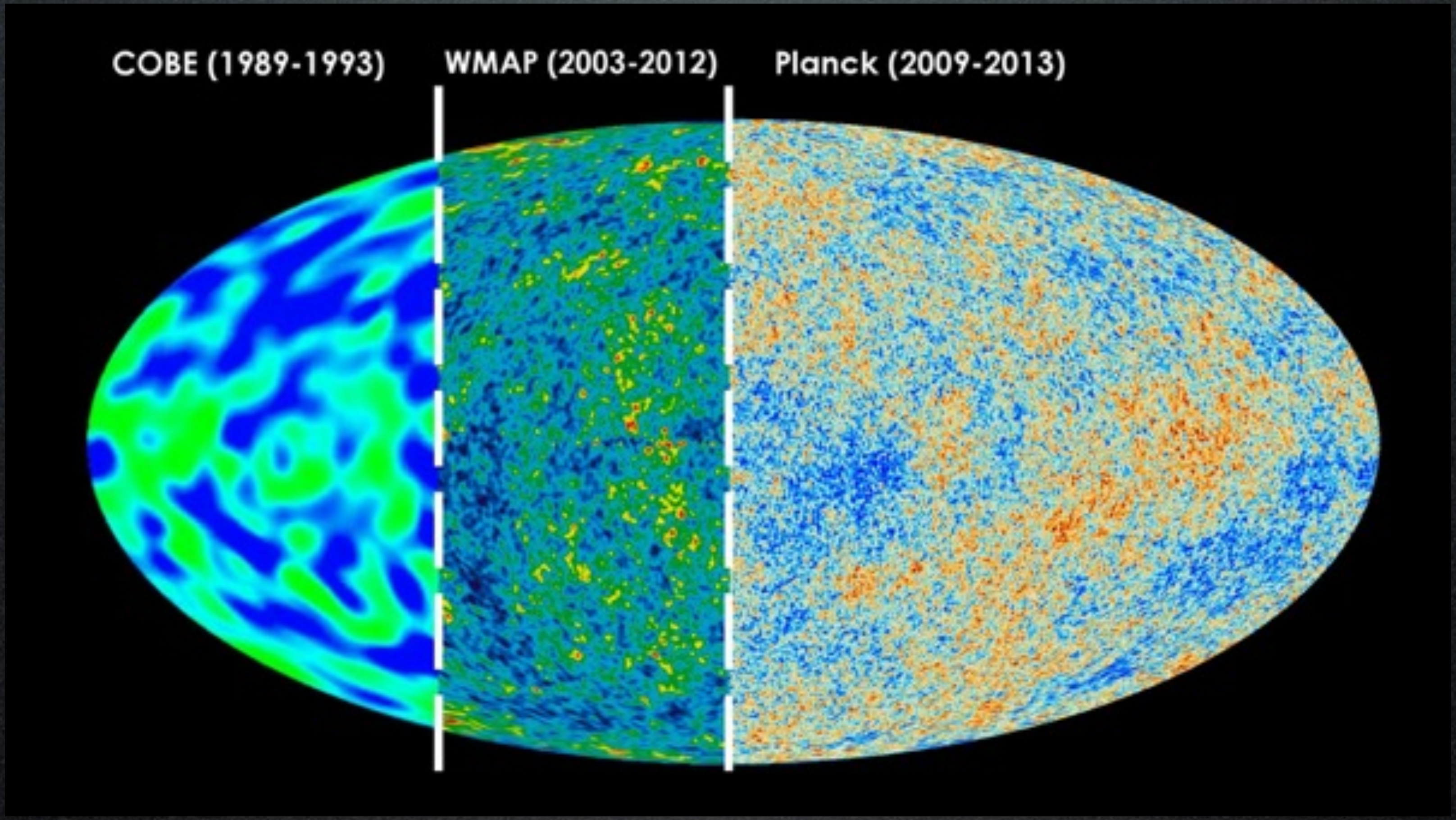

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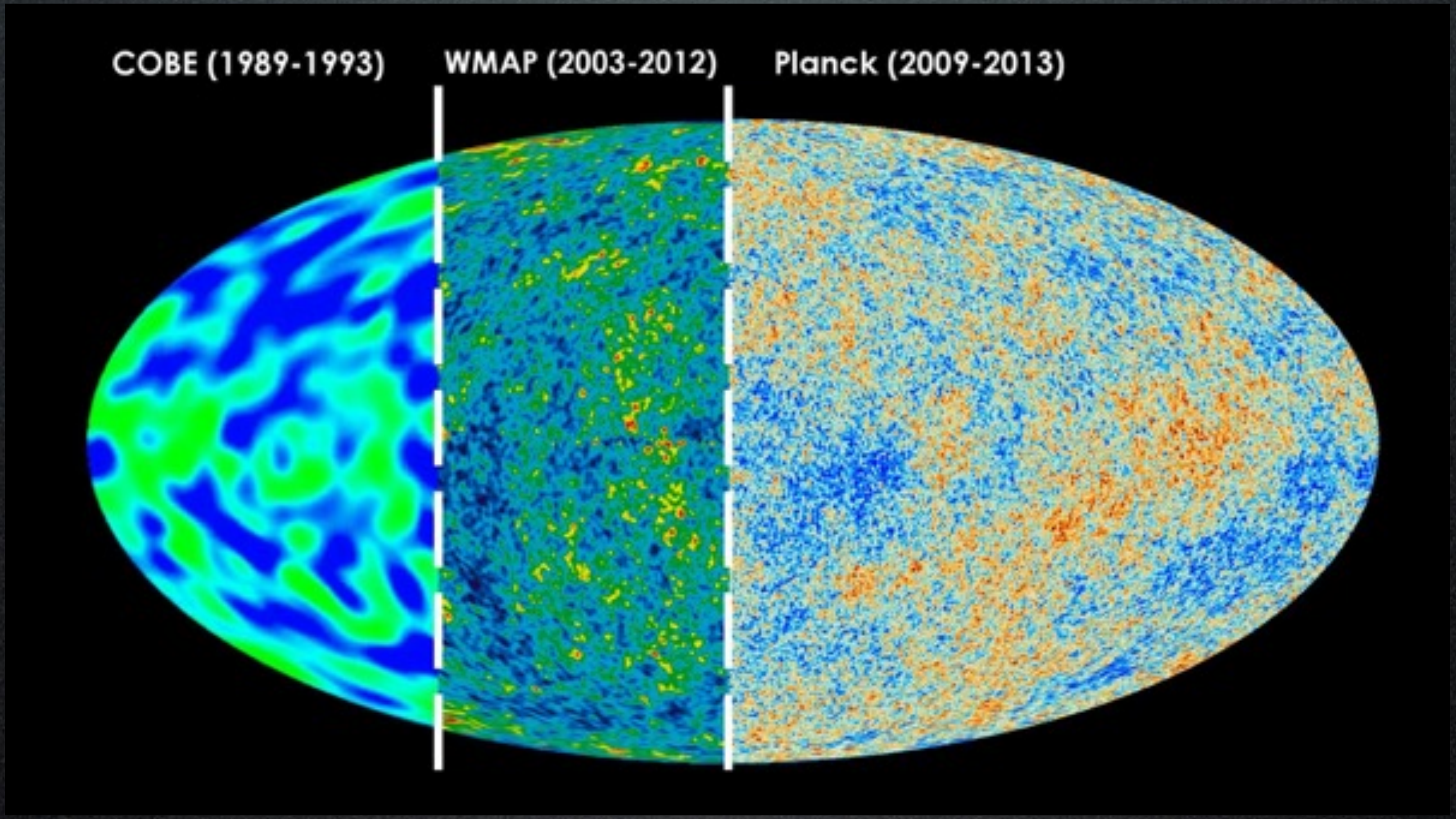

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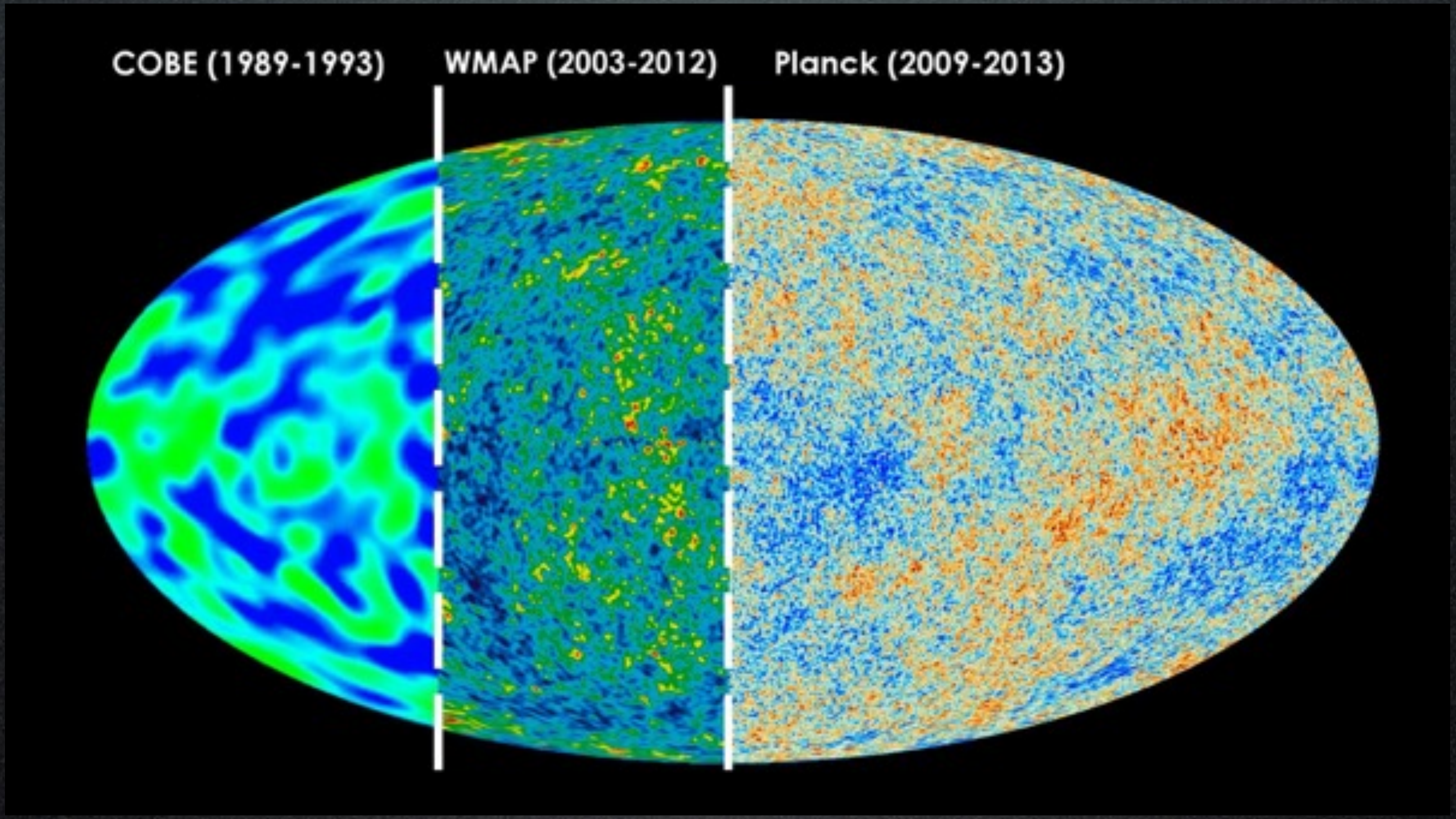

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- Along with better experiments and better data analysis, we will need better modelling!

Inhomogeneous cosmologies

Effects of Inhomogeneity

Inhomogeneous cosmologies

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INHOMOGENEOUS COSMOLOGIES

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  - Averaging becomes highly non-trivial.

Inhomogeneous cosmologies

The Numerical-Relativity Perspective

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  - Large-scale structure: **Anninos, Centrella, McKinney, Wilson (1984, 1985, 1999), Shibata (1999), Bentivegna, Korzynski, Hinder, Bruni (2012-2015), Yoo, Okawa, Nakao (2012-2014), Torres, Alcubierre, Diez-Tejedor, Nunez, de la Macorra (2014-2015), Rekier, Cordero-Carrion, Fuzfa (2015)**

Numerical Relativistic Cosmology

The 3+1 decomposition

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Einstein's equation can be solved exactly by formulating it as an initial-boundary value problem, and integrating numerically. One needs to choose a time coordinate and project the equations accordingly; reducing the system to first-order form, one is left with twelve evolution four constraints equations:

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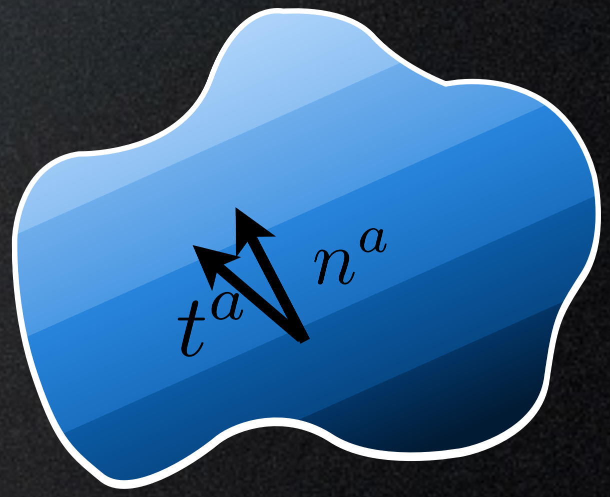

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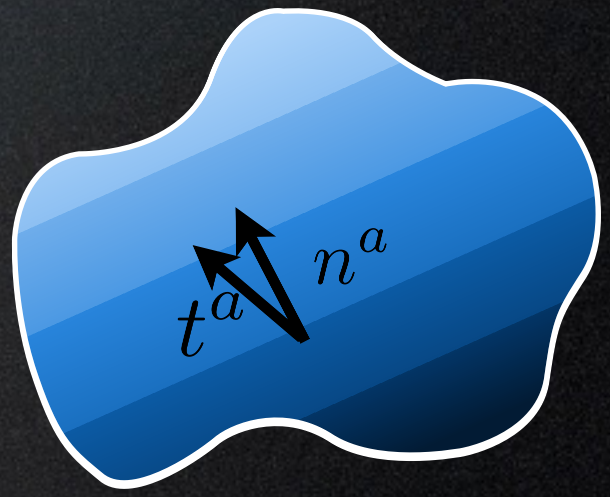

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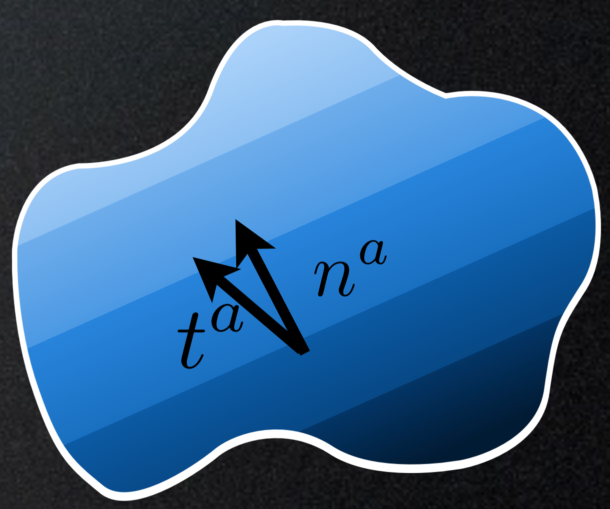

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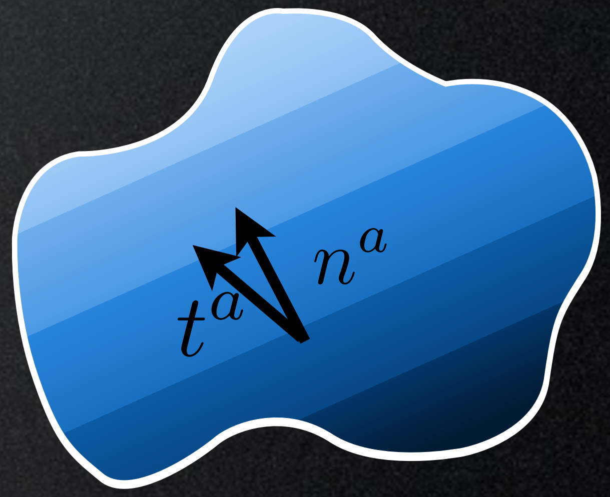

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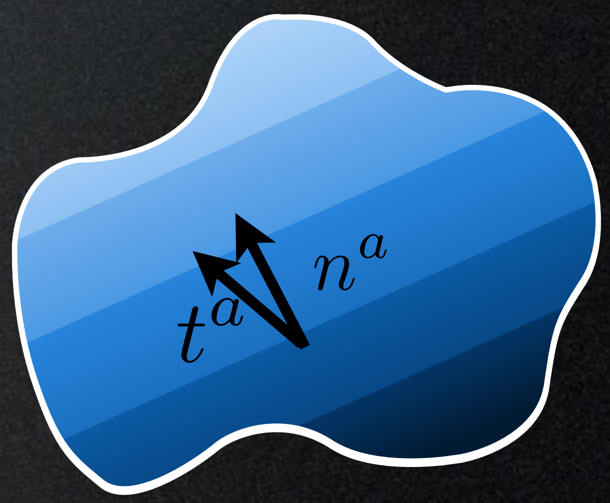


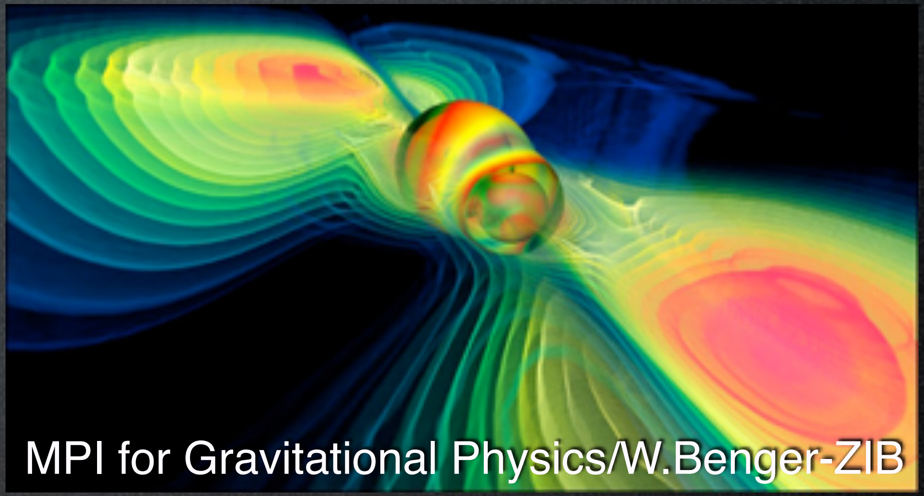
Numerical Relativistic Cosmology

State of the art

Compact objects are the traditional application area of numerical relativity: they play an important observational role, since they drive galactic activity and evolution, are thought to be at the core of a class of Gamma Ray Bursts, and are powerful gravitational-wave emitters when excited.

Thanks to numerical relativity, a number of these scenarios have been under scrutiny in the last ten years, with many more being actively pursued now:

- Black-hole binaries
- Neutron-star binaries
- Mixed binaries
- Gravitational collapse and supernovae
- Black holes surrounded by accretion disks



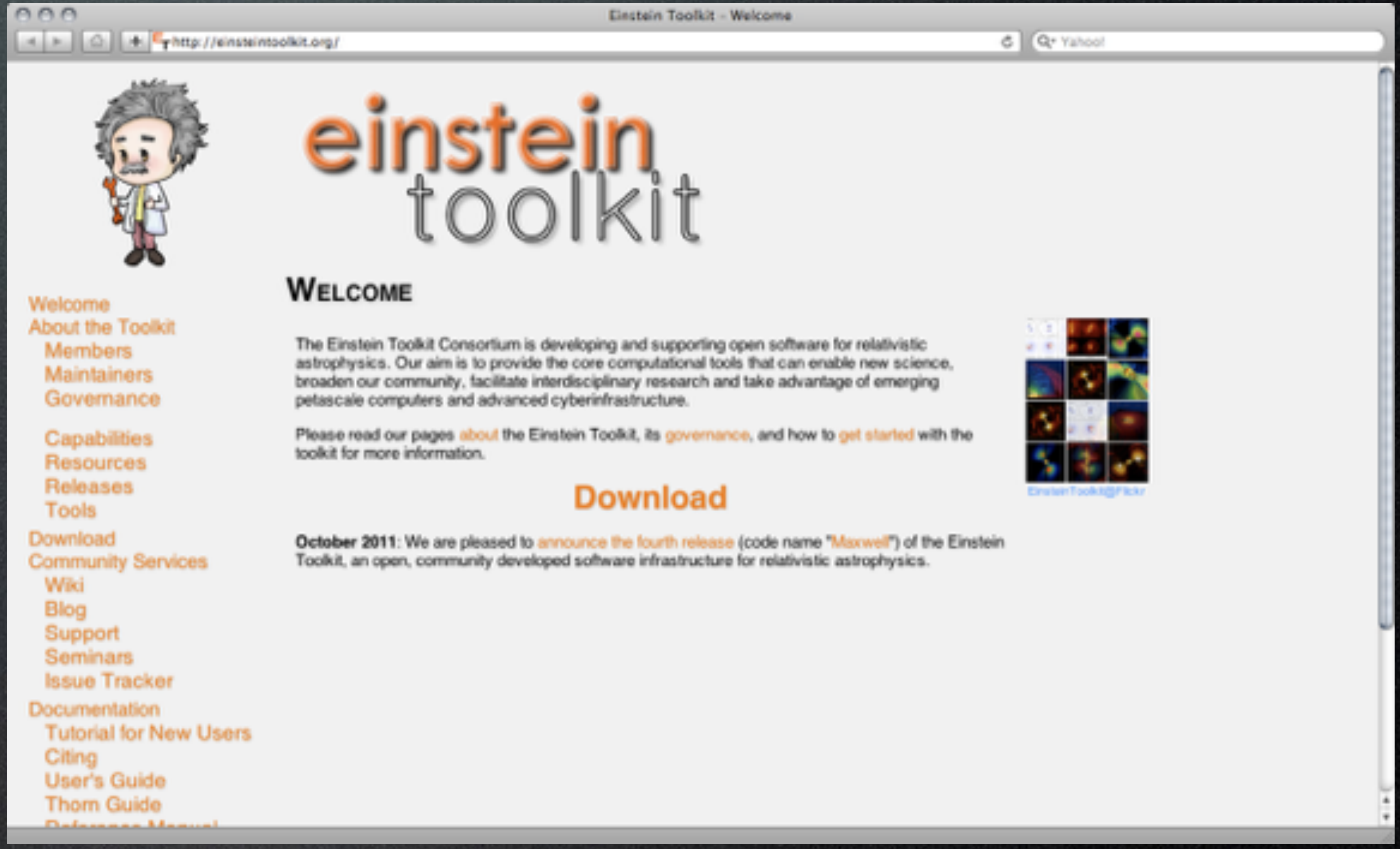
The methods and tools of Numerical Relativity can in principle help construct general, exact spacetimes, thereby providing a laboratory to study any system at will (but beware of assumptions…).

Numerical Relativistic Cosmology

Community Software for Numerical Relativity

The Einstein Toolkit:

- Open-source toolkit;
- One code-generating framework;
- Over one hundred components (evolution of the gravitational field and fluids, analysis of spacetimes, I/O);
- AMR capabilities;
- Leveraging HPC systems worldwide;
- Tutorials and demos for new users — try it out!

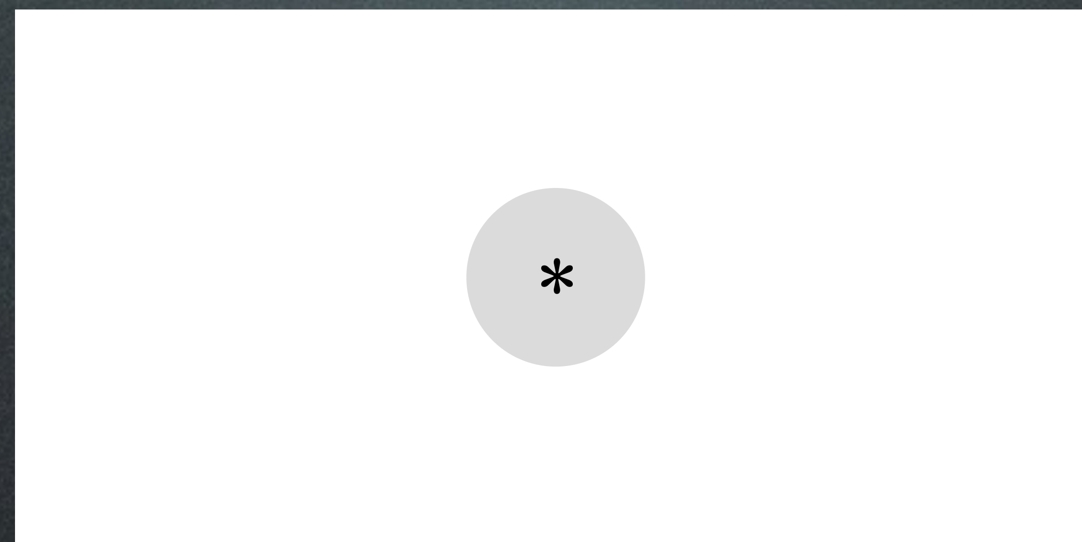


Numerical Relativistic Cosmology

black-hole lattices

NUMERICAL RELATIVISTIC COSMOLOGY

BLACK-HOLE LATTICES



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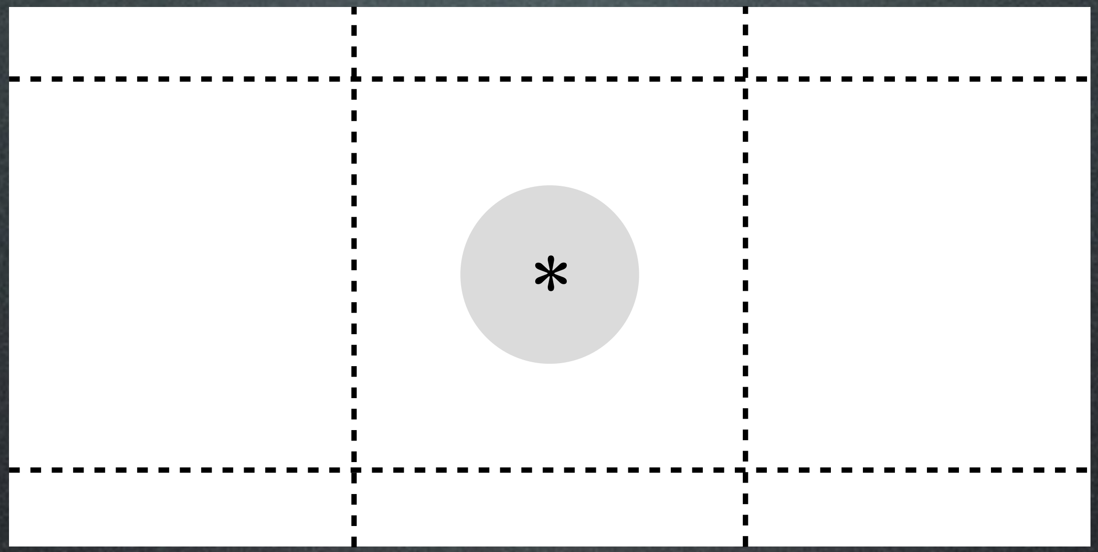

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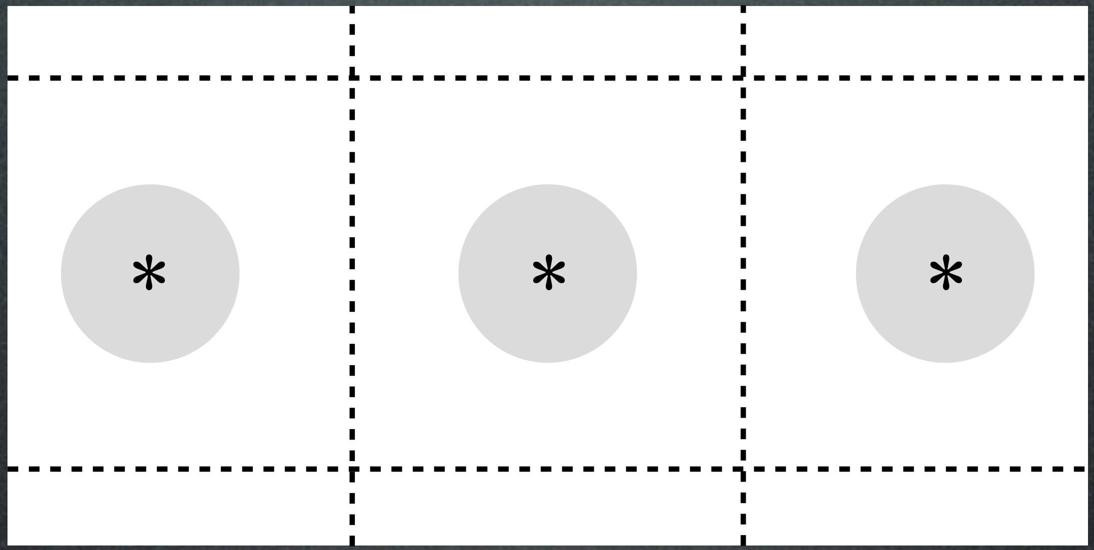
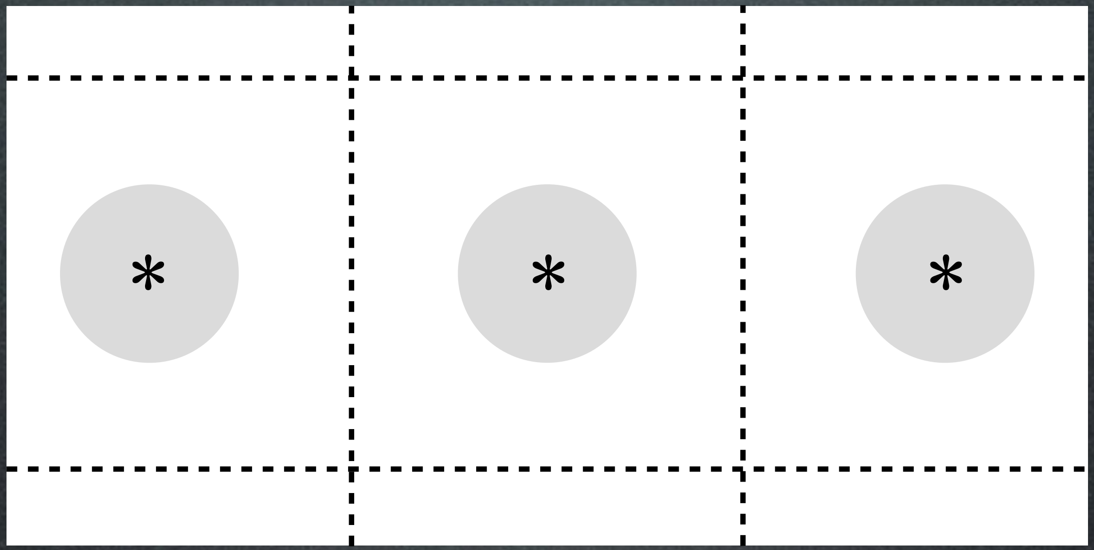


Numerical Relativistic Cosmology

Black-hole lattices



[Lindquist&Wheeler 1957]

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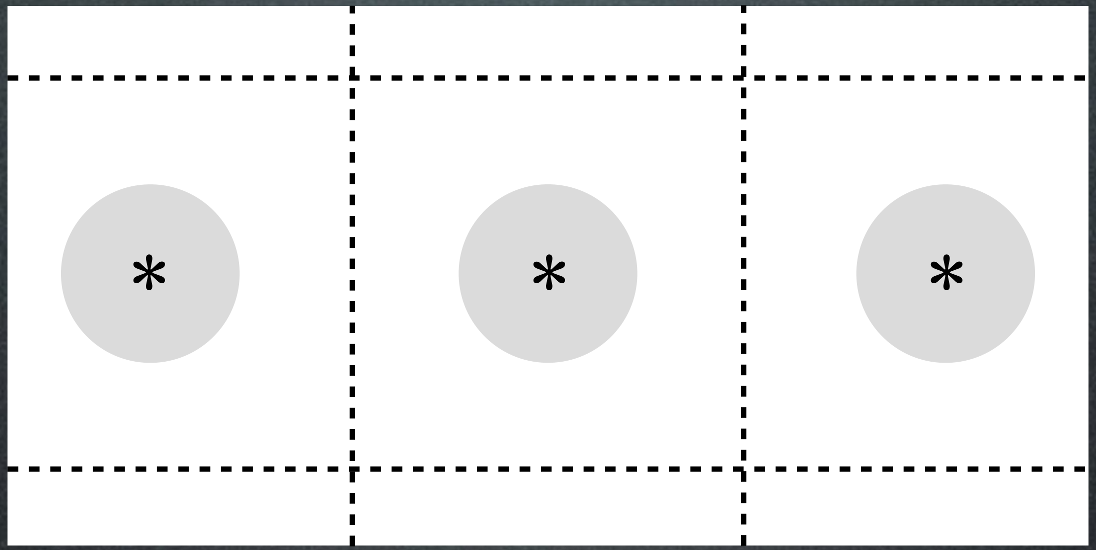

[Lindquist&Wheeler 1957]

Since LW, several roads:

- Junction conditions [Clifton 2009]
- Series expansions [Bruneton&Larena 2012]
- Solving the constraints [Wheeler 1983, Clifton et al. 2012, Yoo et al. 2012, Bentivegna&Korzyński 2012, Yoo et al. 2013, Bentivegna&Korzyński 2013]

Numerical Relativistic Cosmology

Multi-black-hole systems

Numerical Relativistic Cosmology

Multi-black-hole systems

$$\tilde{\Delta}\psi - \frac{\tilde{R}}{8}\psi - \frac{K^2}{12}\psi^5 + \frac{1}{8}\tilde{A}_{ij}\tilde{A}^{ij}\psi^{-7} = -2\pi\rho\psi^5$$

$$\tilde{D}_i\tilde{A}^{ij} - \frac{2}{3}\psi^6\tilde{\gamma}^{ij}\tilde{D}_j K = 8\pi\psi^{10}j^j$$

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Two options:

Numerical Relativistic Cosmology

Multi-black-hole systems

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Requires:

1) Numerical integration;
2) Extreme care with periodic boundaries.

Contracting Black-hole lattices

Initial Data

Hamiltonian constraint:

$$\tilde{\Delta}\psi - \frac{\tilde{R}}{8}\psi = 0$$

Spatial metric given by:

$$\mathrm{d}s^2 = \mathrm{d}\lambda^2 + \sin^2\lambda(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2)$$

$$\lambda \in [0, \pi]$$
$$\theta \in [0, \pi]$$
$$\varphi \in [0, 2\pi]$$

A solution:

$$\psi(\lambda) = \frac{A}{\sin\lambda/2}$$

Contracting Black-hole lattices

Initial Data

Multiple black holes can be obtained by superimposing this fundamental solution. It is convenient to embed this three-sphere in R^4, and to express the solution in this coordinate space:

$$\psi(\bar{X}) = \sum_{i=1}^{N} \frac{A_i}{\sin \lambda_i/2} = \sum_i A_i \sqrt{\frac{2}{1 - \bar{X} \cdot \bar{N}_i}}$$

The parameters A_i and the black-hole centers are arbitrary, but if one is interested in regular lattices these have to be chosen carefully. In particular, the parameters A_i have to be the same, and the centers have to be equidistant from each other.

Curious cases:

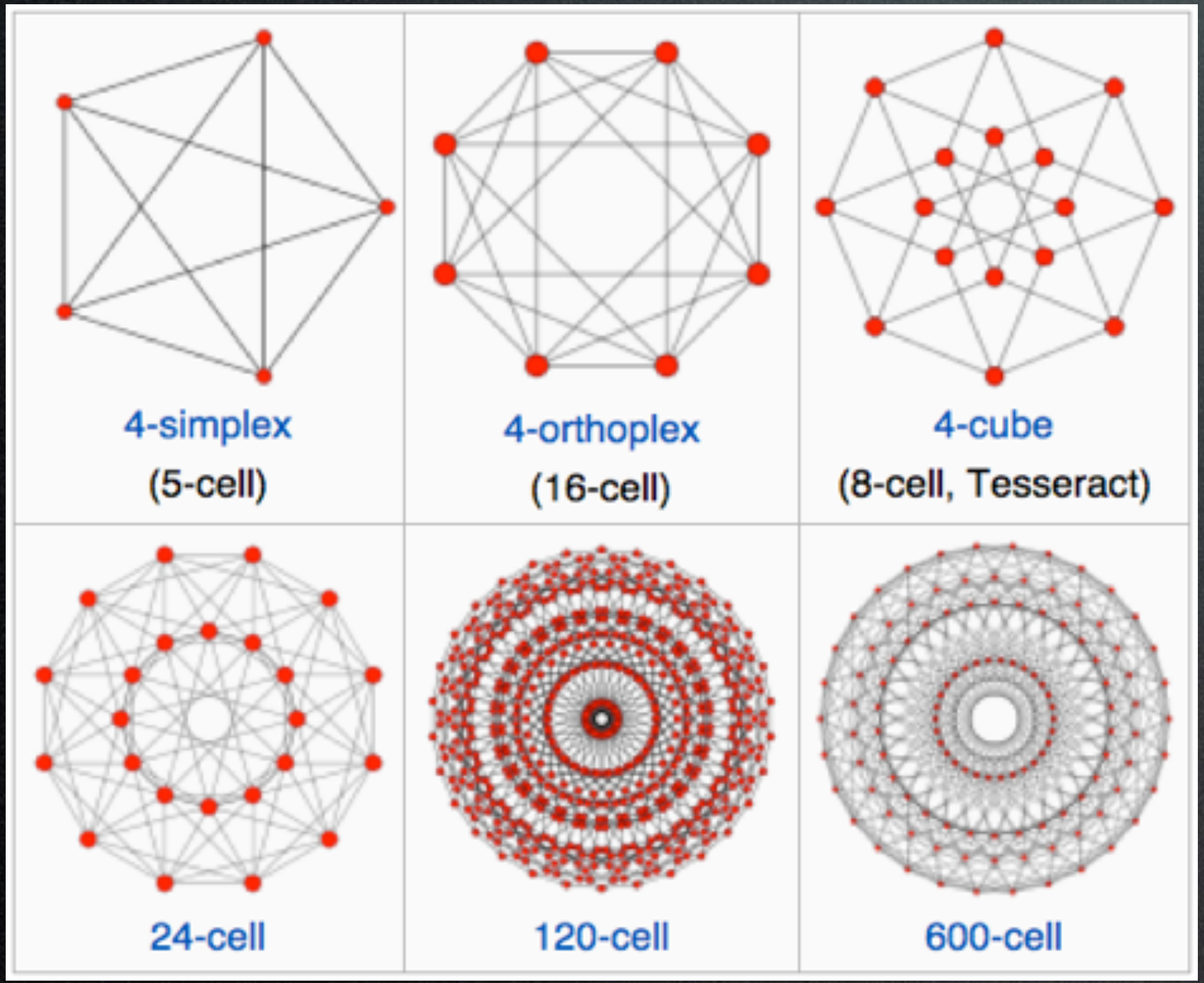
$$N = 1 \quad \mathrm{d}s^2 = \frac{1}{\sin^4 \lambda/2}(\mathrm{d}\lambda^2 + \sin^2 \lambda(\mathrm{d}\theta^2 + \sin^2 \theta \mathrm{d}\phi^2))$$

$$N = 2 \quad \mathrm{d}s^2 = \left(\frac{1}{\sin \lambda/2 + \sin(\lambda - \pi)/2}\right)^4 (\mathrm{d}\lambda^2 + \sin^2 \lambda(\mathrm{d}\theta^2 + \sin^2 \theta \mathrm{d}\phi^2))$$

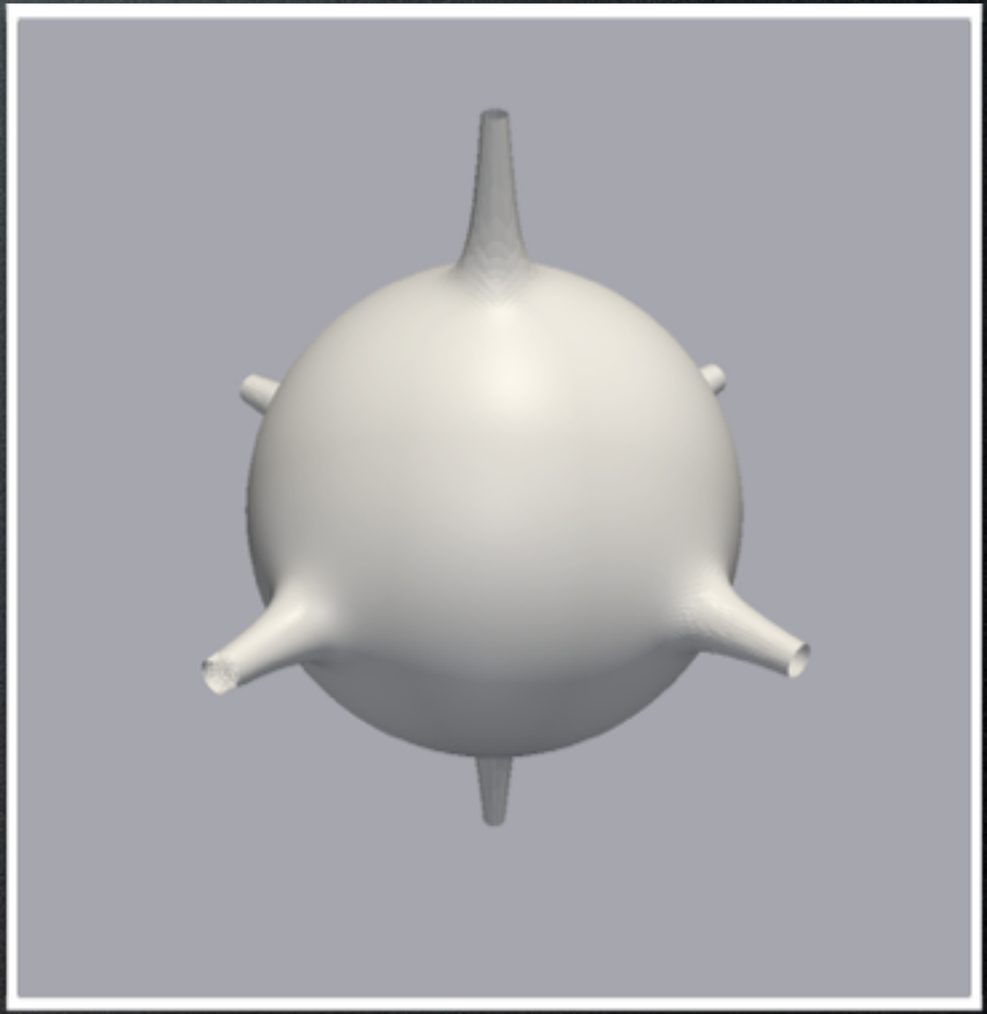
Contracting Black-hole lattices

Initial Data

On a three-sphere, there is only a finite number of “regular” arrangements of points, corresponding to the regular tessellations of S^3. N can be equal to 5, 8, 16, 24, 120, 600.



en.wikipedia.org



Contracting Black-hole lattices

Length scaling

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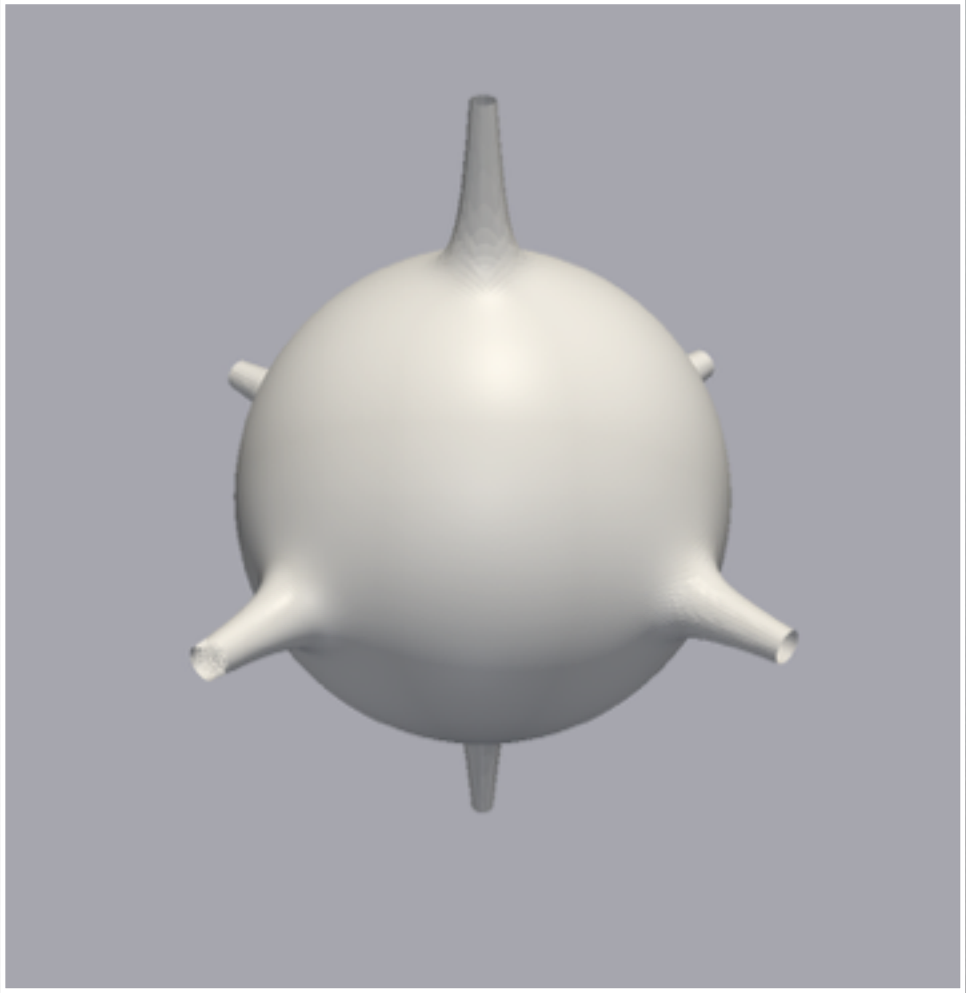

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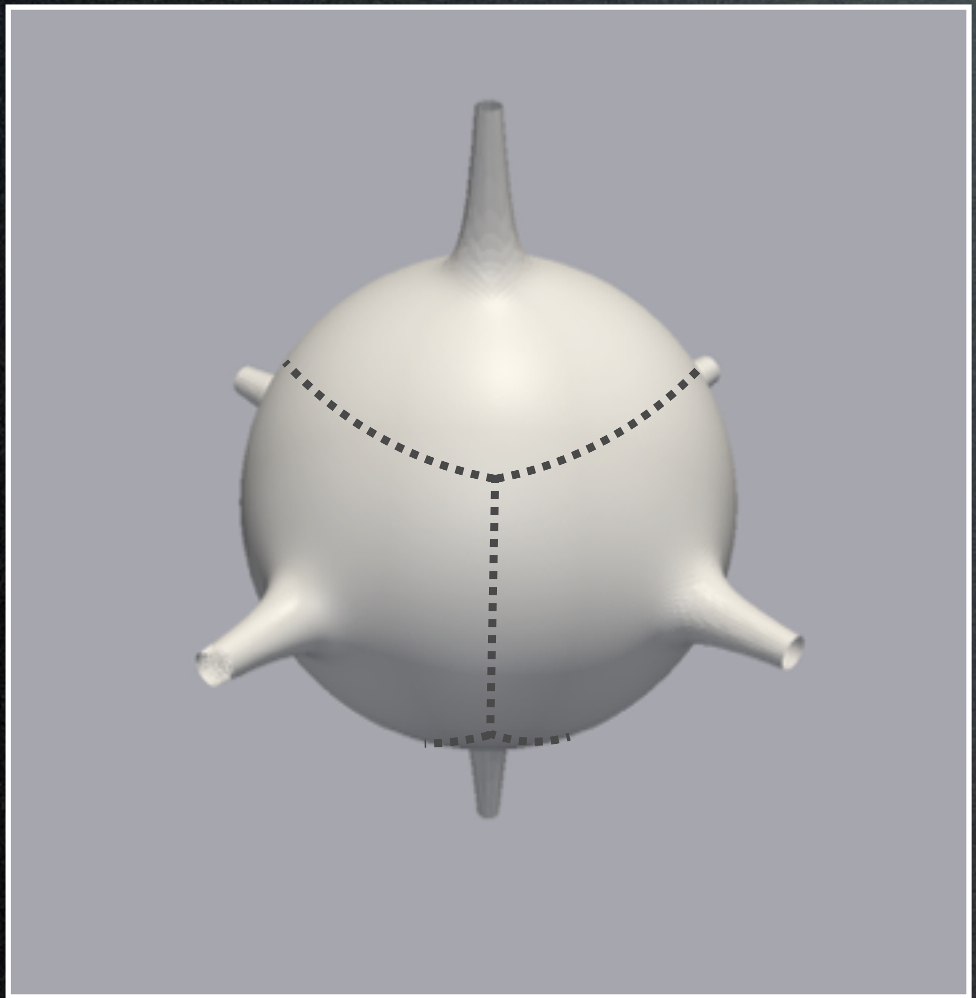
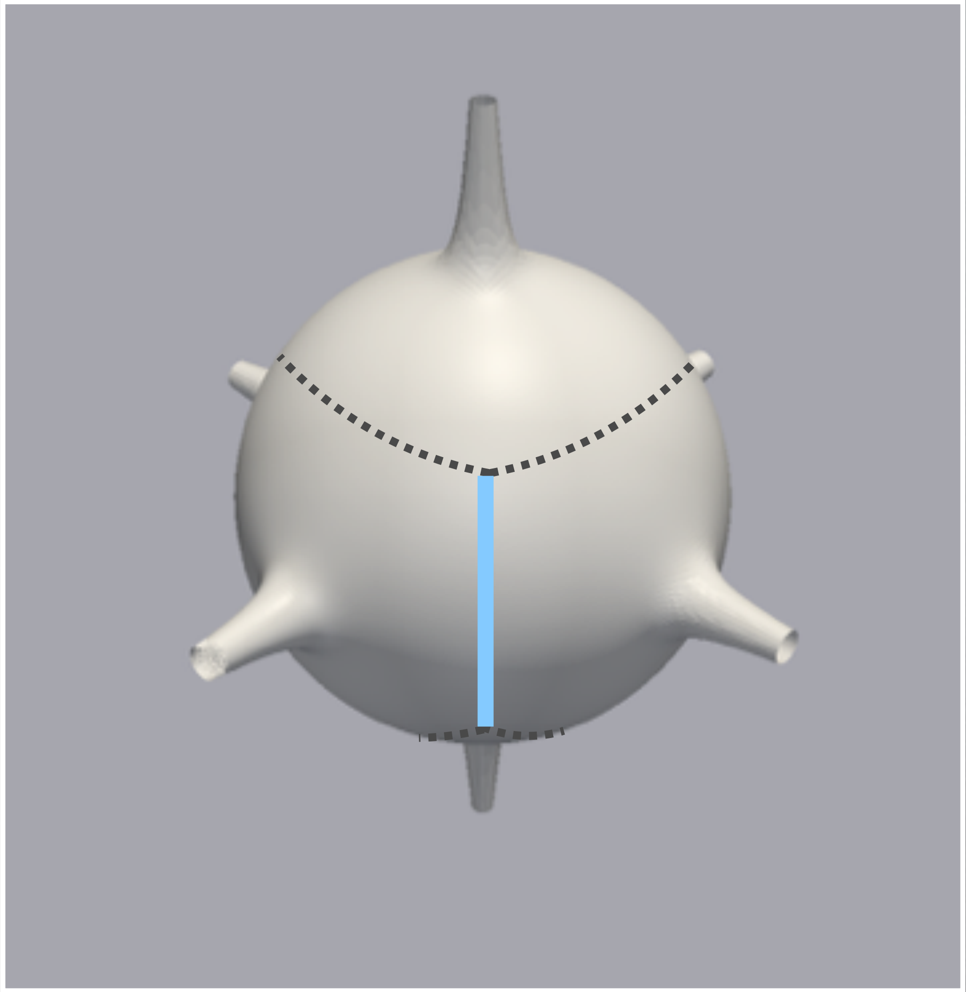


CONTRACTING BLACK-HOLE LATTICES

LENGTH SCALING



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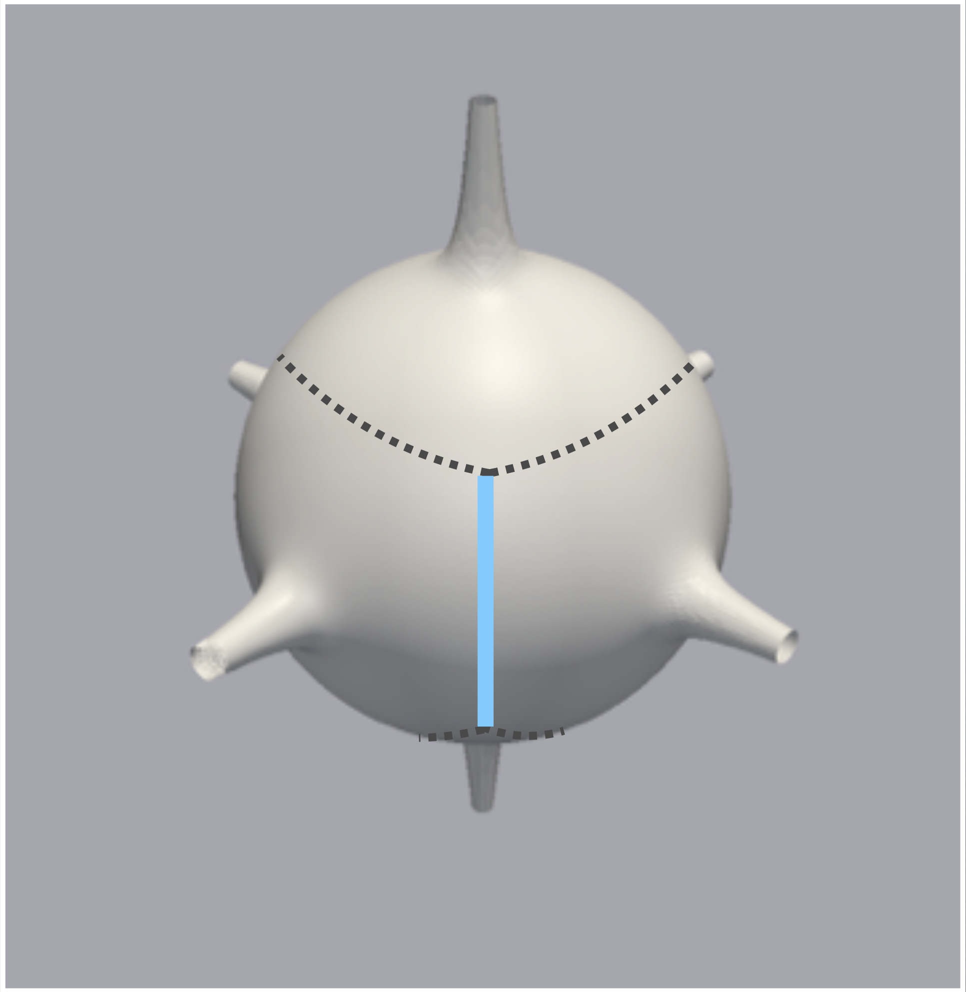

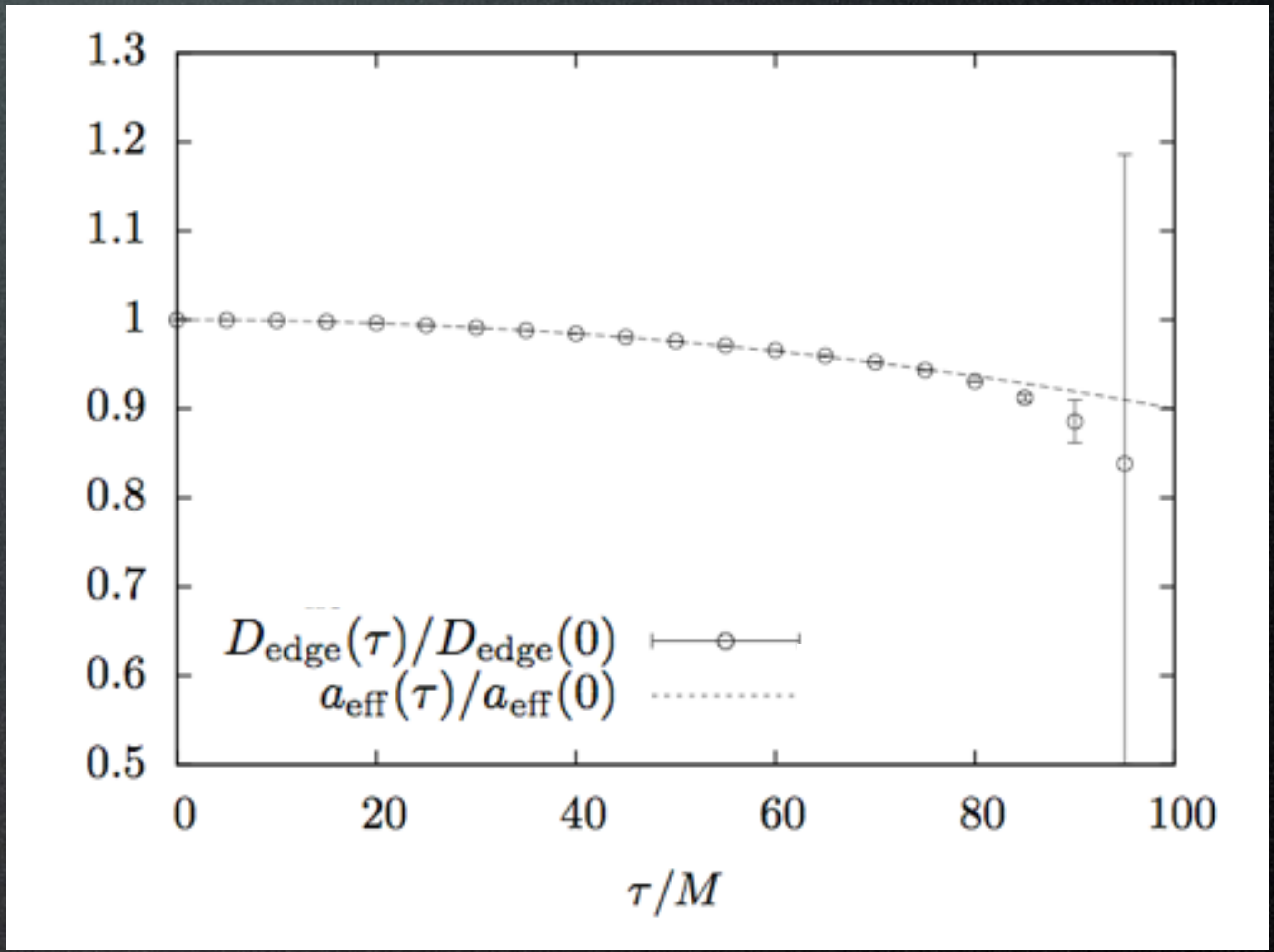

Contracting Black-hole lattices

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$$M_{\rm eff} = \rho_{\rm eff} 2\pi^2 a_{\rm eff}^3 = 378.78, \quad M_{8\rm BH} = 8M_{\rm ADM} = 303.53$$

- In this case, the conformal-data part of the ID plays a rather decisive role in its evolution. Is this always the case?

Expanding Black-hole lattices

Initial Data

Hamiltonian constraint:

$$\Delta\psi - \frac{K^2}{12}\psi^5 + \frac{1}{8}\psi^{-7}\tilde{A}_{ij}\tilde{A}^{ij} = 0$$

It requires numerical integration. If K is not a spatial constant or $\tilde{A}_{ij}$ is not transverse, the momentum constraint has to be solved as well. In all cases, the solution has to include a mechanism to preserve the integrability condition.

In this case, the constraint takes the form:

$$\int_D \left(\frac{K^2}{12}\psi^5 - \frac{1}{8}\psi^{-7}\tilde{A}_{ij}\tilde{A}^{ij}\right)\mathrm{d}V = 2\pi\sum_i m_i$$

This has to be enforced iteratively since it depends on the unknown conformal factor (and potentially on the extrinsic curvature). If this condition is not satisfied, the system does not admit solutions (“singular”)! The extent to which one can reduce the equation residual depends strongly on how well we can satisfy the compatibility condition.

Expanding Black-hole lattices

Initial Data

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A prescription: Yoo et al. (2012) construct an initial-data slice that is asymptotically Schwarzschild (in the static slicing) next to the center, and asymptotically CMC (with a negative mean curvature) next to the cell faces.

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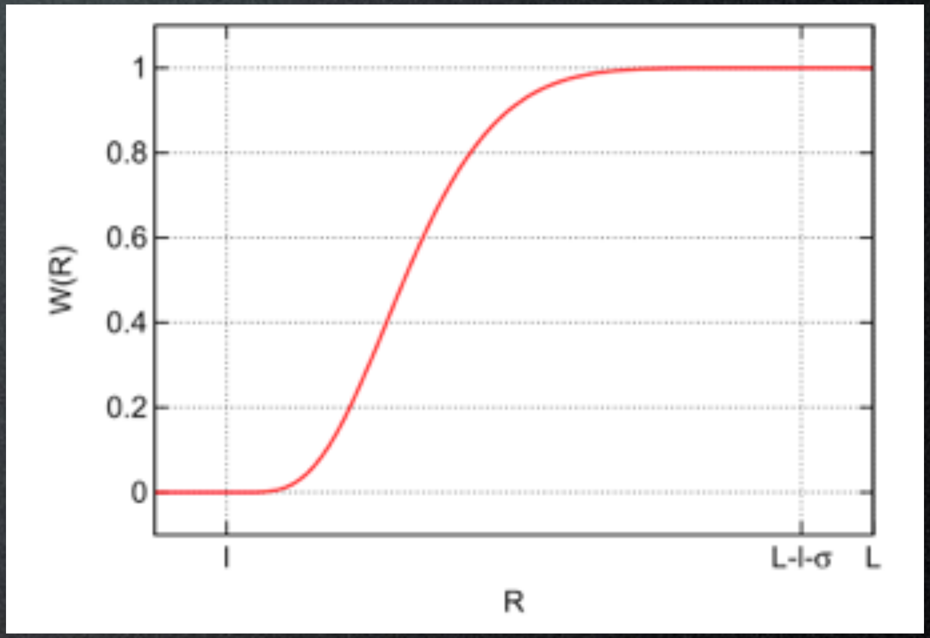

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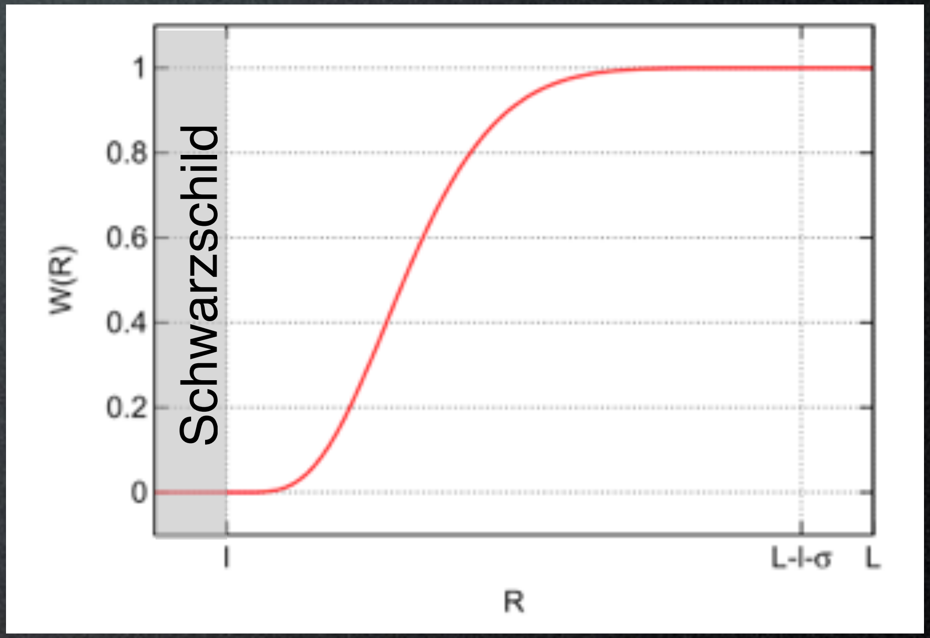

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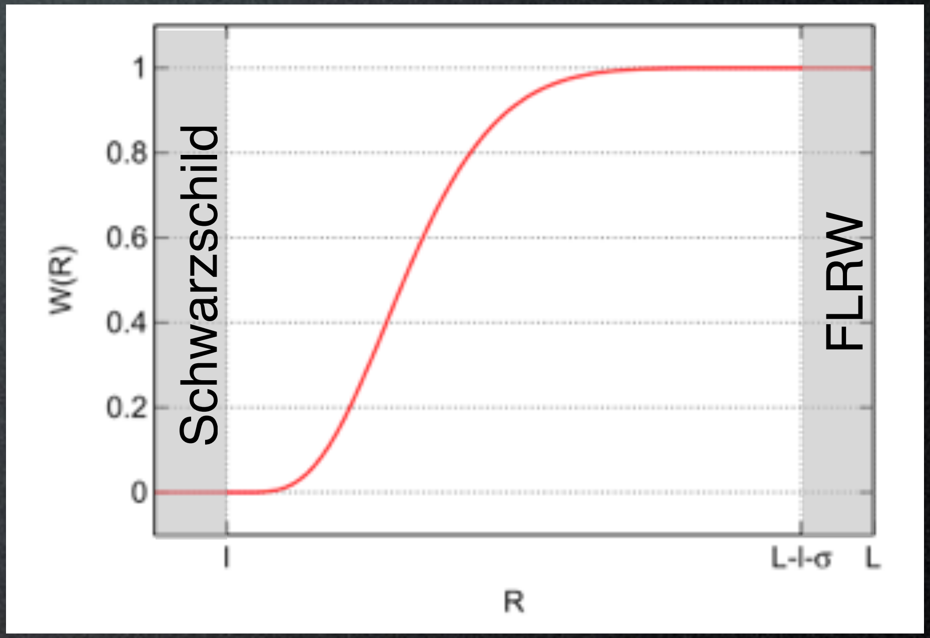

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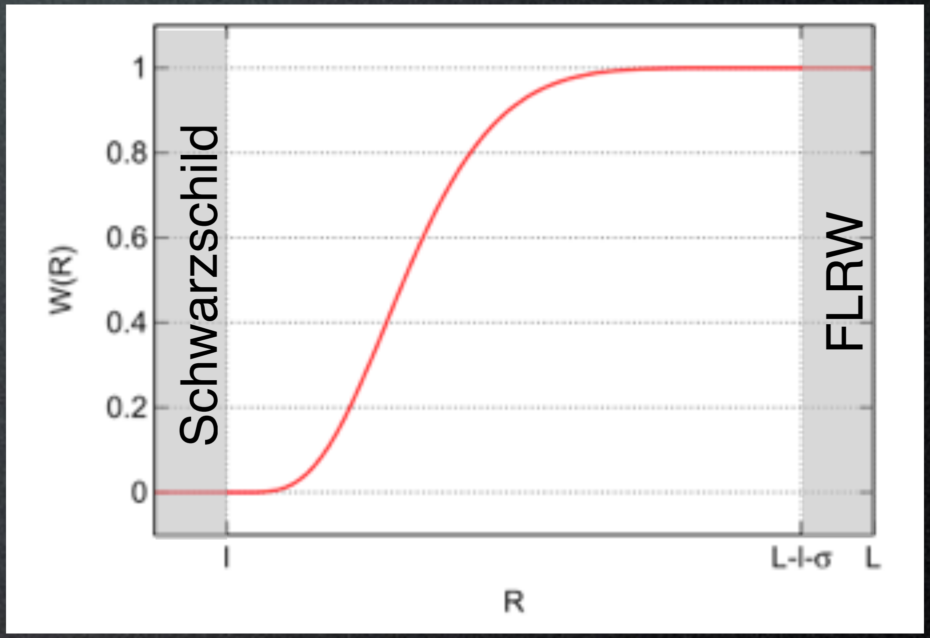

$$\Delta\psi_r - \Delta\left(\frac{m}{2r}T(r)\right) - \frac{K^2}{12}\psi^5 + \frac{1}{8}A_{ij}A^{ij}\psi^{-7} = 0$$
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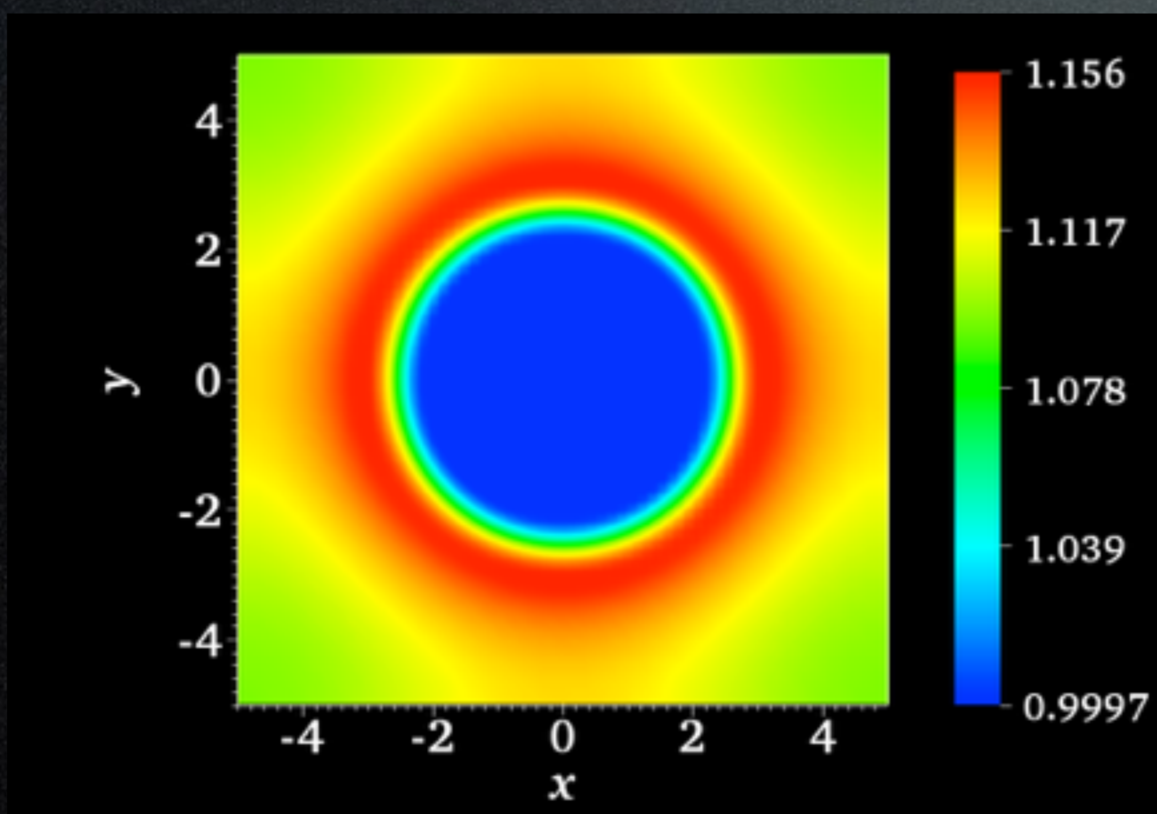
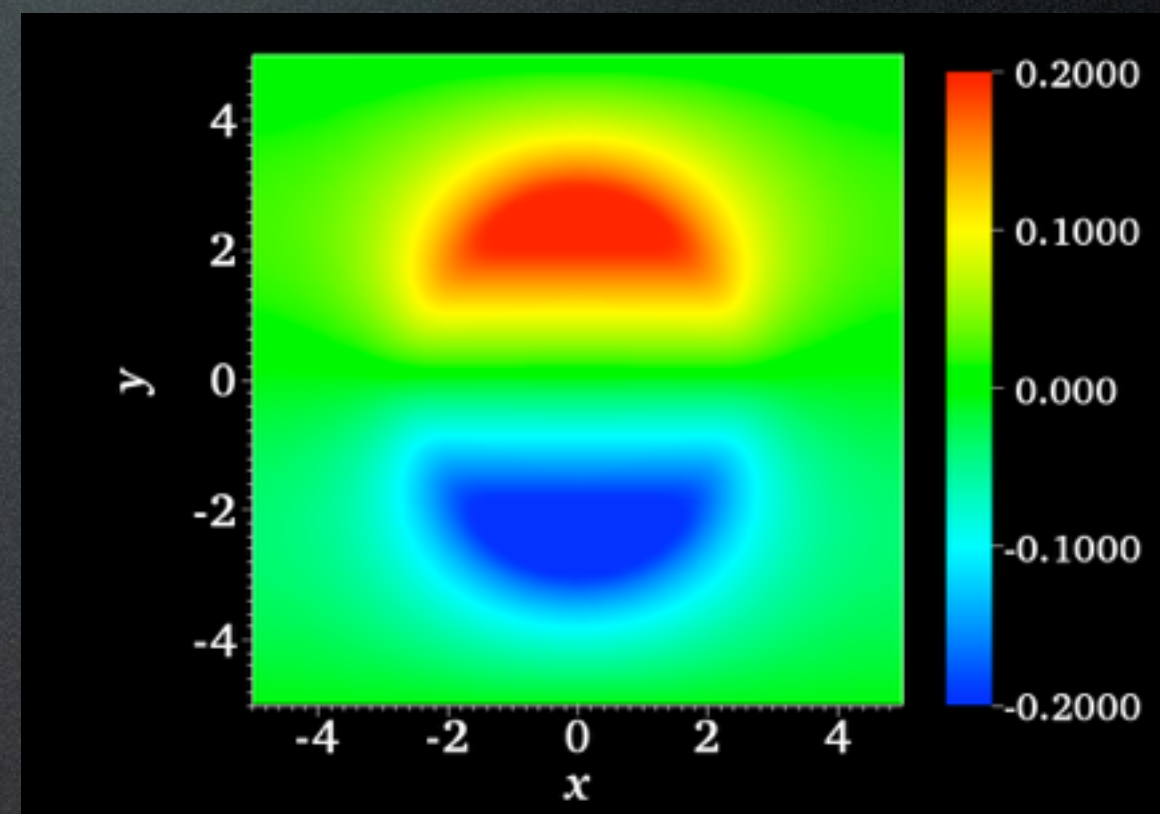


X^x



Expanding Black-hole lattices

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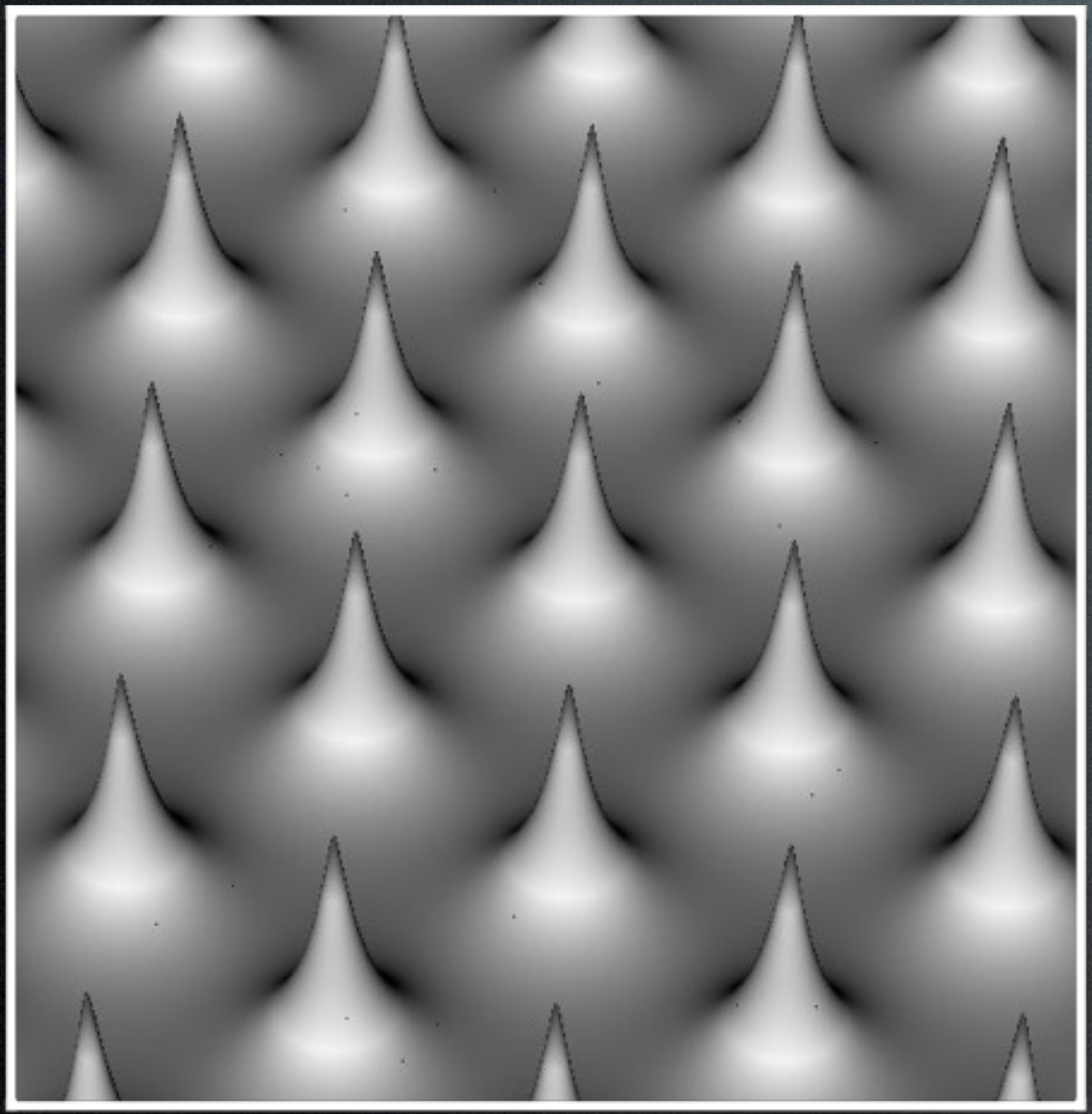

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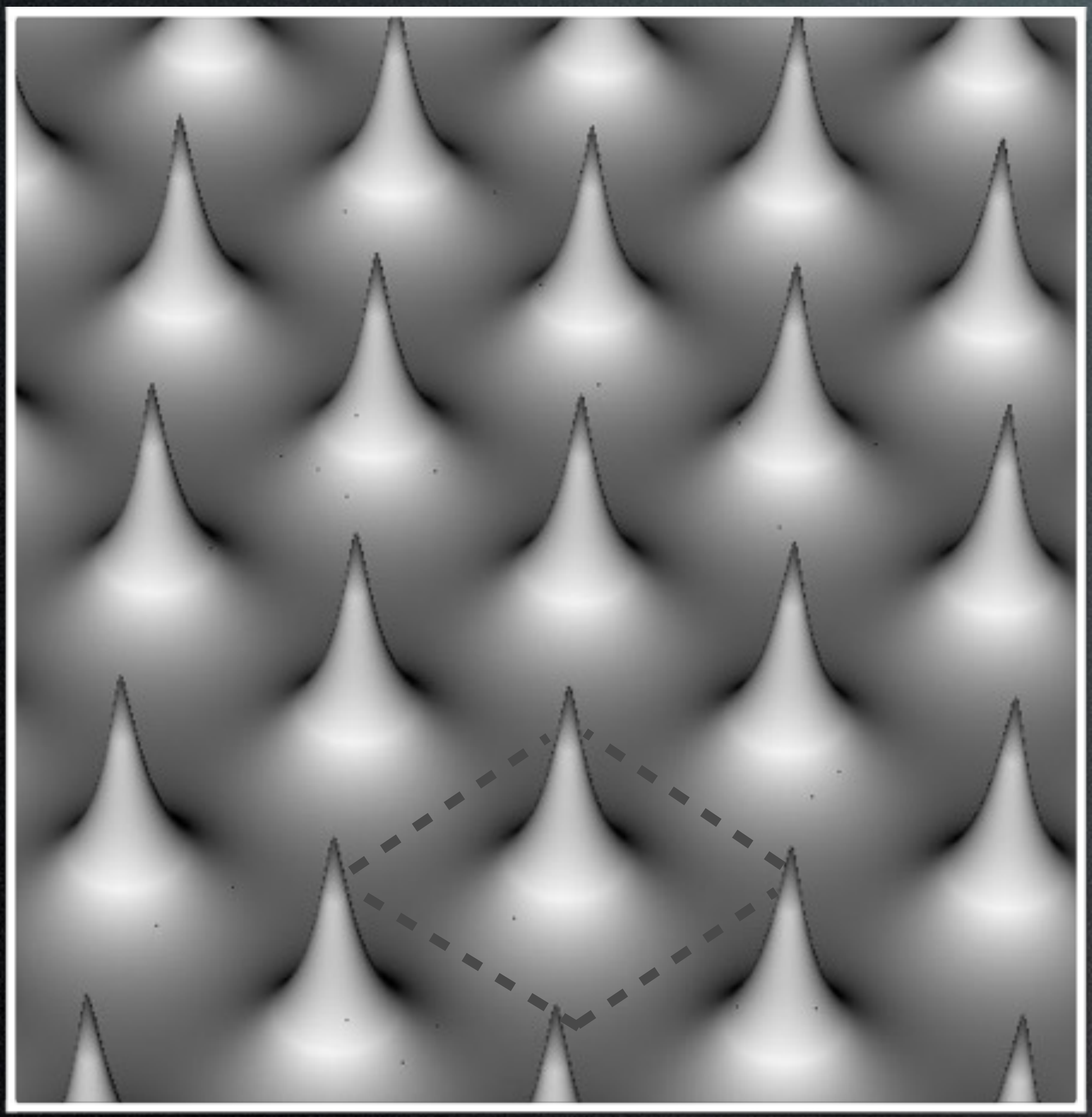

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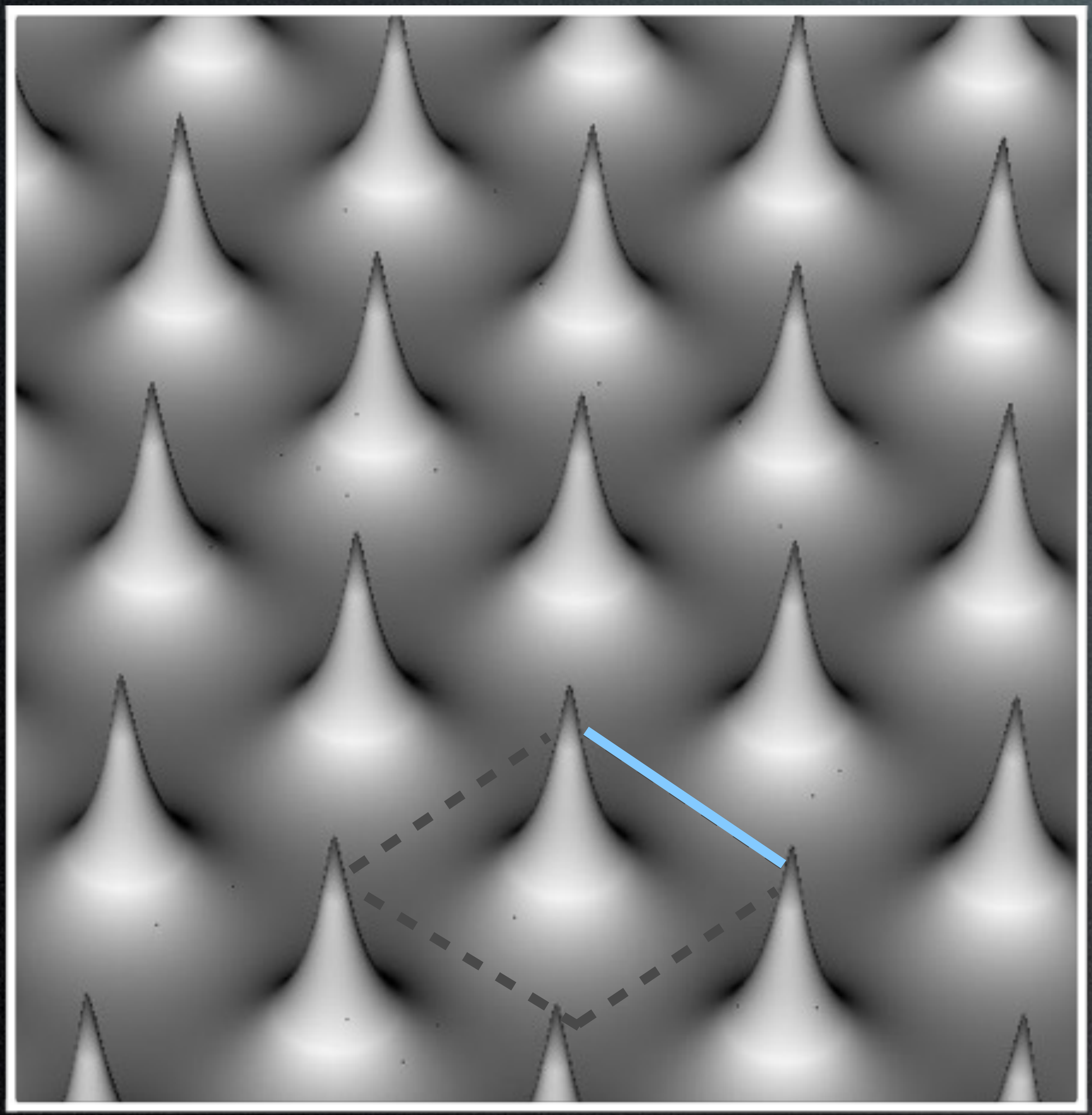

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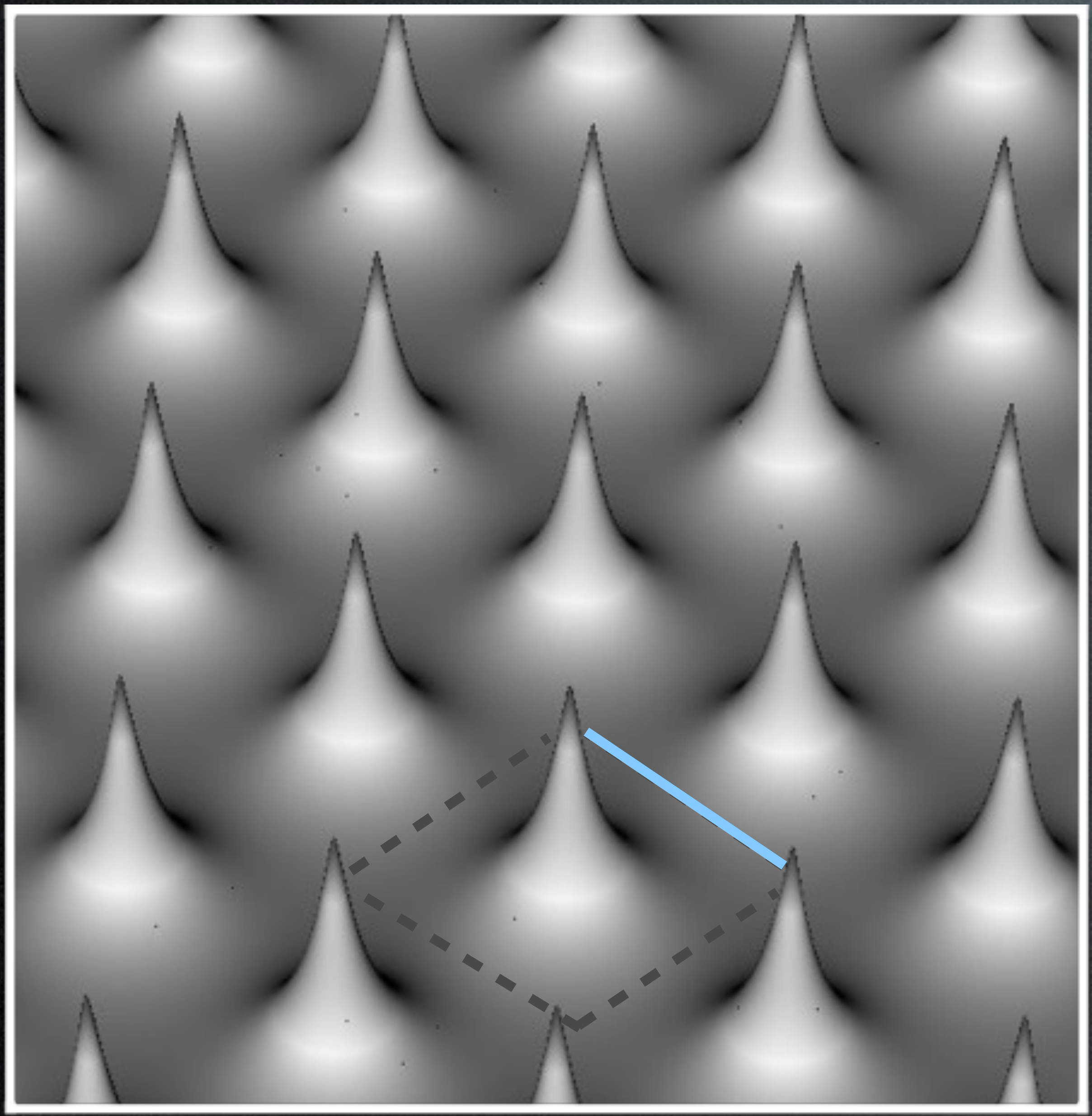

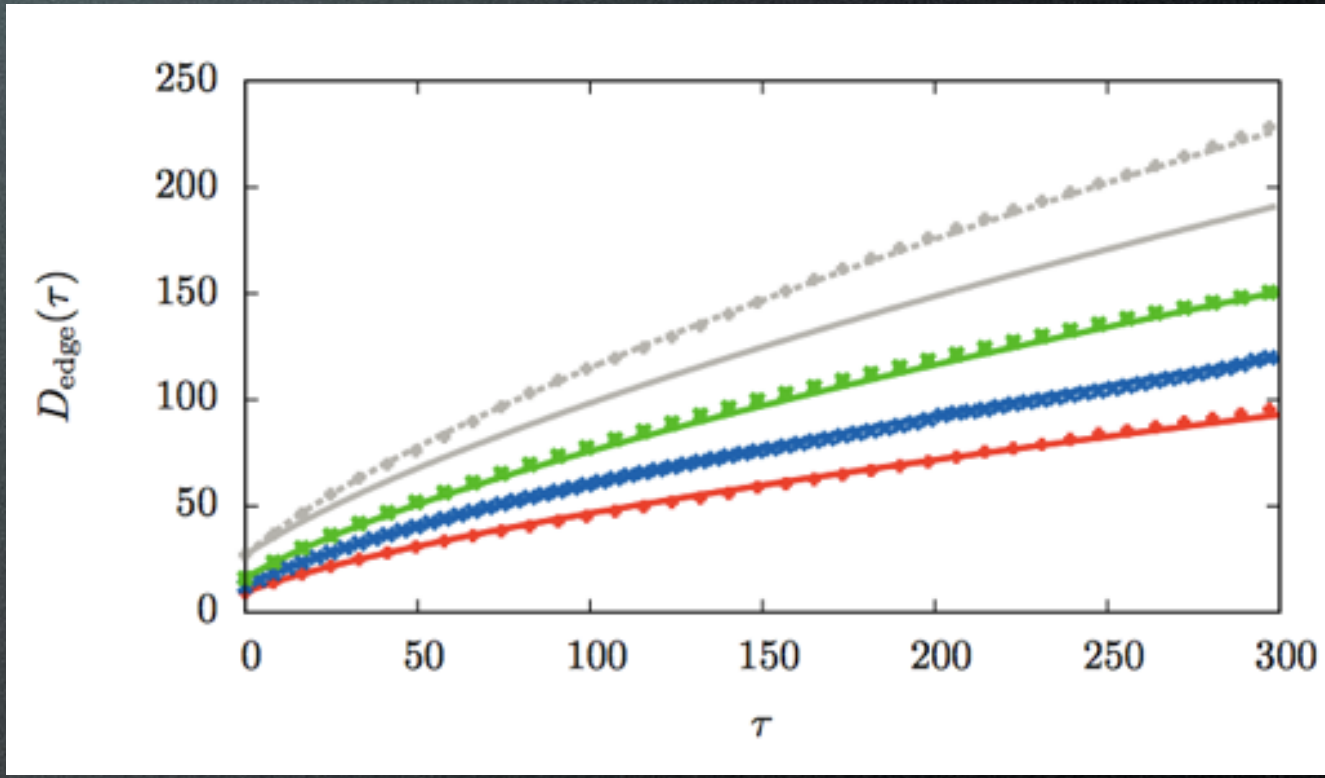

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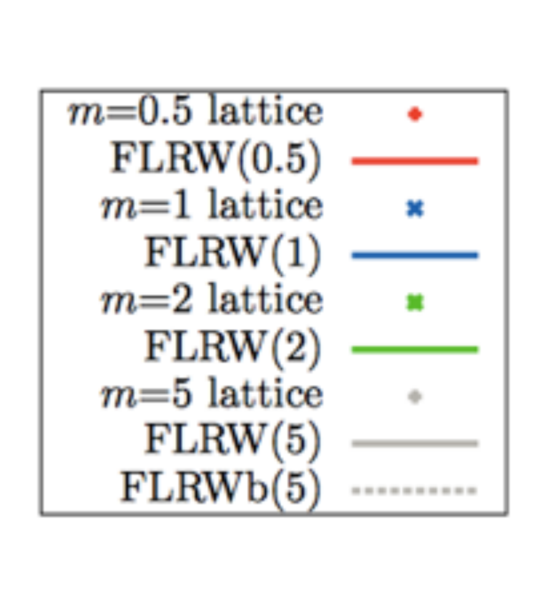

Dust Cosmologies

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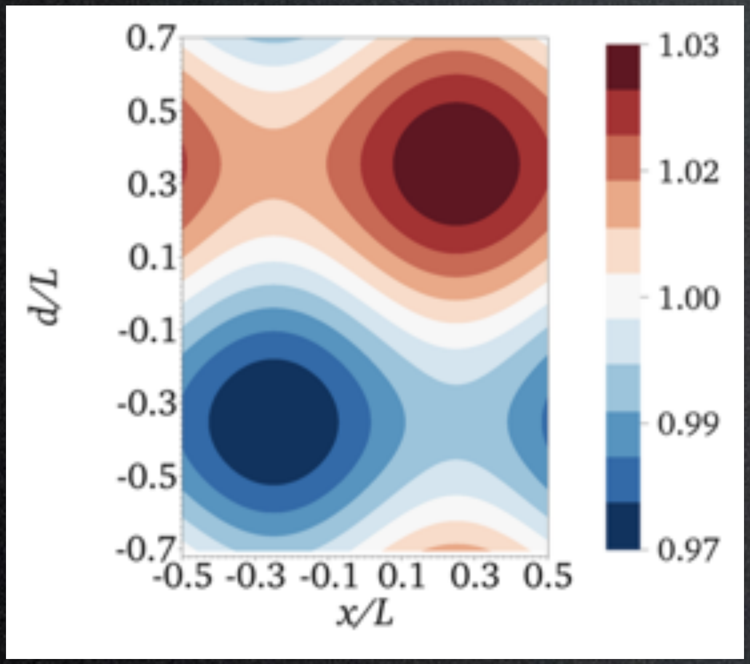

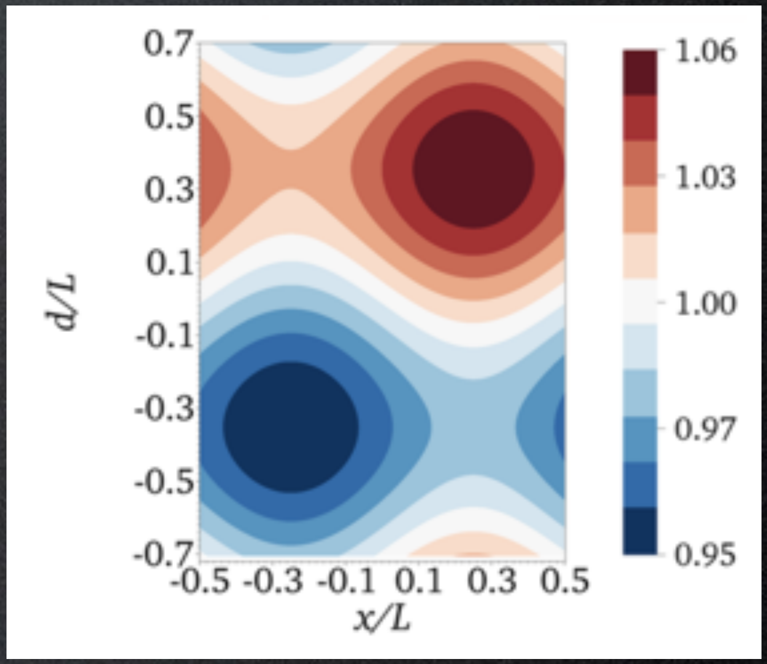

Dust Cosmologies

Analytical Methods

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Dust Cosmologies

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Dust Cosmologies

Full Relativistic 3D Evolution

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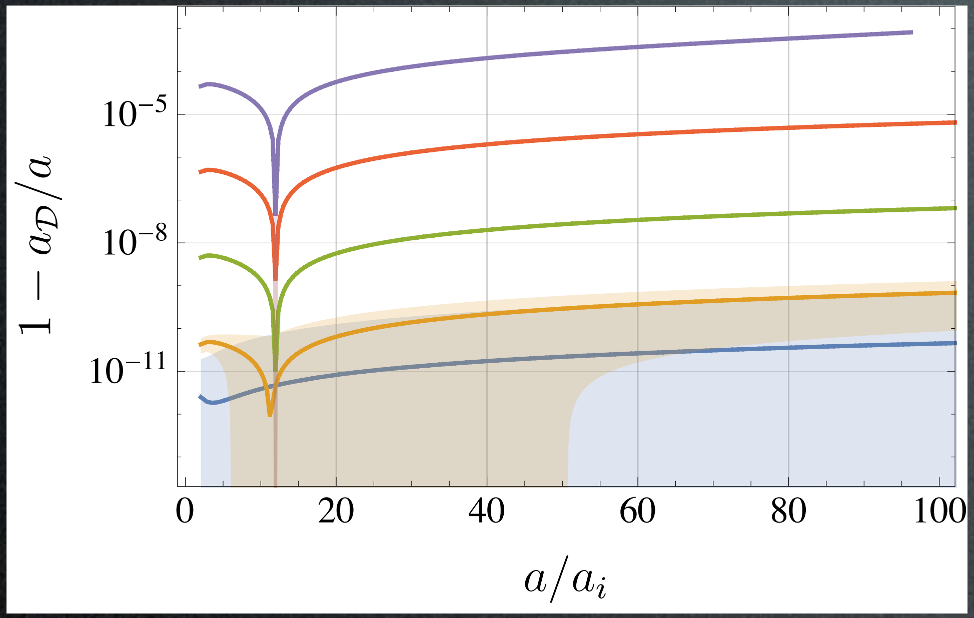
Full Relativistic 3D Evolution

Volume



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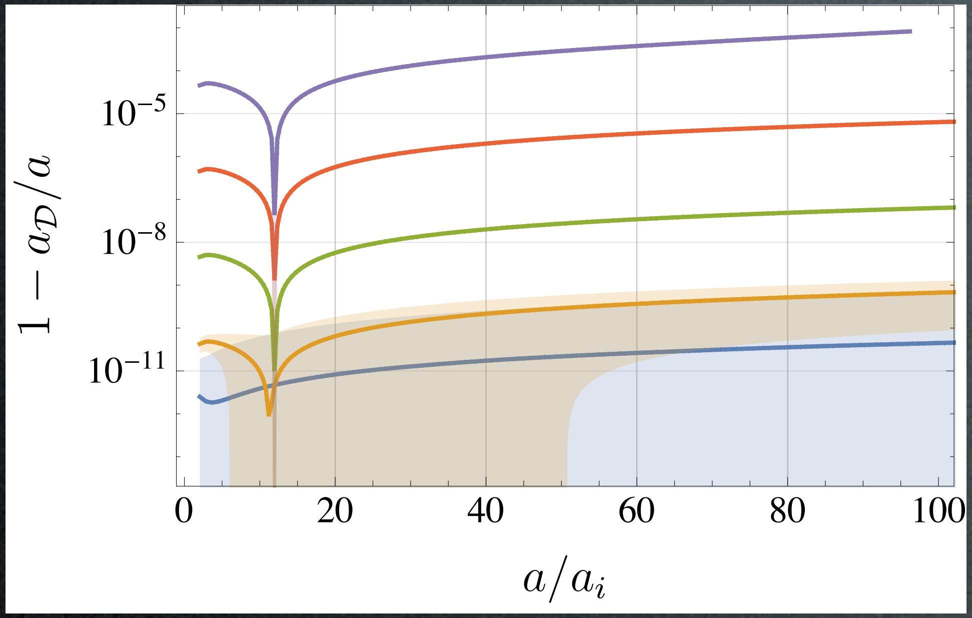
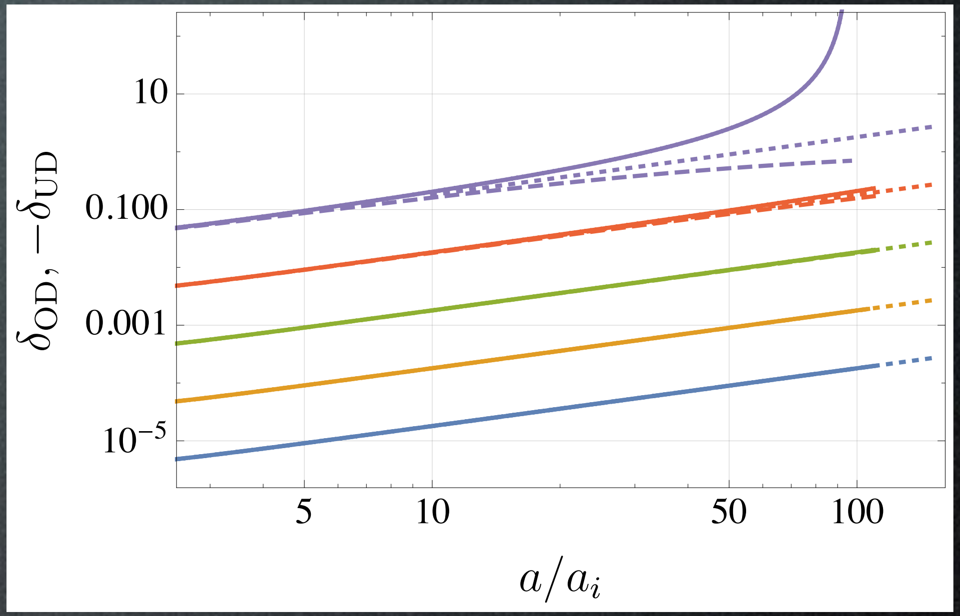


Density Constrast



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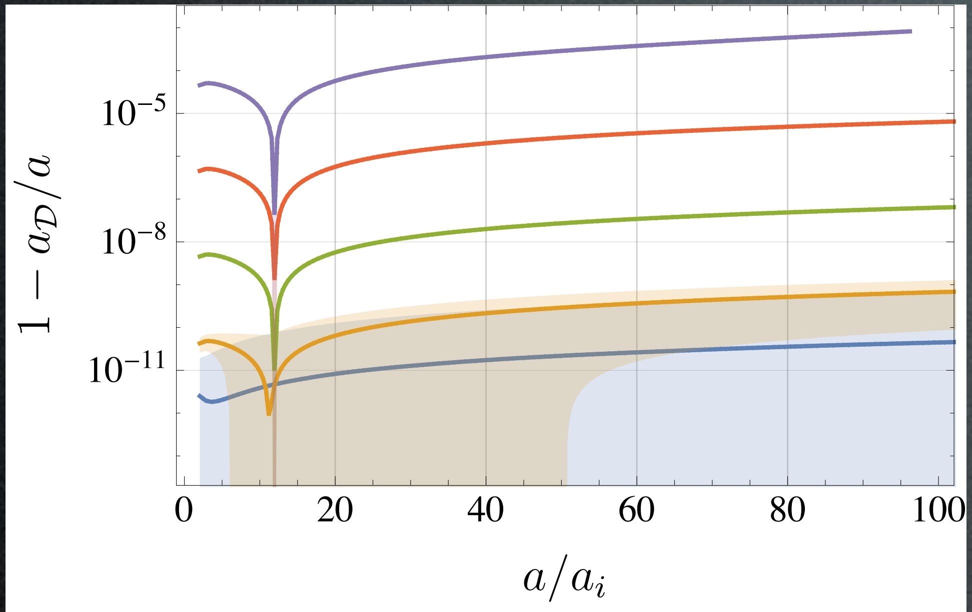

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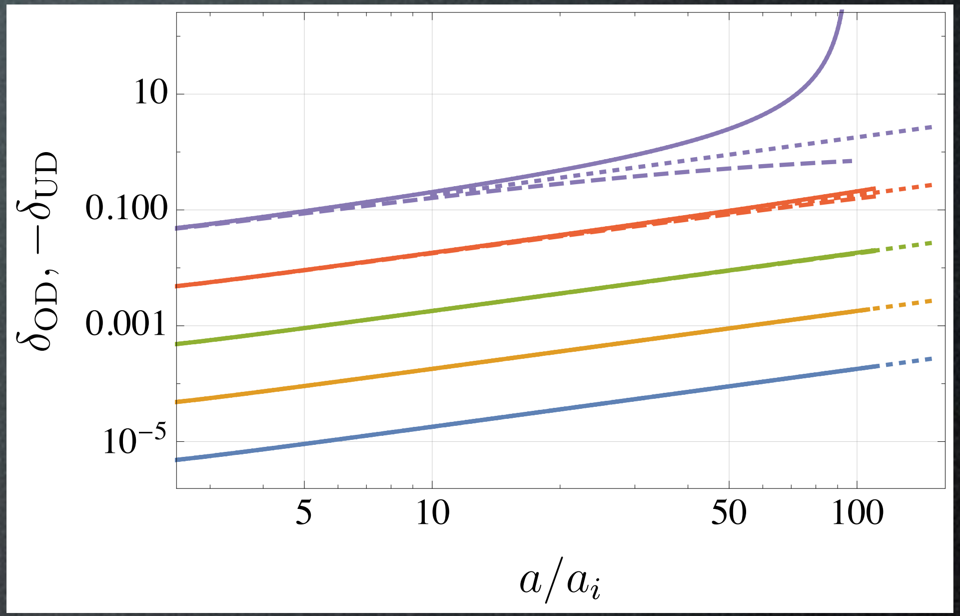
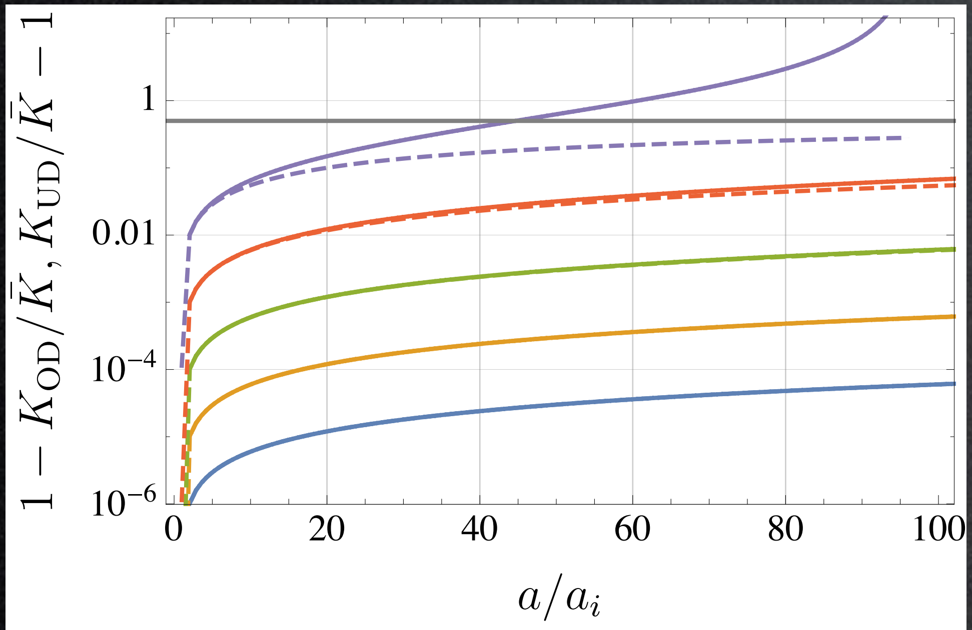


Expansion Rate



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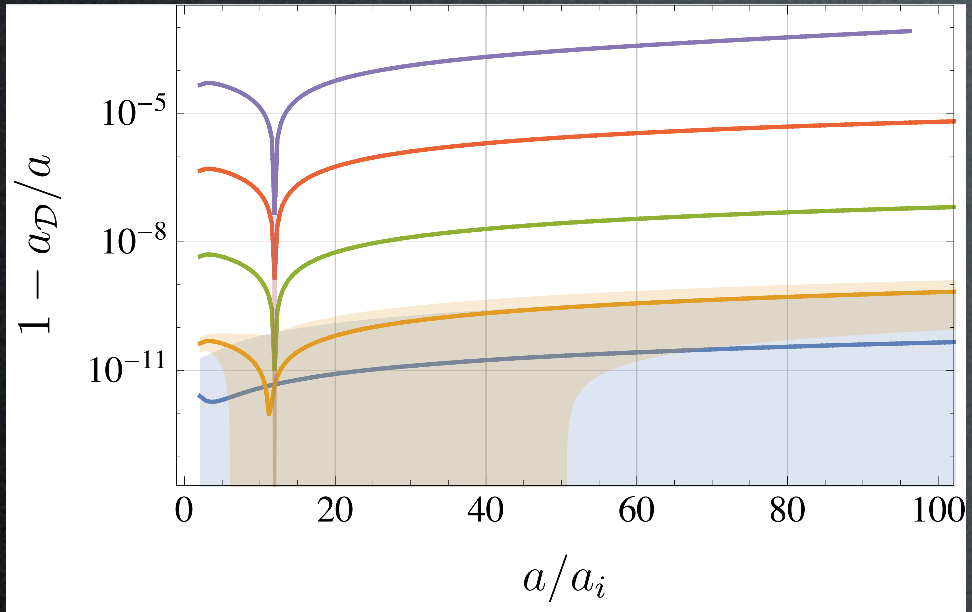

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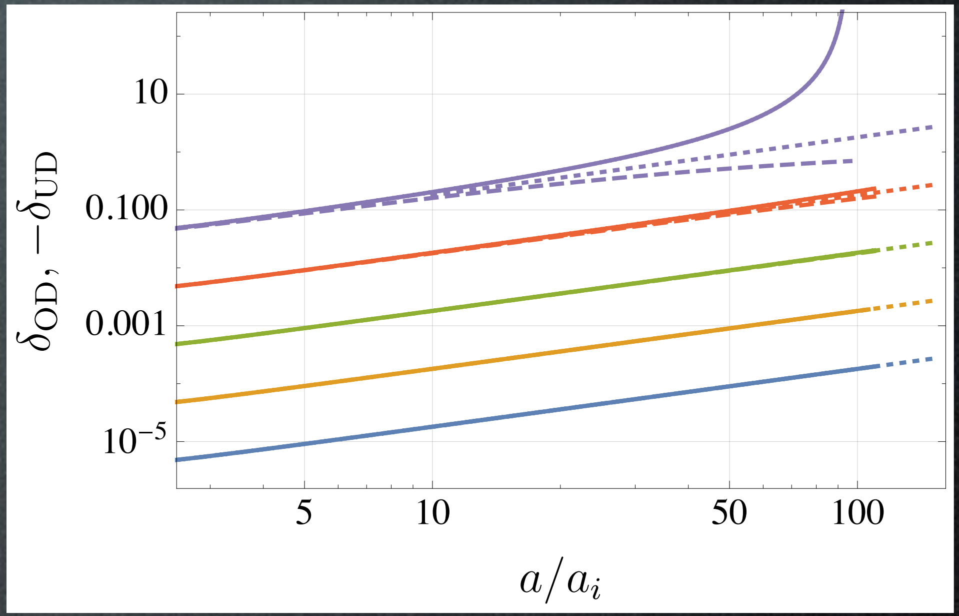

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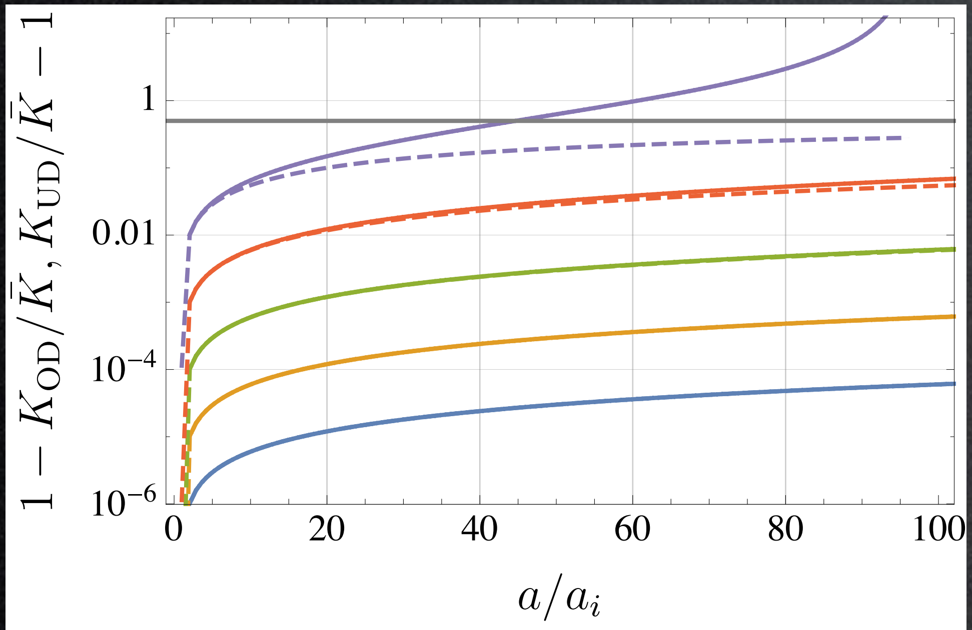
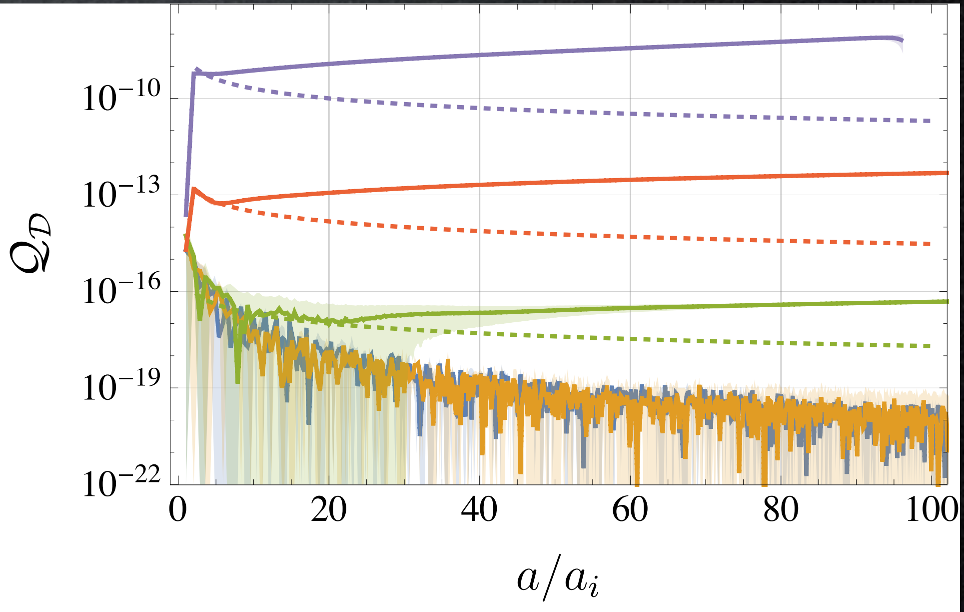


Backreaction



[Bentivegna & Bruni 2015]

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2. A fully 3D, relativistic treatment of simple inhomogeneous cosmologies is possible (some results surprising!).

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3. Status: non-perturbative effects are small globally, but well beyond 1% accuracy at the local level.