Creation of Emergent Universe with Wormhole

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28th Texas Symposium, Geneva, Switzerland
Dec. 14-18, 2015

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Emergent Universe

- A universe which is ever existing, large enough so that space-time may be treated as classical entities.

- No time like singularity

- The universe in the infinite past is in an almost static state but it eventually evolves into an inflationary stage

Advantage of an Emergent Universe Model

An emergent Universe (EU) model, if developed in a consistent way then it is capable of solving the well known conceptual issues of the Big Bang model

Four Fold advantages :

- ▶ Exists forever
- ▶ No Quantum Gravity Region
- ▶ Free from Bigbang problems like HP, FP etc.
- ▶ Accommodates late time acceleration

Gravity Equation in Cosmology

1915, GR : Einstein's Field Equation :

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu} \tag{1}$$

$$\text{Metric}: \quad ds^2 = -dt^2 + a(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right] \tag{2}$$

Einstein's Field Equations

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho \tag{3}$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = -8\pi G p \tag{4}$$

The conservation equation

$$\dot{\rho} + 3H(\rho + p) = 0 \tag{5}$$

$H = \frac{\dot{a}}{a}$ represents Hubble parameter.

BigBang model of the universe and susequently Early Inflationary models came up for model building. Despite its very impressive achievements, the inflationary paradigm leaves some questions unanswered.

- ▶ nature of the inflaton field
- ▶ the state of the universe prior to the commencement of inflation

Several alternative possibilities :

- ▶ The universe quantum mechanically tunnelled into an inflation
- ▶ The universe was dominated by radiation (or some other form of matter) prior to inflation and might therefore have encountered a singularity in its past.
- ▶ The universe underwent a non-singular bounce prior to inflation. Before the bounce the universe was contracting
- ▶ The universe existed eternally in a quasi-static state, out of which inflationary expansion emerged.

In 1967, Harrisson found a cosmological solution in a closed model with radiation and a positive cosmological constant

$$a(t) = a_i \left(1 + e^{\frac{\sqrt{2}t}{a_i}}\right)^{\frac{1}{2}} \tag{6}$$

as $t \rightarrow -\infty$, the universe goes over asymptotically to an Einstein Static Universe, the expansion is given by a finite number of e-foldings

$$N_o = \ln \frac{a_o}{a_i} = \frac{t_o}{\sqrt{2}a_i} \tag{7}$$

.

► Size of the universe is determined by Λ
► Problem : No graceful exit from the deSitter phase.

- Considered a dynamical scalar field to obtain EU in a closed universe ($k = +1$). In the model a minimally coupled scalar field ϕ with a self interacting potential $V(\phi)$ was considered.

- In the case the initial size a_i of the universe is determined by the KE of the field.

- A universe consisting of ordinary matter and minimally coupled homogeneous scalar field was considered.

The Einstein static universe is characterized by $k = 1$ and $a = a_i = constant$ which is realized with

- A simple potential for EU model with scalar field only.

- The potential must have the following characteristics : $V(\phi) \rightarrow V_i$ as $\phi \rightarrow \infty$ and $t \longrightarrow -\infty$.

- But drops towards a minimum at a finite value ϕ_f.

Form of the Potential $V - V_f = (V_i - V_f) \left[Exp \left(\frac{\phi - \phi_i}{\alpha} \right) - 1 \right]^2$

Ellis et al., CQG 21, 233 (2004)
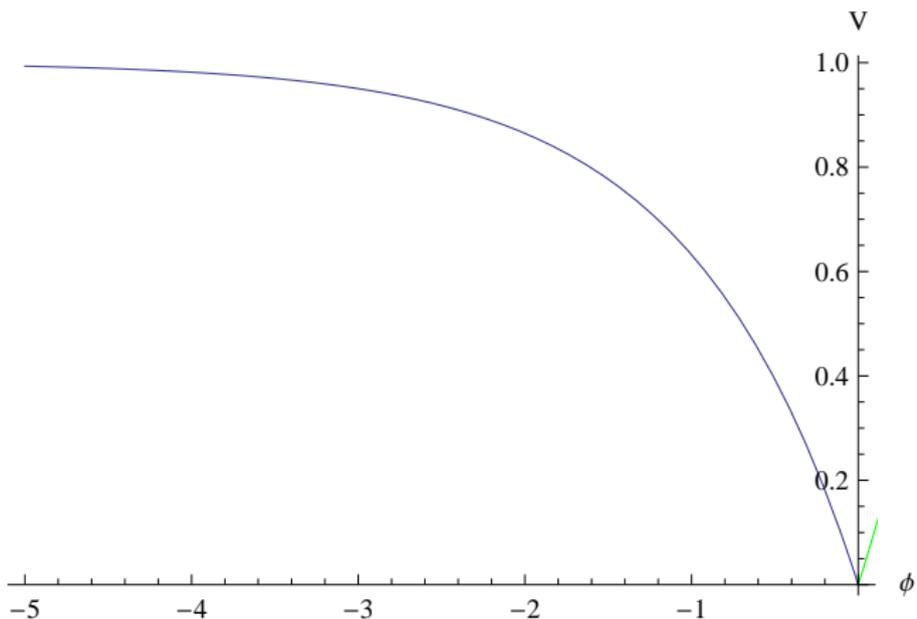
Making use of R^2-modified gravity. The gravitational action

$$I = \int d^4x \sqrt{-g}[R + \alpha R^2] \tag{8}$$

Define a conformal transformation

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \tag{9}$$

Here $\Omega^2 = 1 + 2\alpha R$, one obtains

$$\tilde{R} = \frac{1}{\Omega^2} \left[R - 6g^{\mu\nu}\nabla_\mu \nabla_\nu)(\ln \Omega) - 6g^{\mu\nu}\nabla_\mu(\ln \Omega)\nabla_\nu(\ln \Omega) \right] \tag{10}$$

Defining $\phi = \sqrt{3}\ln(1 + 2\alpha R)$, one obtains

$$I = \int d^4x \sqrt{-g} \left[\tilde{R} - \frac{1}{2}\tilde{g}_{\mu\nu}\nabla_\mu \phi \nabla_\nu \phi - \frac{1}{4\alpha} \left(e^{-\frac{\phi}{\sqrt{3}}} - 1 \right)^2 \right] \tag{11}$$

EU Model in a spatially flat case

S Mukherjee, BCP, N K Dadhich, S D Maharaj, A Beesham, CQG 23, 6927 (2006)

Preamble : In looking for a model of emergent universe, the following features for the universe are assumed:

- ▶ The universe is isotropic and homogeneous at large scales.
- ▶ Spatially flat (WMAP results) :
- ▶ It is ever existing, No singularity
- ▶ The universe is always large enough so that classical description of space-time is adequate.
- ▶ The matter or in general, the source of gravity has to be described by quantum field theory.
- ▶ The universe may contain exotic matter (SEC violated)
- ▶ The universe is accelerating (Type Ia Supernovae data)

EOS

$$p = A\rho - B\sqrt{\rho} \qquad (12)$$

The Einstein equations for a flat universe in RW-metric ($G = \frac{1}{8\pi}$)

$$\rho = 3\frac{\dot{a}^2}{a^2} \qquad (13)$$

$$p = -2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} \qquad (14)$$

Making use of the EOS we obtain

$$2\frac{\ddot{a}}{a} + (3A+1)\frac{\dot{a}^2}{a^2} - \sqrt{3}B\frac{\dot{a}}{a} = 0 \qquad (15)$$

On integration

$$a(t) = \left(\frac{3\kappa(A+1)}{2} \left(\sigma + \frac{2}{\sqrt{3}B} Exp\left[\frac{\sqrt{3}Bt}{2} \right] \right) \right)^{\frac{2}{3(A+1)}} \qquad (16)$$

- ► If $B < 0$ Singularity
- ► If $B > 0$ and $A > -1$ Non-singular (EU)

Composition of the Emergent Universe

Using EOS, in $\frac{d\rho}{dt} + 3(\rho + p)\frac{\dot{a}}{a} = 0$ one obtains

$$\rho(a) = \frac{1}{(A+1)^2}\left(B + \frac{K}{a^{\frac{3(A+1)}{2}}}\right)^2 \tag{17}$$

where K is an integration constant.

This provides us with indications about the components of energy density in EU.

$$\rho(a) = \Sigma_{i=1}^3 \rho_i \quad and \quad p(a) = \Sigma_{i=1}^3 p_i \tag{18}$$

where we denote

$$\rho_1 = \frac{B^2}{(A+1)^2}, \quad \rho_2 = \frac{2KB}{(A+1)^2}\frac{1}{a^{\frac{3(A+1)}{2}}}, \quad \rho_3 = \frac{K^2}{(A+1)^2}\frac{1}{a^{3(A+1)}} \tag{19}$$

$$p_1 == -\frac{B^2}{(A+1)^2}, \quad p_2 = \frac{KB(A-1)}{(A+1)^2}\frac{1}{a^{\frac{3(A+1)}{2}}}, \quad p_3 = \frac{AK^2}{(A+1)^2}\frac{1}{a^{3(A+1)}}. \tag{20}$$

Comparing with the barotropic EoS given by $p_i = \omega_i \rho_i$ one obtains

- $\omega_1 = -1$
- $\omega_2 = \frac{A-1}{2}$
- $\omega_3 = A$

Table-I

A	$\omega_2 = \frac{1}{2}(A-1)$	$\omega_3 = A$	Composition
$\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	DE, Exotic Matter, Radiation
$-\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{1}{3}$	DE, Exotic Matter, Cosmic String
1	0	1	DE, Exotic Matter, Stiff fluid
0	$-\frac{1}{2}$	0	DE, Exotic Matter, Dust

Implementation of EU in Different Gravity theories

- Brane World Scenario
  A Banerjee, T Bandyopadhyay, S Chakraborty : (i) (2007) Grav. Cosmo 13, 290 (ii) (2008) GRG 26, 075017

- Phantom and Tachyon Field : U. Debnath (2008) CQG 25, 205019

- Non-linear Sigma model
  A Beesham, S V Chervon, S D Maharaj (2009) CQG 26, 075017
  A Beesham, S V Chervon, S D Maharaj, A Kubasov (2009) arXiv: 0904.0773

- EU with Gauss-Bonnet terms : B C Paul and S Ghose, (2010) *GRG* **42** 795

- Chiral field in EGB gravity
  S V Chervon, S D Maharaj, A Beesham, A Kubasov (2014) (i) arXiv:1405.7219 (ii) (2013) *Quantum Matter* **2** 388

Observational Aspects

- BCP, P Thakur, S Ghose (2010)
  Mon. Not. Roy. Astron. Soc. **407** 415

- BCP, S Ghose, P Thakur (2011)
  Mon. Not. Roy. Astron. Soc. **413** 686

- S Ghose, P Thakur, BCP (2012)
  Mon. Not. Roy. Astron. Soc. **421** 20

Model I : The two fluids model

Interacting fluids : ρ and ρ' which can exchange energies. One of them is the non-linear EoS for EU

$$p = A\rho - B\rho^{1/2}. \tag{21}$$

and the other fluid is barotropic

$$p' = \omega'\rho' \tag{22}$$

where ω' corresponds to EoS parameter. The Hubble parameter is

$$3H^2 = \rho + \rho'. \tag{23}$$

Assume interaction operative at $t \geq t_i$ and the conservation eqs.

$$\dot{\rho} + 3H(\rho + p) = -\alpha\rho H, \tag{24}$$

$$\dot{\rho'} + 3H(\rho' + p') = \alpha\rho H \tag{25}$$

where α represents a coupling parameter.

The energy density and pressure for the fluid of the first kind are given by

$$\rho = \frac{B^2}{(A+1+\frac{\alpha}{3})^2} + \frac{2KB}{(A+1+\frac{\alpha}{3})^2}\frac{1}{a^{\frac{3(A+1+\frac{\alpha}{3})}{2}}} + \frac{K^2}{(A+1+\frac{\alpha}{3})^2}\frac{1}{a^{3(A+1+\frac{\alpha}{3})}},$$
(26)

$$p = -\frac{B^2}{(A+1+\frac{\alpha}{3})^2} + \frac{KB(A-1+\frac{\alpha}{3})}{(A+1+\frac{\alpha}{3})^2}\frac{1}{a^{\frac{3(A+1+\frac{\alpha}{3})}{2}}} + \frac{(A+\frac{\alpha}{3})K^2}{(A+1+\frac{\alpha}{3})^2}\frac{1}{a^{3(A+1+\frac{\alpha}{3})}}$$
(27)

Considering interaction with pressureless dark fluid *i.e.*, $p' = 0$

$$\rho_{total} = \rho + \rho' = \frac{B^2}{A+1} + \frac{2KB}{(A+1)^2}\frac{1}{a^{\frac{3(A+1)}{2}}} + \frac{K^2}{A+1}\frac{1}{a^{3(A+1)}}, \quad (28)$$

$$p_{total} = p = -\frac{B^2}{A+1} + \frac{KB(A-1)}{(A+1)^2}\frac{1}{a^{\frac{3(A+1)}{2}}} + \frac{AK^2}{(A+1)^2}\frac{1}{a^{3(A+1)}}.$$
(29)

An interesting case emerges when $\alpha = 2$ and $A = \frac{1}{3}$. A universe with DE, EM and radiation initially (*i.e.*, before the interaction sets in) transits to a matter dominated phase (observed universe now).

Model II : The three fluids model

The original emergent universe model is composed of three non-interacting fluids. Here we bring interaction among themselves at $t \geq t_0$,

$$\dot{\rho_1} + 3H(\rho_1 + p_1) = -Q', \qquad (30)$$

$$\dot{\rho_2} + 3H(\rho_2 + p_2) = Q, \qquad (31)$$

$$\dot{\rho_3} + 3H(\rho_3 + p_3) = Q' - Q, \qquad (32)$$

where Q and Q' represent the interaction terms.
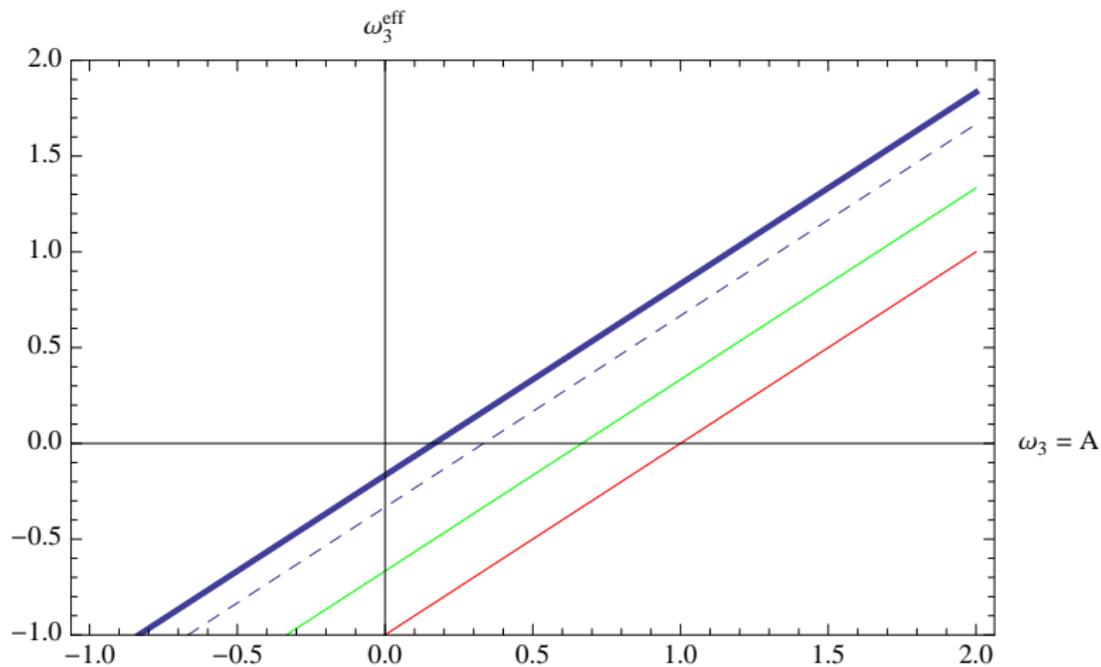Here $\rho_1 \longrightarrow$ DE density, $\rho_2 \longrightarrow$ DM and $\rho_3 \longrightarrow$ normal matter. The equivalent effective uncoupled model is described by the conservation equations:

$$\dot{\rho_i} + 3H(1 + \omega_1^{eff})\rho_i = 0 \qquad (33)$$

where the effective equation of state parameters are

$$\omega_i^{eff} = \omega_i + \frac{Q'}{3H\rho_i}, \qquad (34)$$

For $Q - Q' = -\beta H \rho_3$, the effective state parameter for the normal

Wormhole for EU

▶ Gravitational action for massive Gravity

$$S = -\frac{1}{8\pi} \int \left(\frac{1}{2} R + m^2 L \right) \sqrt{-g} \, d^4 x + S_m \qquad (36)$$

▶ Massive Gravity Lagrangian

$$L = -\frac{1}{2} \left(K^2 - K_\nu^\mu K_\mu^\nu \right) + \frac{c_3}{3!} \epsilon_{\mu\nu\rho\sigma} \epsilon^{\alpha\beta\gamma\sigma} K_\alpha^\mu K_\beta^\nu K_\gamma^\rho + \frac{c_4}{4!} \epsilon_{\mu\nu\rho\sigma} \epsilon^{\alpha\beta\gamma\delta}$$
$$K_\alpha^\mu K_\beta^\nu K_\gamma^\rho K_\delta^\sigma \qquad (37)$$

where c_3, c_4 are constants and $K_\nu^\mu = \delta_\nu^\mu - \gamma_\nu^\mu$, $\gamma_\sigma^\mu \gamma_\nu^\sigma = g^{\mu\nu} f_{\sigma\nu}$, and $f_{\sigma\nu}$ is symmetric tensor.

▶ For Euclidean time

$$\left(\frac{\dot{a}}{a} \right)^2 = -\frac{8\pi G \rho}{3} - \frac{m^2}{3} \left(4c_3 + c_4 - 6 + 3C \frac{3 - 3c_3 - c_4}{a} \right)$$
$$\tag{38}$$
$$-\frac{m^2}{3} \left(3C^2 \frac{c_4 + 2c_3 - 1}{a^2} - C^3 \frac{c_3 + c_4}{a^3} \right)$$

- Conservation equation

$$\dot{\rho} + 3H(\rho + p) = 0 \qquad (39)$$

- The field equation can be rewritten as

$$\dot{a}^2 = V(a) \qquad (40)$$

- $V(a) = -\frac{m^2}{3}\left[(4c_3 + c_4 - 6 + 3\Lambda)a^2 - \frac{c_3 + c_4}{a} + X\right]$

-

$$\Lambda = \frac{8\pi G}{3m^2}\rho$$

$$X = 3(3 - 3c_3 - c_4)a + 3(c_4 + 2c_3 - 1)$$

$$p = A\rho - B\sqrt{\rho} \qquad (41)$$

- we note existence of wormhole solution

$$\dot{a}^2 = 1 - \mu a^2 + \Sigma_{n=1}^{N}\frac{\nu_n}{a^{2n}}; \qquad (42)$$

- The field equation can be rewritten as

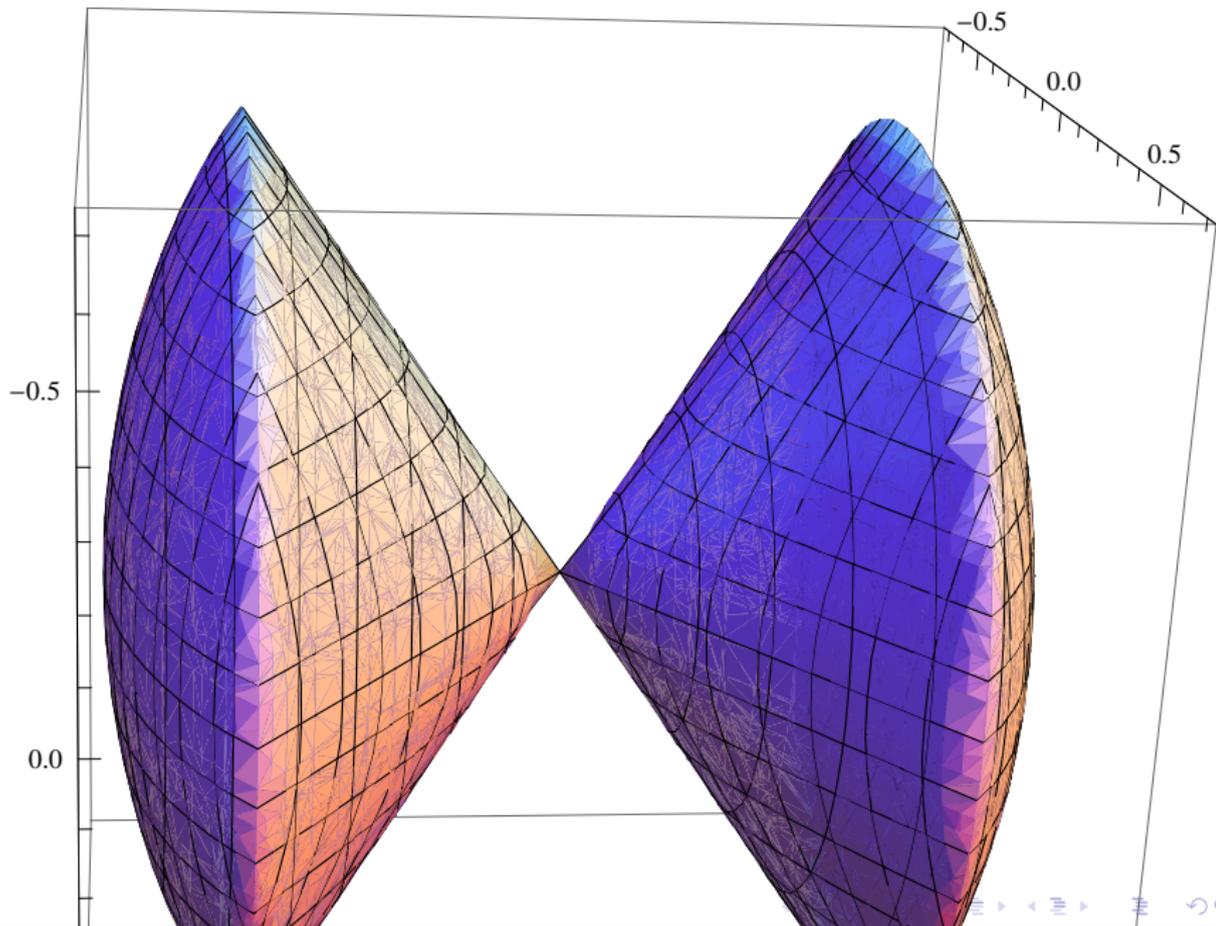$$\dot{a}^2 = \alpha - \beta a^2 - \frac{\gamma}{a^2} \qquad (43)$$

- where

$$\alpha = \frac{1}{2} \left(1 - \frac{4K}{B} \Lambda \right)$$

$$\beta = \frac{1}{2}(2\Lambda - 1) \quad \gamma = \left(\frac{K}{B} \right)^2 \Lambda$$

$$\frac{1}{2} < \Lambda < \frac{B}{4K}$$

- The above differential admits (i) $\dot{a}(\tau) < 0$ (ii) $\dot{a}(\tau) = 0$ (iii) $\dot{a}(\tau) < 0$

- It is found that in GTR with nonlinear EoS $\alpha < 0$ in flat universe. Consequently No WORMHOLE. However, in massive gravity $\alpha \geq 1$, WORMHOLE exists.

- ▶ The wormhole solution we obtain for EU model

$$a^2(\tau) = \frac{\frac{1}{4} - \frac{K}{B}\Lambda}{2\Lambda - 1} + a_o \; Cos\sqrt{2(2\Lambda - 1)} \; \tau \qquad (44)$$

- ▶ where

$$a_o = \frac{\sqrt{\left(\frac{1}{4} - \frac{K}{b}\Lambda\right)^2 - 2\Lambda\left(\frac{K}{B}\right)^2}}{2\Lambda - 1}$$

$$\Lambda = \left(\frac{B}{A+1}\right)^2 \qquad (45)$$

- ▶ Wormhole solution in GTR is not permitted in flat universe

- ▶ Wormhole solution in massive gravity theory is permitted in flat universe case.

Discussion

- Two different cosmological models are presented:
- Model I, assuming interaction of EU fluid with a pressureless barotropic fluid. The interaction sets in at an epoch $t = t_i$.
- Model II, assuming interactions among the three fluids of the emergent universe at time $t = t_0$. Before the epoch EU can be realized without an interaction among the fluids original EU model. The problem of obtaining present universe with matter can be resolved even if starts from radiation.
- The initial static Einstein universe in EU may be considered due to the presence of neck of wormhole from which the present universe might have originated.

THANK YOU