Simulations of the Magnetospheres of Accreting Millisecond Pulsars

torque enhancement,
spin equilibrium, and jet power

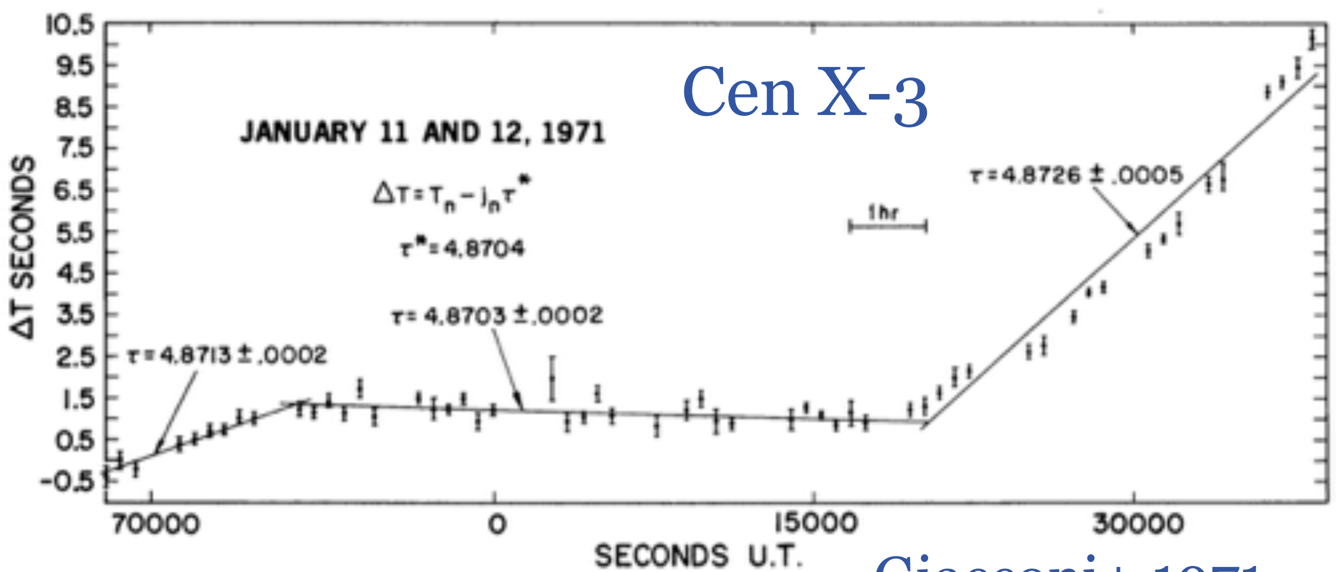
Kyle Parfrey
Lawrence Berkeley Laboratory
with
Anatoly Spitkovsky & Andrei Beloborodov

BERKELEY LAB

Observational puzzles

torques on accreting neutron stars

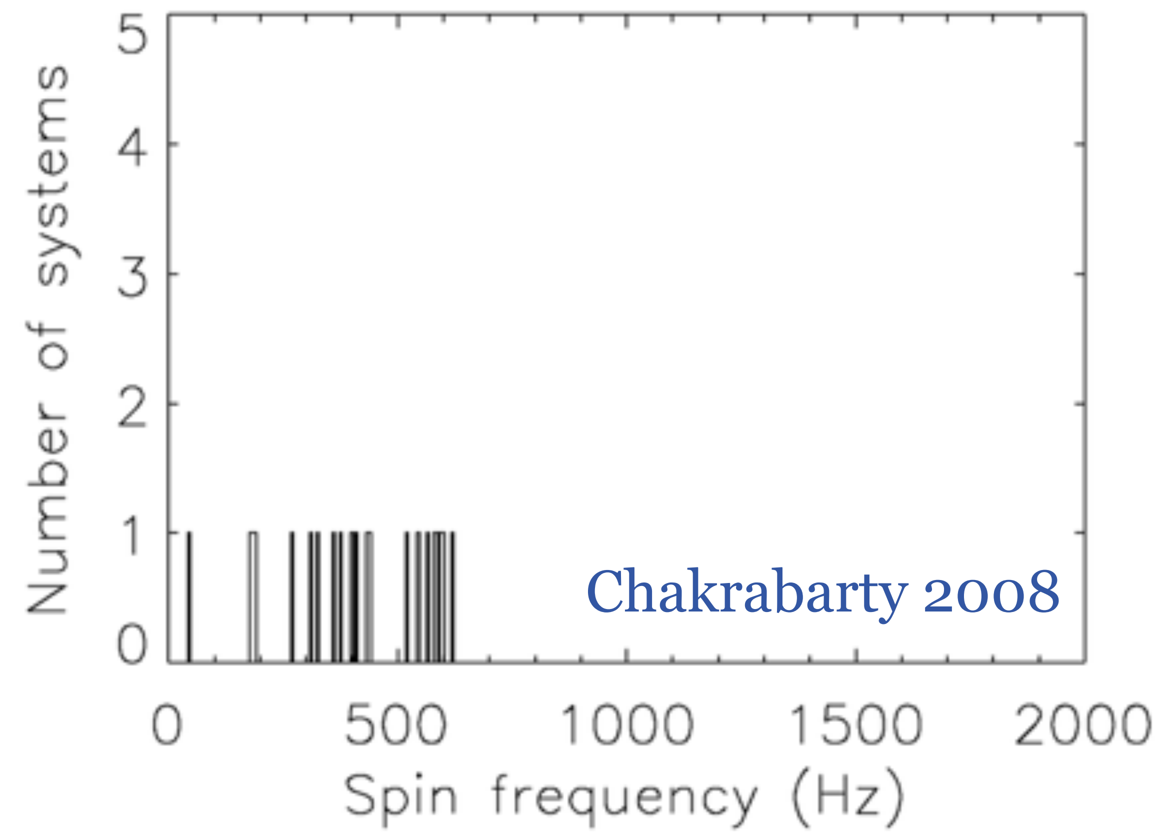
phase residuals



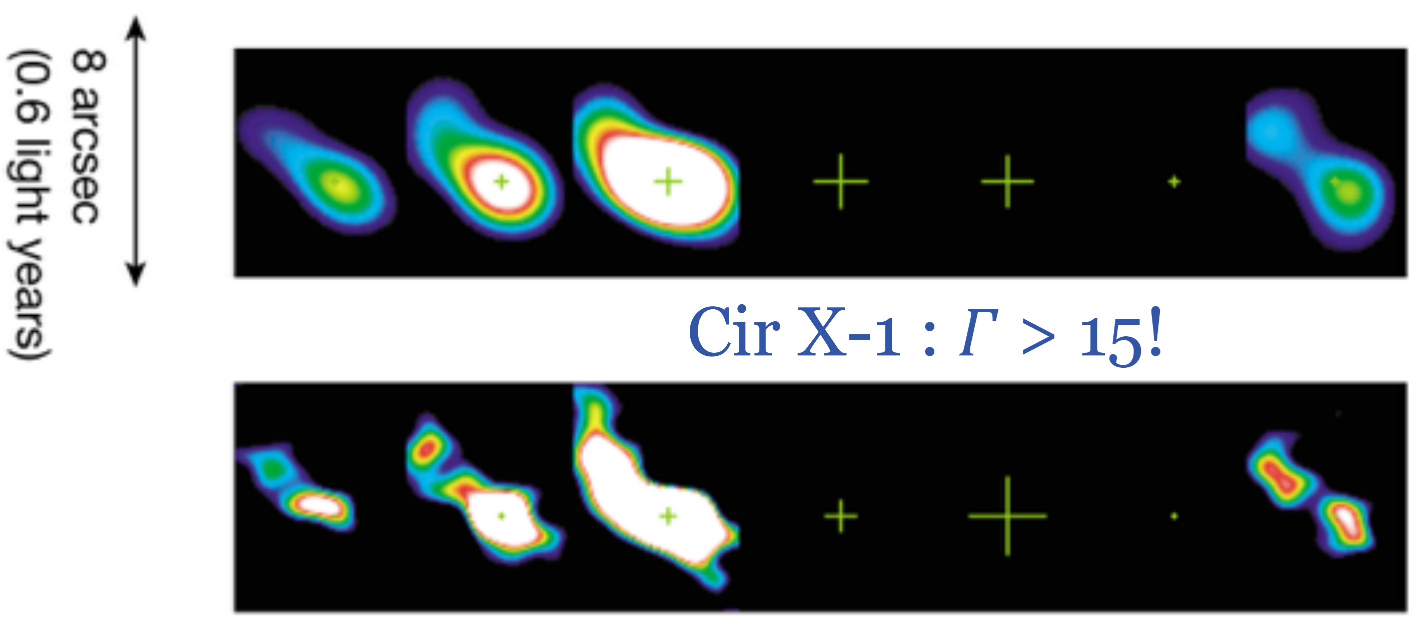
Cen X-3

Giacconi+ 1971

spin cutoff of millisecond X-ray pulsars



Chakrabarty 2008



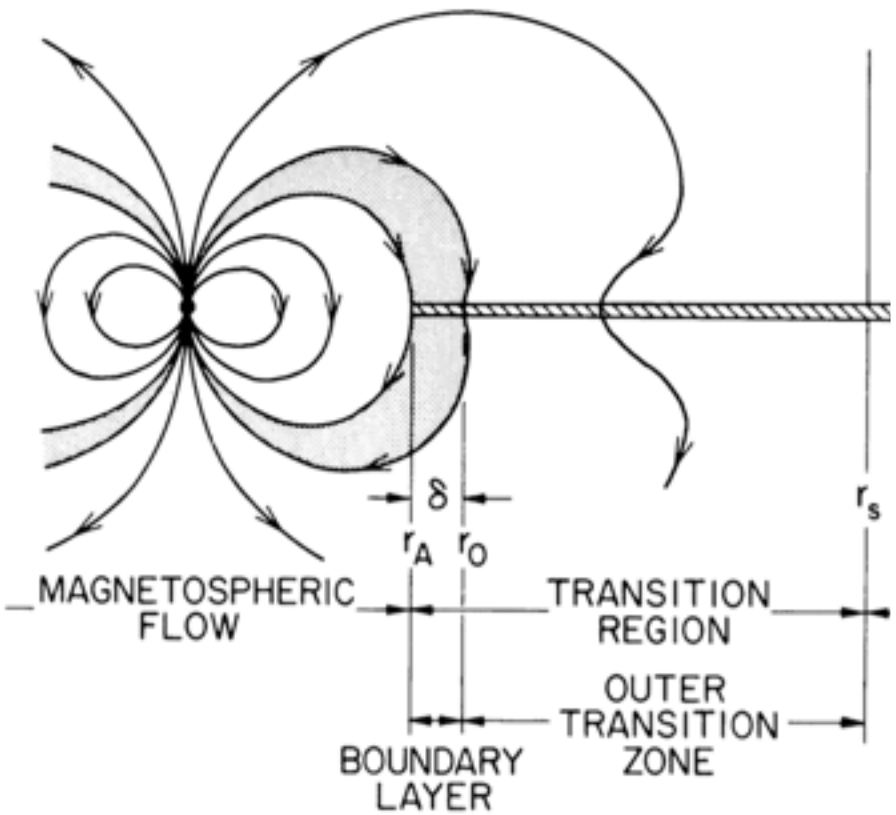
Cir X-1 : $\Gamma > 15$!

Fender+ 2004

neutron star jets

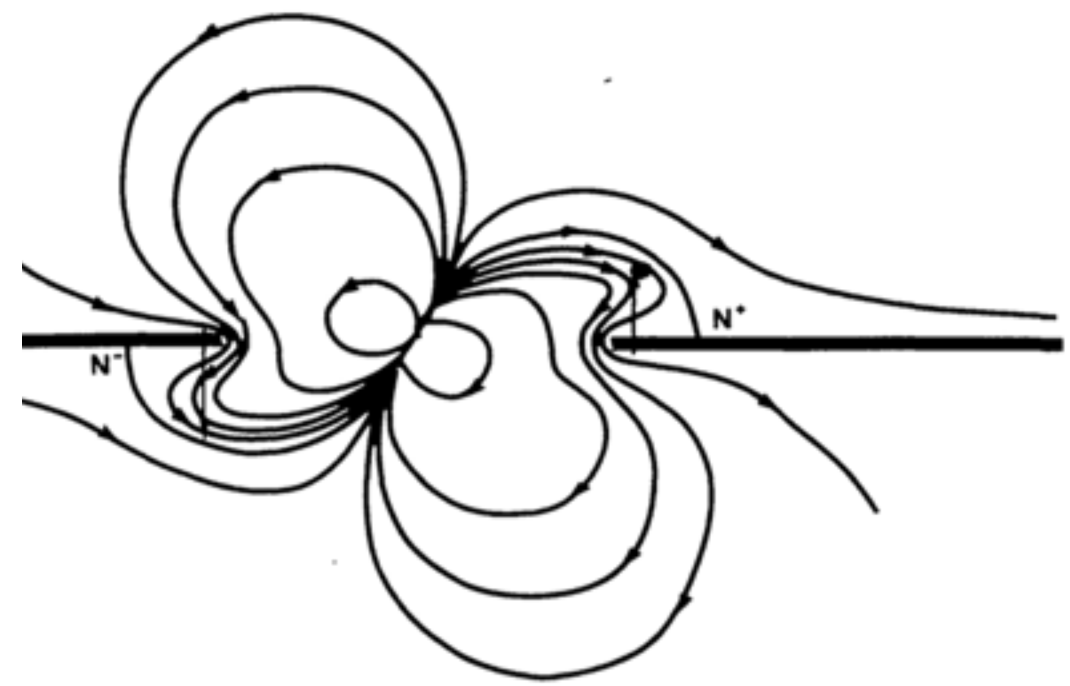
Theory: magnetospheric geometry is central

Closed…



Ghosh & Lamb 1978

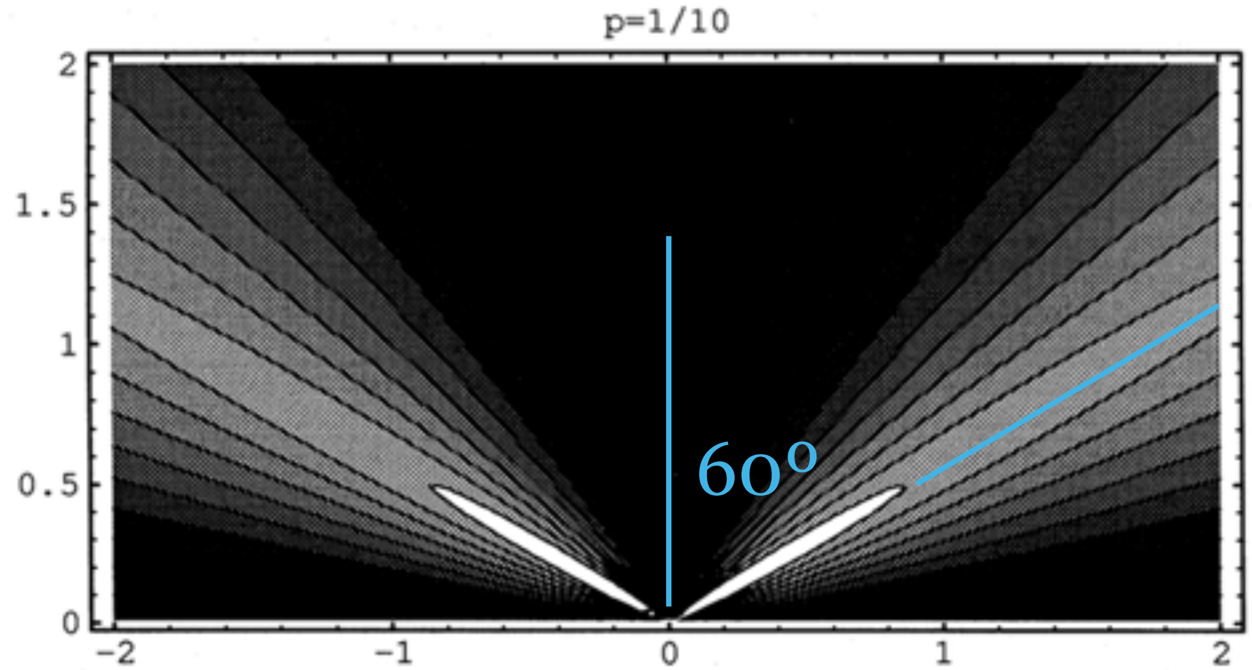
…or open?



Aly 1980

1. Disc exerts torques on the star via the field lines
2. Radio jet may be driven by the stellar rotation + open magnetic flux

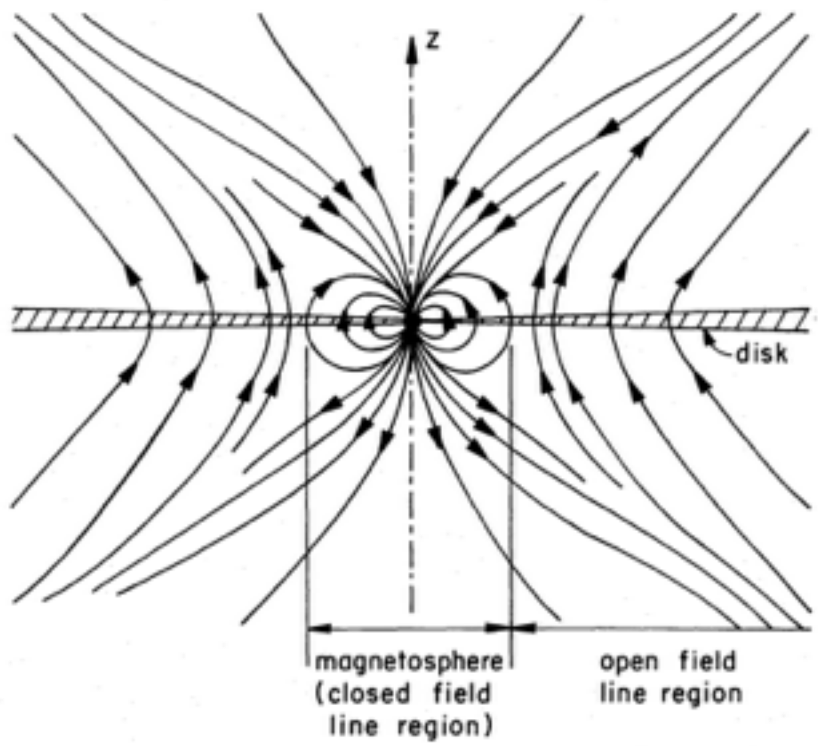
Field lines can be opened by disc



Twisting/winding causes field lines to open radially

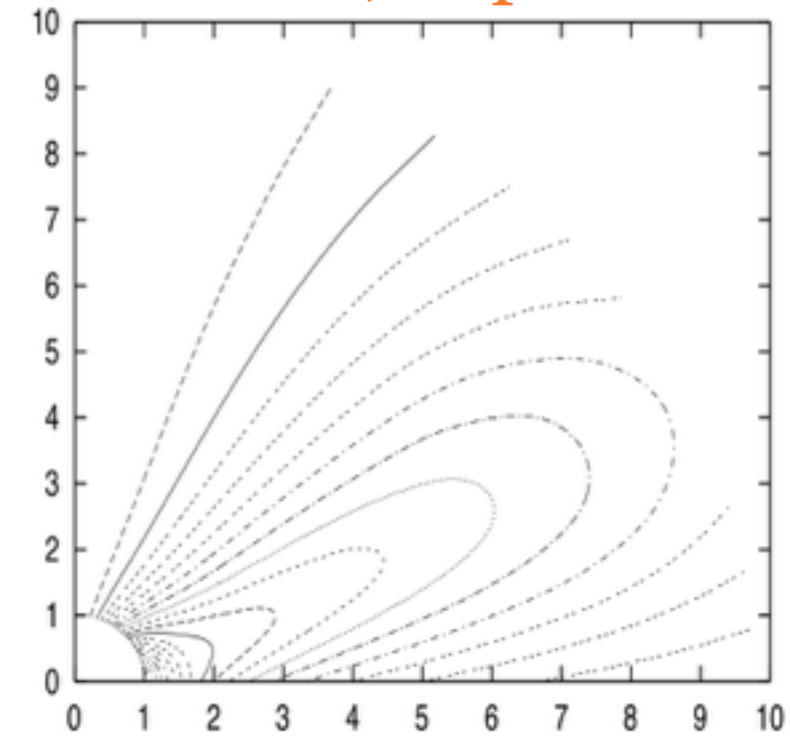
Lynden-Bell & Boily 1994

open field model for accreting star



Lovelace, Romanova, Bisnovatyi-Kogan 1995

steady-state solution at fixed twist; Keplerian disc

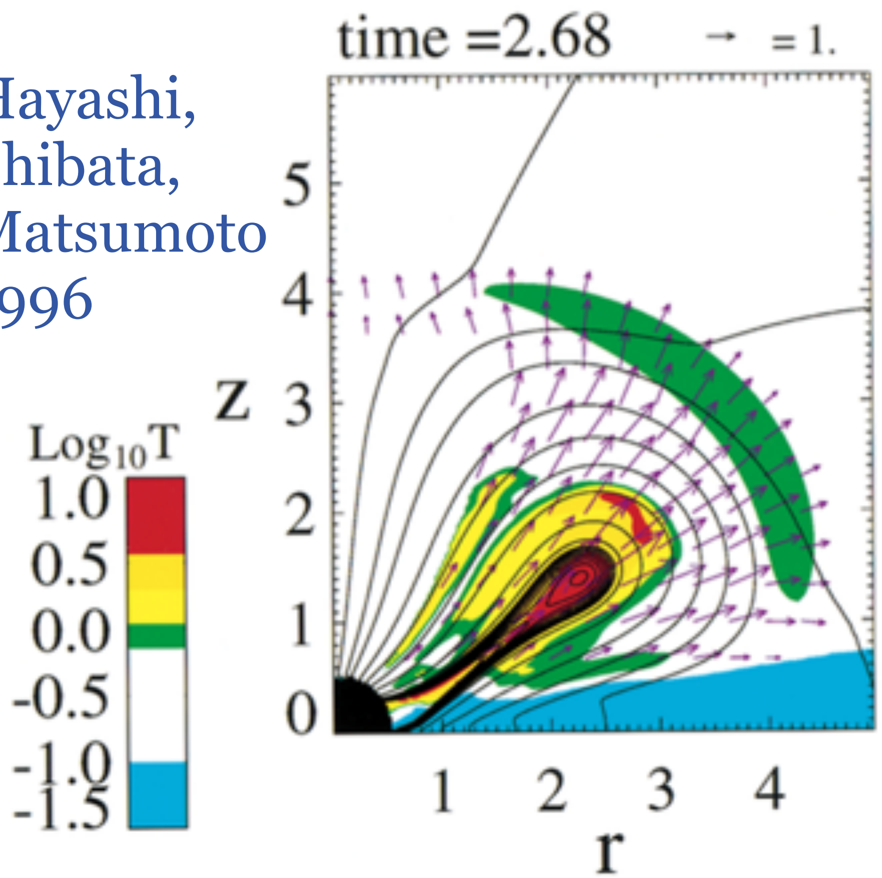


Uzdensky, Koenigl, Litwin 2002

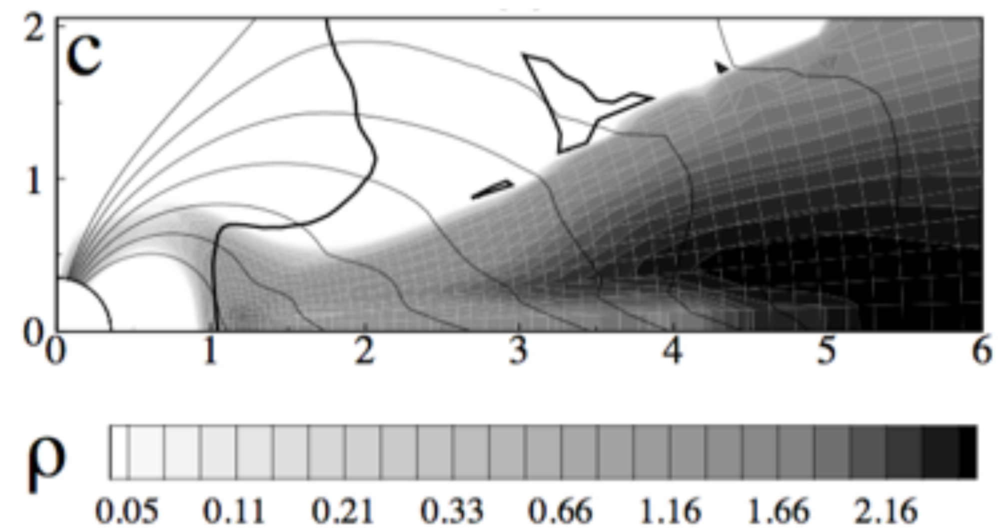
MHD simulations

opening + reconnection: flaring

Hayashi, Shibata, Matsumoto 1996

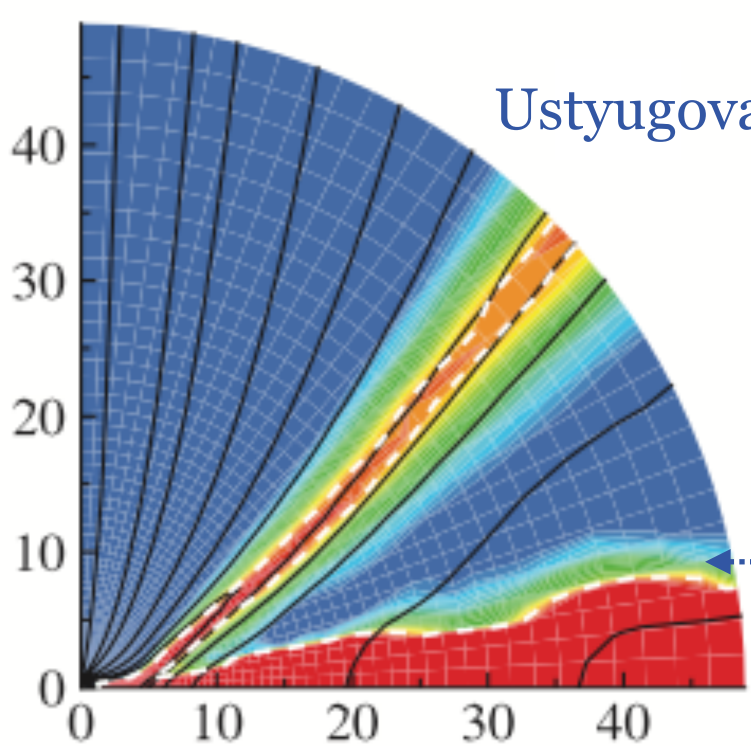


funnel flows & accretion torque



Romanova+ 2002

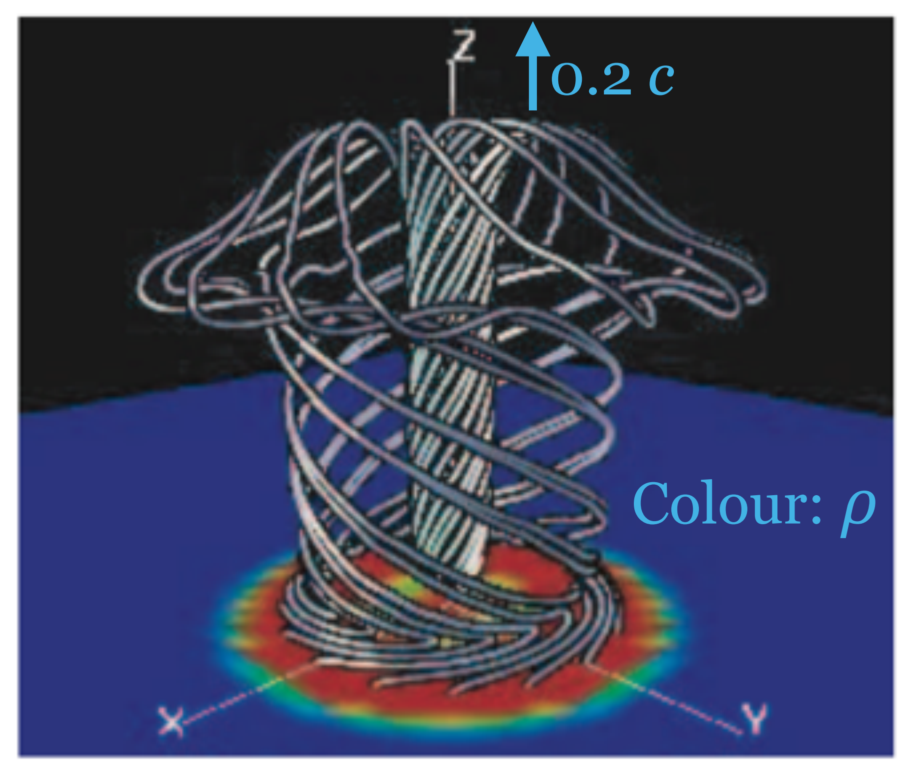
Ustyugova+ 2006



axial magnetised jet

$\beta = 1$ contour

Colour: plasma β



Kato, Hayashi, Matsumoto 2004

Millisecond pulsars: relativistic effects

1. All previous simulations were non-relativistic
2. Coronae/magnetospheres were heavy and fairly (numerically) diffusive

Explore relativistic regime with thinner discs

& lighter, nearly dissipationless coronae

↓

use broken force-free electrodynamics & PHAEDRA spectral code

Parfrey, Beloborodov, Hui 2012

“broken”: FFE + causal resistive corrections

Parfrey 2016, in prep

Simulation set up

Solve Maxwell's equations with current $\boldsymbol{J}$.

Dynamic Corona

Nearly ideal: $4\pi\sigma_0 = 2 \times 10^5 \, c/r_*$

$\boldsymbol{J} = \boldsymbol{J}_{\mathrm{FFE}} + \text{resistive corrections}$

implemented via dynamic resistivity: $\eta = \eta_0 + \eta_1 \left| \frac{\boldsymbol{J}_{\mathrm{FFE}} \cdot \boldsymbol{B}}{B^2 + E^2} \right|$

Kinematic Disc

α-disc model: $\alpha_{\mathrm{SS}} = 0.4 \qquad \mathrm{Pr_m} = 1$

$$v_{\hat{r}} = \alpha_{\mathrm{SS}} \left(\frac{h}{r} \right)^2 v_{\mathrm{Kepler}}$$

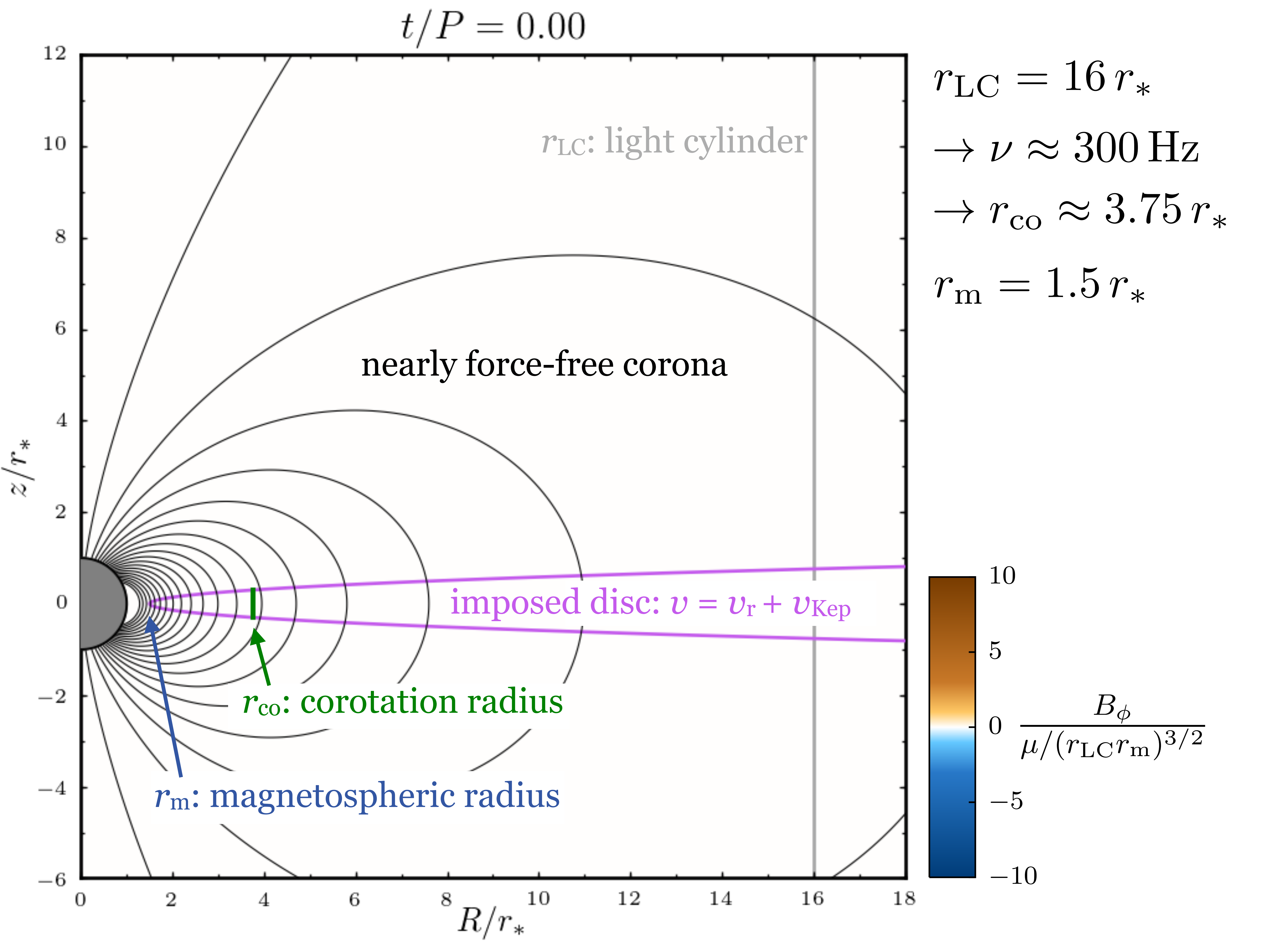
$$v_{\hat{\phi}} = v_{\mathrm{Kepler}}$$

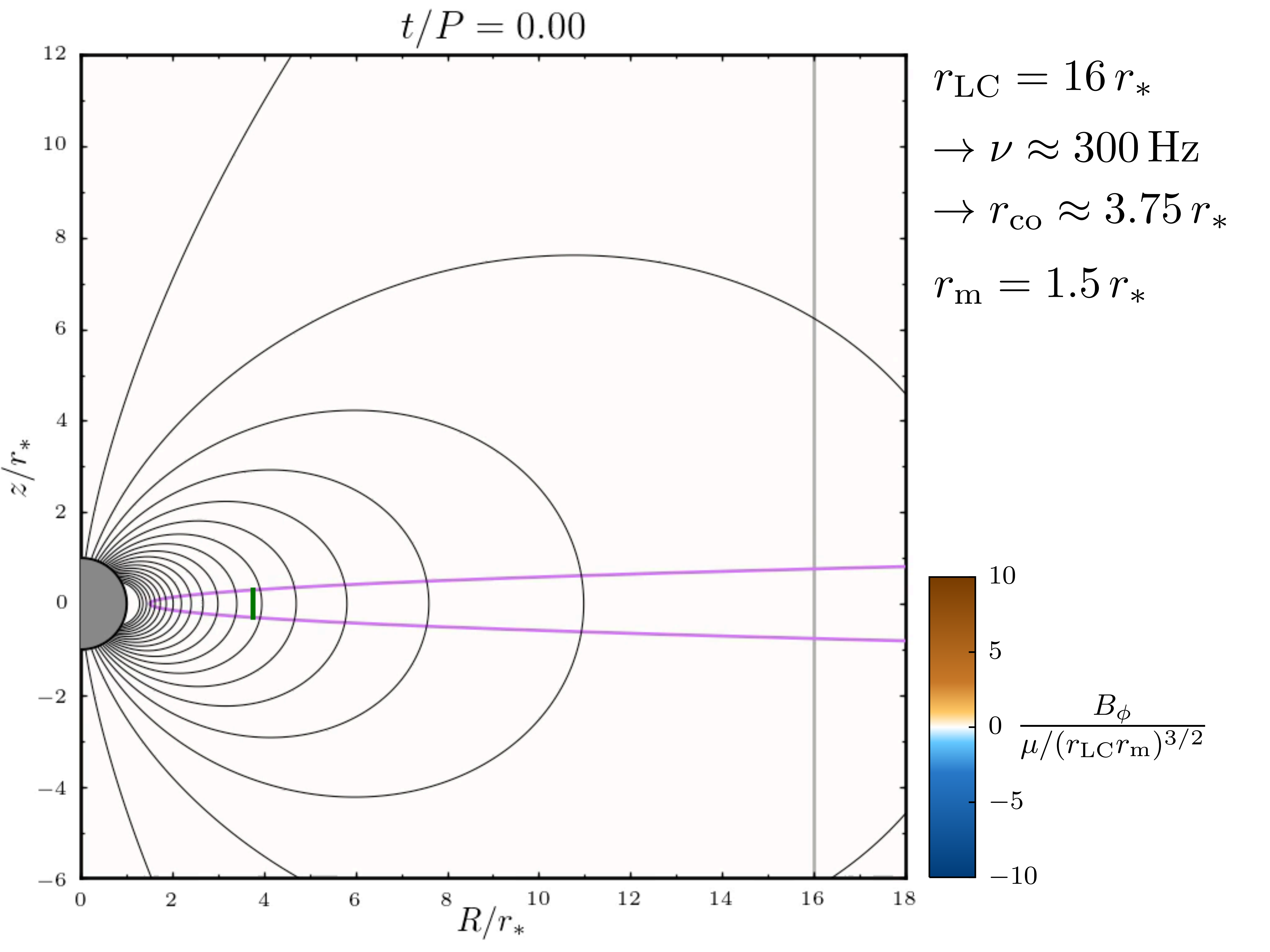
$$4\pi\sigma = c^2 / \left\{ \mathrm{Pr_m} \, \alpha_{\mathrm{SS}} (h^2/r) v_{\mathrm{Kepler}} \right\}$$

$$\sim 500 \, c/r_*$$

$$\boldsymbol{J} = W\sigma \left(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} - [\boldsymbol{v} \cdot \boldsymbol{E}]\boldsymbol{v} \right) + (\nabla \cdot \boldsymbol{E})\boldsymbol{v}$$

$$W = 1/\sqrt{1 - \boldsymbol{v} \cdot \boldsymbol{v}}$$


$t/P = 0.00$
r_{LC}: light cylinder
$r_{LC} = 16\, r_*$
$\to \nu \approx 300\,\mathrm{Hz}$
$\to r_{co} \approx 3.75\, r_*$
$r_m = 1.5\, r_*$
nearly force-free corona
imposed disc: $v = v_r + v_{Kep}$
r_{co}: corotation radius
r_m: magnetospheric radius
z/r_*
R/r_*
$\frac{B_\phi}{\mu/(r_{LC} r_m)^{3/2}}$



$t/P = 0.00$
$r_{\rm LC} = 16\, r_*$
$\rightarrow \nu \approx 300\,\mathrm{Hz}$
$\rightarrow r_{\rm co} \approx 3.75\, r_*$
$r_{\rm m} = 1.5\, r_*$
z/r_*
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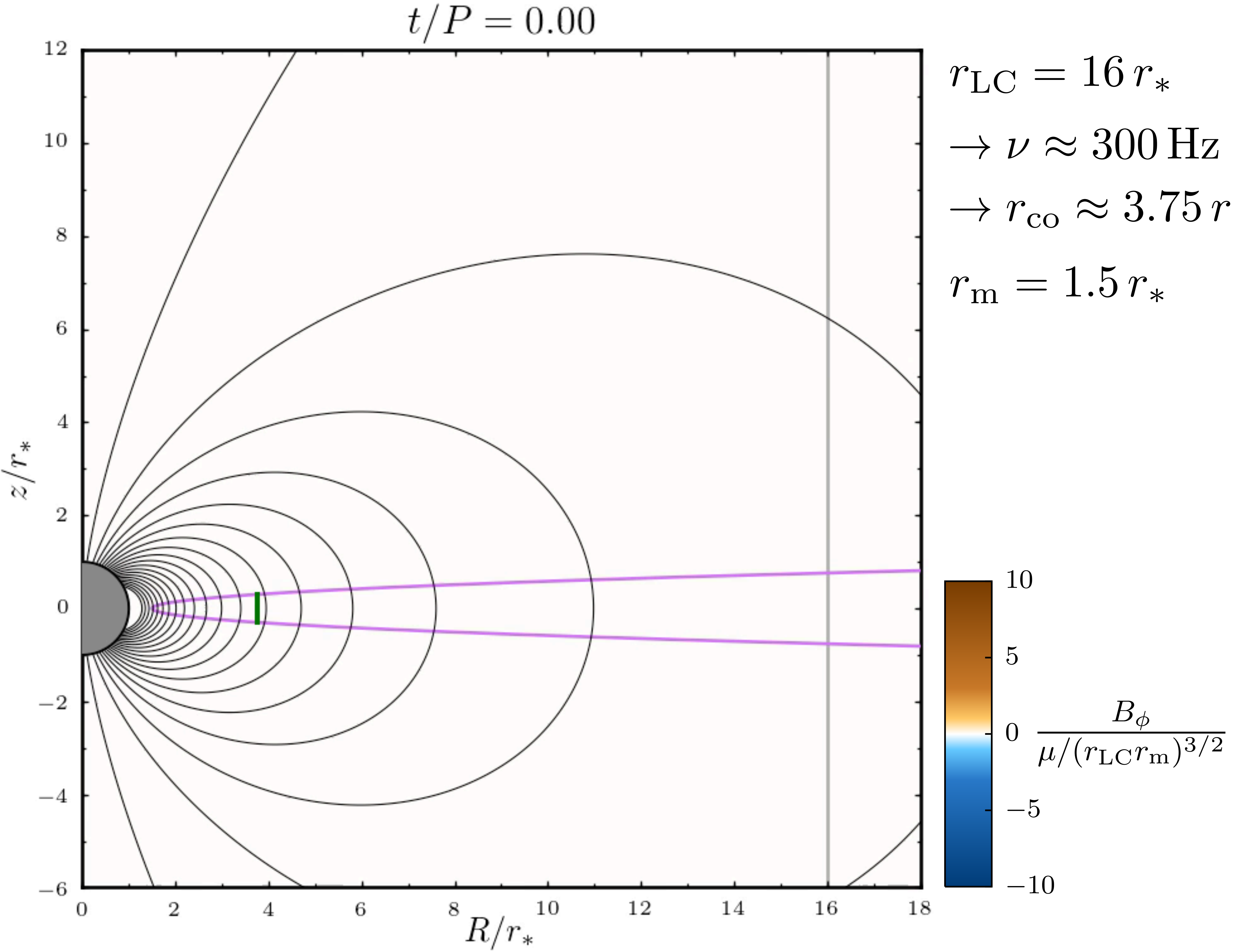


$r_{\rm LC} = 16\, r_*$

$\rightarrow \nu \approx 300\,{\rm Hz}$

$\rightarrow r_{\rm co} \approx 3.75\, r_*$

$r_{\rm m} = 1.5\, r_*$

Field lines: dragged in or "pushed" out?

Get nearly the same final steady state when disc has $v_r = 0$

	$N_{\mathrm{spindown}}/N_0$
v_r from α-disc model	74.2
$v_r = 0$	74.5

N_0: torque on equivalent isolated pulsar

Estimate outward field line speed

→ resistive annihilation of the radial field: $v_\eta \approx \frac{\eta}{h} \tan\theta_{\mathrm{f}}$

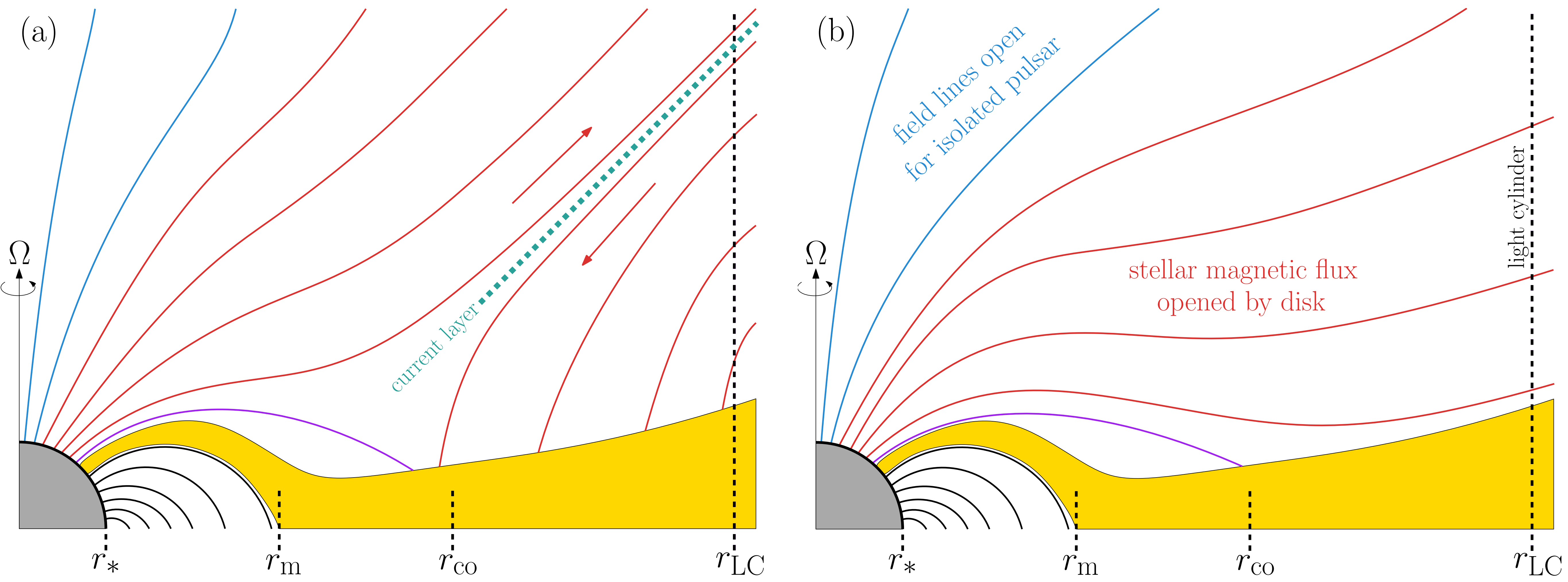
angle at which field lines enter disc

therefore $\frac{v_{r,\alpha}}{v_\eta} \approx \frac{h}{r} \frac{\mathrm{Pr_m}}{\tan\theta_{\mathrm{f}}}$

So for thin discs can ~ neglect disc accretion velocity

Taking stock — a toy model

Approximate all spin-down torque
as coming from open field lines

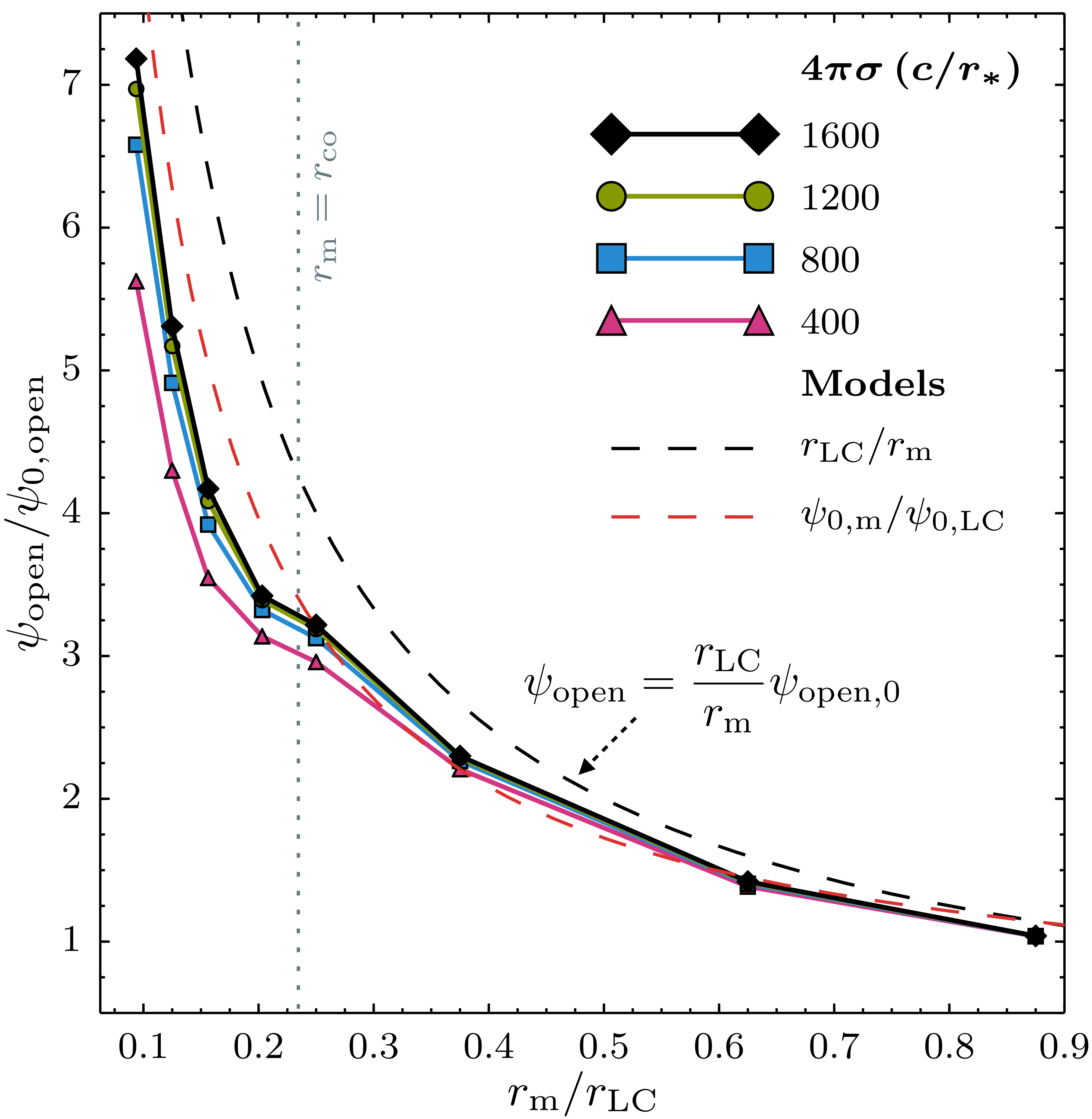


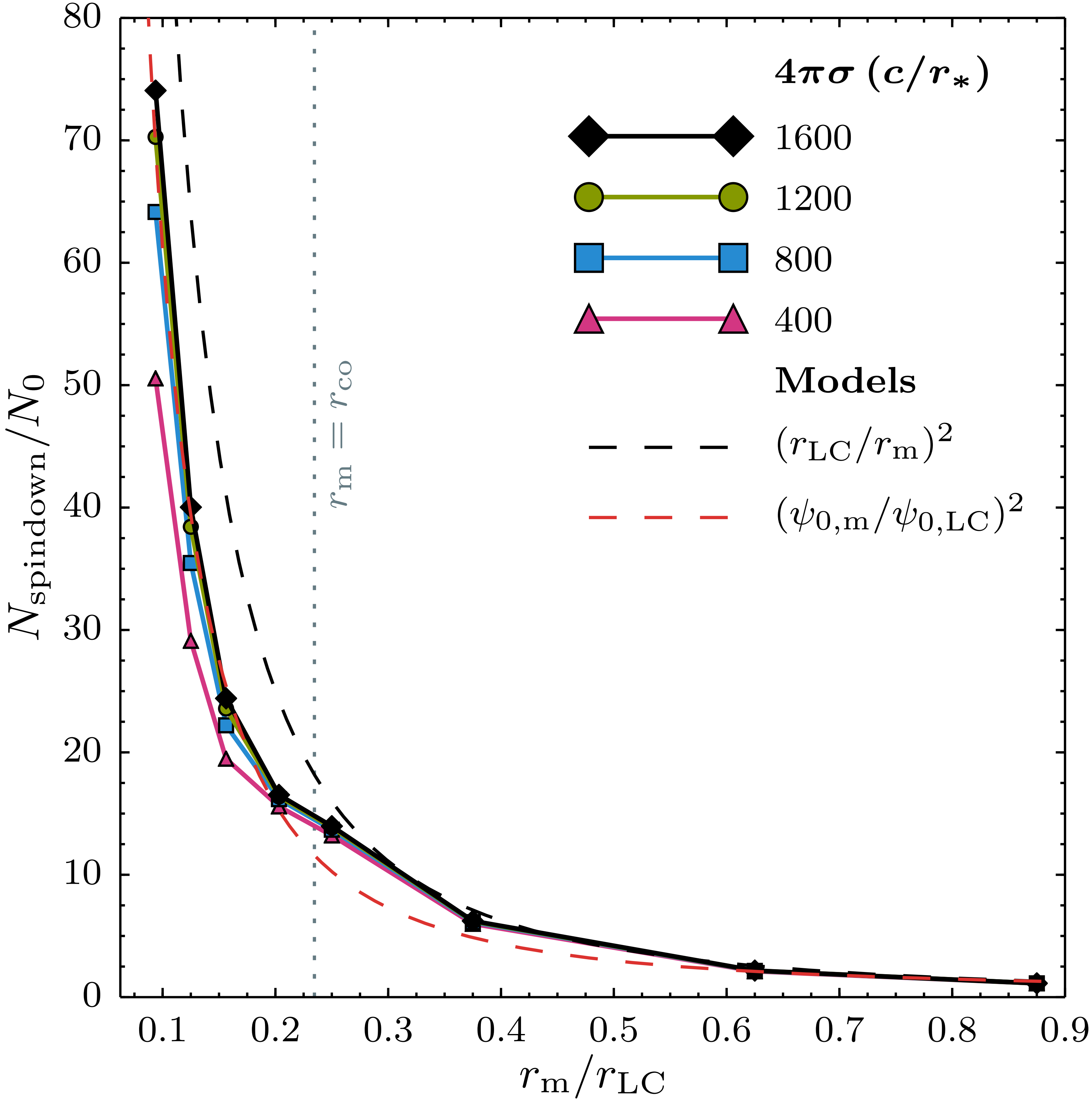
Parfrey, Spitkovsky, Beloborodov 2015

But how much flux is opened? Expect $\psi_{\rm open} \sim \dfrac{r_{\rm LC}}{r_{\rm m}} \psi_{\rm open,0}$

Simulations: grid of simplified models

$v_r = 0$
$\sigma = \text{const}$

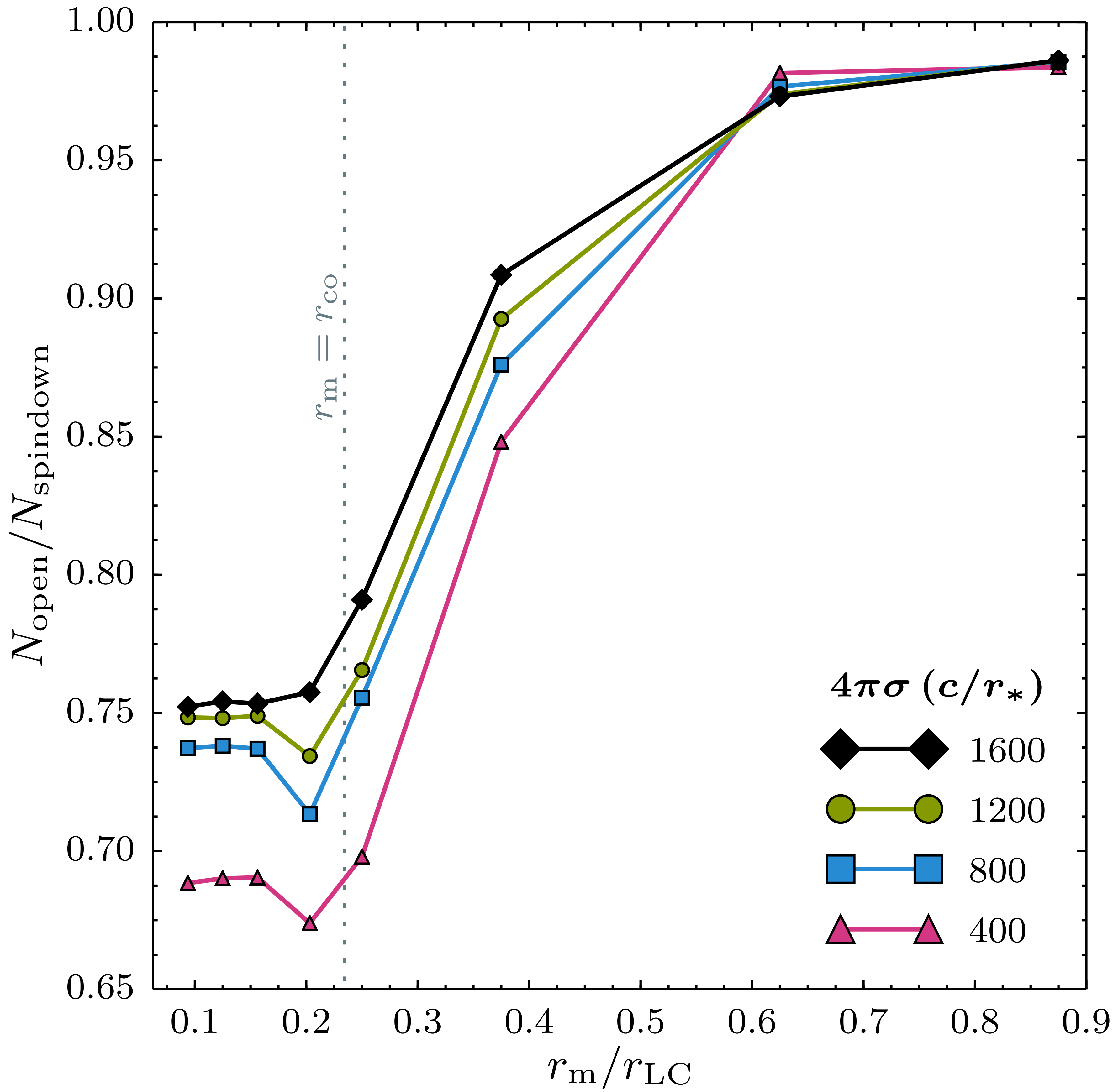




$$N_{\rm spindown} \propto \psi_{\rm open}^2$$

total spin-down torque
vs
magnetospheric
radius

fraction of spin-down torque applied by open field lines



Simple model for torques

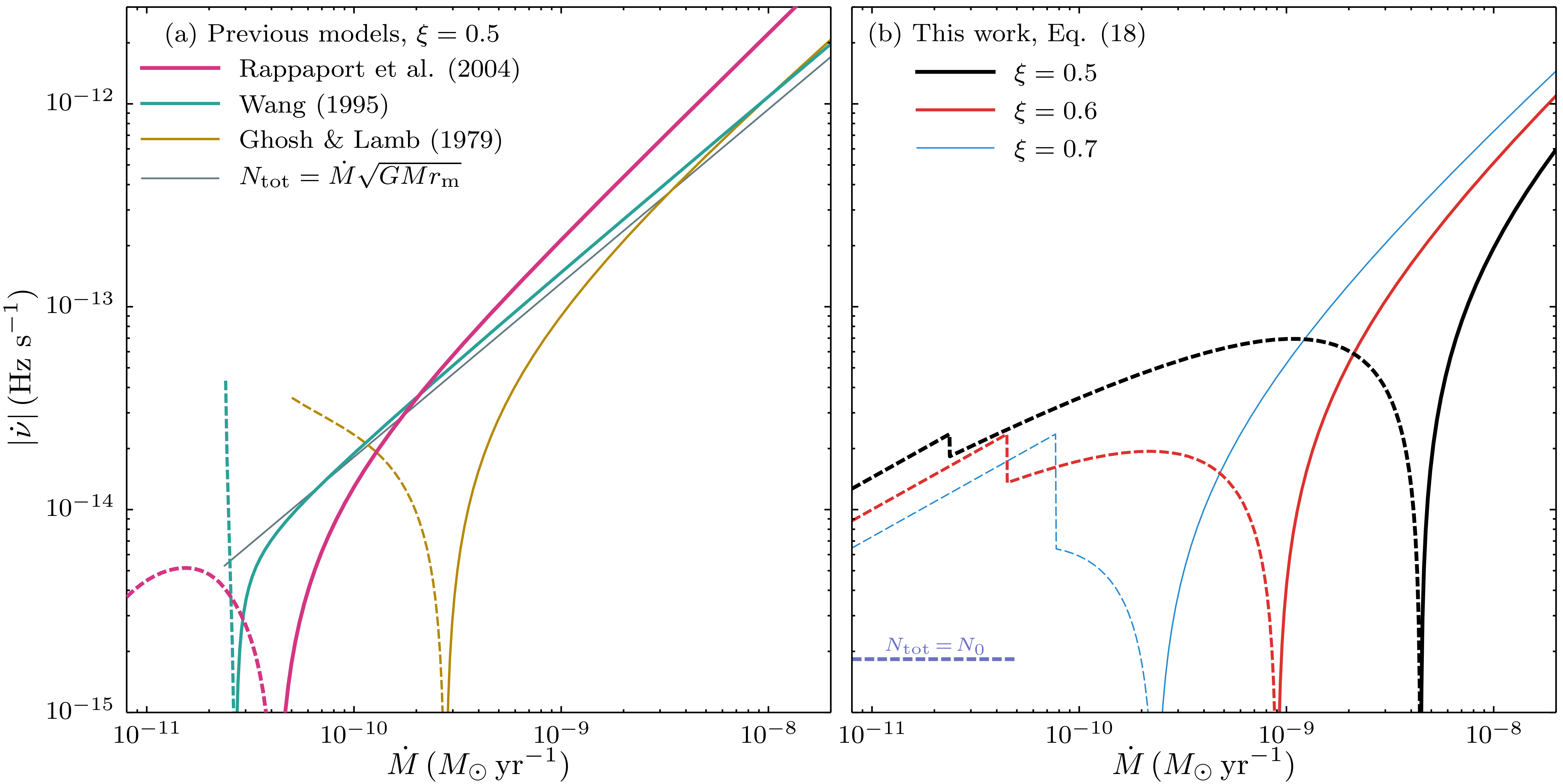
Isolated pulsar: $L_0 = -N_0 \Omega = \mu^2 \frac{\Omega^4}{c^3} \approx \frac{2}{3c} \Omega^2 \psi^2_{\mathrm{open},0}$

Model for open flux: $\psi_{\mathrm{open}} = \zeta \frac{r_{\mathrm{LC}}}{r_{\mathrm{m}}} \psi_{\mathrm{open},0}$

Torque: $N_{\mathrm{down,open}} = \zeta^2 \left(\frac{r_{\mathrm{LC}}}{r_{\mathrm{m}}} \right)^2 N_0$

$$N_{\mathrm{tot}} = \begin{cases} \dot{M}\sqrt{GMr_{\mathrm{m}}} - \zeta^2 \frac{\mu^2}{r_{\mathrm{m}}^2} \frac{\Omega}{c}, & r_{\mathrm{m}} < r_{\mathrm{co}} \\ -\zeta^2 \frac{\mu^2}{r_{\mathrm{m}}^2} \frac{\Omega}{c}, & r_{\mathrm{co}} < r_{\mathrm{m}} < r_{\mathrm{LC}} \\ -\mu^2 \frac{\Omega^3}{c^3}, & r_{\mathrm{m}} > r_{\mathrm{LC}}. \end{cases}$$

Torque models: 500 Hz, 10^8 G star



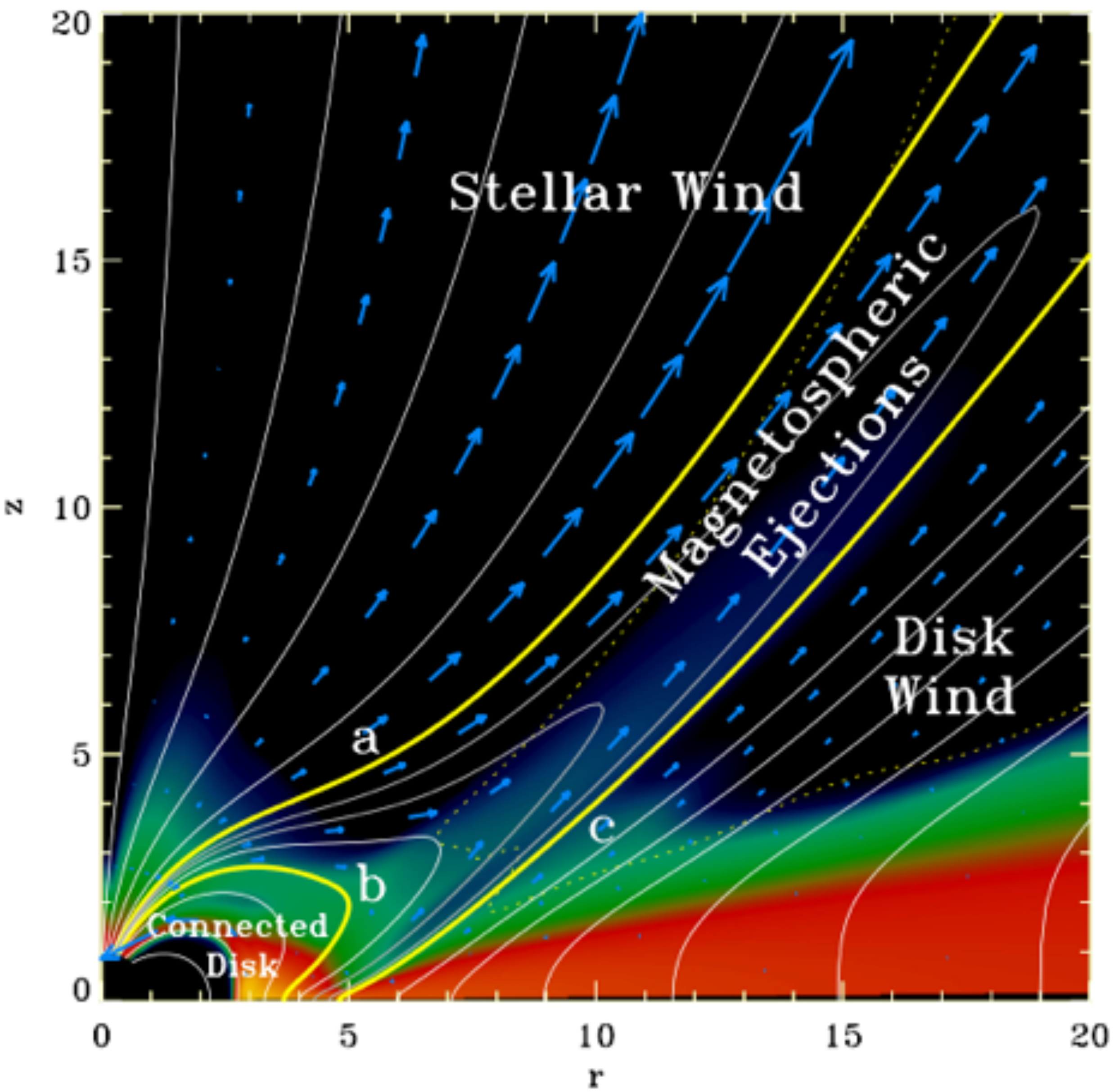
Parfrey, Spitkovsky, Beloborodov 2015

relating $r_{\rm m} = \xi r_{\rm A}$ where Alfvén radius $r_{\rm A} = \left(\frac{\mu^4}{2GM\dot{M}^2}\right)^{1/7}$

Jet power — if open flux is collimated

Scale with open flux in same way: $L_{\rm j} = \zeta^2 \left(\frac{r_{\rm LC}}{r_{\rm m}}\right)^2 L_0$

$$L_{\rm j} = 1.59 \times 10^{36} \left(\frac{\zeta}{\xi}\right)^2 \left(\frac{\nu}{500\,{\rm Hz}}\right)^2 \left(\frac{\mu}{10^{26}\,{\rm G\,cm^3}}\right)^{6/7} \times \left(\frac{M}{1.4\,M_\odot}\right)^{6/7} \left(\frac{\dot{M}}{\dot{M}_{\rm Edd,\odot}}\right)^{4/7} {\rm erg\,s^{-1}}$$



Zanni & Ferreira 2013

colour: mass density

Application 1: Torques on AMSPs

Test torque models when get a magnetic moment estimate via spin measurements during multiple outbursts

For reasonable parameters, can explain lack of detectable spin-up during outbursts of

Haskell & Patruno 2011

SAX J1808.4-3658

$\xi < [0.65,\ 0.61,\ 0.55]$
for $\zeta = [1.0,\ 0.9,\ 0.8]$

XTE J1814-338*

* assuming $B \sim 10^8$ G

$\xi < [0.72,\ 0.67,\ 0.61,\ 0.56]$
for $\zeta = [1.0,\ 0.9,\ 0.8,\ 0.7]$

No enhanced/anomalous spin-down needed for

XTE J1751-305 — Papitto+ 2008, Riggio+ 2011

IGR J00291+5934 — Patruno 2010, Hartman+ 2011, Papitto+ 2011

Application 2: Spin equilibrium

Spin-up from r_m = Spin-down on open flux

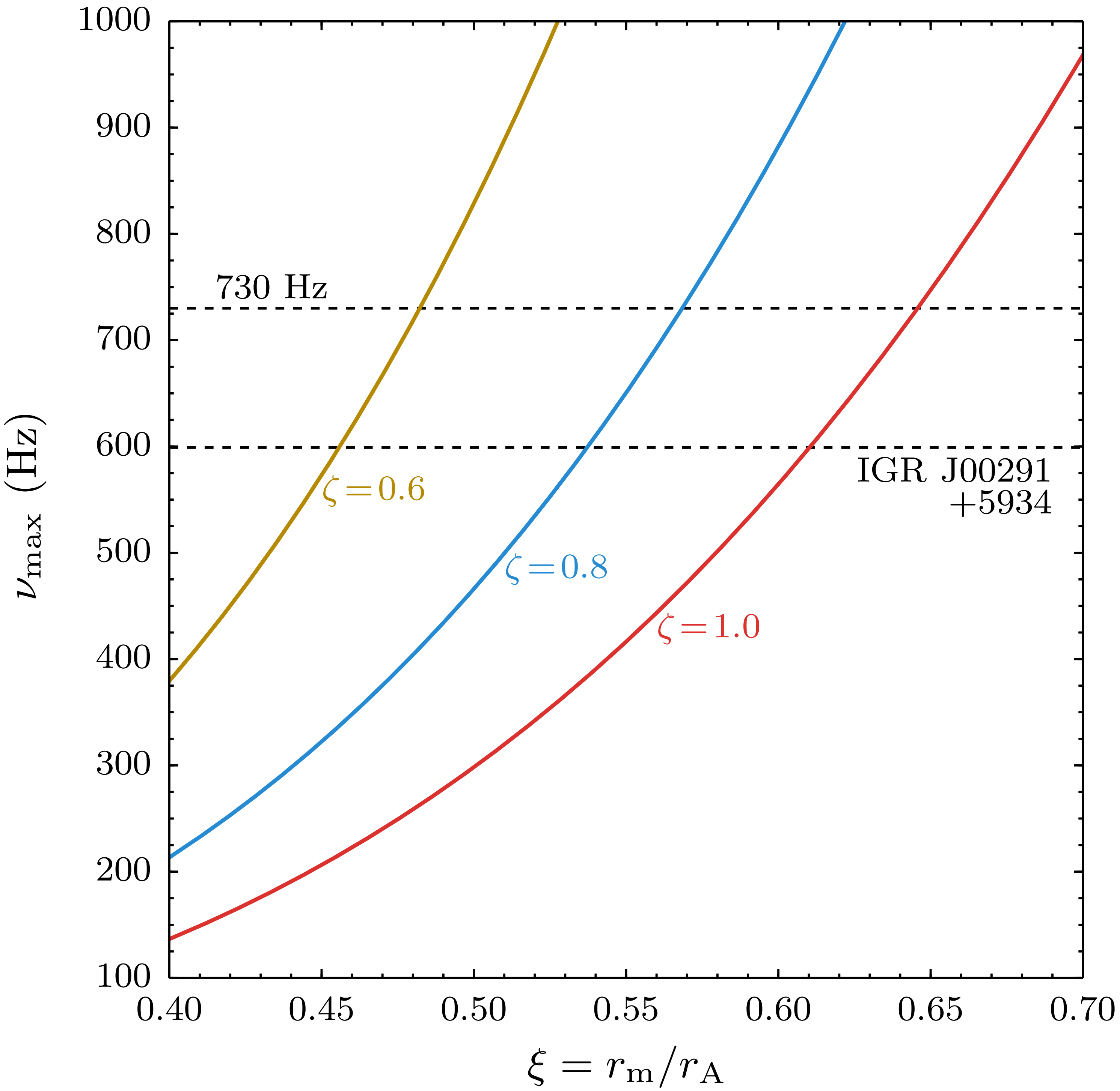
$$\dot{M}\sqrt{GMr_\mathrm{m}} = -\zeta^2 \left(\frac{r_\mathrm{LC}}{r_\mathrm{m}}\right)^2 N_0$$

↓

$$\nu_\mathrm{eqlm} = 956\,\zeta^{-2}\xi^{5/2}\left(\frac{\mu}{10^{26}\,\mathrm{G\,cm^3}}\right)^{-4/7} \times \left(\frac{M}{1.4\,M_\odot}\right)^{1/7}\left(\frac{\dot{M}}{10^{-10}\ M_\odot\,\mathrm{yr}^{-1}}\right)^{2/7}\ \mathrm{Hz}$$

In spin eqlm:

$$\frac{r_{\rm m}}{r_{\rm LC}} = 2^{-1/2}\frac{\xi^{7/2}}{\zeta^2}$$



To see channeled accretion:

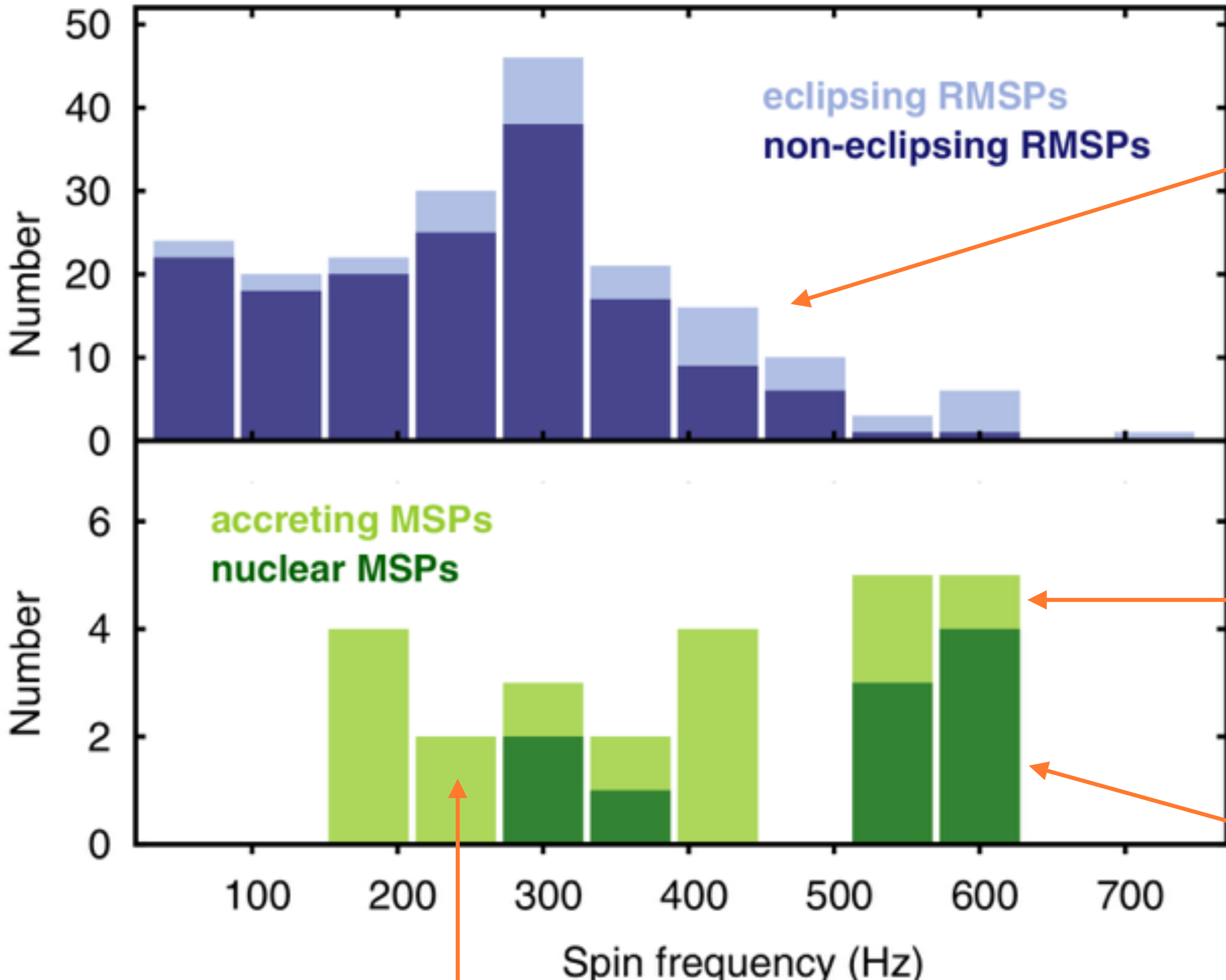
$$r_{\rm m} > r_*$$

Max spin for AMSPs:

$$\nu_{\rm max} = 3374\,\zeta^{-2}\xi^{7/2}\left(\frac{r_*}{10\,{\rm km}}\right)^{-1}\ {\rm Hz}$$

Independent of magnetic moment and accretion rate!

Papitto+ 2014



radio pulsars slower due to strong spin-down during RLDP?
e.g. Tauris 2012

ν_{max} gives AMSP cut-off?

nuclear sources similar if mag. moments not much smaller?

$\nu_{\rm eqlm}(\mu, \dot{M}, M)$ — gives flat-ish distribution?

Application 3: Jets

Sco X-1, Cir X-1 — $L_j > 10^{35}$ erg/s Fomalont+ 2001, Fender+ 2004

Model: $L_j = 4.6 \times 10^{35} (\zeta/\xi)^2 \, \mathrm{erg\,s^{-1}}$ for $\mu = 10^{26}$ G, $\nu = 300$ Hz, $\dot{M} = 0.5\,\dot{M}_{\rm Edd}$

$L_j \propto \dot{M}^{4/7}$ — similar to Aql X-1 [modulo $L_j(L_R)$]

— not similar to 4U 1728-34

May explain why see soft state quenching in some sources

e.g. Aql X-1 Tudose+ 2009, Miller-Jones+ 2010

but not others (most?) Migliari & Fender 2006

→ critical μ for $r_m \rightarrow r_*$ at $\dot{M}_{\rm Edd}$: $\mu_{\rm crit} \sim$ few $\times 10^{26}$ G

Summary

- Differential rotation between star & disc may open nearly all the disc-coupling magnetic flux

- If opening is efficient, significant power can be tapped by high-spin, strongly magnetised objects — e.g. millisecond pulsars

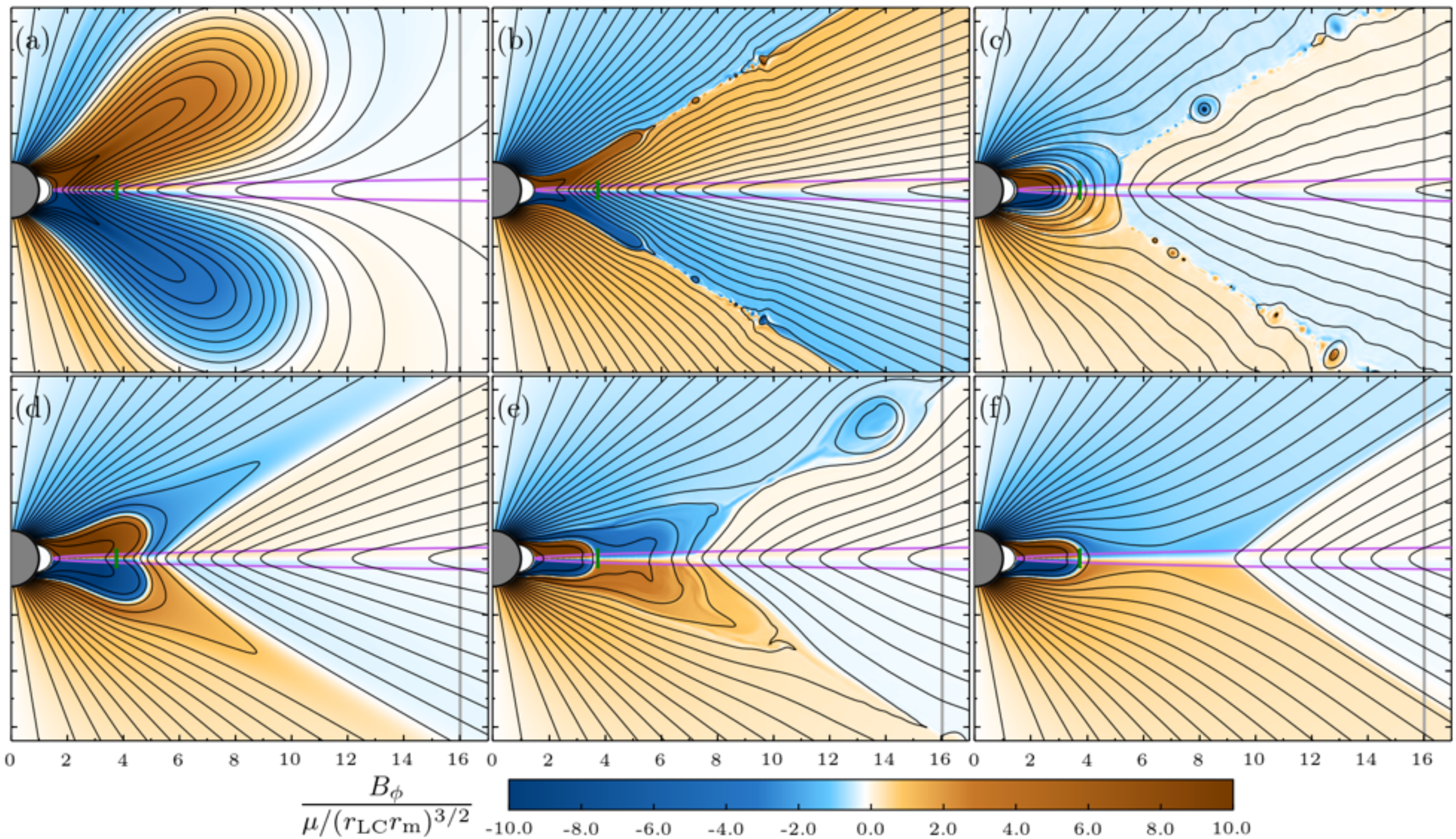
- May be relevant for setting the torque on AMSPs in outburst, their spin distribution, and jets from high-spin neutron stars

- Can transitional MSPs help untangle some of the relationships between magnetic moment, accretion rate, torque, and radio emission?

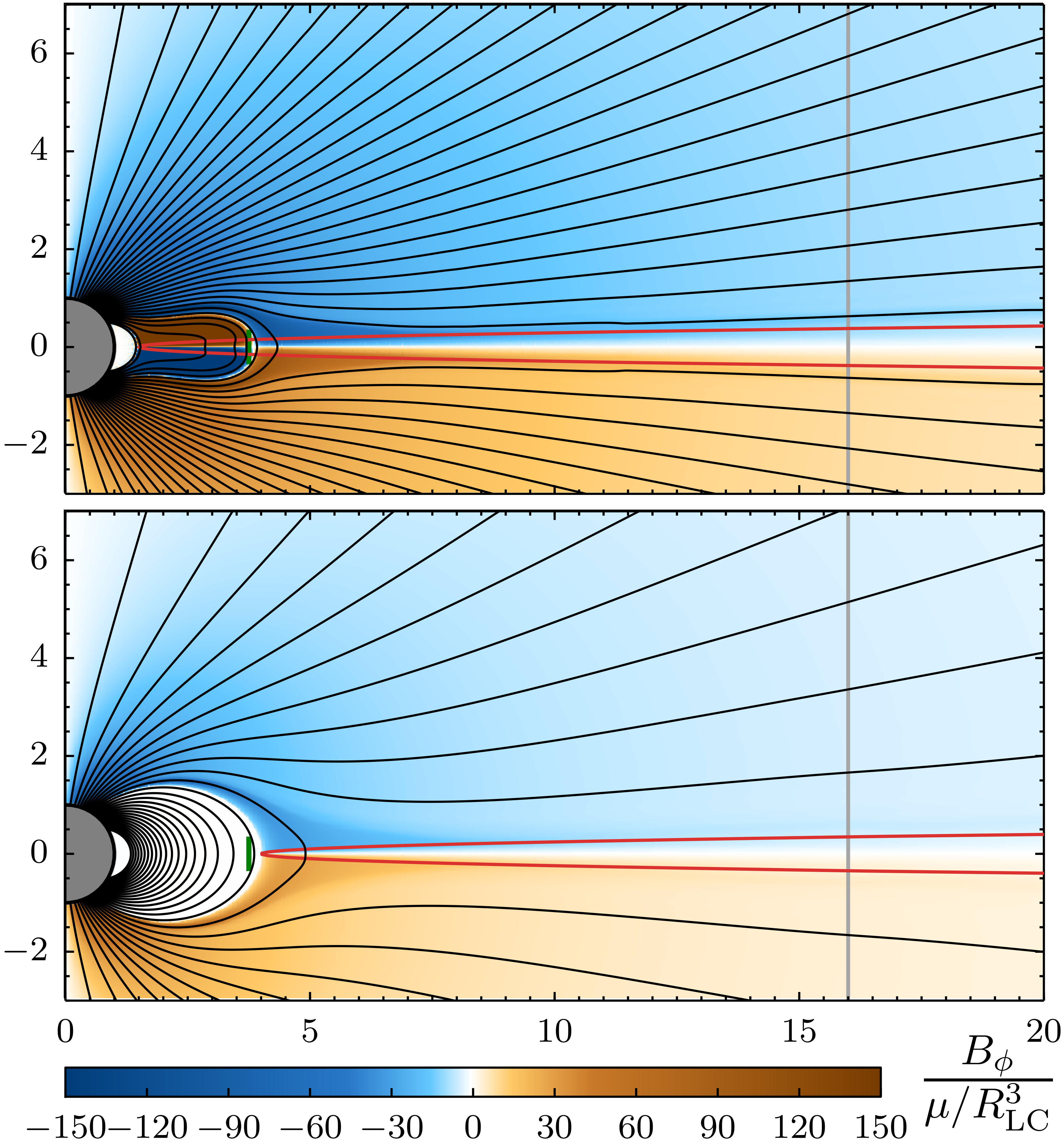
- arXiv:1507.08627 — analytic model & comparison to observations


BONUS

Opening, reconnection, relaxation



Steady States



$r_{\rm m} = 1.5\, r_*$

“accreting”

$4\pi\sigma_{\rm disc} = 1{,}600\, c/r_*$

$r_{\rm m} = 4.0\, r_*$

“propeller regime”

Modifies magnetic moment estimate

$$\mu_{\rm std}^2 = \frac{I\dot{\nu}c^3}{4\pi^2(1+\sin^2\theta)\nu^3}$$ ← force-free relationship

$$\mu_{\rm corr} = \zeta^{-1}\frac{r_{\rm m}}{r_{\rm LC}}\mu_{\rm std} = \left(\frac{\xi}{\zeta}\right)^{7/3}\left(2GM\dot{M}^2\right)^{-1/3}\left[\frac{I\dot{\nu}c}{(1+\sin^2\theta)\nu}\right]^{7/6}$$

Neglecting opening can lead to overestimate of μ