Linear perturbations in massive bigravity: formalism and cosmology

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Massive (bi)gravity

History

- Fierz & Pauli (1939) $m^2(h_{\mu\nu}h^{\mu\nu} - h^2)$
- vDVZ discontinuity (1970)
- Boulware & Deser (1972)
- Vainshtein (1972)
- Ghost freedom: dRGT(2010)
- Dynamical f, Hassan & Rosen (2012)

Aspects

- IR high-pass filter for CC $\sim e^{-mr}$
- Technically natural small m
- Matter couplings
- Tests of gravity

Bigravity Action

$$S = \frac{M_g^2}{2}\int dx^4\sqrt{-\boldsymbol{g}}\; R_g + \frac{M_f^2}{2}\int dx^4\sqrt{-\boldsymbol{f}}\; R_f + \frac{M_g^2}{2}\int dx^4\sqrt{-\boldsymbol{g}}\; U(g,f)$$

$$U(g,f) = -2m^2\sum_{n=0}^{4}\beta_n U_n\left(\sqrt{g^{-1}f}\right)$$

$$\mathbb{X} = \sqrt{g^{-1}f},\;\; \mathbb{X}^\mu_\nu = \sqrt{g^{\mu\lambda}f_{\lambda\nu}}$$

$$U_0 = 1$$

$$U_1 = [\mathbb{X}]$$

$$U_2 = 1/2([\mathbb{X}]^2 - [\mathbb{X}^2])$$

$$U_3 = 1/6([\mathbb{X}]^3 - 3[\mathbb{X}][\mathbb{X}^2] + 2[\mathbb{X}^3])$$

$$U_4 = \det[\mathbb{X}]$$

Bigravity Action

$$S = \frac{M_g^2}{2}\int dx^4\sqrt{-\boldsymbol{g}}\ R_g + \frac{M_f^2}{2}\int dx^4\sqrt{-\boldsymbol{f}}\ R_f + \frac{M_g^2}{2}\int dx^4\sqrt{-\boldsymbol{g}}\ U(g,f)$$

Absorb m^2 into β_n

$$U(g,f) = -2 \sum_{n=0}^{4} \beta_n U_n\left(\sqrt{g^{-1}f}\right)$$

$$\mathbb{X} = \sqrt{g^{-1}f},\ \ \mathbb{X}^{\mu}_{\nu} = \sqrt{g^{\mu\lambda}f_{\lambda\nu}}$$

$$U_0 = 1$$

$$U_1 = [\mathbb{X}]$$

$$U_2 = 1/2([\mathbb{X}]^2 - [\mathbb{X}^2])$$

$$U_3 = 1/6([\mathbb{X}]^3 - 3[\mathbb{X}][\mathbb{X}^2] + 2[\mathbb{X}^3])$$

$$U_4 = \det[\mathbb{X}]$$

Goal:

Write the second order action for bigravity on a generic background

$$S_m^{(2)} = -\frac{M_g^2}{2}\int dx^4\sqrt{-\boldsymbol{g}}\left[\mathcal{M}_{gg}^{\mu\nu\alpha\beta}\, h_{\mu\nu}h_{\alpha\beta} + \mathcal{M}_{ff}^{\mu\nu\alpha\beta}\, l_{\mu\nu}l_{\alpha\beta} + \mathcal{M}_{gf}^{\mu\nu\alpha\beta}\, h_{\mu\nu}l_{\alpha\beta}\right]$$

Matrix eigenvalues

$$U_0 = 1$$

$$U_1 = [\mathbb{X}]$$

$$U_2 = 1/2([\mathbb{X}]^2 - [\mathbb{X}^2])$$

$$U_3 = 1/6([\mathbb{X}]^3 - 3[\mathbb{X}][\mathbb{X}^2] + 2[\mathbb{X}^3])$$

$$U_4 = \det[\mathbb{X}]$$

$$[A\,] = \sum_i a_i$$

$$U_0 = 1$$

$$U_1 = \sum_i \mathbb{x}_i$$

$$U_2 = \sum_{i<k} \mathbb{x}_i \mathbb{x}_k$$

$$U_3 = \sum_{i<k<l} \mathbb{x}_i \mathbb{x}_k \mathbb{x}_l$$

$$U_4 = \mathbb{x}_1 \mathbb{x}_2 \mathbb{x}_3 \mathbb{x}_4$$

Action is symmetric under exchange

$g \leftrightarrow f, M_g \leftrightarrow M_f, \beta_n \leftrightarrow \beta_{n-4}$

Matrix eigenvalues

$$[A^n] = \sum_i a_i^n$$

$$s_1 \equiv U_1(g^{-1}f) = \sum_i \lambda_i$$

$$s_2 \equiv U_2(g^{-1}f) = \sum_{i<k} \lambda_i \lambda_k$$

$$s_3 \equiv U_1(g^{-1}f) = \sum_{i<k<l} \lambda_i \lambda_k \lambda_l$$

$$s_4 \equiv U_1(g^{-1}f) = \lambda_1 \lambda_2 \lambda_3 \lambda_4$$

$$t_1 \equiv U_1\left(\sqrt{g^{-1}f}\right) = \sum_i \lambda_i^{1/2}$$

$$t_2 \equiv U_2\left(\sqrt{g^{-1}f}\right) = \sum_{i<k} \lambda_i^{1/2} \lambda_k^{1/2}$$

$$\mathrm{t}_3 \equiv U_1\left(\sqrt{g^{-1}f}\right) = \sum_{i<k<l} \lambda_i^{1/2} \lambda_k^{1/2} \lambda_l^{1/2}$$

$$t_4 \equiv U_1\left(\sqrt{g^{-1}f}\right) = \sqrt{\lambda_1 \lambda_2 \lambda_3 \lambda_4}$$

Matrix Eigenvalues

Relate potential $U_i\left(\sqrt{g^{-1}f}\right)$ to $U_i(g^{-1}f)$, which I know how to perturb

$$t_1^2 = s_1 + 2t_2 \qquad \Rightarrow \qquad \left[U_1\left(\sqrt{g^{-1}f}\right)\right]^2 = U_1(g^{-1}f) + 2\,U_2\left(\sqrt{g^{-1}f}\right)$$

$$t_2^2 = s_2 - 2t_4 + 2t_1t_3$$

$$t_3^2 = s_3 + 2t_2t_4$$

$$t_4^2 = s_4$$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$
$$f_{\mu\nu} = \bar{f}_{\mu\nu} + l_{\mu\nu}$$

$$t_{g,i} = U_i\left(\sqrt{g^{-1}f}\right)_{,g} \qquad s_{g,i} = U_i(g^{-1}f)_{,g}$$

$$t_{gg,i} = \frac{1}{2}U_i\left(\sqrt{g^{-1}f}\right)_{,g,g} \qquad s_{gg,i} = \frac{1}{2}U_i(g^{-1}f)_{,g,g}$$

Second order massive action

$$S_m^{(2)} = -\frac{M_g^2}{2}\int dx^4\sqrt{-\boldsymbol{g}}\left[\mathcal{M}_{gg}^{\mu\nu\alpha\beta}\, h_{\mu\nu}h_{\alpha\beta} + \mathcal{M}_{ff}^{\mu\nu\alpha\beta}\, l_{\mu\nu}l_{\alpha\beta} + \mathcal{M}_{gf}^{\mu\nu\alpha\beta}\, h_{\mu\nu}l_{\alpha\beta}\right]$$

$$\mathcal{M}_{gg} = \frac{1}{2}\frac{1}{\sqrt{-\boldsymbol{g}}}\left[(\sqrt{-\boldsymbol{g}})_{,g,g}\, t_i + 2(\sqrt{-\boldsymbol{g}})_{,g}\, t_{g,i} + \sqrt{-\boldsymbol{g}}\, t_{gg,i}\right]$$

$$\mathcal{M}_{gf} = \frac{1}{2}\frac{1}{\sqrt{-\boldsymbol{g}}}\left[(\sqrt{-\boldsymbol{g}})_{,g}\, t_{f,i} + \sqrt{-\boldsymbol{g}}\, t_{gf,i}\right]$$

$$\mathcal{M}_{ff} = \frac{1}{2}\frac{1}{\sqrt{-\boldsymbol{g}}}\left[\sqrt{-\boldsymbol{g}}\, t_{ff,i}\right]$$

$$\mathcal{M}_{gg}^{\mu\nu\alpha\beta} = \frac{1}{2}\frac{1}{\sqrt{-\boldsymbol{g}}}\frac{\partial^2\left[\sqrt{-\boldsymbol{g}}\; U\left(\sqrt{g^{-1}f}\right)\right]}{\partial g_{\mu\nu}\partial g_{\alpha\beta}}\Bigg|_{g=\bar{g},f=\bar{f}}$$

Mass term

$$t^{\mu\nu}_{\bullet,1} = A\left\{\bar{t}_4\left(\bar{t}_1\bar{t}_4 - \bar{t}_2\bar{t}_3\right) s^{\mu\nu}_{\bullet,1} - \bar{t}_3\bar{t}_4 s^{\mu\nu}_{\bullet,2} - \bar{t}_1\bar{t}_4 s^{\mu\nu}_{\bullet,3} + \left(\bar{t}_3 - \bar{t}_1\bar{t}_2\right) s^{\mu\nu}_{\bullet,4}\right\} ,$$

$$t^{\mu\nu}_{\bullet,2} = A\left\{-\bar{t}_3^2\bar{t}_4 s^{\mu\nu}_{\bullet,1} - \bar{t}_3\bar{t}_4\bar{t}_1 s^{\mu\nu}_{\bullet,2} - \bar{t}_4\bar{t}_1^2 s^{\mu\nu}_{\bullet,3} + \left(\bar{t}_3 - \bar{t}_1\bar{t}_2\right)\bar{t}_1 s^{\mu\nu}_{\bullet,4}\right\} ,$$

$$t^{\mu\nu}_{\bullet,3} = A\left\{-\bar{t}_3\bar{t}_4^2 s^{\mu\nu}_{\bullet,1} - \bar{t}_1\bar{t}_4^2 s^{\mu\nu}_{\bullet,2} + \left(\bar{t}_3 - \bar{t}_1\bar{t}_2\right)\bar{t}_4 s^{\mu\nu}_{\bullet,3} + \left(\bar{t}_2\bar{t}_3 + \bar{t}_1\left(\bar{t}_4 - \bar{t}_2^2\right)\right) s^{\mu\nu}_{\bullet,4}\right\} ,$$

$$t^{\mu\nu}_{\bullet,4} = \frac{s^{\mu\nu}_{\bullet,4}}{2\bar{t}_4} ,$$

$$t^{\mu\nu\alpha\beta}_{\bullet\bullet,1} = A\left\{\bar{t}_4\left(\bar{t}_1\bar{t}_4 - \bar{t}_2\bar{t}_3\right) S^{\mu\nu\alpha\beta}_{\bullet\bullet,1} - \bar{t}_3\bar{t}_4 S^{\mu\nu\alpha\beta}_{\bullet\bullet,2} - \bar{t}_1\bar{t}_4 S^{\mu\nu\alpha\beta}_{\bullet\bullet,3} + \left(\bar{t}_3 - \bar{t}_1\bar{t}_2\right) S^{\mu\nu\alpha\beta}_{\bullet\bullet,4}\right\} ,$$

$$t^{\mu\nu\alpha\beta}_{\bullet\bullet,2} = A\left\{-\bar{t}_3^2\bar{t}_4 S^{\mu\nu\alpha\beta}_{\bullet\bullet,1} - \bar{t}_1\bar{t}_3\bar{t}_4 S^{\mu\nu\alpha\beta}_{\bullet\bullet,2} - \bar{t}_1^2\bar{t}_4 S^{\mu\nu\alpha\beta}_{\bullet\bullet,3} + \bar{t}_1\left(\bar{t}_3 - \bar{t}_1\bar{t}_2\right) S^{\mu\nu\alpha\beta}_{\bullet\bullet,4}\right\} ,$$

$$t^{\mu\nu\alpha\beta}_{\bullet\bullet,3} = A\left\{-\bar{t}_3\bar{t}_4^2 S^{\mu\nu\alpha\beta}_{\bullet\bullet,1} - \bar{t}_1\bar{t}_4^2 S^{\mu\nu\alpha\beta}_{\bullet\bullet,2} + \left(\bar{t}_3 - \bar{t}_1\bar{t}_2\right)\bar{t}_4 S^{\mu\nu\alpha\beta}_{\bullet\bullet,3} + \left[\bar{t}_2\bar{t}_3 + \bar{t}_1\left(\bar{t}_4 - \bar{t}_2^2\right)\right] S^{\mu\nu\alpha\beta}_{\bullet\bullet,4}\right\} ,$$

$$t^{\mu\nu\alpha\beta}_{\bullet\bullet,4} = \frac{S^{\mu\nu\alpha\beta}_{\bullet\bullet,4}}{2\bar{t}_4} ,$$

Mass term

$$
\begin{aligned}
\bar{s}_1 &= [\bar{\Sigma}] \,, \\
s^{\mu\nu}_{h,1} &= -\bar{\Sigma}^{\mu\nu} \,, \\
s^{\mu\nu}_{\ell,1} &= \bar{g}^{\mu\nu} \,, \\
s^{\mu\nu\alpha\beta}_{hh,1} &= \frac{1}{8} \left\{ \left[\bar{g}^{\alpha\nu}\bar{\Sigma}^{\mu\beta} + (\mu \leftrightarrow \nu) + (\alpha \leftrightarrow \beta) + (\mu \leftrightarrow \nu)(\alpha \leftrightarrow \beta) \right] + [\cdots] \left((\mu,\nu) \leftrightarrow (\alpha,\beta) \right) \right\} \\
&\equiv \mathrm{Sym} \left\{ \bar{g}^{\alpha\nu}\bar{\Sigma}^{\mu\beta} \right\} \\
s^{\mu\nu\alpha\beta}_{h\ell,1} &= -\frac{1}{2} \left(\bar{g}^{\mu\alpha}\bar{g}^{\nu\beta} + \bar{g}^{\mu\beta}\bar{g}^{\nu\alpha} \right) \,, \\
s^{\mu\nu\alpha\beta}_{\ell\ell,1} &= 0 \,,
\end{aligned}
$$

$$
\begin{aligned}
\bar{s}_2 &= \frac{1}{2} \left([\bar{\Sigma}]^2 - [\bar{\Sigma}^2] \right) \,, \\
s^{\mu\nu}_{h,2} &= (\bar{\Sigma}^2)^{\mu\nu} - [\bar{\Sigma}]\, \bar{\Sigma}^{\mu\nu} \,, \\
s^{\mu\nu}_{\ell,2} &= \bar{g}^{\mu\nu}\, [\bar{\Sigma}] - \bar{\Sigma}^{\mu\nu} \,, \\
s^{\mu\nu\alpha\beta}_{hh,2} &= \mathrm{Sym} \left\{ \bar{g}^{\mu\alpha} \left([\bar{\Sigma}]\, \bar{\Sigma}^{\beta\nu} - (\bar{\Sigma}^2)^{\beta\nu} \right) + \frac{1}{2}\bar{\Sigma}^{\mu\nu}\bar{\Sigma}^{\alpha\beta} - \frac{1}{2}\bar{\Sigma}^{\alpha\mu}\bar{\Sigma}^{\beta\nu} \right\} \,, \\
s^{\mu\nu\alpha\beta}_{h\ell,2} &= \frac{1}{4} \left[\bar{g}^{\beta\mu}\bar{\Sigma}^{\alpha\nu} + \bar{g}^{\alpha\nu}\bar{\Sigma}^{\beta\mu} - \bar{g}^{\alpha\beta}\bar{\Sigma}^{\mu\nu} - \bar{g}^{\alpha\mu}\bar{g}^{\beta\nu}\, [\bar{\Sigma}] + (\mu \leftrightarrow \nu) + (\alpha \leftrightarrow \beta) + (\mu \leftrightarrow \nu)(\alpha \leftrightarrow \beta) \right] \\
&\equiv \mathrm{sym} \left\{ \bar{g}^{\beta\mu}\bar{\Sigma}^{\alpha\nu} + \bar{g}^{\alpha\nu}\bar{\Sigma}^{\beta\mu} - \bar{g}^{\alpha\beta}\bar{\Sigma}^{\mu\nu} - \bar{g}^{\alpha\mu}\bar{g}^{\beta\nu}\, [\bar{\Sigma}] \right\} \,, \\
s^{\mu\nu\alpha\beta}_{\ell\ell,2} &= \frac{1}{2}\bar{g}^{\alpha\beta}\bar{g}^{\mu\nu} - \frac{1}{4} \left(\bar{g}^{\alpha\nu}\bar{g}^{\beta\mu} + \bar{g}^{\alpha\mu}\bar{g}^{\beta\nu} \right) \,,
\end{aligned}
$$

$$
\begin{aligned}
\bar{s}_3 &= \frac{1}{6} \left([\bar{\Sigma}]^3 - 3\, [\bar{\Sigma}]\, [\bar{\Sigma}^2] + 2\, [\bar{\Sigma}^3] \right) \,, \\
s^{\mu\nu}_{h,3} &= [\bar{\Sigma}]\, (\bar{\Sigma}^2)^{\mu\nu} - (\bar{\Sigma}^3)^{\mu\nu} + \frac{1}{2}\bar{\Sigma}^{\mu\nu} \left([\bar{\Sigma}^2] - [\bar{\Sigma}]^2 \right) \,, \\
s^{\mu\nu}_{\ell,3} &= (\bar{\Sigma}^2)^{\mu\nu} - \bar{\Sigma}^{\mu\nu}\, [\bar{\Sigma}] + \frac{1}{2}\bar{g}^{\mu\nu} \left([\bar{\Sigma}]^2 - [\bar{\Sigma}^2] \right) \,, \\
s^{\mu\nu\alpha\beta}_{hh,3} &= \mathrm{Sym} \Big\{ \bar{\Sigma}^{\mu\alpha}\, (\bar{\Sigma}^2)^{\beta\nu} - (\bar{\Sigma}^2)^{\mu\nu}\, \bar{\Sigma}^{\alpha\beta} + \frac{1}{2}\, [\bar{\Sigma}]\, \left(\bar{\Sigma}^{\mu\nu}\bar{\Sigma}^{\alpha\beta} - \bar{\Sigma}^{\mu\alpha}\bar{\Sigma}^{\nu\beta} \right) \\
&\qquad + \bar{g}^{\nu\alpha} \left((\bar{\Sigma}^3)^{\mu\beta} - (\bar{\Sigma}^2)^{\mu\beta}\, [\bar{\Sigma}] \right) + \frac{1}{2}\bar{g}^{\mu\alpha}\bar{\Sigma}^{\nu\beta} \left([\bar{\Sigma}]^2 - [\bar{\Sigma}^2] \right) \Big\} \,, \\
s^{\mu\nu\alpha\beta}_{h\ell,3} &= \mathrm{sym} \Big\{ 2\bar{g}^{\nu\beta} \left(\bar{\Sigma}^{\mu\alpha}\, [\bar{\Sigma}] - (\bar{\Sigma}^2)^{\mu\alpha} \right) + \bar{g}^{\alpha\beta} \left((\bar{\Sigma}^2)^{\mu\nu} - \bar{\Sigma}^{\mu\nu}[\bar{\Sigma}] \right) + \bar{\Sigma}^{\mu\nu}\bar{\Sigma}^{\alpha\beta} - \bar{\Sigma}^{\mu\beta}\bar{\Sigma}^{\alpha\nu} \\
&\qquad + \frac{1}{2}\bar{g}^{\mu\alpha}\bar{g}^{\nu\beta} \left([\bar{\Sigma}^2] - [\bar{\Sigma}]^2 \right) \Big\} \,, \\
s^{\mu\nu\alpha\beta}_{\ell\ell,3} &= \mathrm{Sym} \left\{ \frac{1}{2}\bar{g}^{\mu\nu}\bar{g}^{\alpha\beta}\, [\bar{\Sigma}] - \frac{1}{2}\bar{g}^{\mu\beta}\bar{g}^{\nu\alpha}\, [\bar{\Sigma}] + \bar{g}^{\mu\alpha}\bar{\Sigma}^{\nu\beta} - \bar{g}^{\alpha\beta}\bar{\Sigma}^{\mu\nu} \right\} \,,
\end{aligned}
$$

$$
\begin{aligned}
\bar{s}_4 &= \det \left(\bar{g}^{-1}\bar{f} \right) \,, \\
s^{\mu\nu}_{h,4} &= -\bar{s}_4\bar{g}^{\mu\nu} \,, \\
s^{\mu\nu}_{\ell,4} &= \bar{s}_4\bar{f}^{\mu\nu} \,, \\
s^{\mu\nu\alpha\beta}_{hh,4} &= \frac{\bar{s}_4}{2} \left(\bar{g}^{\mu\nu}\bar{g}^{\alpha\beta} + \frac{1}{2}\bar{g}^{\mu\alpha}\bar{g}^{\nu\beta} + \frac{1}{2}\bar{g}^{\nu\alpha}\bar{g}^{\mu\beta} \right) \,, \\
s^{\mu\nu\alpha\beta}_{h\ell,4} &= -\bar{s}_4\bar{g}^{\mu\nu}\bar{f}^{\alpha\beta} \,, \\
s^{\mu\nu\alpha\beta}_{\ell\ell,4} &= \frac{\bar{s}_4}{2} \left(\bar{f}^{\mu\nu}\bar{f}^{\alpha\beta} - \frac{1}{2}\bar{f}^{\mu\alpha}\bar{f}^{\nu\beta} - \frac{1}{2}\bar{f}^{\mu\beta}\bar{f}^{\nu\alpha} \right) \,.
\end{aligned}
$$

Parametrization for cosmological backgrounds

$$\mathcal{M}_{gf}^{0000} = a^{-4}\boldsymbol{\alpha}_{gf},$$

$$\mathcal{M}_{gf}^{ij00} = a^{-4}\boldsymbol{\gamma}_{gf}\,\delta^{ij},$$

$$\mathcal{M}_{gf}^{00ij} = a^{-4}\boldsymbol{\gamma}_{fg}\,\delta^{ij},$$

$$\mathcal{M}_{gf}^{i0j0} = a^{-4}\boldsymbol{\epsilon}_{gf}\,\delta^{ij},$$

$$\mathcal{M}_{gf}^{ijkl} = a^{-4}\left(\boldsymbol{\eta}_{gf}\,\delta^{ij}\delta^{kl} + \boldsymbol{\sigma}_{gf}\,\frac{(\delta^{ik}\delta^{jl}+\delta^{il}\delta^{jk})}{2}\right).$$

Parametrization for cosmological backgrounds
Example: FRW

$$g_{\mu\nu}dx^\mu dx^\nu = a^2\left(d\tau^2 + \delta_{ij}dx^i dx^j\right),$$

$$f_{\mu\nu}dx^\mu dx^\nu = b^2\left(\boldsymbol{c^2}d\tau^2 + \delta_{ij}dx^i dx^j\right),$$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}\,, \qquad f_{\mu\nu} = \bar{f}_{\mu\nu} + l_{\mu\nu}\,.$$

$$r = b/a,$$

$$\mathcal{H} = \frac{a'}{a}, \quad \mathcal{H}_f = \frac{b'}{cb}.$$

Background Bianchi Identities:

$$\nabla_\mu \bar{T}_m^{\mu\nu} = 0$$

$$\boldsymbol{\sigma_1}\left(\mathcal{H} - \mathcal{H}_f\right) = 0.$$

Algebraic Branch:

$$\boldsymbol{\sigma_1} = \beta_1 + 2\beta_2 r + \beta_3 r^2 = 0,$$

Dynamical Branch:

$$\mathcal{H} = \mathcal{H}_f.$$

Parametrization for cosmological backgrounds

Example: FRW

Friedmann equations

$$3\mathcal{H}^2 = a^2\left(\rho_m + \boldsymbol{\rho_g}\right)$$

$$\mathcal{H}^2 + 2\mathcal{H}' = -a^2(p_m + \boldsymbol{p_g})$$

$$3\mathcal{H}_f^2 = a^2 r^2\, \boldsymbol{\rho_f}$$

$$c\mathcal{H}_f^2 + 2\mathcal{H}_f' = -a^2 r^2\, \boldsymbol{p_f}$$

All 16 functions

$$\{\alpha_., \gamma_., \epsilon_., \eta_., \sigma_.\}$$

can be expressed in terms of

$$\{\sigma_1, \sigma_2, \rho_g, \rho_f, p_g, p_f\}$$

$$\boldsymbol{\sigma_1} = \beta_1 + 2\beta_2 r + \beta_3 r^2$$

$$\boldsymbol{\sigma_2} = \beta_1 + (c+1)\beta_2 + \beta_3 c r^2$$

Parametrization for cosmological backgrounds

Tests of gravity

- Detecting modifications of gravity.
- Identifying common parameters in scalar and tensor sectors. [Saltas et al (2014)]
- No-go theorem for gravitational slip. [Amendola, Kunz, Motta, Saltas, Sawicki, to appear!]

$$ds^2 = a^2\{-(1+\phi_g)dt^2 + 2B_{g,i}\,dtdx^i + [1+2\psi_g\delta_{ij} + E_{g,ij}]dx^i dx^j\}$$

$$ds^2 = b^2\{-c^2(1+\phi_f)dt^2 + 2cB_{f,i}\,dtdx^i + [1+2\psi_f\delta_{ij} + E_{f,ij}]dx^i dx^j\}$$

Gravitational waves (g −sector):

$$h'' + 2\mathcal{H}h' + k^2h^2 + a^2\boldsymbol{\sigma_2}r\,(h-l)^2 = 0$$

Slip equation (g −sector):

$$\phi_g - \psi_g = a^2 r\boldsymbol{\sigma_2}\left(E_f - E_g\right)$$

Cosmological application: *Algebraic Branch*

Algebraic Branch:

$$\boldsymbol{\sigma_1} = \beta_1 + 2\beta_2 r + \beta_3 r^2 = 0,$$

$$r = const.$$

Coupling Parameters:

$\alpha_{gf} = \gamma_{gf} = \epsilon_{gf} = 0,$

$\rho_{gf} = -\frac{\boldsymbol{\sigma_2}}{2\ r}$

$\sigma_{gf} = -\rho_{gf}.$

Infinite Strong-coupling

Ex: $\quad \mathcal{L}_{Gal} = -\frac{1}{2}(\partial\pi)^2 - \frac{1}{\Lambda^3}(\partial\pi)^2\Box\pi + \frac{1}{M_{Pl}}\pi T$

- Difficult treatment: non-linearities
- Usually sick: non linear instabilities.

Strong coupling scale: $\quad \Lambda = \Lambda_3 = m^2 M_{pl}$

Beyond Λ_3 π no longer grasps the behaviour of the helicity-0, which does not behave as a scalar.

Tensors in Algebraic Branch:

Would be minimal model of bigravity?[De Felice & Mukohyama (2015)]

$$Q_h \equiv M_g a\, h, \qquad Q_l \equiv M_g a \frac{r}{\sqrt{c}} l$$

$$S^2 \equiv \frac{1}{2}\int dx^4 \left[\begin{pmatrix} Q_h' & Q_l' \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} Q_h' \\ Q_l' \end{pmatrix} - \begin{pmatrix} Q_h & Q_l \end{pmatrix} \boldsymbol{\mathcal{M}} \begin{pmatrix} Q_h \\ Q_l \end{pmatrix} \right]$$

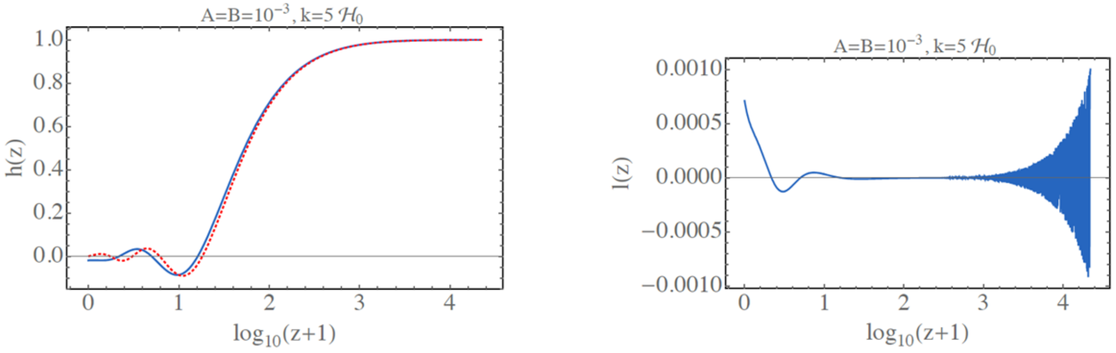
$$\boldsymbol{\mathcal{M}} = a^2 \begin{pmatrix} \mathcal{E}(a) + k^2 + \boldsymbol{\sigma_{hl}} & -\frac{\sigma_2 \sqrt{c}}{2r^2} \\ -\frac{\sigma_2 \sqrt{c}}{2r^2} & \mathcal{F}(a) + k^2 c^2 + \frac{a^2}{r^2} c\, \boldsymbol{\sigma_{hl}} \end{pmatrix}$$

Stability condition:

$$a = \tau^\alpha$$

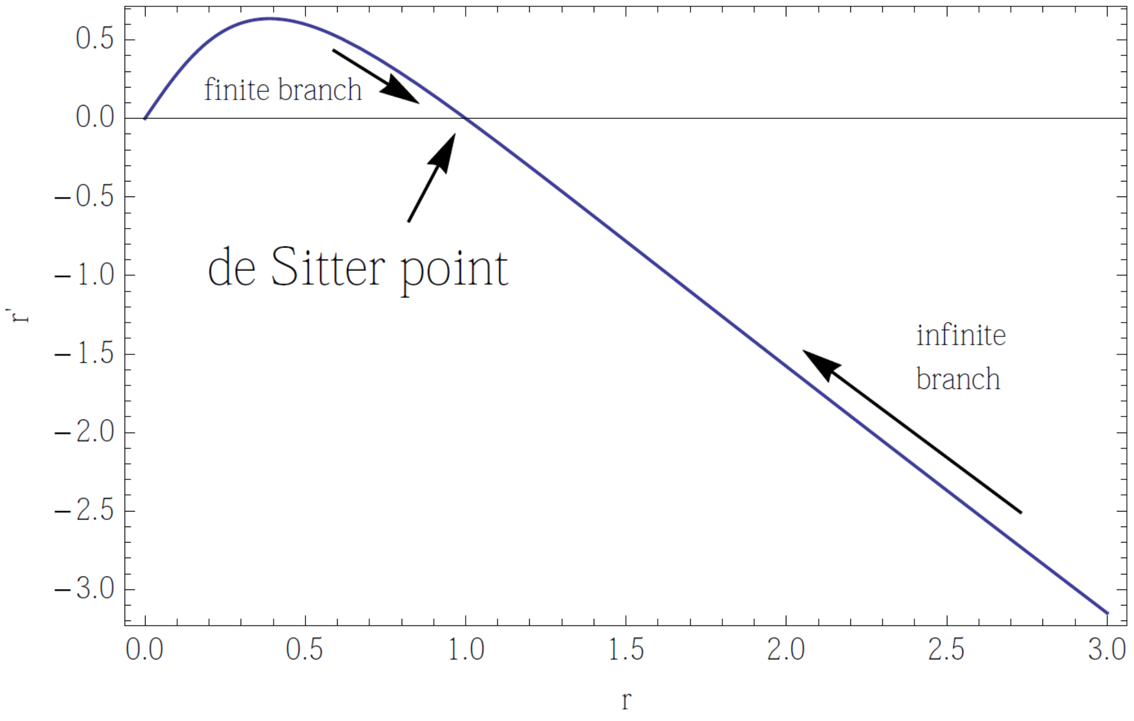
$$k^2 - \frac{\alpha(\alpha-1)}{\tau^2} \le k^2 c^2 - \frac{359\alpha^2 - 2\alpha + 3/4}{\tau^2}$$

- Tachionic instability at late times for large scales on both sectors



Cosmological application: *Dynamical Branch*

Dynamical Branch: $\mathcal{H} = \mathcal{H}_f = 0,$



Scalar sector:

- Infinite branch: always plagued by Higuchi ghost

Scalar sector: $r' \geq 0$, $(\boldsymbol{\sigma_1} > 0)$ [Könnig, 2015]

Tensor sector: $c \geq 0$

- Finite branch: viable background → free of Higuchi ghost

- Higuchi ghost or Gradient instabilitiy: $\frac{\partial \boldsymbol{\sigma_1}}{\partial r}$

Gradient instabilities: breakdown of linear perturbation theory, nonlinear treatment

Summary

- General mass term for bigravity on general backgrounds.
- Convenient parametrization on cosmological brackgrounds.
- Algebraic Branch: Tensor as in minimal model for bigravity.
- Cosmology of tensors: tachionic instability.
- Incorporate double matter couplings.

Thank you!