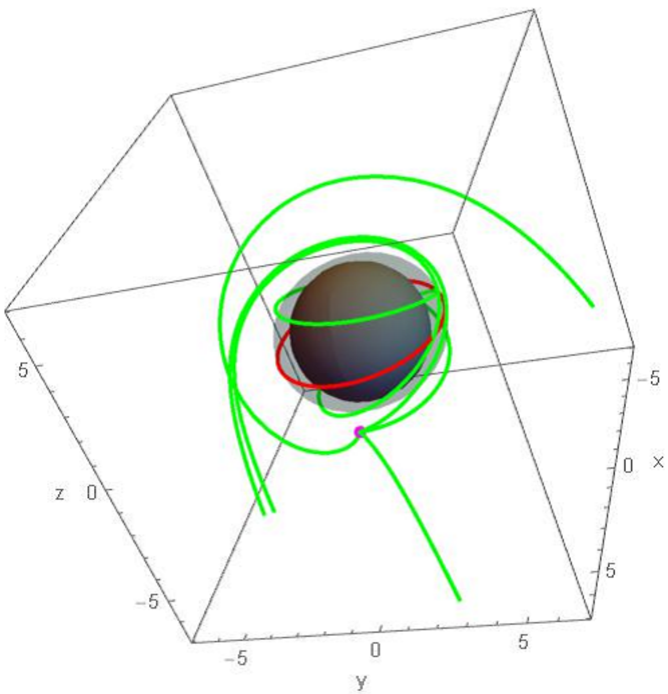
How the obs. quantities of strong grav. lens effect depend on BH's mass and spin



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1. Introduction : Basic idea

1.1 From candidate to itself

- Best observational knowledge of BH at present

  $\rightarrow$ BH candidates by Newtonian gravity

  $\Updownarrow$ Large Gap in Physics !!

- BH is a general relativistic (GR) object

  $\rightarrow$ The method to find "BH itself" is at least a direct detection of the GR effect of BH.

What is it? How can we do it?

1.2 Meaning of BH detection in GR context

- Theoretical (mathematical) fact in GR

Uniqueness Theorem

Asymptotic flat BH spacetime is uniquely specified by 3 parameters:

$$
\begin{array}{rcl}
M_{\rm BH} & : & \text{mass} \\
J_{\rm BH} & : & \text{spin angular momentum} \\
Q_{\rm BH} & : & \text{electric charge}
\end{array}
$$

◇ $Q_{\rm BH} = 0$ is expected for real situations.

$\rightarrow$ **BH is specified by $M_{\rm BH}$ and $J_{\rm BH}$.** (Kerr BH)

- Define the meaning of "direct" detection of BH

BH Detection is …

To measure the parameters M and χ by detecting the GR effect of BH.

◇ Mass in length scale: $M = \dfrac{G M_{\rm BH}}{c^2}$ [cm]

◇ Dimensionless spin parameter: $\chi = \dfrac{a}{M}$ [no-dim]

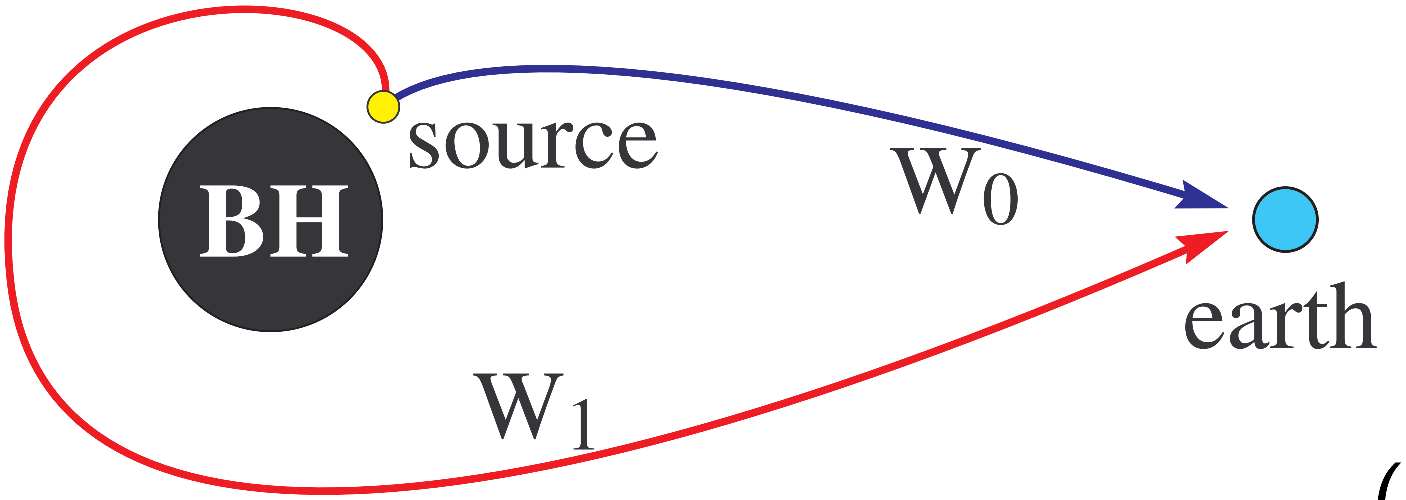
(usual spin parameter: $a = J_{\rm BH}/(M_{\rm BH}\, c)$ [cm])

◇ Kerr BH horizon radius: $r_{\rm BH} = M\left[1 + \sqrt{1-\chi^2}\,\right]$

$$\Rightarrow 0 \le \chi < 1$$

1.3 GR effect of BH as our target

- Target : **Strong Gravitational Lens** (SGL) effect
- An ideal situation we want to observe:
  - ◇ Clear environment around BH except the source
  - ◇ Burst-like and spherical emission seen from the source



W_0 : "0th ray" (primary ray)

W_1 : "1st ray" (secondary ray)

$$\Downarrow$$

Basic fact in our situation

Observing two quantities of SGL

$$\left\{\begin{array}{ll} \Delta t_{\mathrm{obs}} & : \text{ Time delay} \\ \mathcal{R}_{\mathrm{obs}} = \dfrac{F_1}{F_0} & : \text{ Flux ratio} \end{array}\right. \quad \text{of } \mathrm{W}_0 \text{ and } \mathrm{W}_1 \,,$$

gives the BH parameters $(M\,,\,\chi)$,

if the inclination angle θ_{obs},

the source's motion $(\vec{x}_{\mathrm{s}}\,,\vec{u}_{\mathrm{s}})$,

and the source's emission spectrum $I_{\mathrm{s}}(\nu_{\mathrm{s}})$

are known.

$\rightarrow$ What should we do with observation?

- Steps for extracting (M, χ) from observation.

(a) Theory:
Prepare numerically the date set of $(\Delta t_{\rm obs}, \mathcal{R}_{\rm obs})$ with various values of $(M, \chi\, ;\, \theta_{\rm obs}, \vec{x}_{\rm s}, \vec{u}_{\rm s}, I_{\rm s})$.

(b) Observation:
Observe the target (BH candidate) and take the data $(\Delta t_{\rm obs}, \mathcal{R}_{\rm obs})$ as many as possible.

(c) Comparison:
Make the table from (a) and (b).

$\rightarrow$ See the next page $\cdots$

◇ If this table is obtained by steps (a), (b) and (c),

obs. data $(\Delta t_{\rm obs}\,,\,\mathcal{R}_{\rm obs})$	corresponding theoretical data by step (a) $(M,\chi\,;\,{\rm C})\;,\;\; {\rm C}=(\theta_{\rm obs}\,,\vec{x}_{\rm s},\vec{u}_{\rm s},I_{\rm s})$
(1.32 , 0.27)	$(9.0\,,\,0.1\,;\,{\rm C}_0)\;,\;(\boxed{3.2\,,\,0.8}\,;\,{\rm C}'_0)\;,\;(5.8\,,\,0.8\,;\,{\rm C}''_0)\;,\;\cdots$
(4.05 , 0.03)	$(\boxed{3.2\,,\,0.8}\,;\,{\rm C}_1)\;,\;(2.1\,,\,0.9\,;\,{\rm C}'_1)\;,\;(1.9\,,\,0.5\,;\,{\rm C}''_1)\;,\;\cdots$
(7.94 , 1.04)	$(0.8\,,\,0.3\,;\,{\rm C}_2)\;,\;(7.4\,,\,0.9\,;\,{\rm C}'_2)\;,\;(\boxed{3.2\,,\,0.8}\,;\,{\rm C}''_2)\;,\;\cdots$
(9.28 , 0.44)	$(\boxed{3.2\,,\,0.8}\,;\,{\rm C}_3)\;,\;(4.5\,,\,0.5\,;\,{\rm C}'_3)\;,\;(1.9\,,\,0.5\,;\,{\rm C}''_3)\;,\;\cdots$

$\rightarrow$ then we suggest $\underline{(M,\chi)=(3.2\,,\,0.8)}$

This talk discusses the steps (a) and (b)

2. SGL's Observable Quantities

2.1 Setup for numerical calculation

- Input parameters: M , χ , θ_{obs} , $\vec{x}_{\mathrm{s}}$, $\vec{u}_{\mathrm{s}}$, $I_{\mathrm{s}}(\nu_{\mathrm{s}})$
- Output parameters: Δt_{obs} , $\mathcal{R}_{\mathrm{obs}}$ $\leftarrow$ I calculate these quant.
- Back Ground: Kerr spacetime

$$\mathrm{d}s^2 = g_{tt}\,\mathrm{d}t^2 + 2g_{t\varphi}\,\mathrm{d}t\,\mathrm{d}\varphi + g_{rr}\,\mathrm{d}r^2 + g_{\theta\theta}\,\mathrm{d}\theta^2 + g_{\varphi\varphi}\,\mathrm{d}\varphi^2$$

$$\begin{cases} g_{\mu\nu} = g_{\mu\nu}(r,\theta\,;\,M,\chi) & \text{determined by } M\,,\,\chi \\ x^{\mu} = (\,t\,,\,r\,,\,\theta\,,\,\varphi\,) & \text{Boyer-Lindquist coord.} \end{cases}$$

2.2 Steps to calculate $(\Delta t_{\rm obs}\,,\;\mathcal{R}_{\rm obs})$

Step1. Solve Null Geodesic Eq. which connects the source and observer (shooting)

$\rightarrow$ Time delay Δt is obtained.

Step2. Solve Geodesic Deviation Eq.

$\rightarrow$ Visible solid-angle $\Delta\Omega$ is obtained.

Step3. Specify the source's velocity $\vec{u}_{\rm s}$ and specific intensity $I_{\rm s}(\nu_{\rm s})$ $[{\rm erg/s\,cm^2\,Hz\,\Omega}]$.

$\rightarrow$ Flux ratio $\mathcal{R}_{\rm obs}$ is obtained.

2.3 Step1: Null geodesics and $\Delta t_{\rm obs}$

- Some notes on Kerr BH :

◇ BH horizon at t =const. is the sphere of radius $r_{\rm BH}$

$$r_{\rm BH} = M\left[1+\sqrt{1-\chi^2}\right]\ [{\rm cm}]$$

◇ Ergo-surface : $r_{\rm erg} = M\left[1+\sqrt{1-\chi^2\cos^2\theta}\right]$

→ Radial motion $(\theta, \varphi =$ const.) is

impossible in the ergo-region $r < r_{\rm erg}$.

→ Any object rotates with BH spin in "$r \le r_{\rm erg}$".

◇ Geodesic motion is "three-dimensional" in general, except for on the equatorial plane $\theta = \pi/2$.

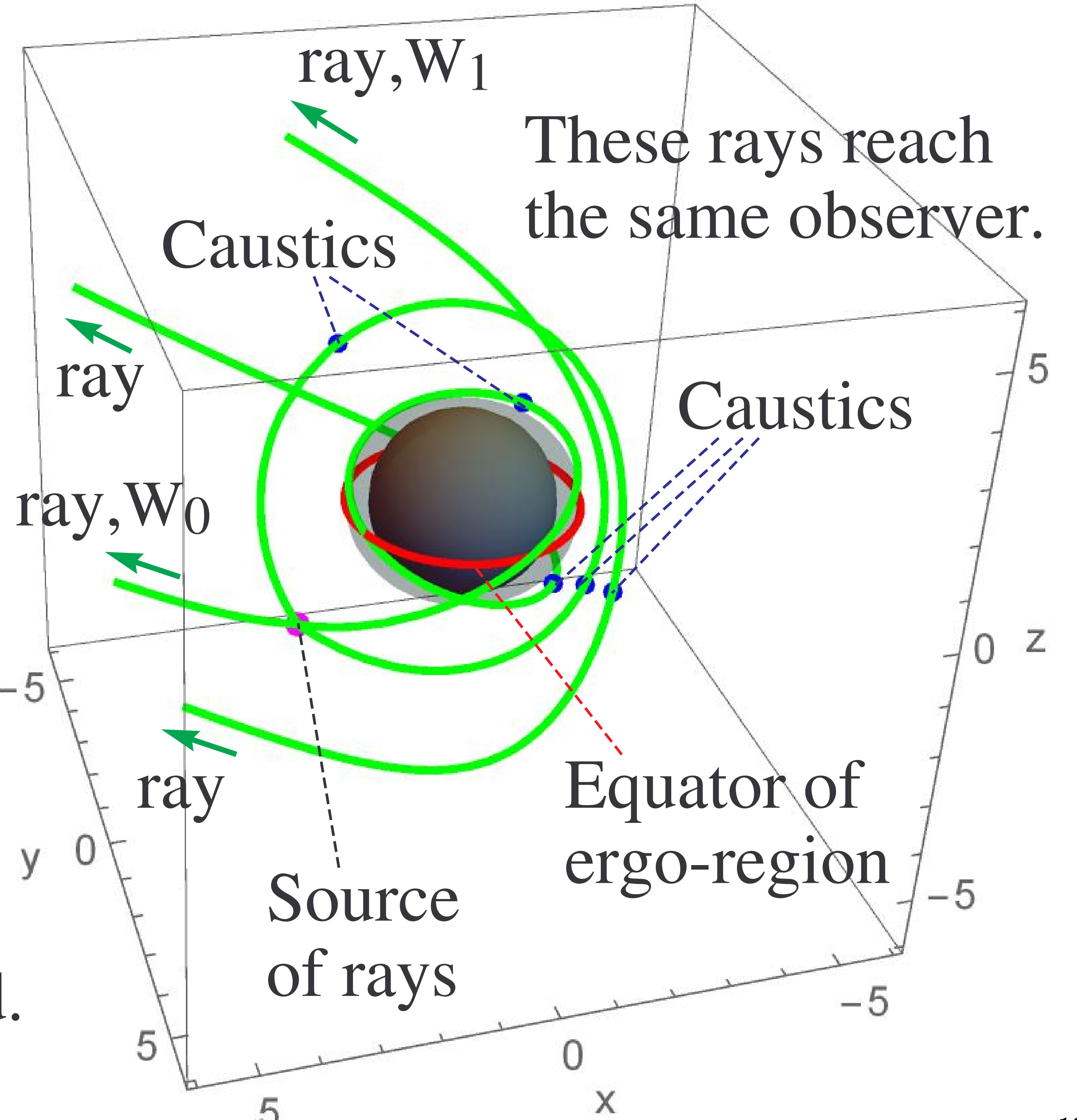
Some examples of light rays:

$M = 1.0$
$\chi = 0.8$

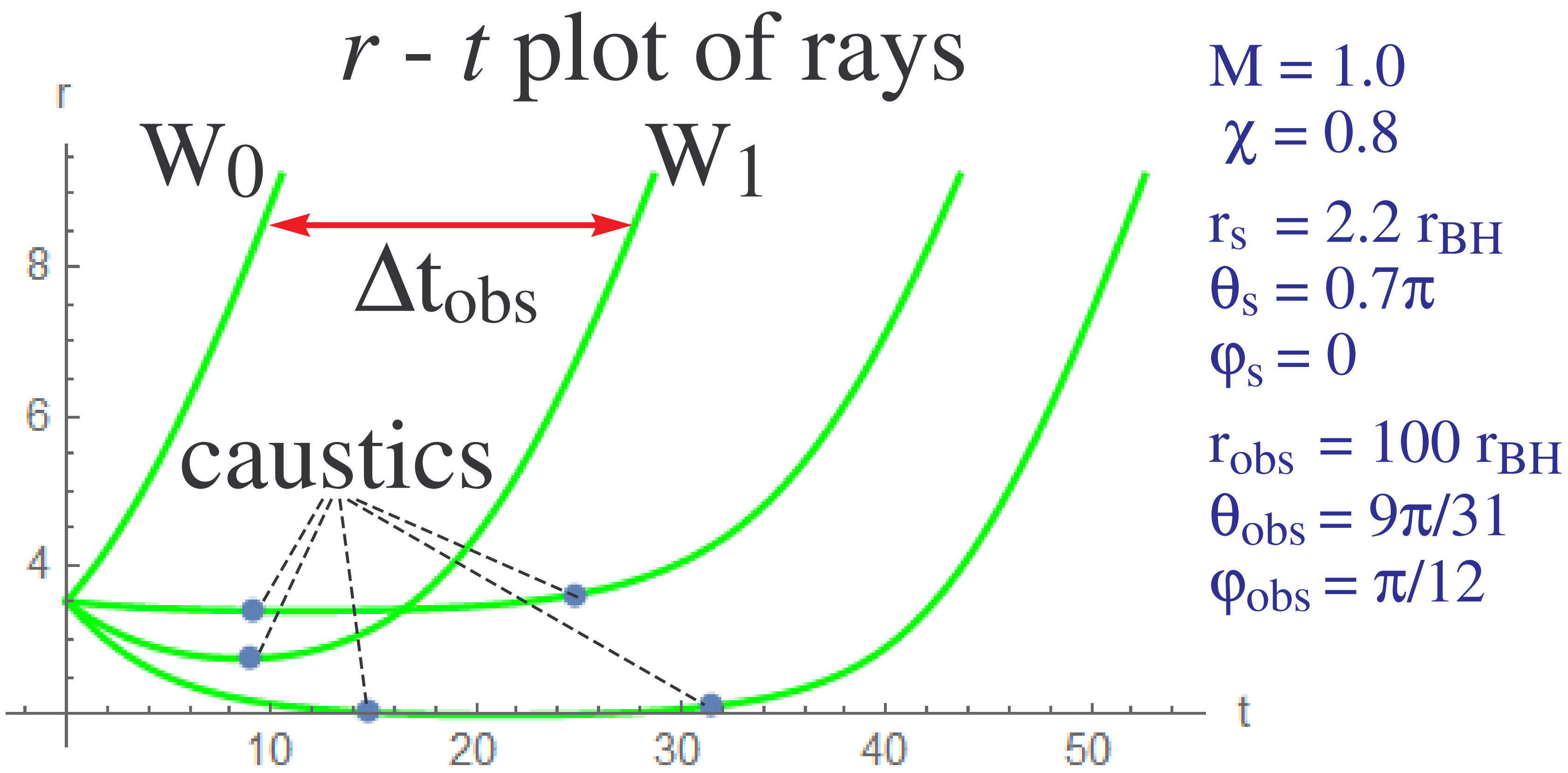
$r_s = 2.2\, r_{BH}$
$\theta_s = 0.7\pi$
$\varphi_s = 0$

$r_{obs} = 100\, r_{BH}$
$\theta_{obs} = 9\pi/31$
$\varphi_{obs} = \pi/12$

Higher winding rays are omitted.



- Time delay $\Delta t_{\rm obs}$ is read from the "r-t plot" of the primary ray W_0 and secondary ray W_1.

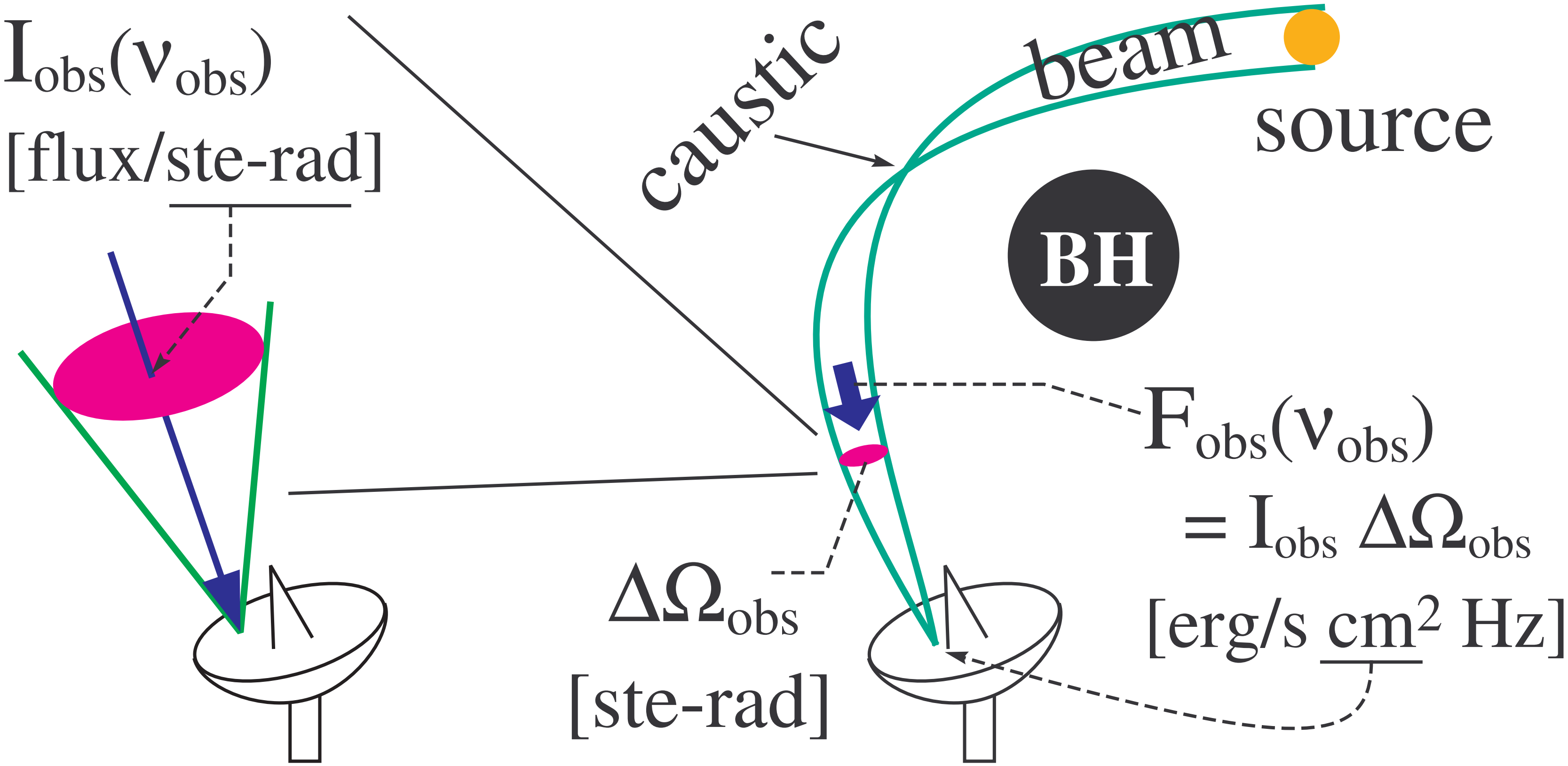


- Total Doppler effect: $\dfrac{\nu_{\rm s}}{\nu_{\rm obs}} = \dfrac{k_\mu u^\mu_{\rm s}}{k_\mu u^\mu_{\rm obs}}$ (k^μ : null vector)

2.4 Steps2 & 3: Geodesic deviation and $R_{\rm obs}$

Liouville's Thm:

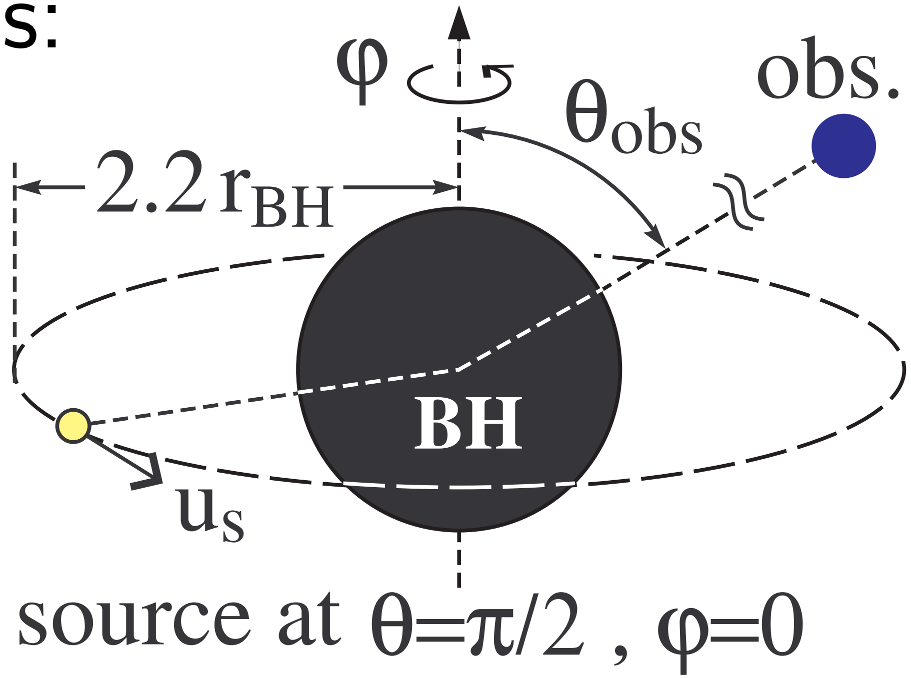
$$\frac{I_{\rm obs}}{\nu_{\rm obs}^3} = \frac{I_{\rm s}}{\nu_{\rm s}^3}$$



- Geodesic deviation eq. $\Rightarrow$ Cross section of beam $\Rightarrow$ Visible solid-angle $\Delta\Omega_{\rm obs}$ $\Rightarrow$ Obs. Flux $F_{\rm obs}$

2.5 Ex. of numerical results: preliminary

- Parameters of next figures:
  - Configuration: $\longrightarrow$
  - BH: $(M, \chi) = (1.0\,,\, 0.8)$
  - Source velocity:
    $u_{\rm s}^{\mu} = (1.49\,,\, 0\,,\, 0\,,\, 0.05)$
  - Inclination: $\theta_{\rm obs} = \dfrac{12}{31}\pi$
  - Line emission: $I_{\rm s}(\nu_{\rm s}) = \delta(\nu_{\rm s} - \nu_{\rm c})$, $\nu_{\rm c}$ is const.
  - Emission at $\nu_{\rm c}$ $\rightarrow$ Obs. with $\nu_{\rm obs(0)}$ and $\nu_{\rm obs(1)}$

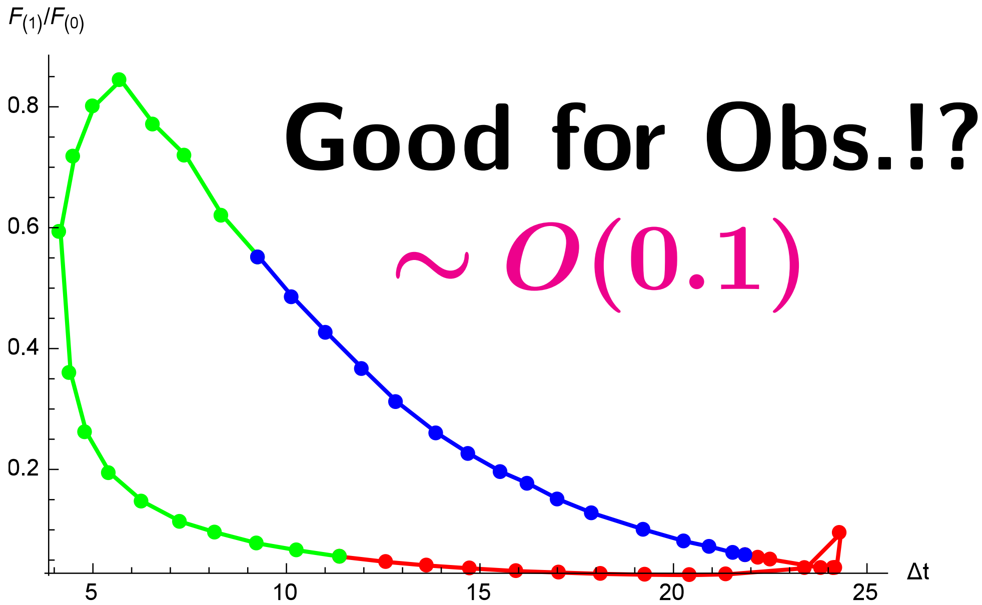


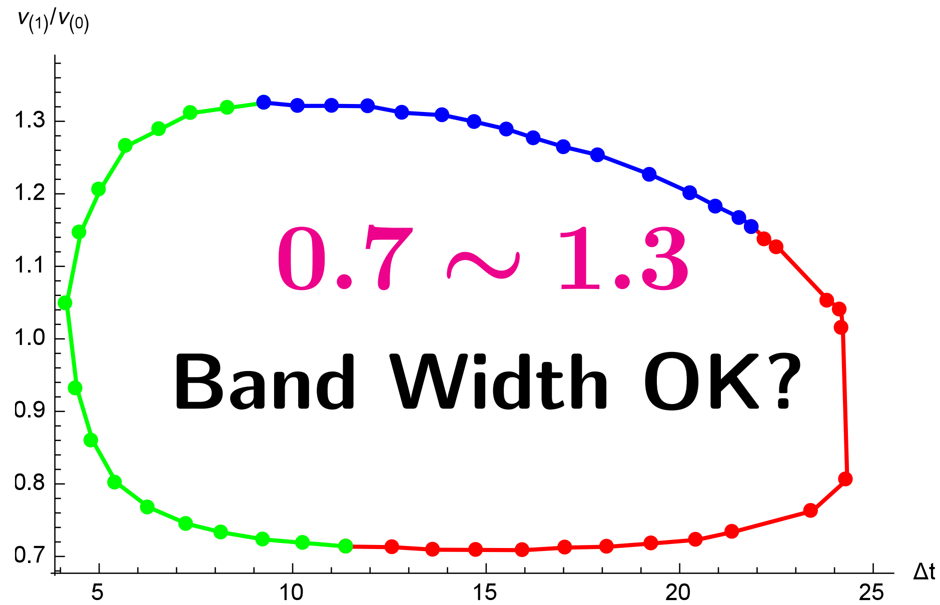
- Calculate $\Delta t_{\rm obs}$, $F_{\rm obs(1)}/F_{\rm obs(0)}$, $\nu_{\rm obs(1)}/\nu_{\rm obs(0)}$ at every azimuthal angle of obs. $\varphi_{\rm obs}$

◇ coloring of $\varphi_{\rm obs}$:

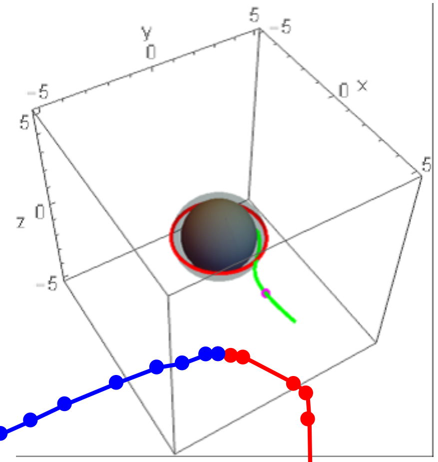
$\varphi_{\rm obs}$	$\mathbf{0} \longrightarrow \mathbf{2\pi}$
color	**R** → **G** → **B**

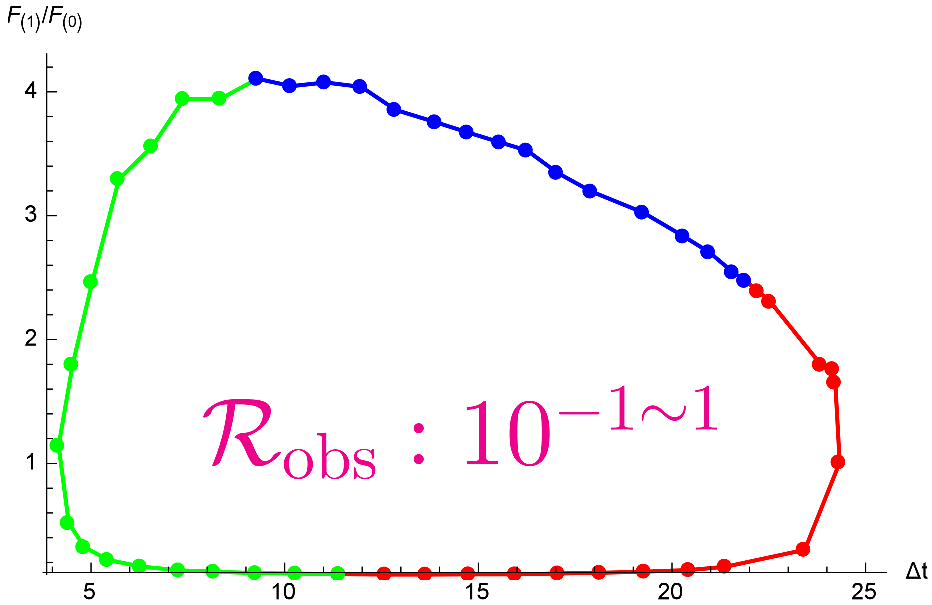
◇ hor. : $\Delta t_{\rm obs}/M$ — ver. : F_1/F_0(left) , ν_1/ν_0(right)

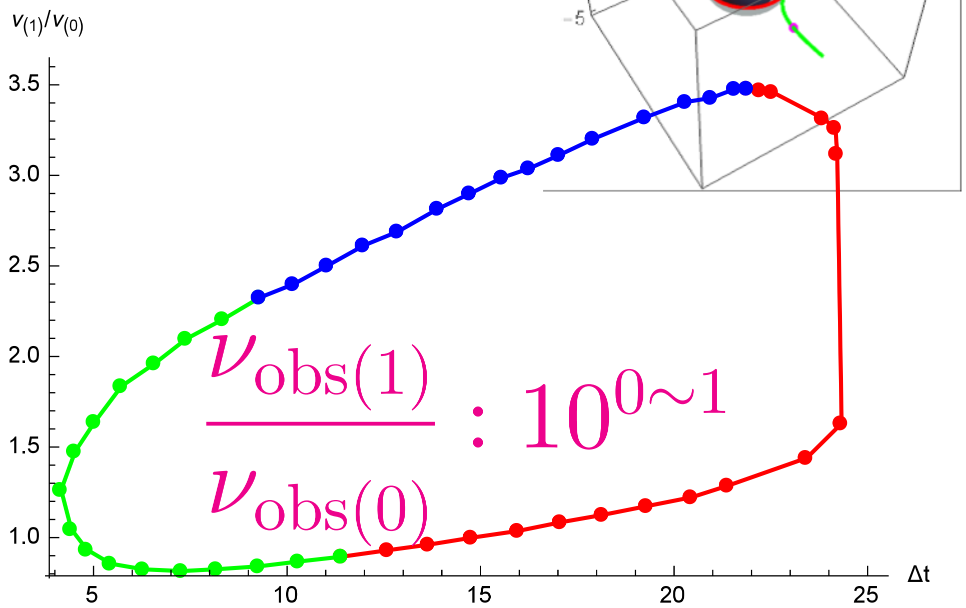




- Replace with $u_{\rm s}^{\mu} = (2.7\,,\,-1\,,\,0\,,\,0)$
  the other parameters are the same







◇ Various values of $\mathcal{R}_{\rm obs}$ is possible!

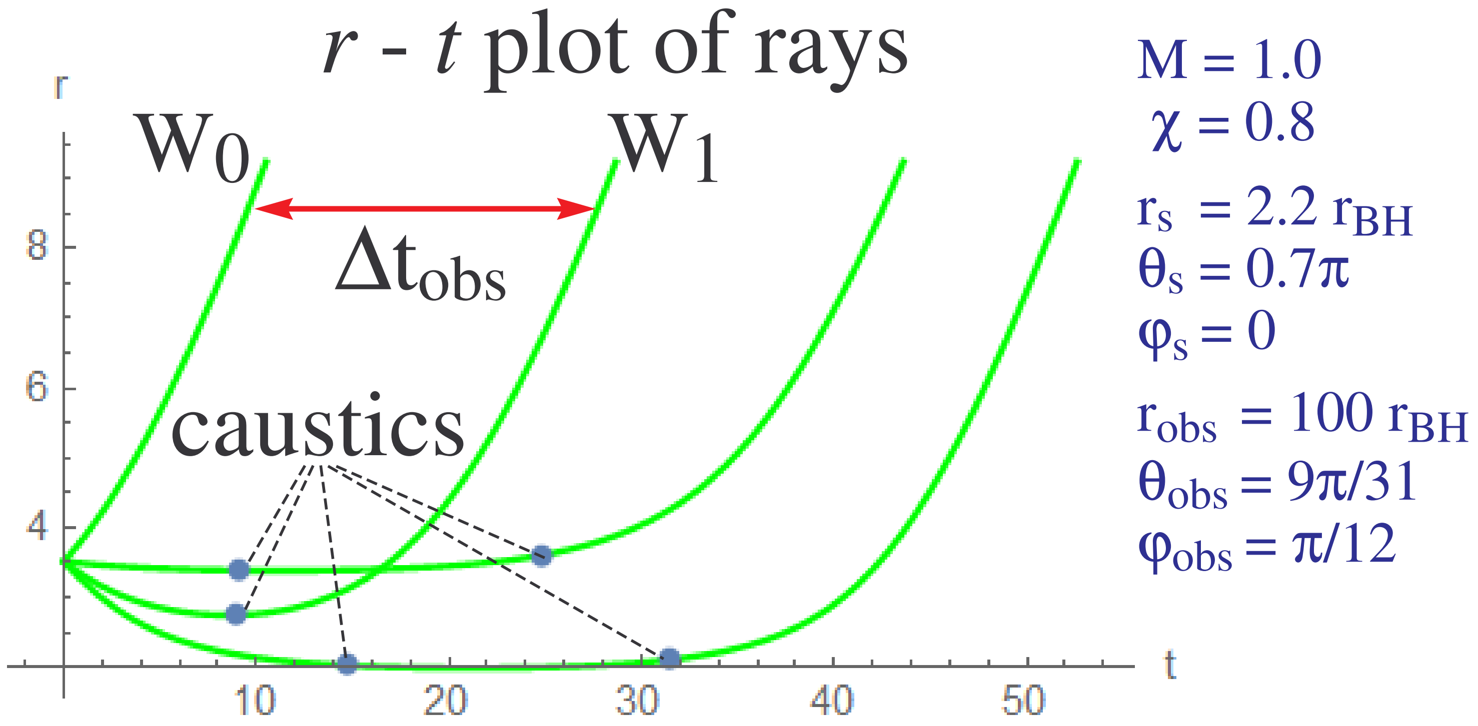
◇ Typically $O(\Delta t_{\rm obs}) \sim O(\pi r_{\rm s})$ ($= 10$ for this case)

$\Rightarrow$ There should be the case which is
detectable by the present telescope capability!

3. SGL in the Light Curve

3.1 Indication by sec.2

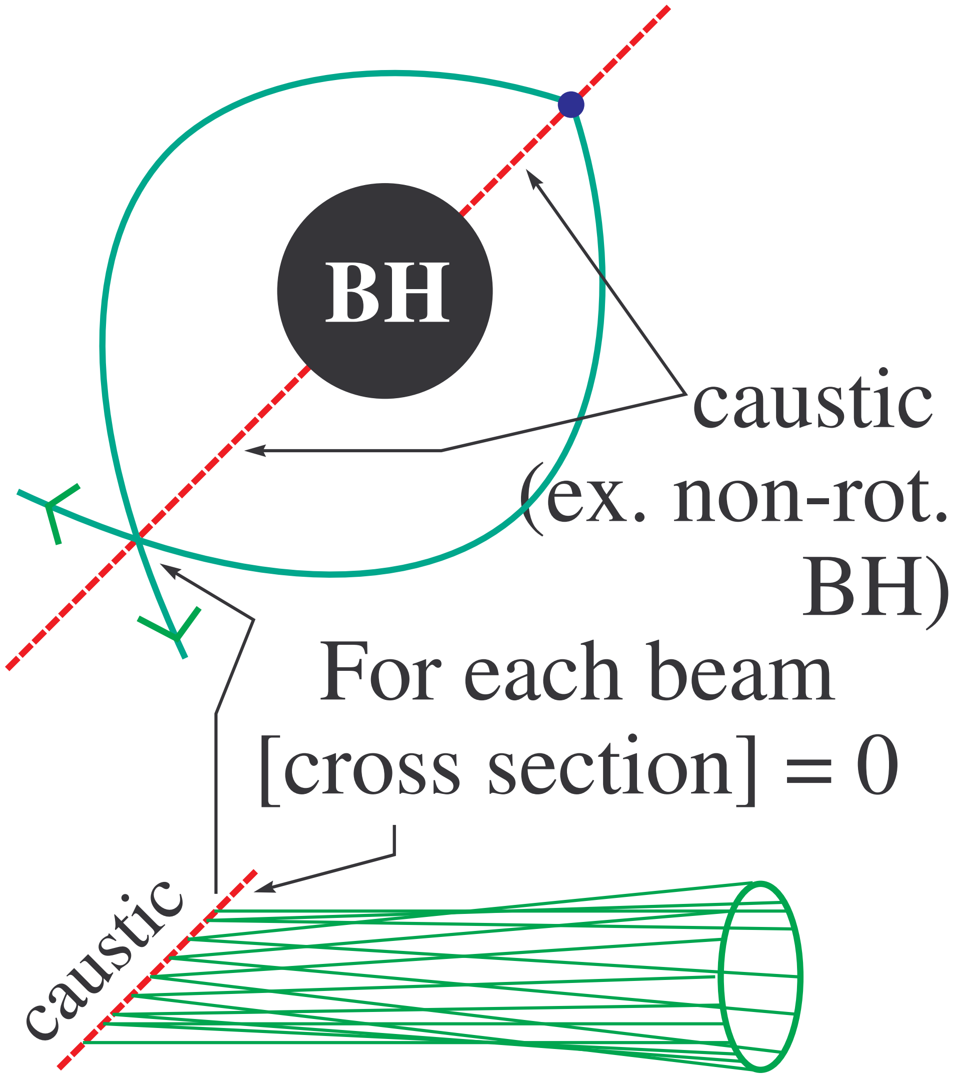
- The ray W_0 passes no causitic.
- The ray W_1 passes one caustic.



$\rightarrow$ The effect of caustic on the light curve may be important for the observation.

3.2 Gouy phase shift: wave optics issue (not GR)

Phase shift of waves when passing the caustic (an interference effect)



◇ positive freq. mode

→ phase shift by $-\pi/2$

◇ negative freq. mode

→ phase shift by $+\pi/2$

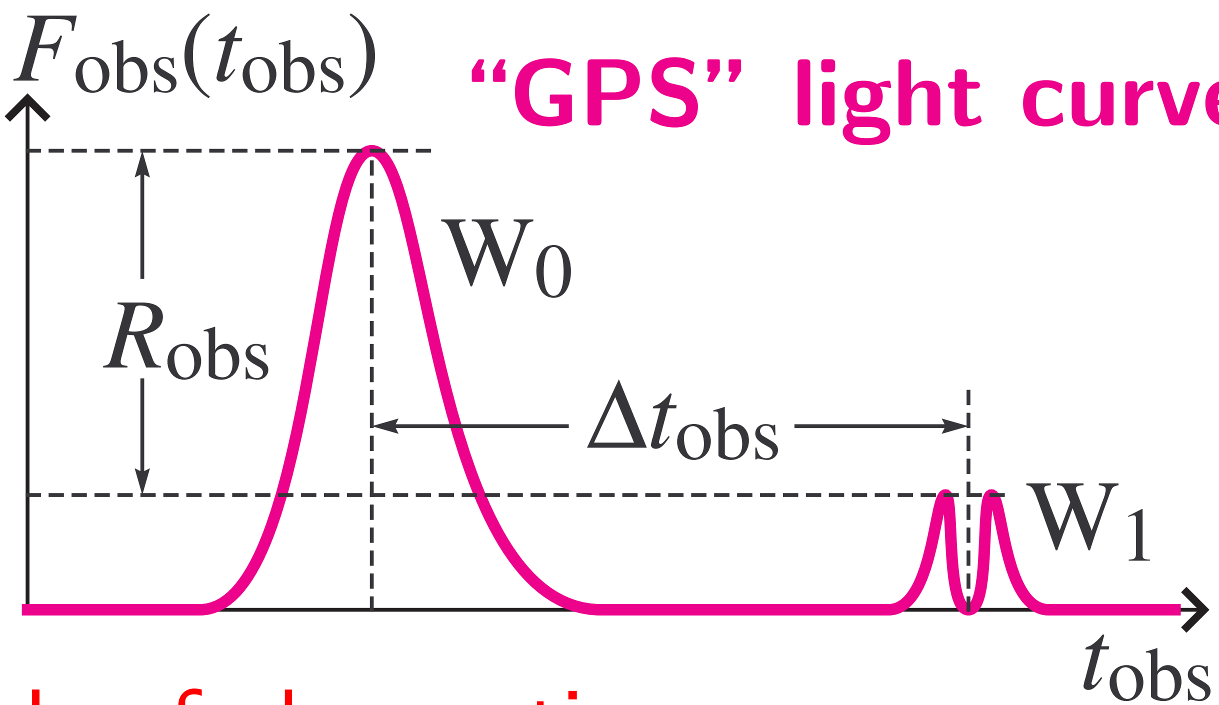
⇒ ex. $\cos(\omega t) \leftrightarrow \sin(\omega t)$

⇒ This is the Hilbert transformation of wave form.

3.3 Expected feature of light curve

- Gouy Phase Shift
  ⇔ Hilbert trans.
  of wave form
- Observed Flux
  $F_{\rm obs} \propto |E_{\rm obs}|^2$
  ($E_{\rm obs}$: amplitude)

"GPS" light curve



Principle of observation

Find the GPS (Gous Phase Shifted) light curve from the time series data taken by a telescope.
Then, the delay $\Delta t_{\rm obs}$ and ratio $\mathcal{R}_{\rm obs}$ are obtained.

4. Summary

- "Direct" BH detection is
  to measure M , χ through GR effects.
- Focus on the Strong Gravitational Lens (SGL)
- Obs. quantities $(\Delta t_{\rm obs}\,,\,F_1/F_0)$ seem to be
  detectable by the present telescope capability !?
  $\rightarrow$ Already estimated for a radio telescope in Japan.
  How about X-ray telescope ?
- Light curve $\rightarrow$ the Gouy effect may appear.
- If $\nu_{(1)}/\nu_{(0)}$ is also an observable, it is useful.