Acoustic production of gravitational waves at phase transitions

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Mark Hindmarsh[1,2]

with

Stephan Huber[1], Kari Rummukainen[2], David Weir[3]

1. Department of Physics & Astronomy, University of Sussex
2. Helsinki Institute of Physics & Dept of Physics, University of Helsinki
3. Institute of Mathematics & Natural Sciences, University of Stavanger

Gravitational waves in the early universe

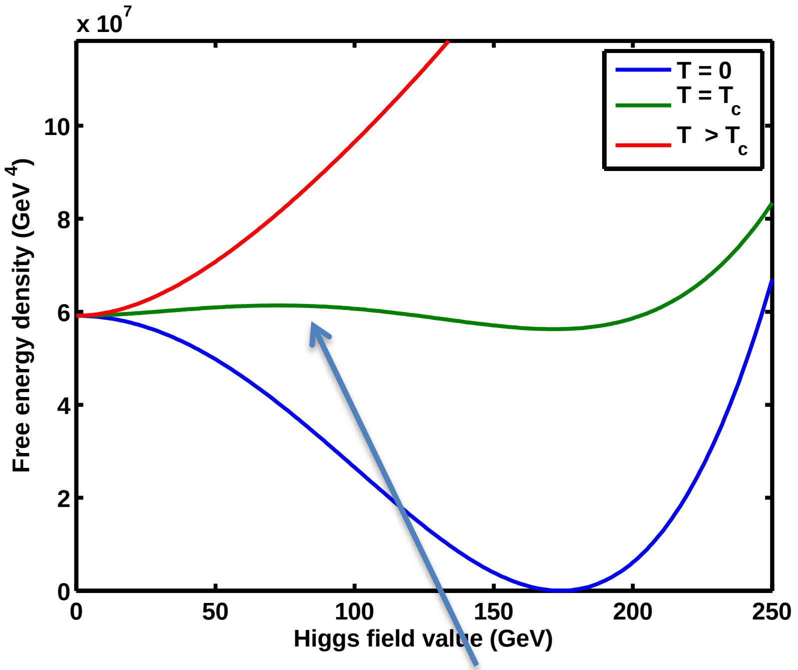
- Sources
  - Inflation
  - Preheating after inflation
  - Topological defects
  - First-order phase transitions
- Early Universe is transparent to GWs
  - Unique probe of physics at high energies

Summary of this talk

- Previous models of GW production at first order phase transitions missed an important source …
- … GWs from acoustic waves, generated by the nucleating bubbles of the Higgs phase
- The GW power has been underestimated by two orders of magnitude or more (weak transitions)
- Good news for eLISA

Electroweak phase transition

- Free energy density of plasma depends on
  - Temperature T
  - Particle masses $m_i(\phi)$
- High T: reduce free energy by forcing Higgs ϕ to zero
- Phase transition in weakly coupled gauge theories: Kirzhnits 1972
- Electroweak phase transition: $T_c \approx v_{EW} \approx 100$ GeV

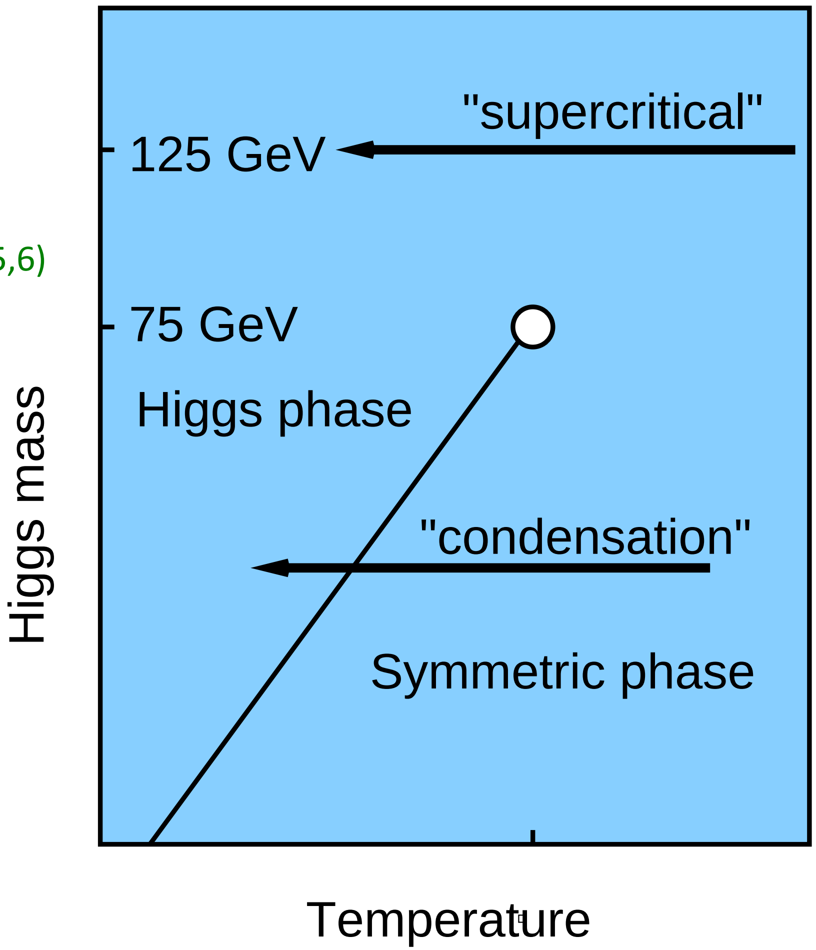


Potential barrier from cubic term in perturbative high-T expansion. First order transition?

- High T (>> $m_i(\phi)$): $V(\phi,T) = \frac{1}{2}\gamma(T^2 - T_0^2)\phi^2 - \frac{1}{3}AT\phi^3 + \frac{1}{4}\lambda\phi^4.$

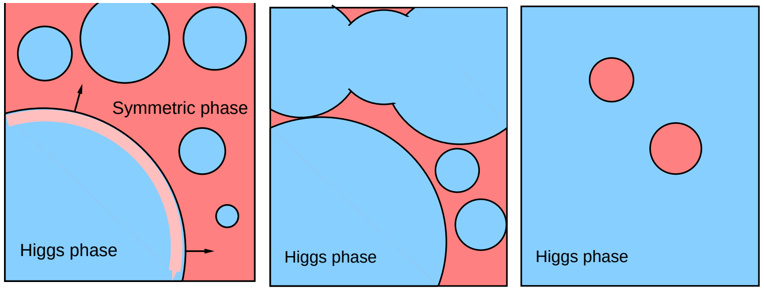
Standard Model electroweak phases

- SM is not weakly coupled at high T
- Non-perturbative techniques:
  - Dimensional reduction + effective field theory + 3D lattice
    Kajantie, Laine, Rummukainen, Shaposhnikov (1995,6)
  - 4D lattice
    Czikor, Fodor, Heitger (1998)
- SM transition at $m_h \approx 125$ GeV like a supercritical fluid
- 1st order transition = beyond the Standard Model physics

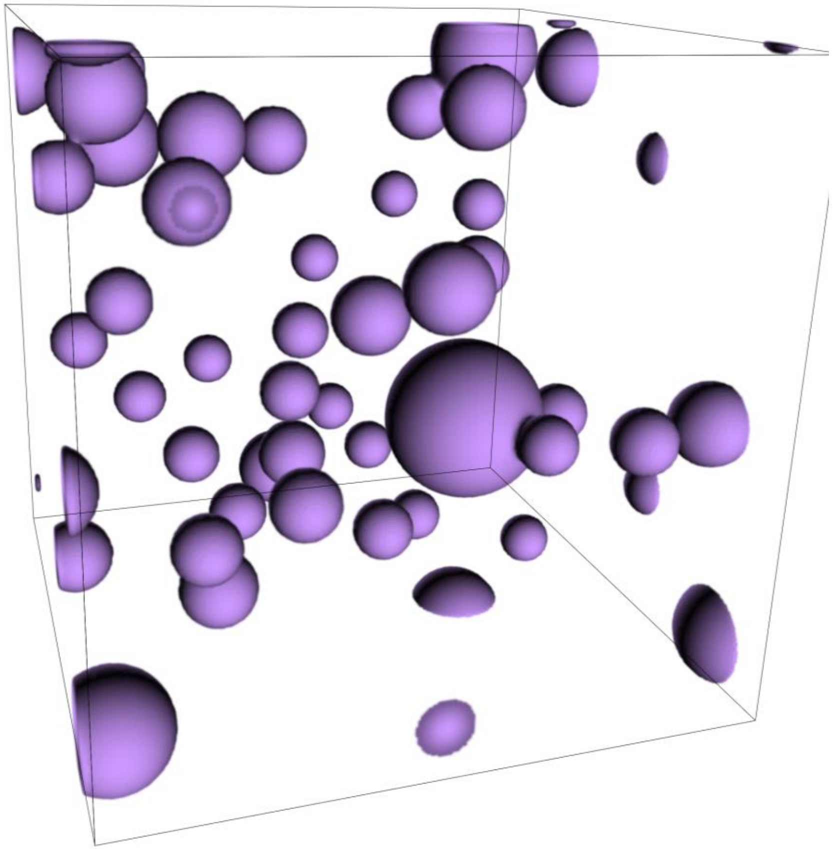


Little bangs in the Big Bang

- 1st order transition proceeds by nucleation of bubbles of Higgs phase
- Expanding bubbles generate shear stresses in hot fluid
- Detectable gravitational waves?



Steinhardt (1982); Gyulassy et al (1984); Witten (1984); Enqvist et al (1992);



MH, Huber, Rummukainen, Weir (2013)
Scalar only: Child, Giblin (2012)

First order phase transitions

- Bubble nucleation rate/volume Linde (1983)

$$p(t) = \Gamma(T)e^{-S(T)} \simeq \Gamma(T_N)e^{-S(t_\mathrm{N})+\beta(t-t_\mathrm{N})},$$

- Thermal EW transition:

$$S(t_\mathrm{N}) \sim 10^2; \quad \frac{\beta}{H_\mathrm{N}} \simeq \frac{2S(t_\mathrm{N})}{1-T_\mathrm{N}/T_\mathrm{c}}$$

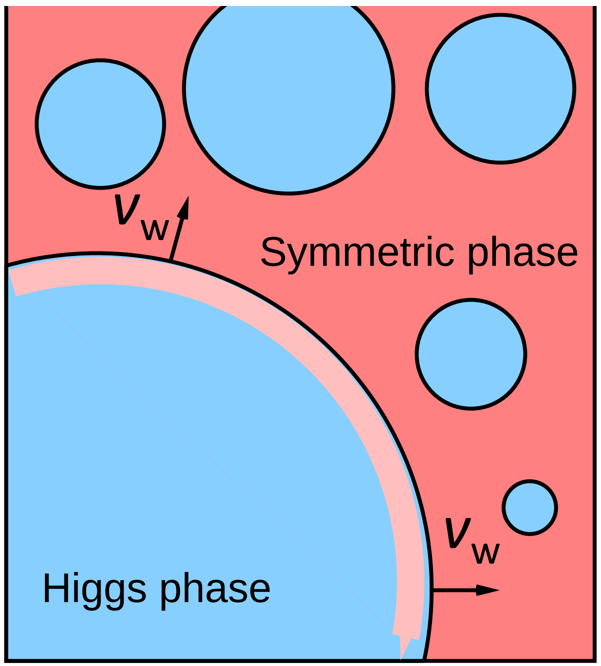
Hogan (1983) ; Moore & Rummukainen (2000)

- Duration of transition: β^{-1}
- Average bubble separation

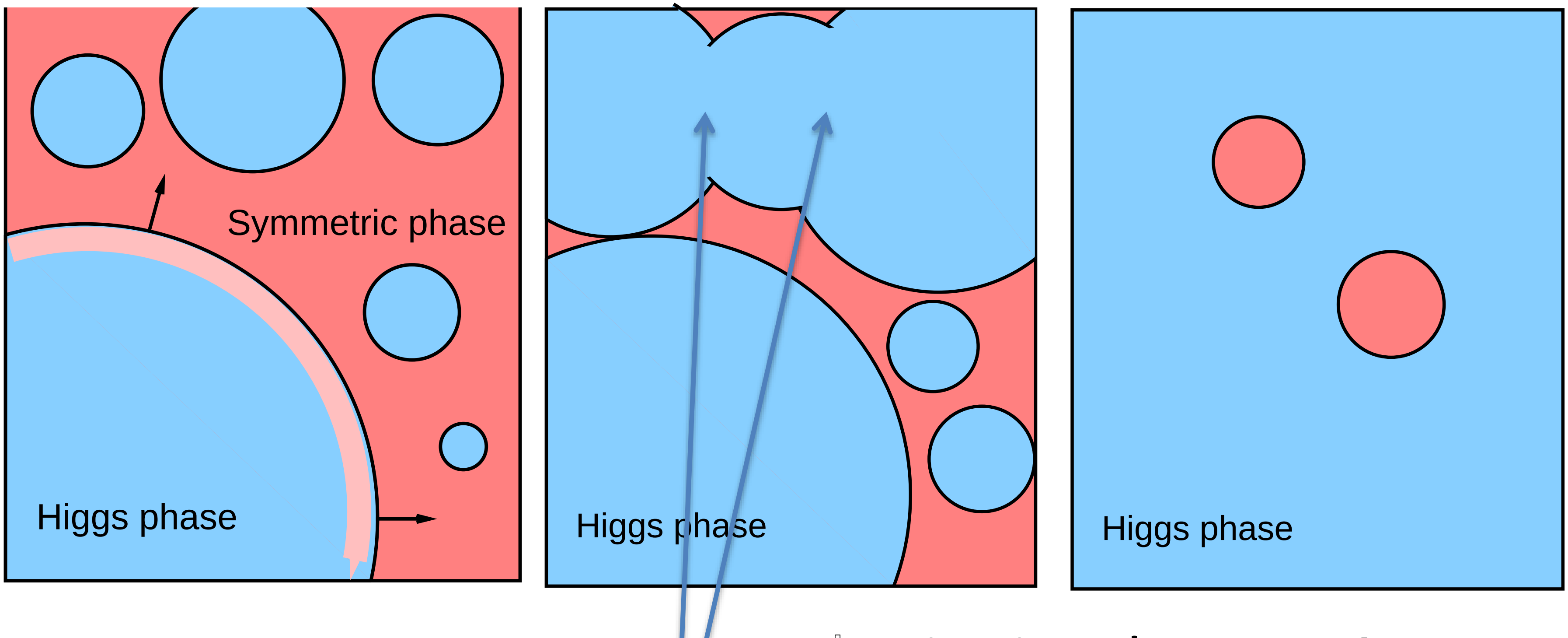
$$R_* \sim v_\mathrm{w}/\beta$$

- Wall speed v_w
  - $< c_s$ deflagration
  - $\sim c_s$ Jouguet/hybrid
  - $> c_s$ detonation

Steinhardt (1982)



The old picture: envelope approximation



Spherical distributions do not radiate

Assume: Gravitational waves generated by thin annihilating shells
(Kosowski, Turner, Watkins 1992
Kamionkowski, Kosowsky, Turner 1994)

The old picture: envelope approximation

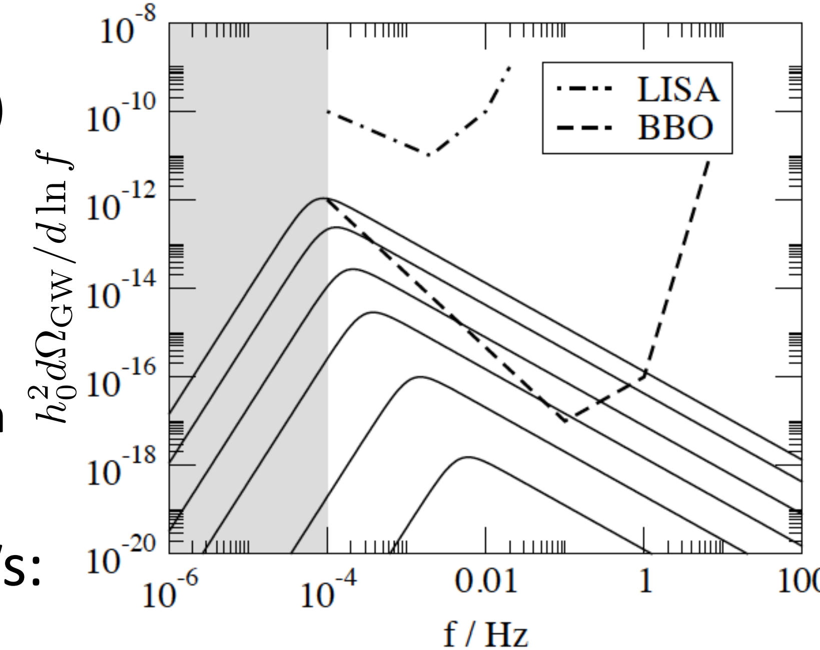
Kosowsky, Turner, Watkins 1992; Kamionkowski, Kamionkowsky, Turner 1994

- Parametrise transition:
  - α = (Vacuum energy)/(Total energy)
  - v_w = Bubble wall speed
  - H_* = Hubble rate at transition
  - β = bubble nucleation rate
  - κ = conversion efficiency of vacuum energy to fluid kinetic energy
    Espinosa et al 2008
- Fraction of energy density in GWs:

$$\Omega_{\rm GW} = \frac{\rho_{\rm GW}}{\rho_{\rm Tot}}$$

$$\Omega_{\rm GW}^{\rm ea} \simeq \frac{0.11 v_{\rm w}^3}{0.42 + v_{\rm w}^2} \left(\frac{H_*}{\beta}\right)^2 \frac{\kappa^2\alpha^2}{(\alpha+1)^2},$$

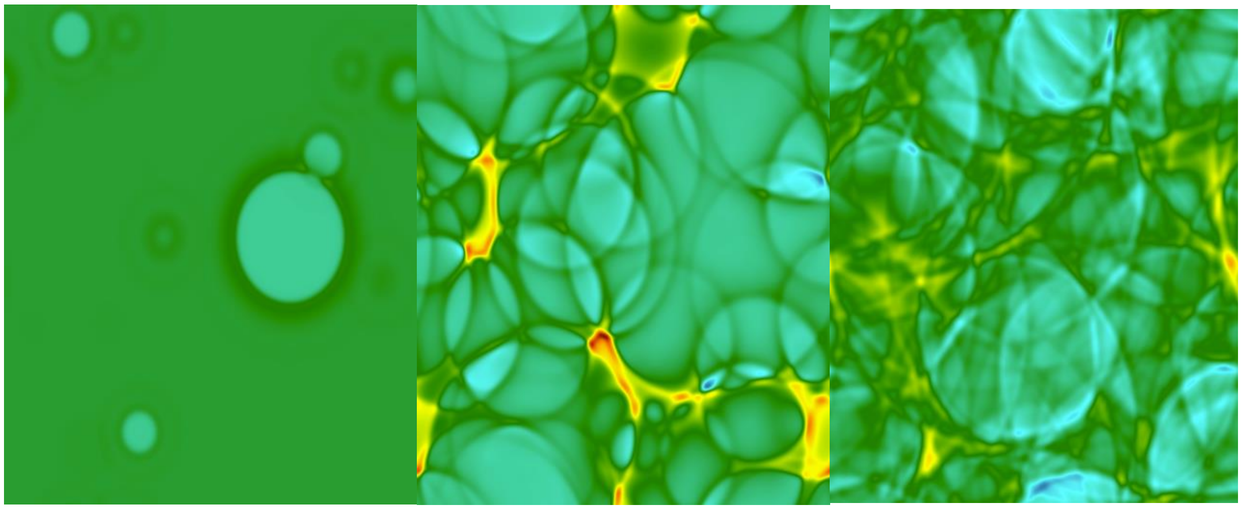
Huber, Konstandin 2008



Huber, Konstandin 2008

GW power spectrum:
α = 0.2 … 0.03
H_*/β = 0.1 … 0.003

Hydrodynamics of phase transitions: acoustic production of GWs



Fluid energy density

MH, Huber, Rummukainen, Weir (2013,2015)
See also Giblin, Mertens (2013)

- Latent heat of transition goes into fluid compression waves – **sound** Hogan (1986)
- Sound waves remain long after transition is complete MH, Huber, Rummukainen, Weir (2013)

Lifetime of sound waves

- Sound waves damped by viscosity
  - Shear viscosity $\eta_{\rm s} \sim e^4 T^3/\ln(1/e),$
  - Bulk viscosity $\zeta \simeq 15(\frac{1}{3} - c_{\rm s}^2)\eta_{\rm s}$
- Lifetime of scale *R*: $\tau_\eta(R) \sim R^2\epsilon/\eta_{\rm s} \sim \ln(1/e) R^2 T/e^4.$
- Damping time longer than Hubble time for scales

$$R \gg \frac{v_{\rm w}}{H_*}\left(\frac{T_{\rm c}}{m_{\rm Pl} e^4}\right) \sim 10^{-11}\frac{v_{\rm w}}{H_*}\left(\frac{T_{\rm c}}{100\ {\rm GeV}}\right),$$

- Viscous damping not important
- Timescale for non-linear effects $t_{\rm nl} \sim R/\sqrt{\kappa\alpha}$
- Non-linearities not important for weak transitions

Generation of gravitational waves

- Metric perturbation $$\ddot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G T_{ij}^{\mathrm{TT}}$$
- Metric projected from u_{ij} $$\ddot{u}_{ij} - \nabla^2 u_{ij} = 16\pi G \tau_{ij}$$

Garcia-Bellido, Figueroa, Sastre (2008)

- Ingredients:
  - Higgs field $$\tau_{ij}^{\phi} = \partial_i \phi \partial_j \phi$$
  - Relativistic fluid $$\tau_{ij}^{\mathrm{f}} = W^2 (\epsilon + p) V_i V_j$$

ϵ	energy density
p	pressure
V_i	velocity
W	gamma-factor

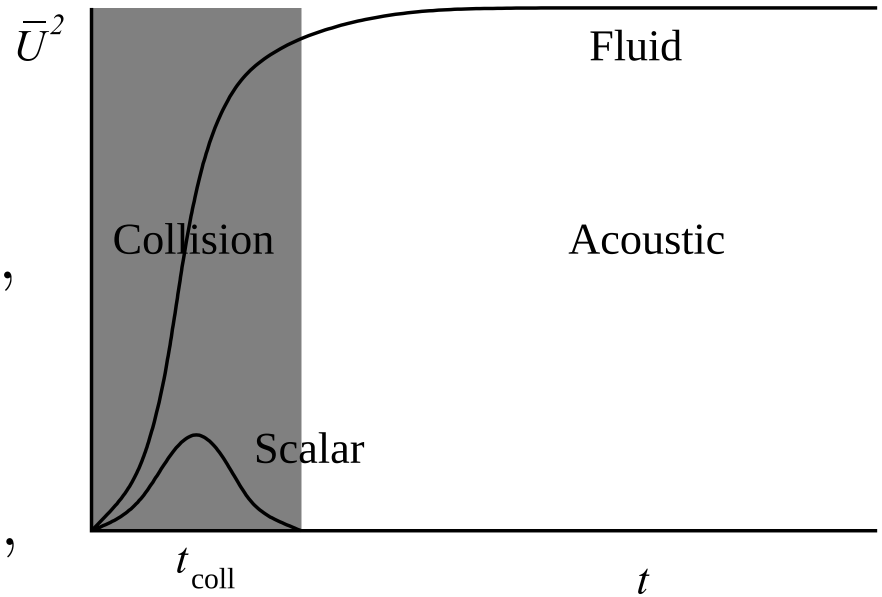
Source of gravitational waves

- Look at
  - rms fluid velocity $\bar{U}_\mathrm{f}$

$$(\bar{\epsilon}+\bar{p})\bar{U}_\mathrm{f}^2 = \frac{1}{V}\int d^3x\tau_{ii}^\mathrm{f},$$

  - equivalent field quantity

$$(\bar{\epsilon}+\bar{p})\bar{U}_\phi^2 = \frac{1}{V}\int d^3x\tau_{ii}^\phi,$$



- Ratio $$\frac{\bar{U}_\mathrm{f}^2}{\bar{U}_\phi^2} \sim R_* T_\mathrm{c} \sim v_\mathrm{w}\left(\frac{H_*}{\beta}\right)\left(\frac{T_\mathrm{c}}{H_*}\right)$$

- Approx 10^{12} for electroweak
- **Source of GWs is fluid**

Exceptions:

- (near-) vacuum transitions
- Run-away walls

Bodeker, Moore (2009)

Acoustic GW production (Minkowski)

- Spectral density of gravitational radiation:

$$P_{\dot{h}}(k,t) = (16\pi G)^2 \int_0^t dt_1 dt_2 \frac{\cos[k(t_1 - t_2)]}{2} \Pi^2(k,t_1,t_2).$$

  - Π^2 – shear stress UETC

Caprini, Durrer, Konstandin, Servant (2009)

$$\langle T_{ij}^{TT}(\mathbf{k},t_1) T_{ij}^{TT*}(\mathbf{k}',t_2)\rangle = \Pi^2(k,t_1,t_2)(2\pi)^3\delta^3(\mathbf{k}-\mathbf{k}')$$

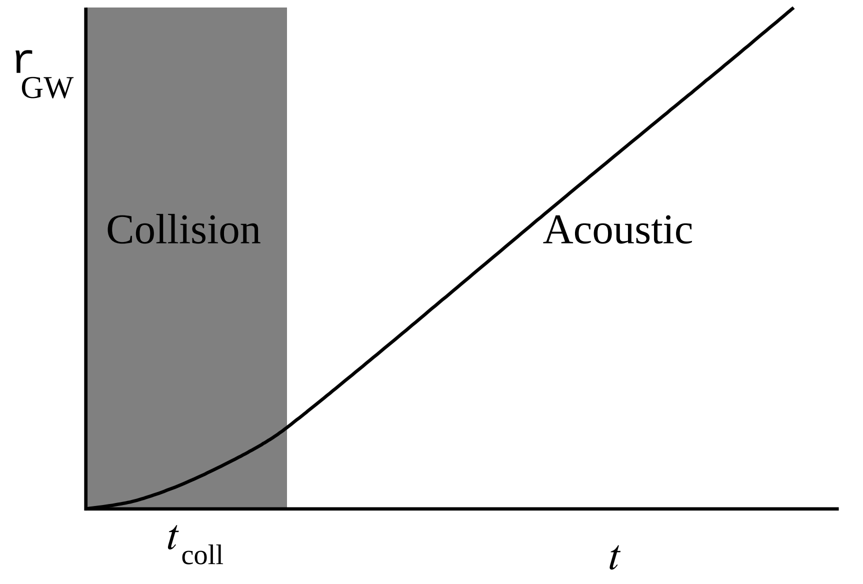
  - $\tilde{\Pi}^2$ – dimensionless shear stress UETC

$$\Pi^2(k,t_1,t_2) = [(\bar{\epsilon}+\bar{p})\bar{U}_{\rm f}]^2 L_{\rm f}^3 \tilde{\Pi}^2$$

- Can show: $\rho_{\rm GW} \propto t[G L_{\rm f}(\bar{\epsilon}+\bar{p})^2 \bar{U}_{\rm f}^4]$
  - Fluid flow length scale $L_{\rm f}$

GW energy density

- Fluid sources GWs continuously
- Source goes as (energy density)^2
- GW energy density grows with time



$$\rho_{\rm GW} = t[GL_{\rm f}(\bar{\epsilon}+\bar{p})^2\bar{U}_{\rm f}^4(8\pi\tilde{\Omega}_{\rm GW})]$$

- Fluid length scale L_f
  - E.g. integral scale $\xi_{\rm f} = \frac{1}{\langle V^2\rangle}\int \frac{d^3k}{(2\pi)^3}|k|^{-1}P_V(k),$
- Numerical simulations $8\pi\tilde{\Omega}_{\rm GW} = 0.8 \pm 0.1$

Acoustic GW production (FLRW)

- Spectral density of gravitational radiation:

$$P_{\dot{h}}(k,t) = (16\pi G)^2 a_*^2 \int_0^t dt_1 dt_2 \frac{\sin[k(t-t_1)]}{a(t_1)} \frac{\sin[k(t-t_2)]}{a(t_2)} \Pi^2(k,t_1,t_2).$$

- Assume
  - decorrelation time << Hubble time
  - Fluid flow length scale L_f

$$\frac{d\Omega_{\mathrm{GW}}}{d\ln(k)} = 3(1+w)^2 \bar{U}_\mathrm{f}^4 (H_* L_\mathrm{f}) \tilde{P}_{\mathrm{GW}}(kL_\mathrm{f})$$

- Fits to simulations: $\tilde{P}_{\mathrm{GW}}(x) \propto \frac{x^3}{(1+x^2)^{(p+3)/2}}$

A new model for GW production

- Acoustic production:

$$\Omega_{\rm GW} = 3(1+w)^2\bar{U}_{\rm f}^4(H_* L_{\rm f})(8\pi\tilde{\Omega}_{\rm GW}),$$

  - Fluid source "on" for a Hubble time
  - Fluid source length scale L_f (~ bubble separation R_*)
  - Fluid kinetic energy density fraction $(1+w)\bar{U}_{\rm f}^2$
  - Dimensionless constant $8\pi\tilde{\Omega}_{\rm GW} = 0.8 \pm 0.1$

- Compare with envelope approximation:

$$\Omega_{\rm GW}^{\rm ea} \simeq \frac{0.11 v_{\rm w}^3}{0.42+v_{\rm w}^2}\left(\frac{H_*}{\beta}\right)^2 \frac{\kappa^2\alpha^2}{(\alpha+1)^2},$$

Note $(1+w)\bar{U}_{\rm f}^2 = \dfrac{\kappa\alpha}{\alpha+1}$

Acoustic GW signal boost

- Ratio of acoustic production to envelope approximation
  NB $R_* = (8\pi)^{1/3} v_w/\beta$

$$\frac{\Omega_{\rm GW}}{\Omega_{\rm GW}^{\rm ea}} \simeq \frac{3(8\pi)^{\frac{1}{3}}\tilde{\Omega}_{\rm GW}}{0.11 v_{\rm w}^2(0.42 + v_{\rm w}^2)} \left(\frac{\beta}{H_*}\right).$$

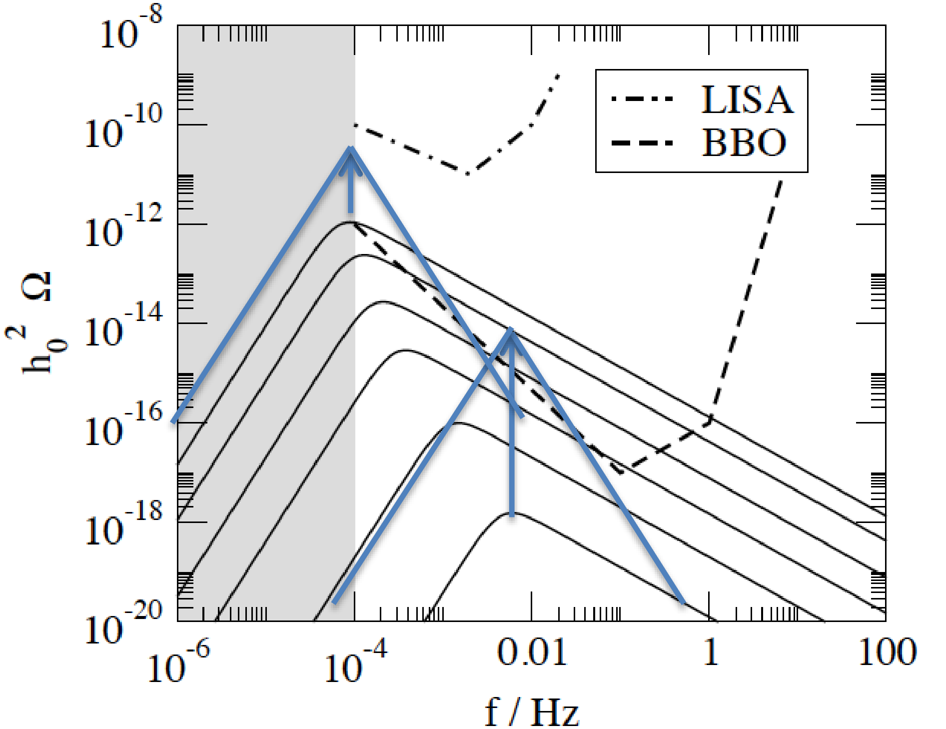
- Wall speed $v_w < 1$
  - Weak transition (linear fluid flow)

$$\frac{\Omega_{\rm GW}}{\Omega_{\rm GW}^{\rm ea}} \gtrsim 60\tilde{\Omega}_{\rm GW}\frac{\beta}{H_*}.$$

- GWs are parametrically larger by a factor which is at least $O(10^2)$ for a thermal electroweak transition

Implications for GW detection

- Preliminary sketch:
  - Peak amplitude increases
  - High frequency GW power spectrum ~ f^{-p} (p>3?)



Summary

- Main source (weak transitions): **sound waves** from the nucleating droplets of the low temperature phase
- GWs at least O(10^2) larger than old estimates (weak transitions)

$$\frac{d\Omega_{\rm GW}}{d\ln(k)} = 3(1+w)^2 \bar{U}_{\rm f}^4 (H_* L_{\rm f}) \tilde{P}_{\rm GW}(kL_{\rm f})$$

$$\tilde{P}_{\rm GW}(x) \propto \frac{x^3}{(1+x^2)^{(p+3)/2}}$$

- At high *k*, approx k^{-p} spectrum, p > 3 (?)
- c.f. envelope approximation k^{-1}
(OK for vacuum transition, runaway?)

Outlook

- Larger simulations needed
  - high-k power law(s)
  - Peak structure
  - Fluid Length scale L_f
  - 18M CPU-hours PRACE Tier 0 [see next talk by D Weir]
- Stronger transitions?
  - Shock formation Pen, Turok (2015)
  - Turbulence Kahniashvili et al; Caprini Durrer Servant (2009)
- Implications for eLISA 2034,
  - [Caprini et al, arXiV soon]