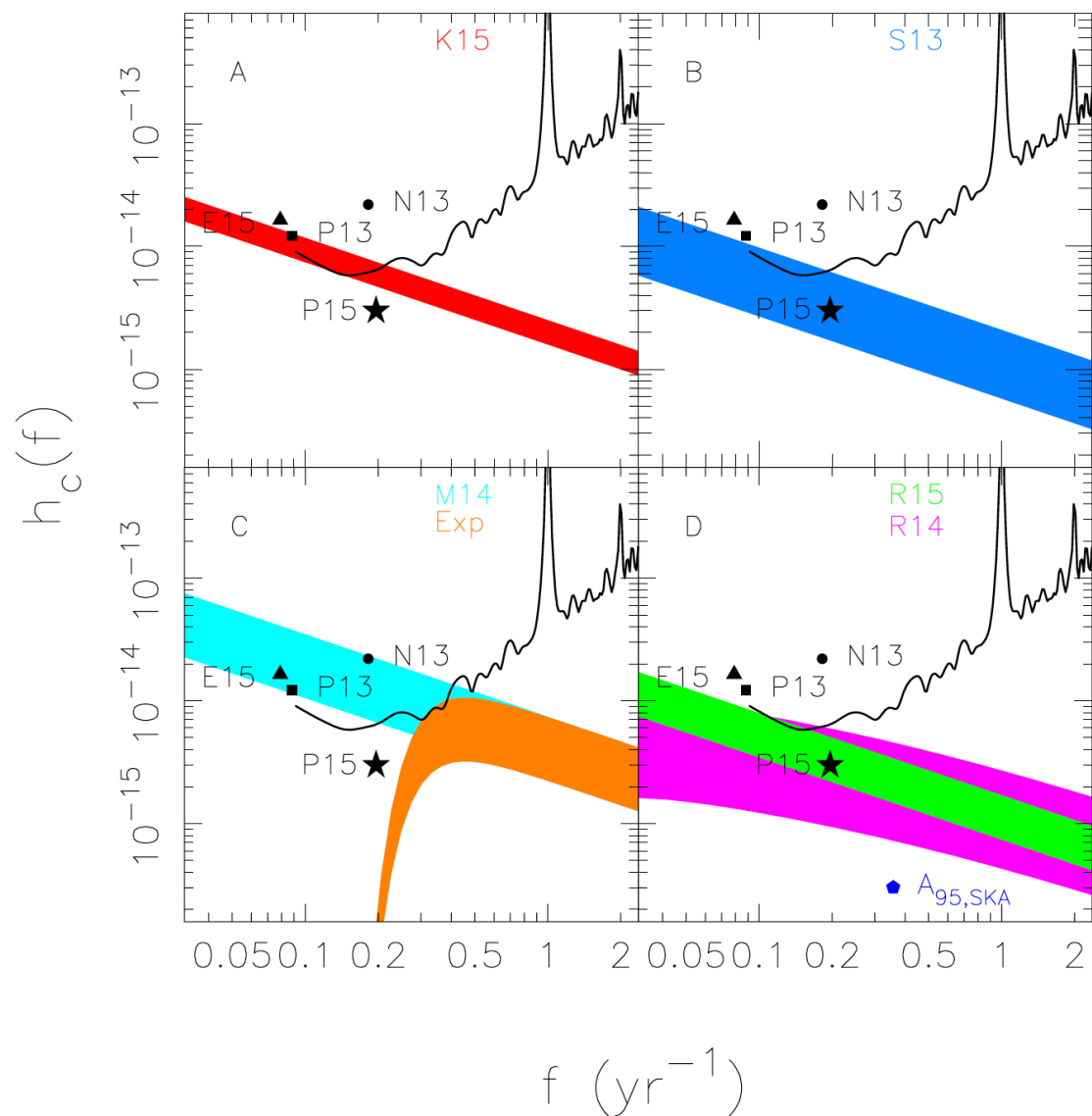


PULSAR TIMING DETECTION OF  
GRAVITATIONAL WAVES FROM SUPERMASSIVE  
BLACK HOLE BINARIES IN STELLAR  
ENVIRONMENTS

Alexander Rasskazov & David Merritt

*(image credit: NASA)*

# CURRENT PTA CONSTRAINTS



- Current PTA limits are in tension with galaxy merger models

- Possible explanation: the effects of stellar environment

# DYNAMICAL EVOLUTION OF BINARY SUPERMASSIVE BLACK HOLES

1. Unbound SMBHs, energy loss due to dynamical friction
2. Bound SMBHs, energy loss due to interactions with stars/gas
3. Tightly bound SMBHs, energy loss due to GW emission
4. Coalescence
5. Recoil

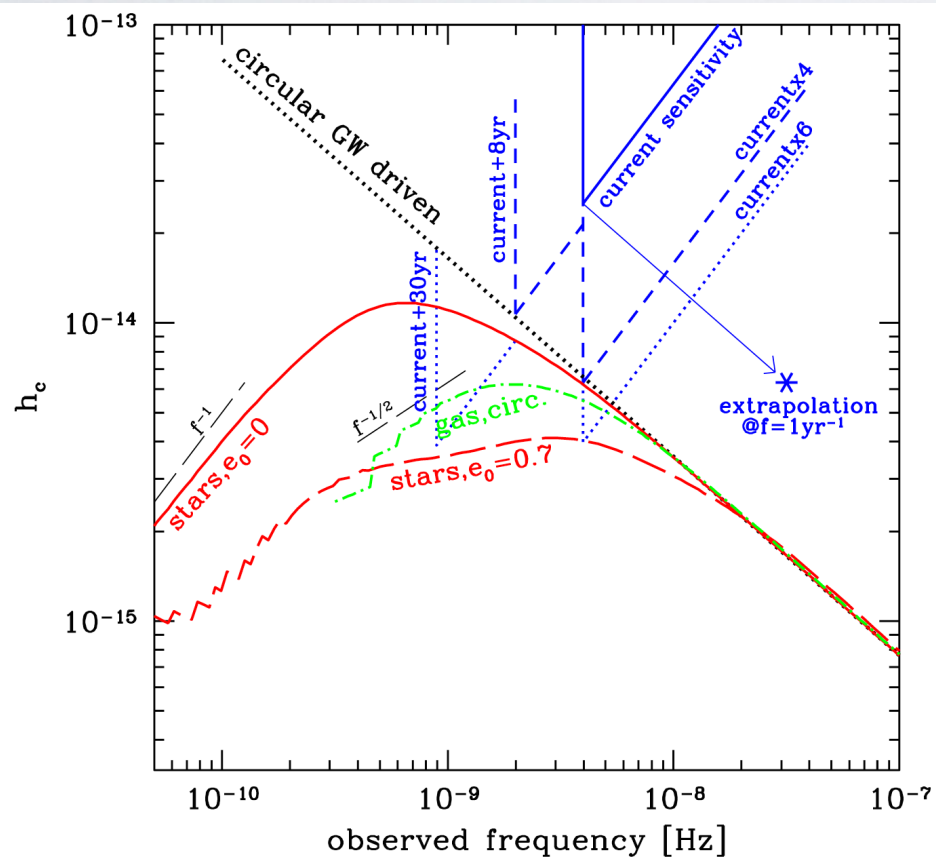
# SUPERMASSIVE BINARY'S DYNAMICAL EVOLUTION

1. Unbound SMBHs, energy loss due to dynamical friction
2. Bound SMBHs, energy loss due to interactions with stars/gas
3. Very close SMBHs, energy loss due to GW emission
4. Coalescence
5. Recoil

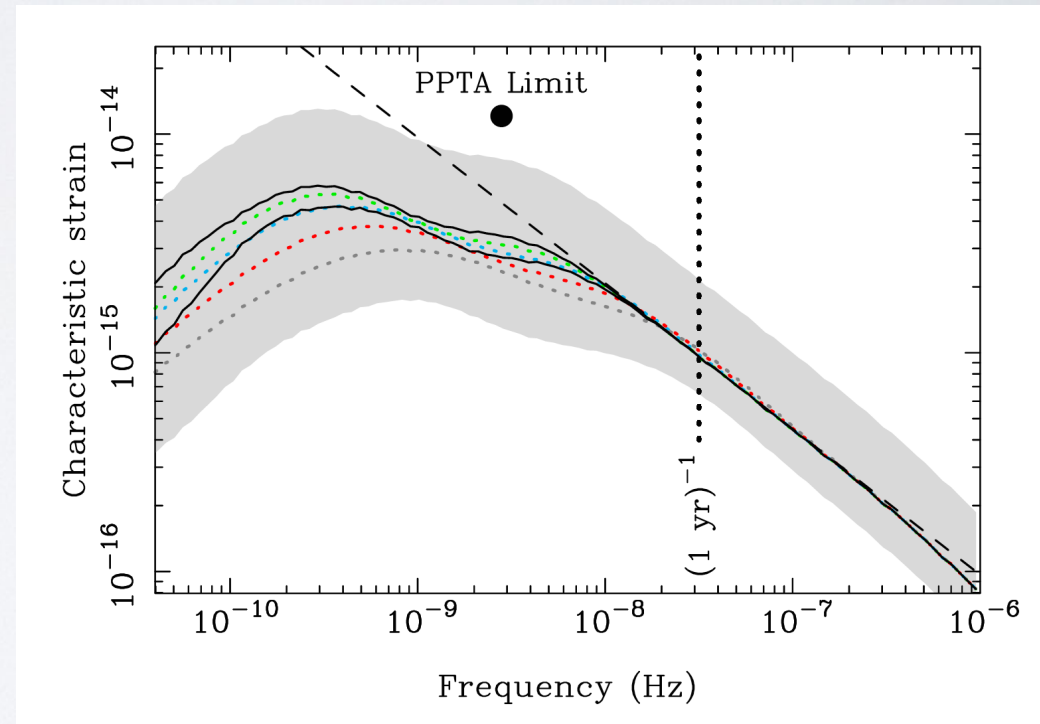
detectable GW emission

# PREVIOUS RESULTS

Sesana (2013)



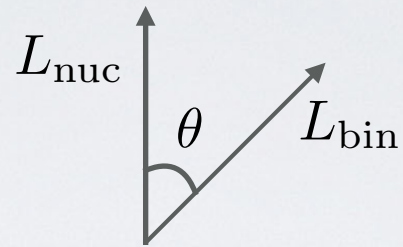
Ravi et al. (2014)



# NUCLEAR ROTATION MATTERS

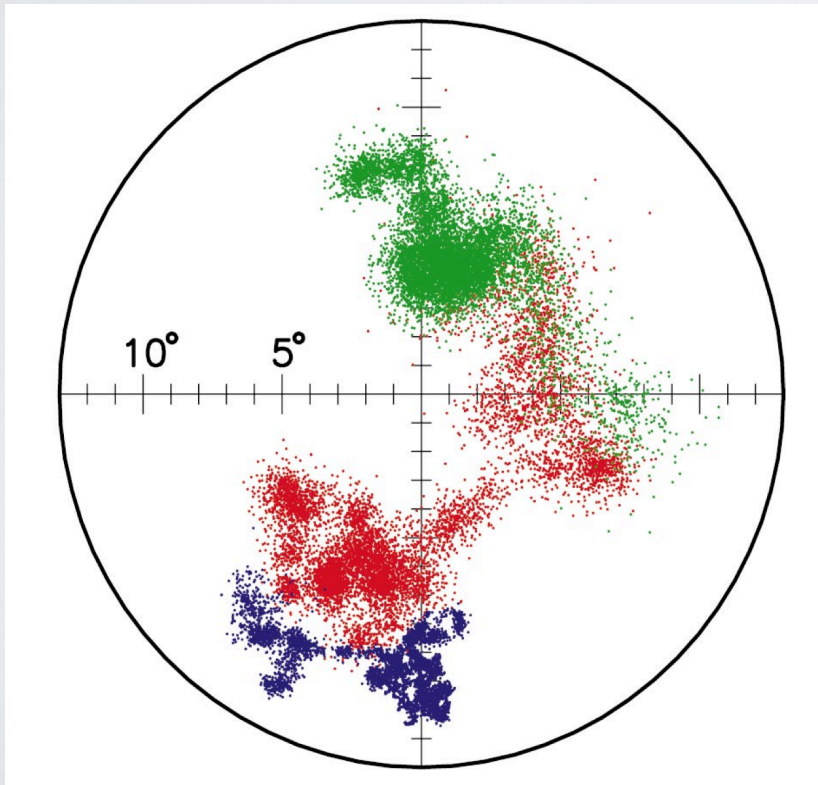
NON-ROTATING NUCLEUS

$$\frac{d\theta}{dt} \approx 0$$

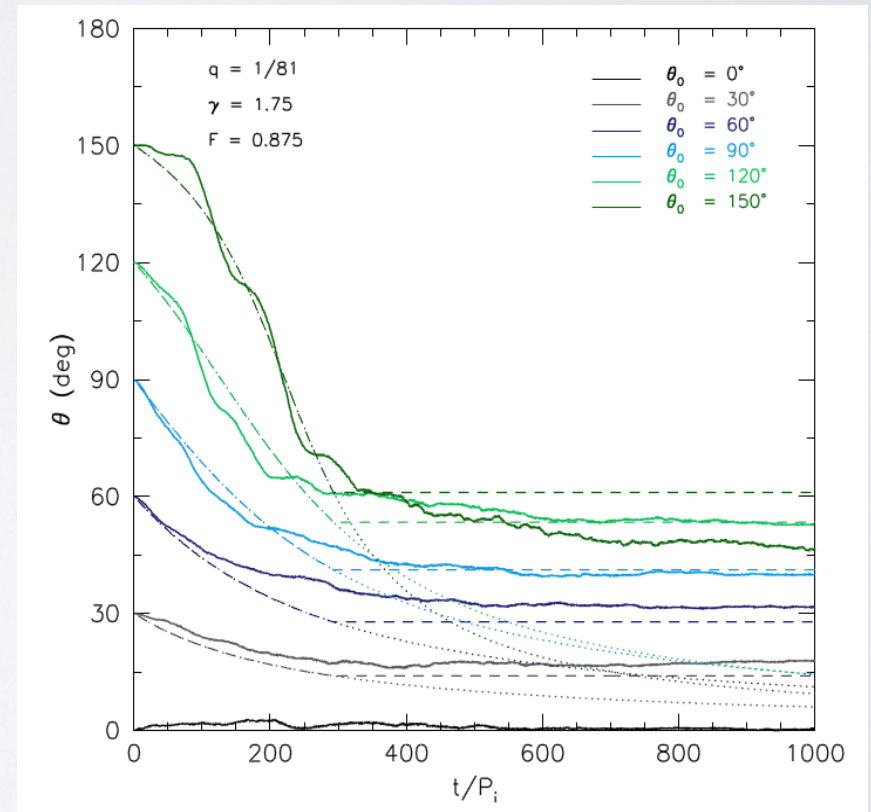


ROTATING NUCLEUS

$$\frac{d\theta}{dt} < 0$$



Milosavljevic & Merritt (2001)

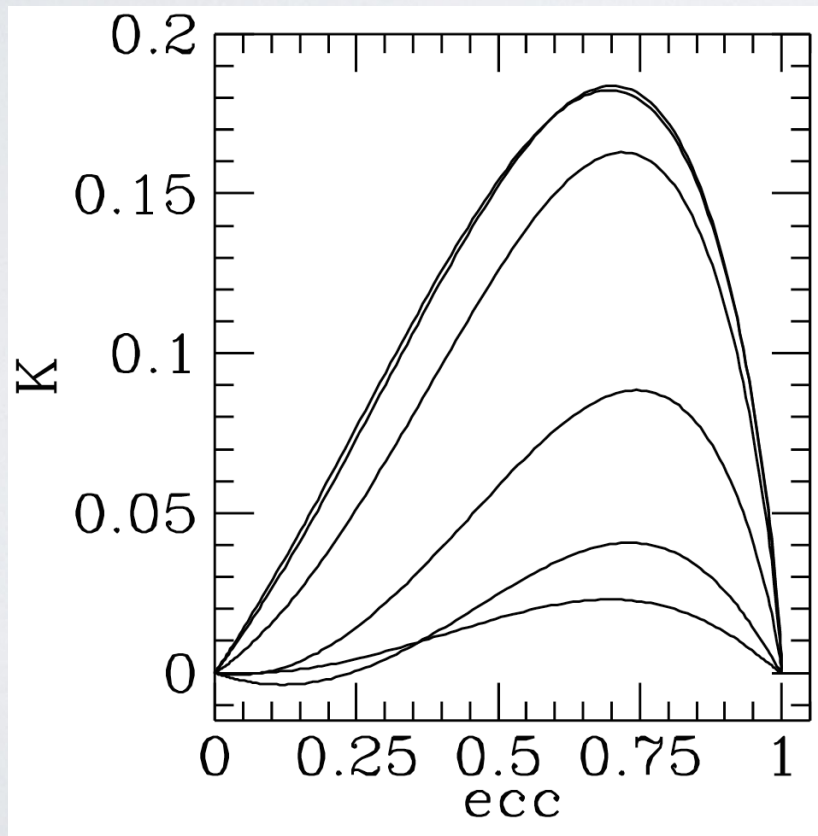


Gualandris et al. (2012)

# NUCLEAR ROTATION MATTERS

NON-ROTATING NUCLEUS

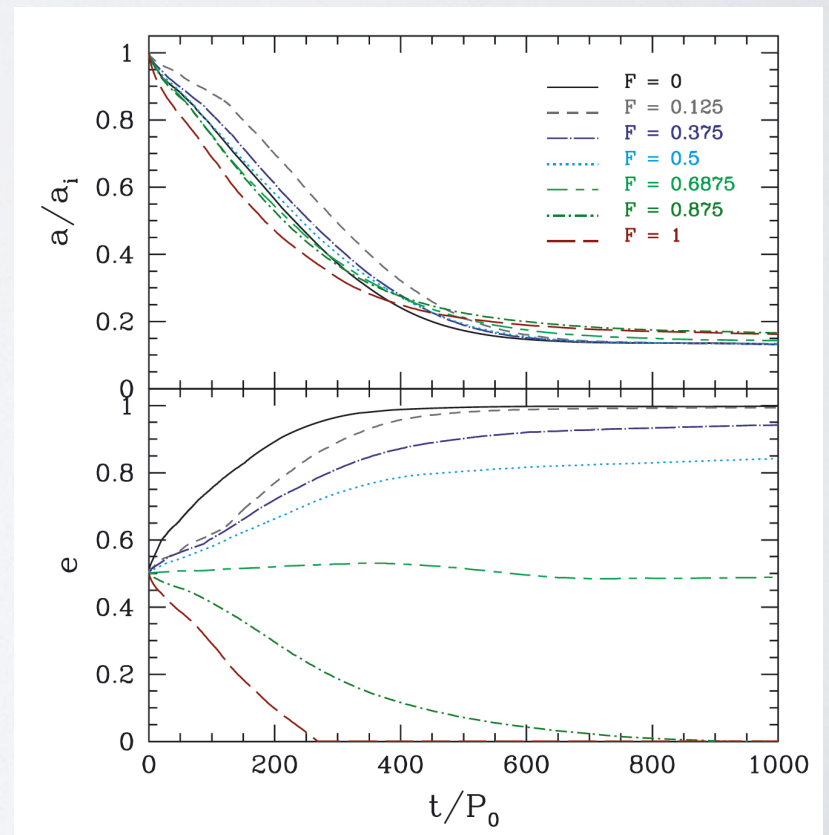
$$\frac{de}{dt} > 0$$



Quinlan (1996)

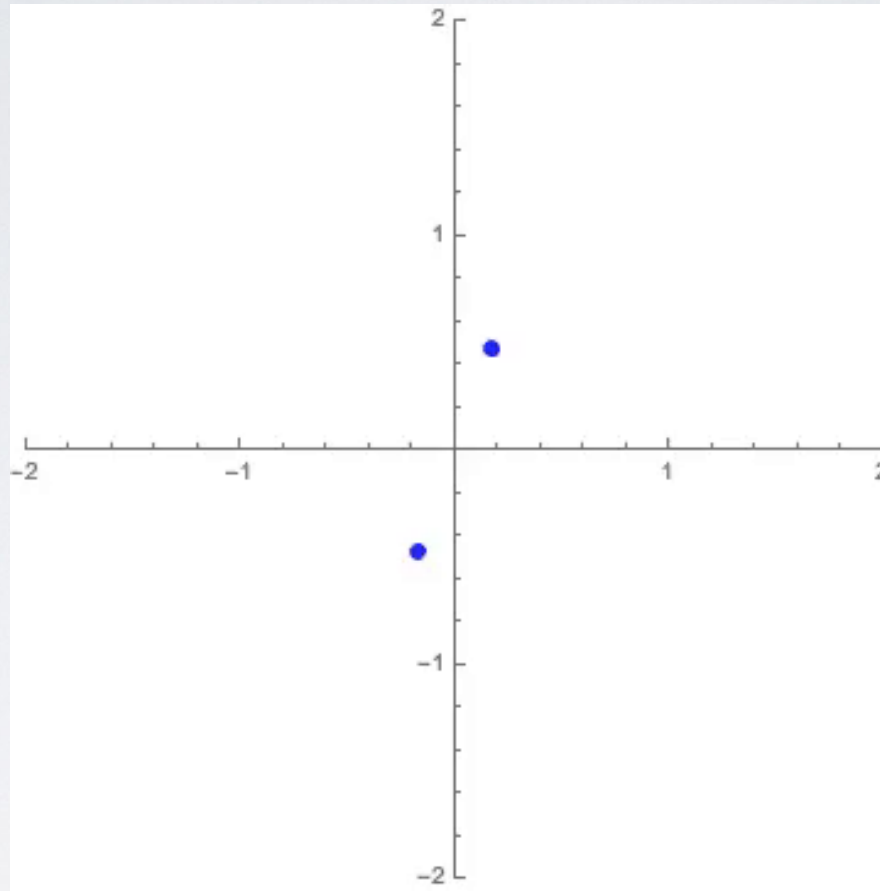
ROTATING NUCLEUS

$$\frac{de}{dt} < 0, \theta \approx 0$$
$$\frac{de}{dt} > 0, \theta \approx \pi$$



Sesana et al. (2011)

# SCATTERING EXPERIMENTS



$$\frac{da}{dt}, \frac{de}{dt}, \frac{d\theta}{dt} = \text{average of } \delta a, \delta e, \delta \theta \text{ over all interactions}$$



# NEW FEATURES COMPARED TO THE PREVIOUS STUDIES\*

- Allowing the binary to evolve out of a fixed plane, which can strongly impact the eccentricity evolution:

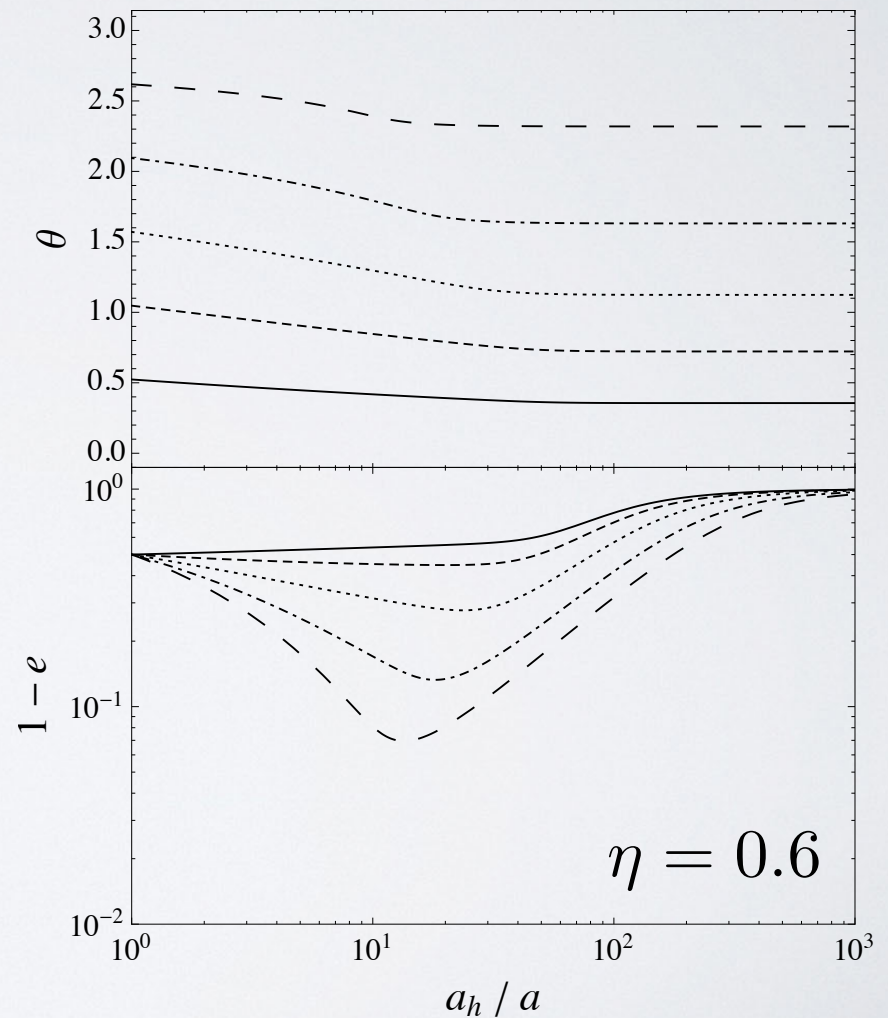
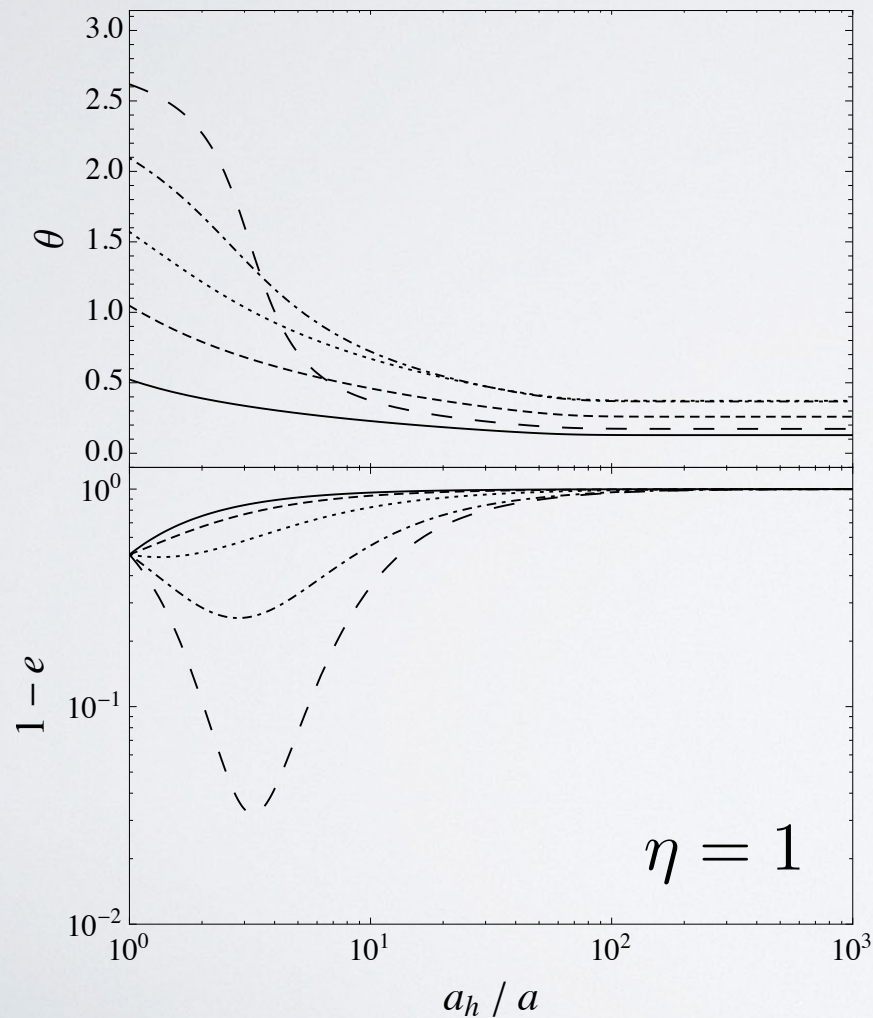
$$\frac{d\theta}{dt} \sim (2\eta - 1) \sqrt{\frac{1+e}{1-e}} \sin \theta$$

$$\frac{de}{d \ln a} = 1.5 e (1 - e^2)^{0.7} [0.15 - (2\eta - 1) \cos \theta]$$

\*Sesana (2013), Ravi et al. (2014), McWilliams et al. (2014) etc.

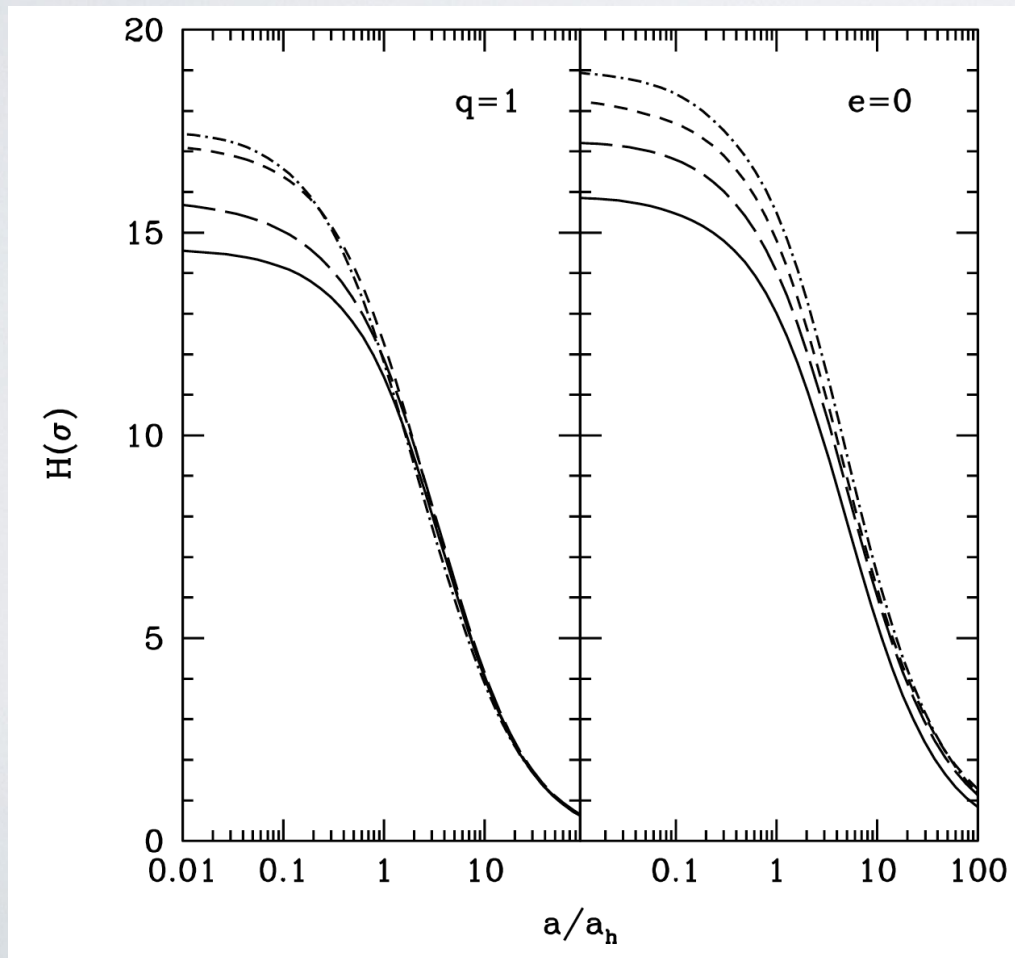
# NEW FEATURES COMPARED TO THE PREVIOUS STUDIES

- GW emission doesn't depend on  $\theta$ , but eccentricity evolution does



# NEW FEATURES COMPARED TO THE PREVIOUS STUDIES

- Previous papers assumed infinite and homogenous stellar medium



$$H = \frac{\sigma}{G\rho} \frac{d(1/a)}{dt} \approx \text{const}$$

# NEW FEATURES COMPARED TO THE PREVIOUS STUDIES

- Incorporating the Vasiliev, Antonini & Merritt (2015) solution to the “final-parsec problem”

$$S \equiv \frac{d(1/a)}{dt} = \mu S_{\text{infl}} \left( \frac{a}{a_h} \right)^\nu,$$

$$S_{\text{infl}} \equiv 4 \sqrt{\frac{GM}{r_{\text{infl}}^5}}$$

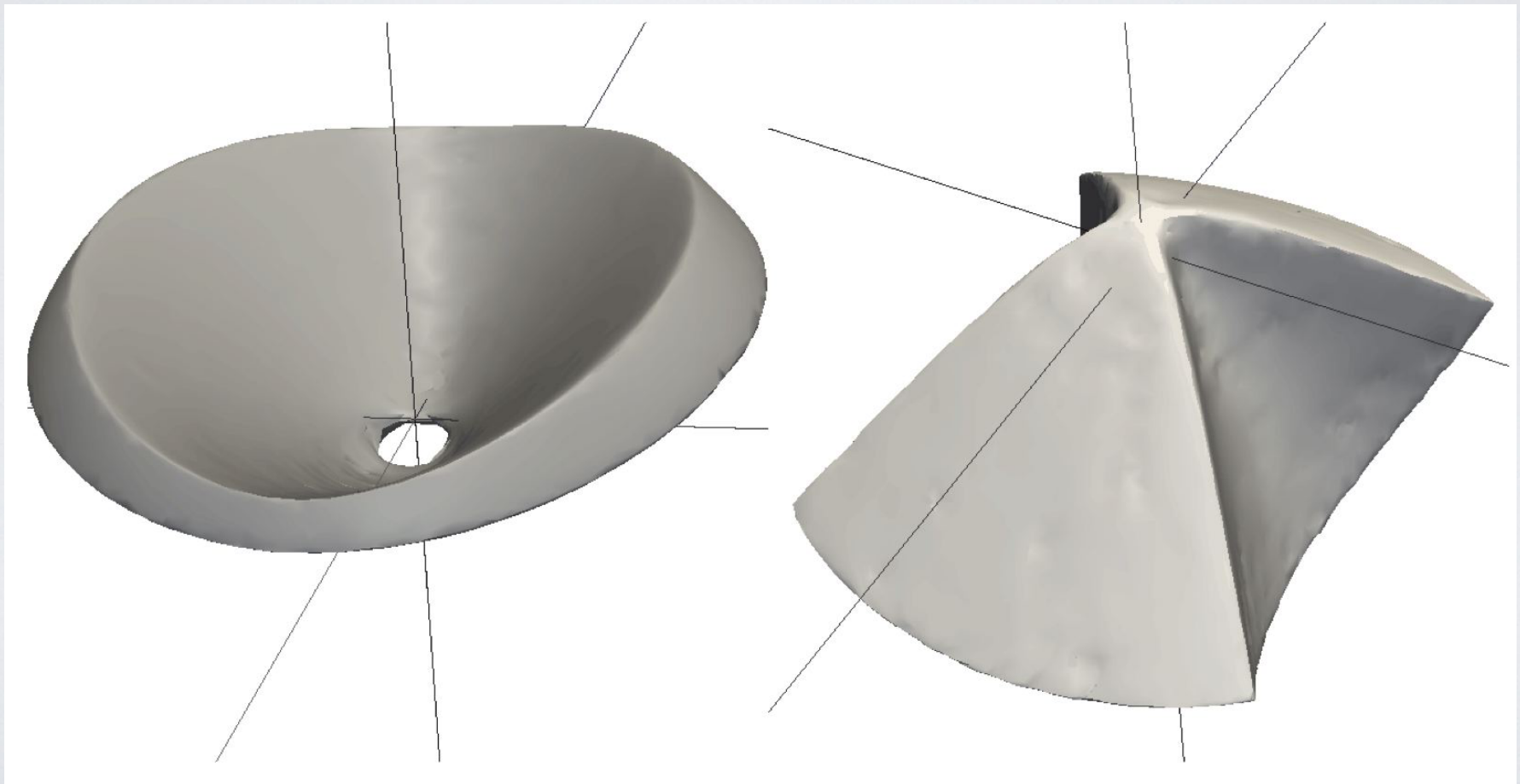
Spherical :  $\nu \simeq 0$ ,  $\mu \simeq (N_\star/10^5)^{-1}$

Axisymmetric :  $\nu \simeq 0$ ,  $\mu \simeq (N_\star/10^5)^{-1/2}$

Triaxial :  $\nu \simeq 0.3 \dots 0.6$ ,  $\mu \sim 1$

# NEW FEATURES COMPARED TO THE PREVIOUS STUDIES

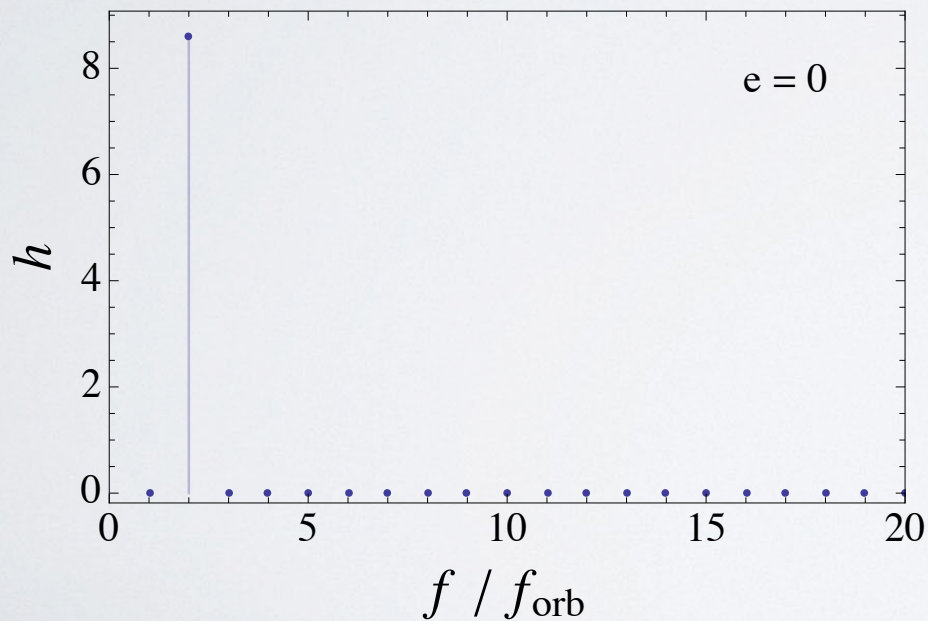
Orbits around SBHs in non-spherical galaxies



# GW SPECTRUM OF A BINARY BLACK HOLE

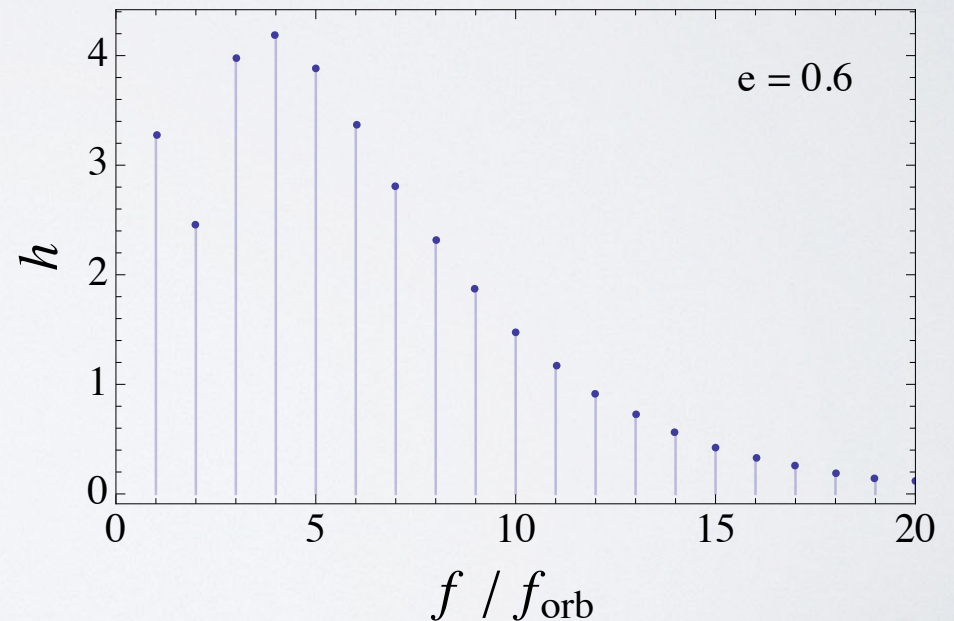
Circular

$$h(f) \sim \delta(f - 2f_{\text{orb}})$$



Eccentric

$$h(f) \sim \sum_{n=1}^{\infty} g(n, e) \delta(f - n f_{\text{orb}})$$



# TOTAL GW BACKGROUND

$N(a, t)da$  = number of binaries per comoving volume  
with semimajor axes  $a \dots a + da$

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial a} (N\dot{a}) = 0$$

$$N(a, t) = \frac{\dot{N}_m(t_f)}{|\dot{a}(a)|}, \quad \dot{N}_m \sim (1+z)^2$$

*(Xu et al. 2012)*

Galaxy merger rate  
at the moment  
when BSBH was  
formed

Generalizing Phinney (2001):

$$\Omega_{\text{GW}} \rho_c c^2 = \frac{32}{5} \frac{G^{7/3}}{c^5} \mathcal{M}^{10/3} f \int_{t(\infty)}^0 dt \cdot \sum_{a_c < a_n < a_h} \left( n \left| \frac{df_{\text{orb}}}{da}(a_n) \right| \right)^{-1} \left( \frac{2\pi f_r}{n} \right)^{10/3} g(n, e(a_n)) \frac{\dot{N}_m(t_f(a_n, t))}{|\dot{a}(a_n)|}$$

# TOTAL GW BACKGROUND

$N(a, t)da$  = number of binaries per comoving volume  
with semimajor axes  $a \dots a + da$

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial a} (N\dot{a}) = 0$$

$$N(a, t) = \frac{\dot{N}_m(t_f)}{|\dot{a}(a)|}, \quad \dot{N}_m \sim (1+z)^2$$

Following Phinney (2001):

$$\Omega_{\text{GW}} \rho_c c^2 = \frac{32}{5} \frac{G^{7/3}}{c^5} \mathcal{M}^{10/3} f \int_{t(\infty)}^0 dt.$$

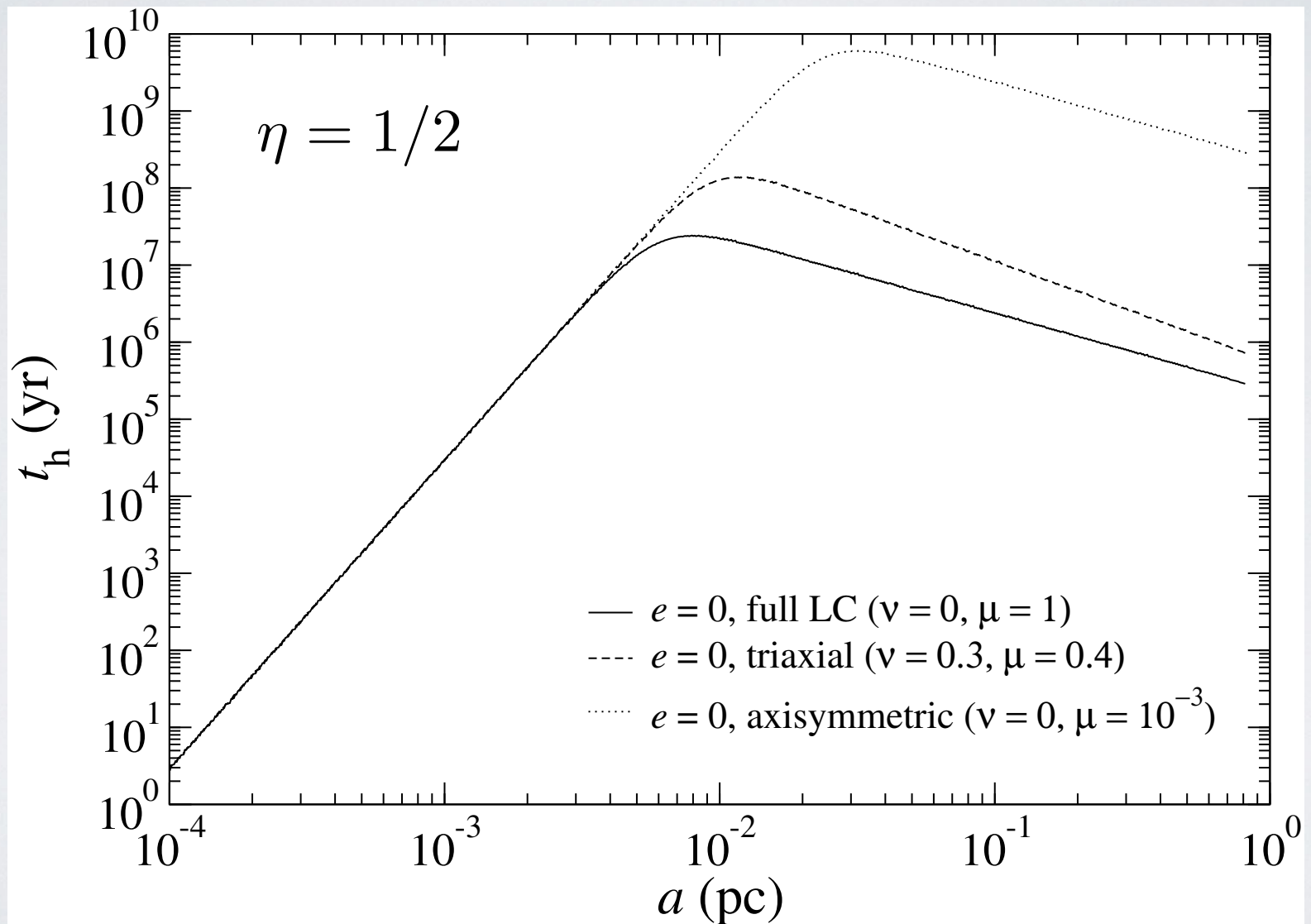
$$\cdot \sum_{a_c < a_n < a_h} \left( n \left| \frac{df_{\text{orb}}}{da}(a_n) \right| \right)^{-1} \left( \frac{2\pi f_r}{n} \right)^{10/3} g(n, e(a_n)) \frac{\dot{N}_m(t_f(a_n, t))}{|\dot{a}(a_n)|}$$

Lifetime of the binary ( $t_f - t$ )  
can be long

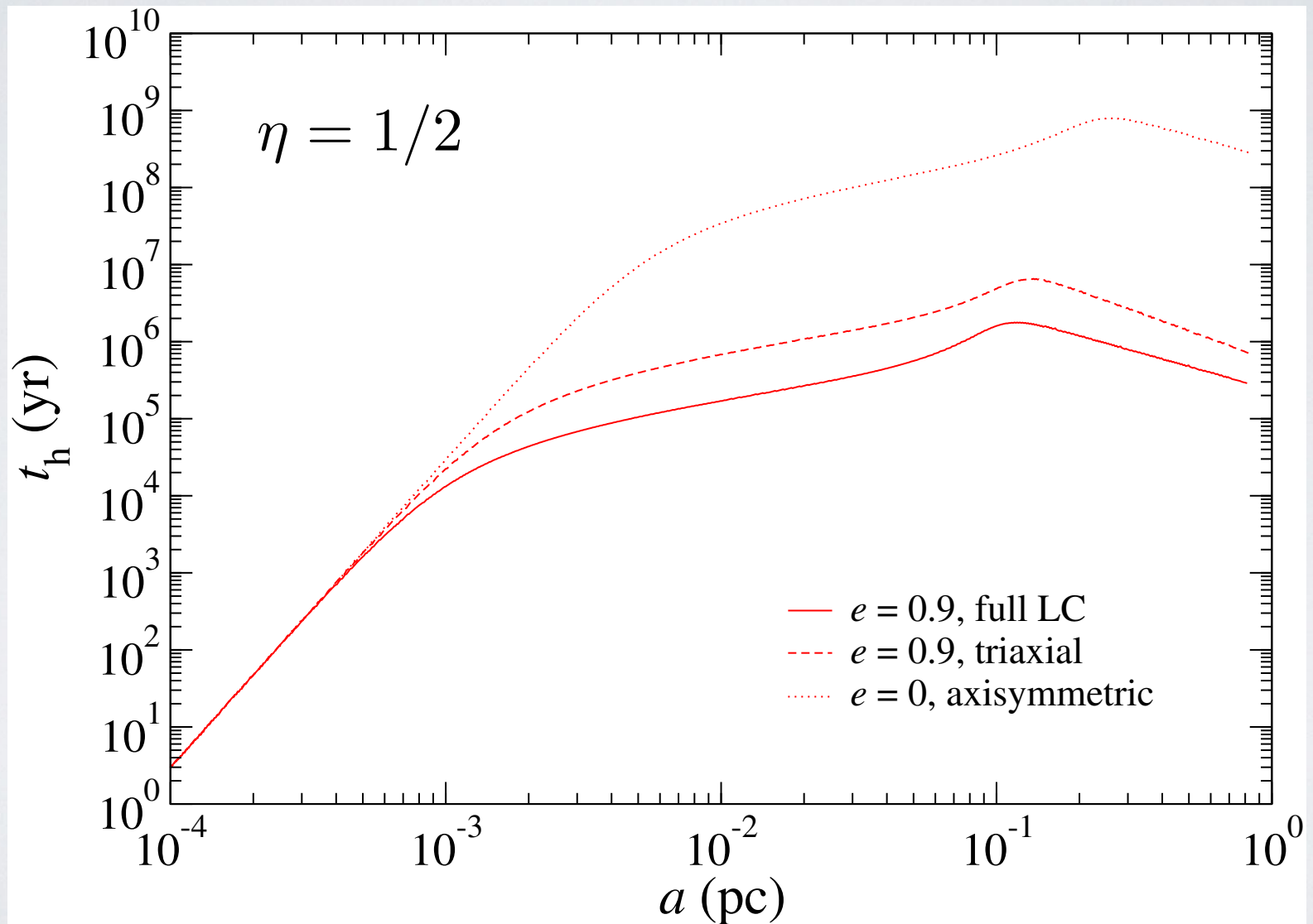




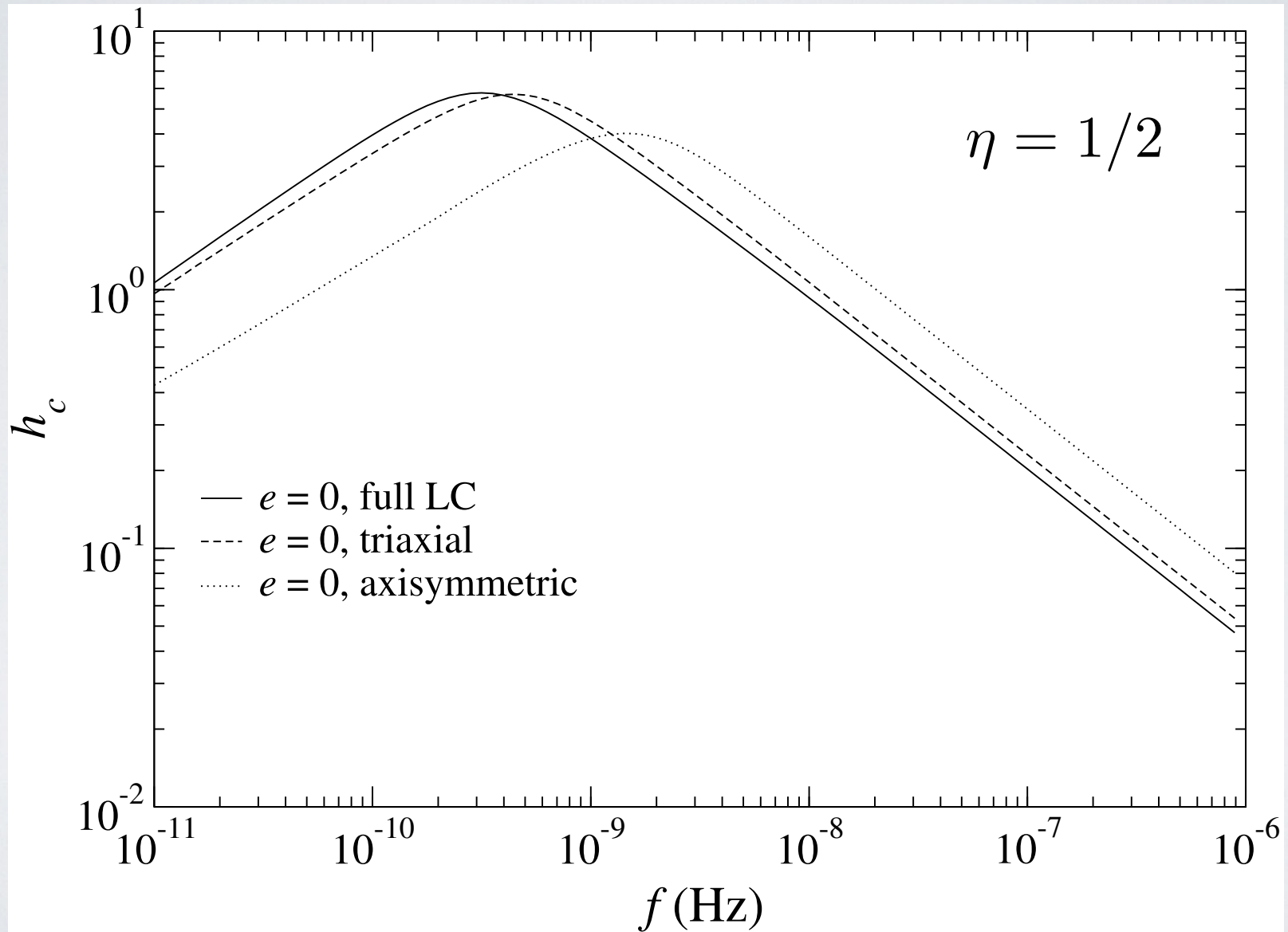
# HARDENING TIMESCALES



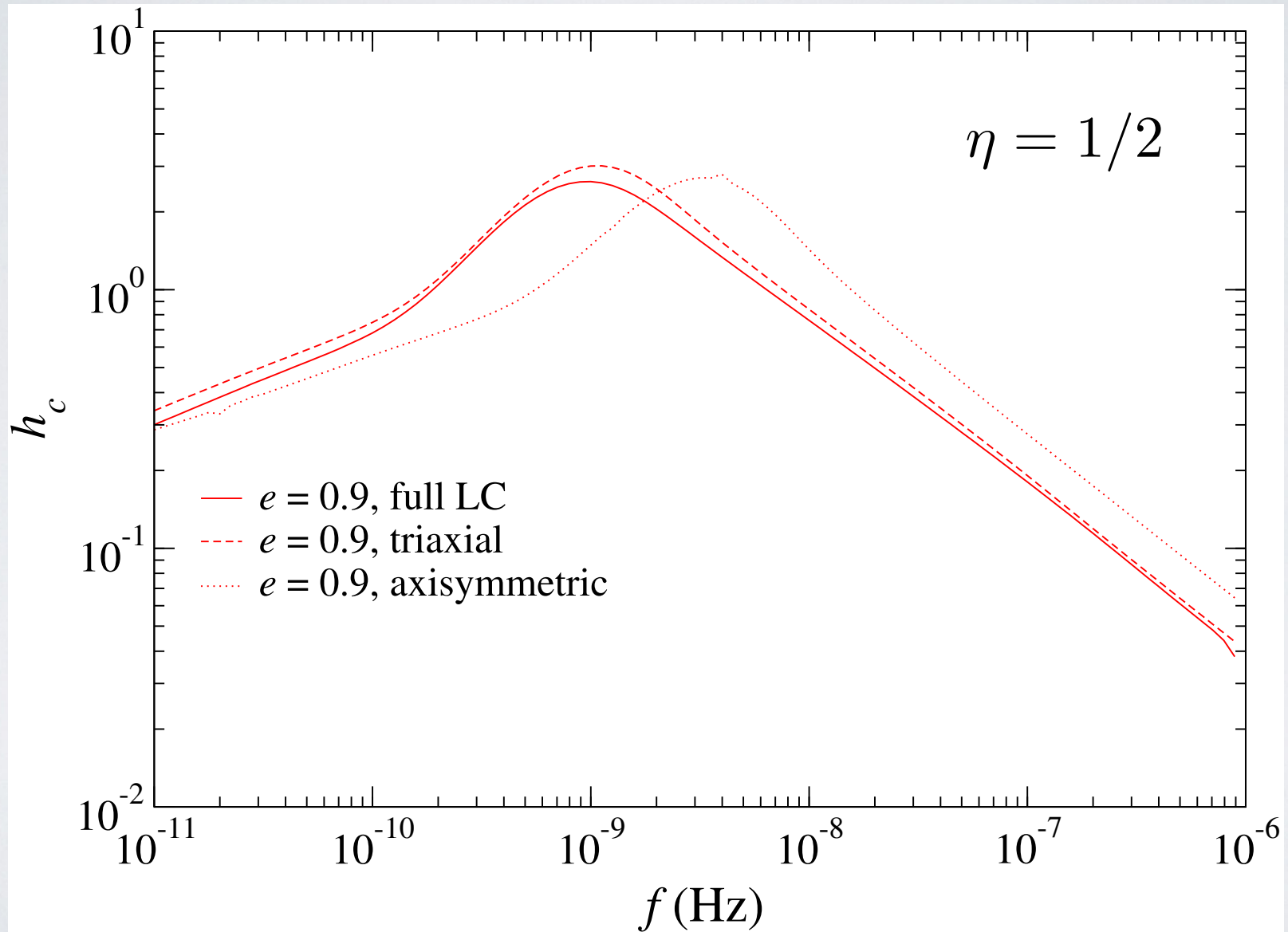
# HARDENING TIMESCALES



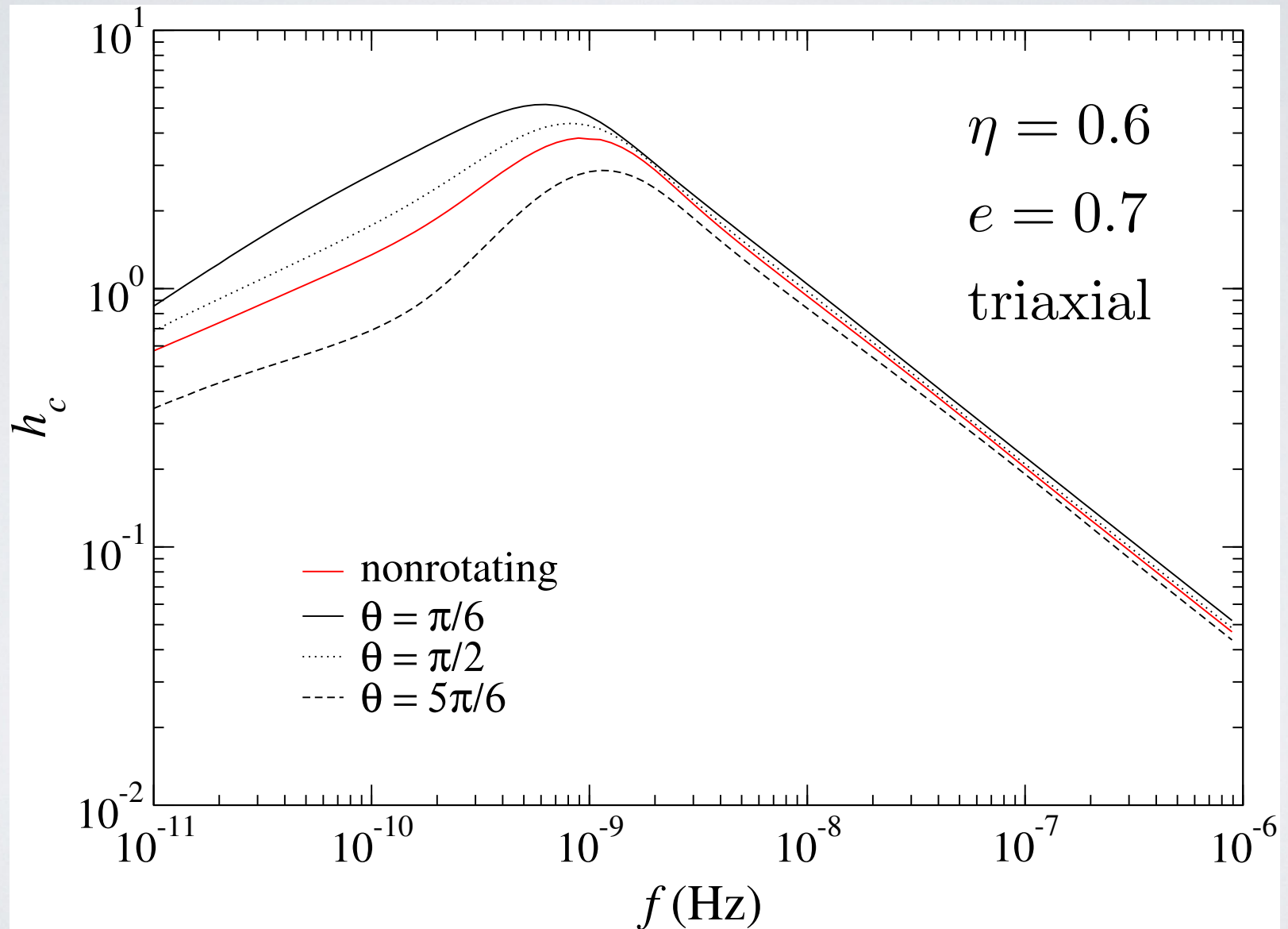
# STOCHASTIC GW BACKGROUND



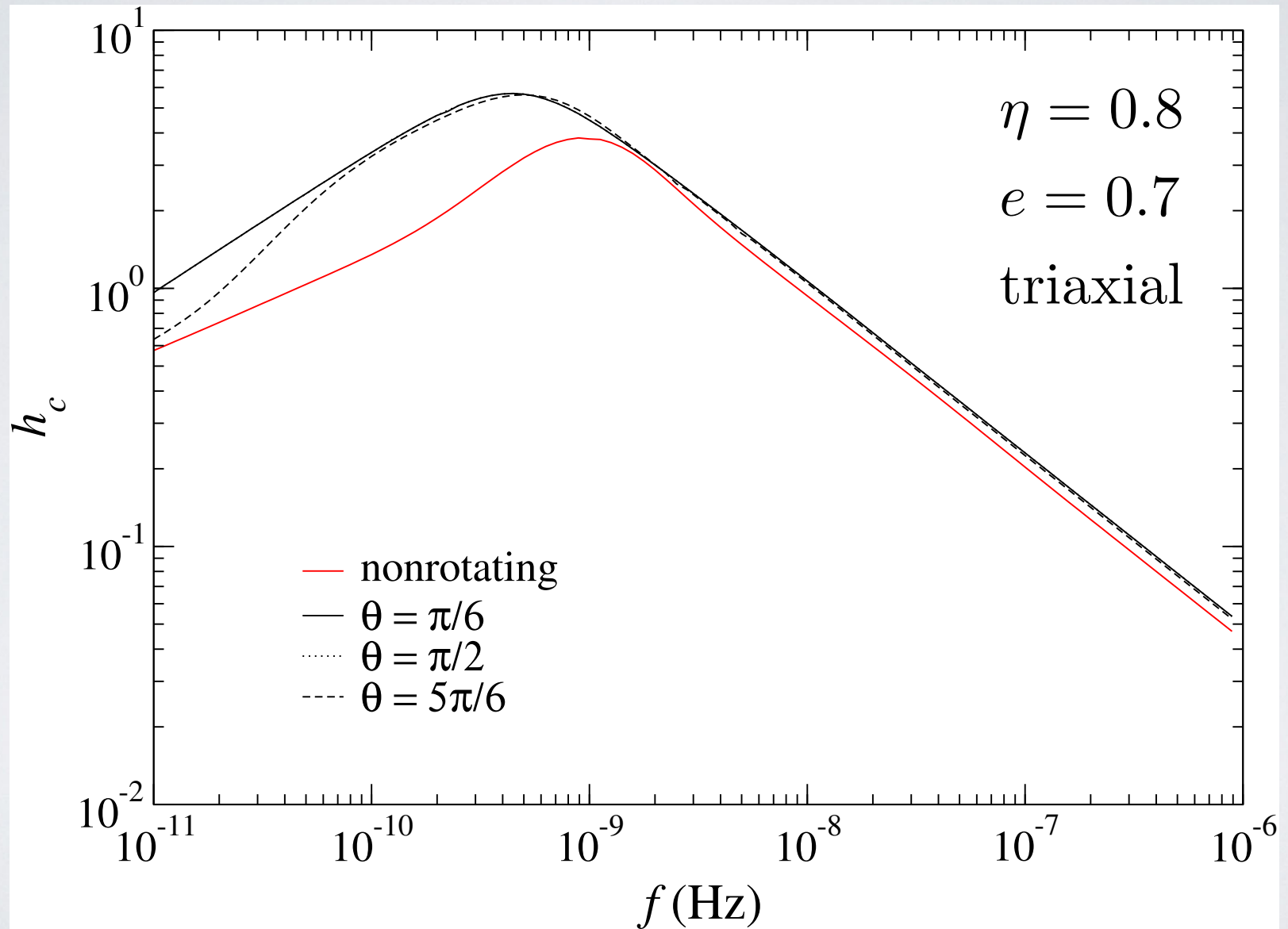
# STOCHASTIC GW BACKGROUND



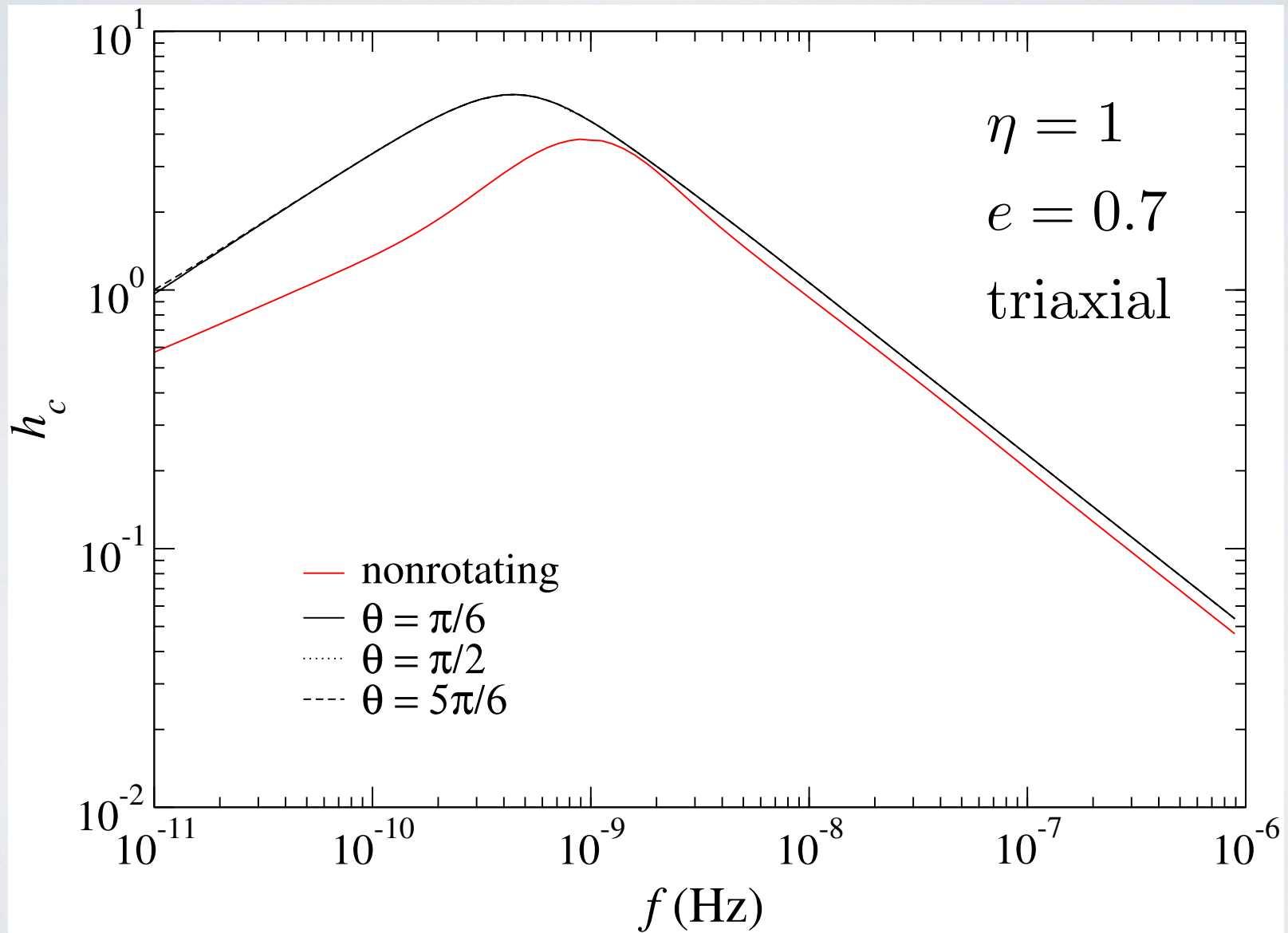
# STOCHASTIC GW BACKGROUND



# STOCHASTIC GW BACKGROUND



# STOCHASTIC GW BACKGROUND



# SUMMARY

- Previous calculations of stochastic GW background are generalized to include:
  - Evolution of stellar nucleus
  - Rotation of stellar nucleus
- Effects of nuclear evolution:
  - Triaxial or axisymmetric - the spectrum is shifted towards higher frequencies (especially for axisymmetric)
  - Spherical - BSBH fail to produce GW emission due to “final parsec problem”



# SUMMARY

- Effects of nuclear rotation:
  - Corotating - initial eccentricity quickly falls to zero and has no impact on GW spectrum
  - Counterrotating - eccentricity can grow significantly (especially in case of slow rotation) and reduce GW emission at low frequencies
- Plans for future work: calculate total GW background for the distribution of binary masses, initial conditions etc.

A visualization of gravitational waves from a binary black hole merger. Two black holes, one purple and one white, are shown merging into a single black hole. The spacetime around them is distorted, creating concentric ripples that propagate outwards. The background is a dark field of stars.

THANK YOU!

*(image credit: NASA)*