Evolution of primordial magnetic fields

Cosmological magnetic fields
Turbulent decay
Nonuniversality of MHD
Weak and strong turbulence
Helical, nonhelical, hydro

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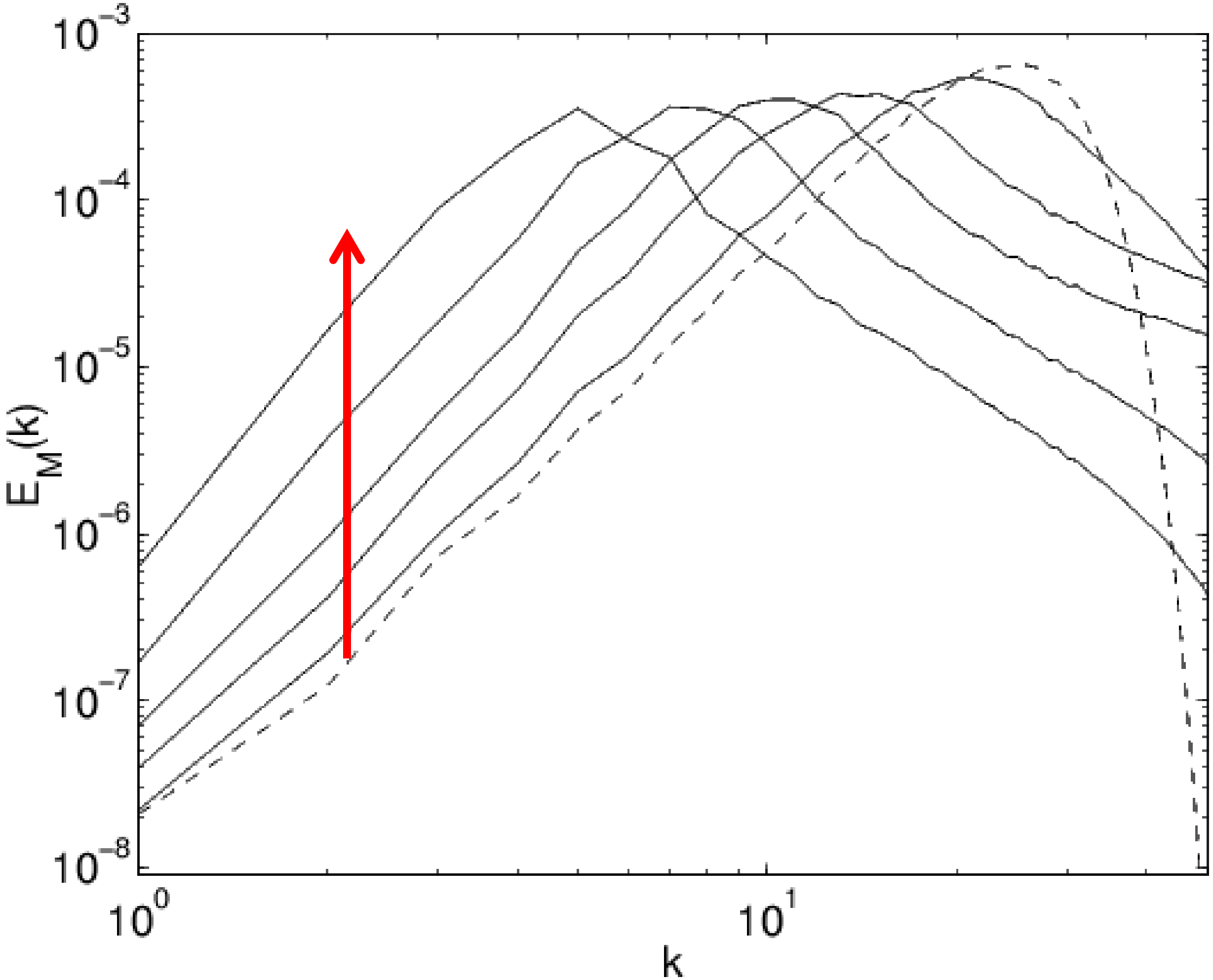
with T. Kahniashvili and A. Tevzadze

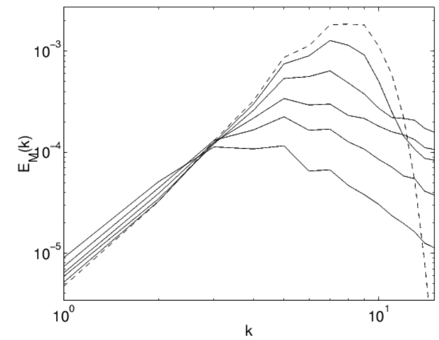
PRL 114, 075001 (2015)

Helical vs nonhelical 3-D decay



Initial slope
$E{\sim}k^4$

helical vs
nonhelical



Christensson et al. (2001, PRE 64, 056405)

Helical decay law:
Biskamp & Müller (1999)

$$H = EL = \mathrm{const}$$

$$\varepsilon = U^3 / L = E^{3/2} / L$$

$$\varepsilon = -\mathrm{d}E / \mathrm{d}t$$

$$\varepsilon = -\mathrm{d}E / \mathrm{d}t = E^{3/2} / L = E^{5/2} / H$$

$$E \propto t^{-2/3}$$

$$L \propto t^{+2/3}$$

Nonhelical Inverse Transfer of a Decaying Turbulent Magnetic Field

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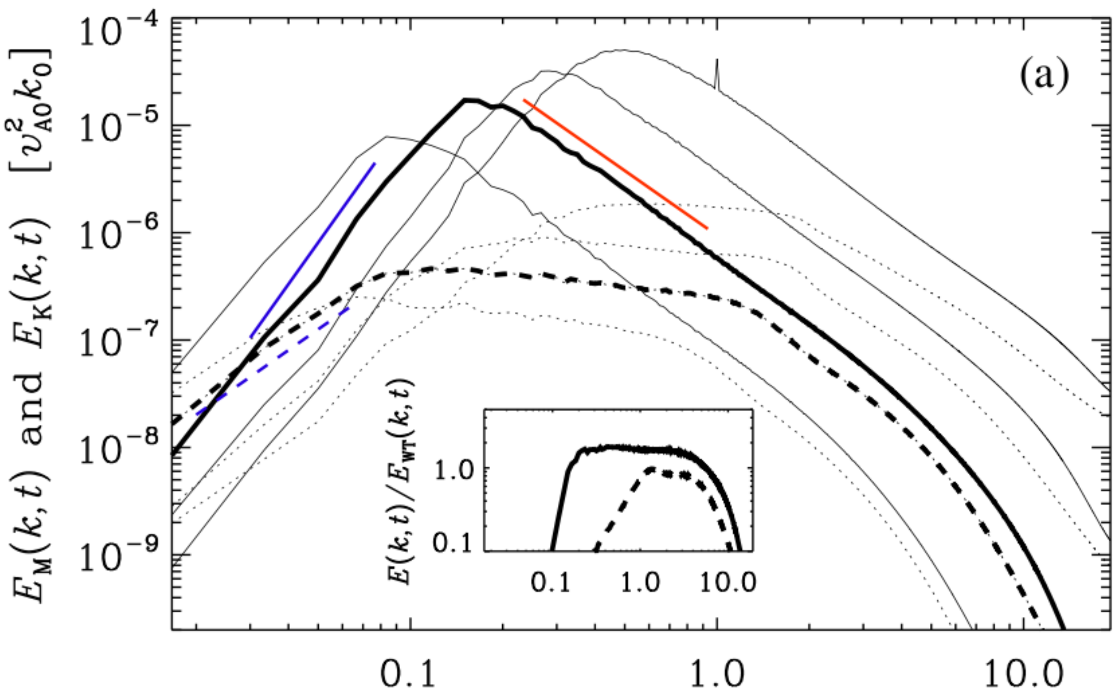
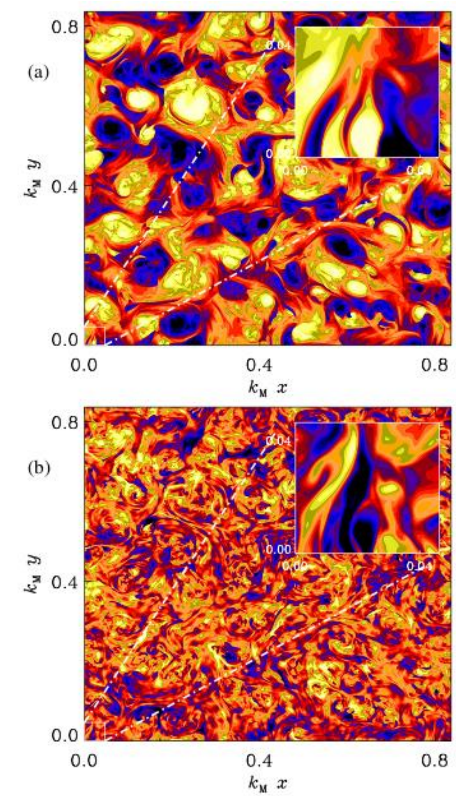
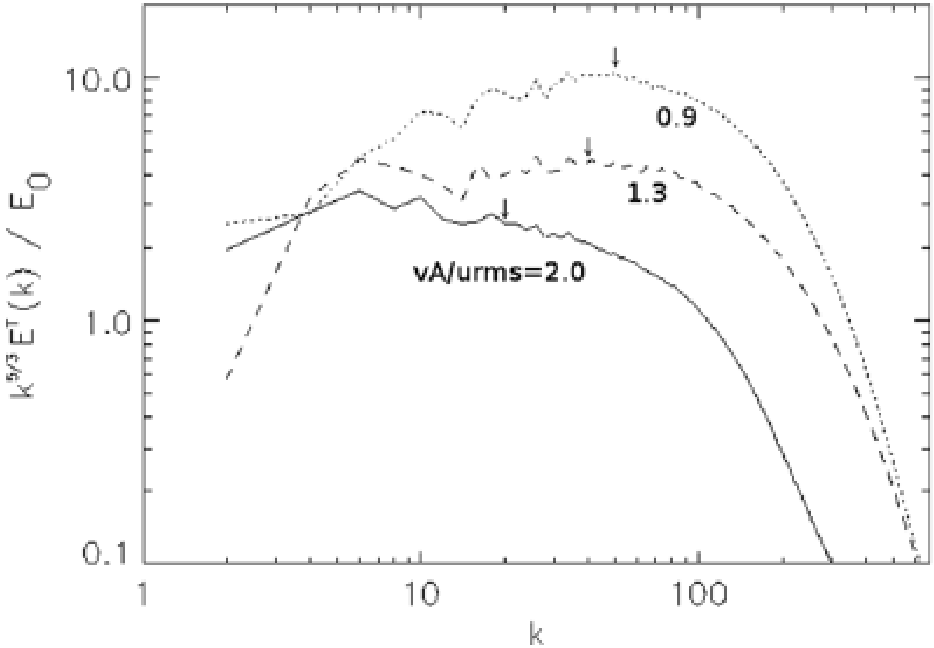


$$E_{\rm WT}(k,t) = C_{\rm WT}(\epsilon v_A k_M)^{1/2} k^{-2}$$



Weak MHD turbulence, because B strong

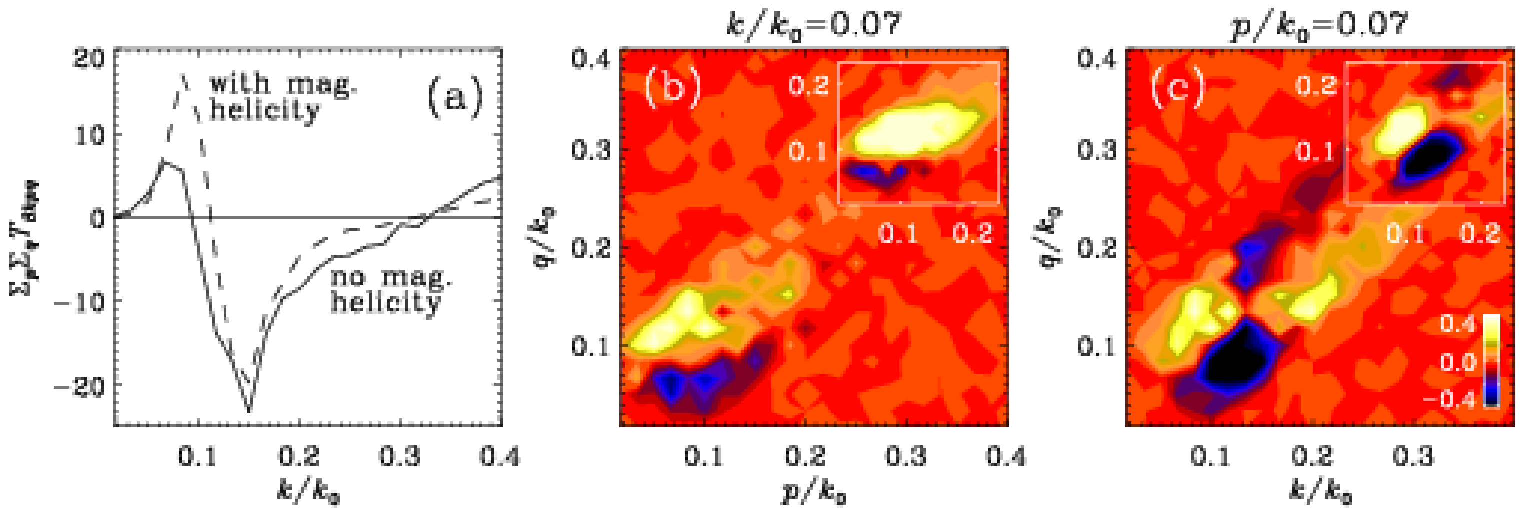


Lee, Brachet, Pouquet, Mininni, Rosenberg (2010)

	Iroshnikov–Kraichnan (isotropic, sub-equip.)	Strong turbulence (critically balanced)	Weak turbulence (wave turbulence)
$v_A/u_{rms} \sim \chi^{-1}$	<1	~ 1	>1
τ_{casc}	$\chi^{-2}\tau_A$ (with $k_\perp = k_\parallel$)	$\chi^{-1}\tau_A (= \tau_{NL})$	$\chi^{-2}\tau_A$
ϵ	$z_k^4 k / v_A$	$z_{k_\perp}^3 k_\perp$	$z_{k_\perp}^4 k_\perp^2 / v_A k_\parallel$
$k_\perp / k_\parallel$	1	$\propto k_\perp^{1/3}$	$\to \infty$
$E(k_\perp, k_\parallel)$	$(\epsilon v_A)^{1/2} k^{-3/2}$	$\epsilon^{2/3} k_\perp^{-5/3}$	$(\epsilon v_A k_\parallel)^{1/2} k_\perp^{-2}$

Inverse transfer similar to helical MHD

$$T_{kpq} = \left\langle \mathbf{J}^k \cdot \left(\mathbf{u}^p \times \mathbf{B}^q \right) \right\rangle = -\left\langle \mathbf{u}^p \cdot \left(\mathbf{J}^k \times \mathbf{B}^q \right) \right\rangle$$



Nonhelical gain
½ of helical case

Gain from SS ***B***
Mediated by LS ***u***

Kinetic gain
From ***B*** field

On Inverse Cascades and Primordial Magnetic Fields

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Next we want to use the well known self-similarity property of the non-relativistic Navier-Stokes or MHD-equations,

$$\mathbf{x}\to l\mathbf{x},\quad t\to l^{1-h}t,\quad \mathbf{v}\to l^{h}\,\mathbf{v},\quad \nu\to l^{1+h}\nu,\quad \mathbf{B}\to l^{h}\,\mathbf{B},\ \eta\to l^{1+h}\eta, \tag{5}$$

where ν is the kinetic and η is the Ohmic diffusion. Using the substitutions $\mathbf{x}=l\mathbf{x}'$ and $\mathbf{y}=l\mathbf{y}'$, we obtain from eqs. (2) and (5)

$$\begin{aligned} E(k/l,l^{1-h}t,Ll,K/l) &= l^4\frac{2\pi k^2}{(2\pi)^3}\int_{2\pi/K}^{L} d^3x'd^3y'\ e^{i\mathbf{k}(\mathbf{x}'-\mathbf{y}')}\ <\mathbf{v}(l\mathbf{x}',l^{1-h}t)\ \mathbf{v}(l\mathbf{y}',l^{1-h}t)> \\ &= l^{4+2h}\ \tilde{E}(k,t,L,K). \end{aligned} \tag{6}$$

primordial magnetic fields, and for the effect of diffusion. In general, if the initial spectrum is k^{α}, then in the "inertial" range, for $\alpha>-3$ there is an inverse cascade, whereas for $\alpha<-3$ there is a forward cascade.

Does initial spectrum determine decay?

$$E_{\rm K}(k,t) \sim E_{\rm M}(k,t) \sim k^\alpha \psi(k^{(3+\alpha)/2} t). \qquad (4)$$

Integrating over k yields the decay law of the energies as

$$\mathcal{E}_{\rm K}(t) = \int E_{\rm K}(k,t)\,dk \sim \int k^\alpha \psi(k^{(3+\alpha)/2} t)\,dk. \qquad (5)$$

Introducing $\kappa = kt^q$ with $q = 2/(3+\alpha)$, we have

$$\mathcal{E}_{\rm K}(t) \sim t^p \int \kappa^\alpha \psi(\kappa)\,d\kappa, \qquad (6)$$

where $p = (1+\alpha)q$. The integral scales like $k_K \sim t^q$ with $q = 2/(3+\alpha)$. Several parameter combinations are given in Table II.

$$q = 1 - p/2$$

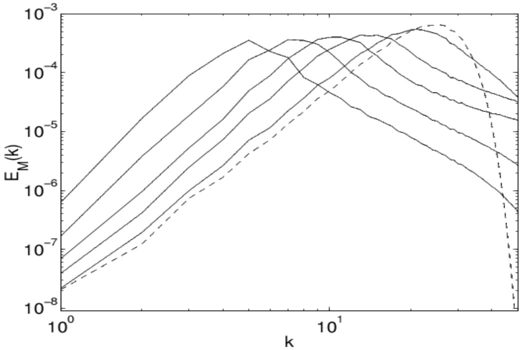
α	p	q
4	10/7	2/7
3	8/6	2/6
2	6/5	2/5
1	4/4	2/4
0	2/3	2/3

Rescaled spectra: self-similar

Alternative interpretation of Olesen's scaling relation

Christensson et al.
(2001, PRE 64, 056405)

Initial slope
$E \sim k^4$



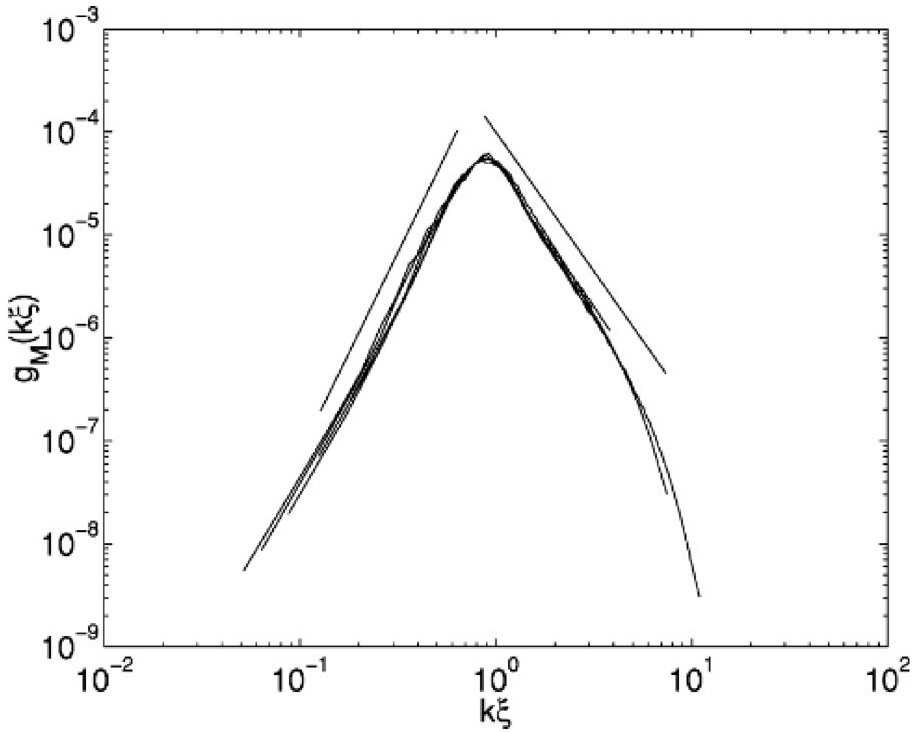


FIG. 2. The magnetic scaling function $g_M(k\xi)$ described in the text, Eq. (13), versus $k\xi$. The straight lines indicate the power laws $\propto (k\xi)^{4.0}$ and $\propto (k\xi)^{-2.5}$, respectively.

$$E_M(k,t) = \xi(t)^{-q} g_M(k\xi). \tag{13}$$

Revised interpretation

$$E(k,t) = \xi^{-\beta}\phi(k\xi)$$

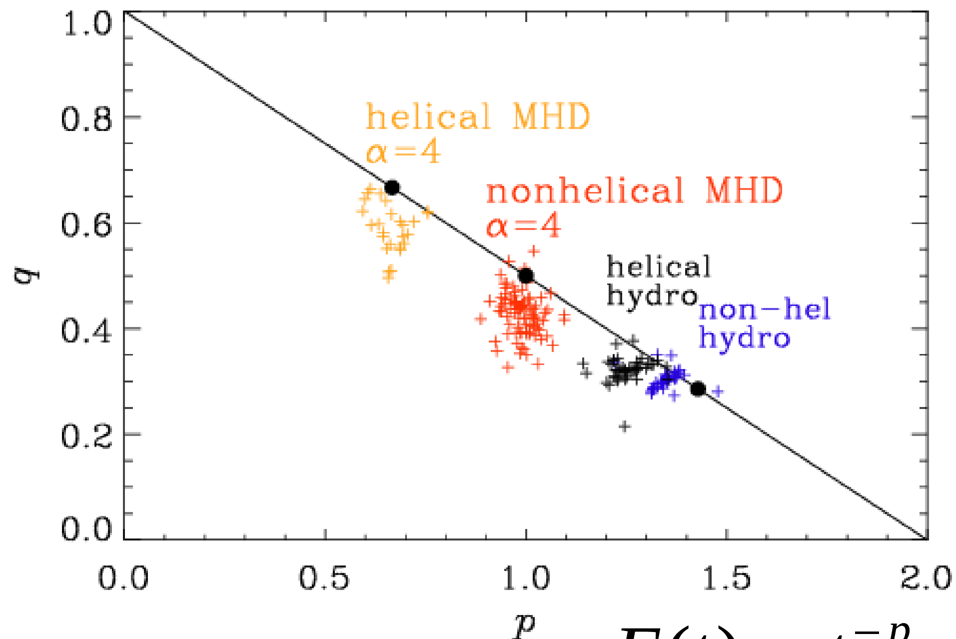
with integral scale ξ

$$\xi = \xi(t) \propto t^{q}$$

and q determined by physics

$$\beta = -3 + 2/q$$

from dimensional arguments



$$E(t) \propto t^{-p}$$

β	p	q	physics
4	10/7	2/7	$\int u^2 r^4 \,\mathrm{d}r = \mathcal{L} = \text{const} \sim \ell^7 \tau^{-2}$ (Loitsiankii)
3	8/6	2/6	
2	6/5	2/5	$\int u^2 r \,\mathrm{d}r = \mathcal{C} = \text{const} \sim \ell^5 \tau^{-2}$ (Saffman)
1	4/4	2/4	$\langle \boldsymbol{A}_{2\mathrm{D}}^2 \rangle = \text{const} \sim \ell^4 \tau^{-2}$ (to be confirmed)
0	2/3	2/3	$\langle \boldsymbol{A} \cdot \boldsymbol{B} \rangle = \text{const} \sim \ell^3 \tau^{-2}$ (Biskamp & Müller)

Scaling relations

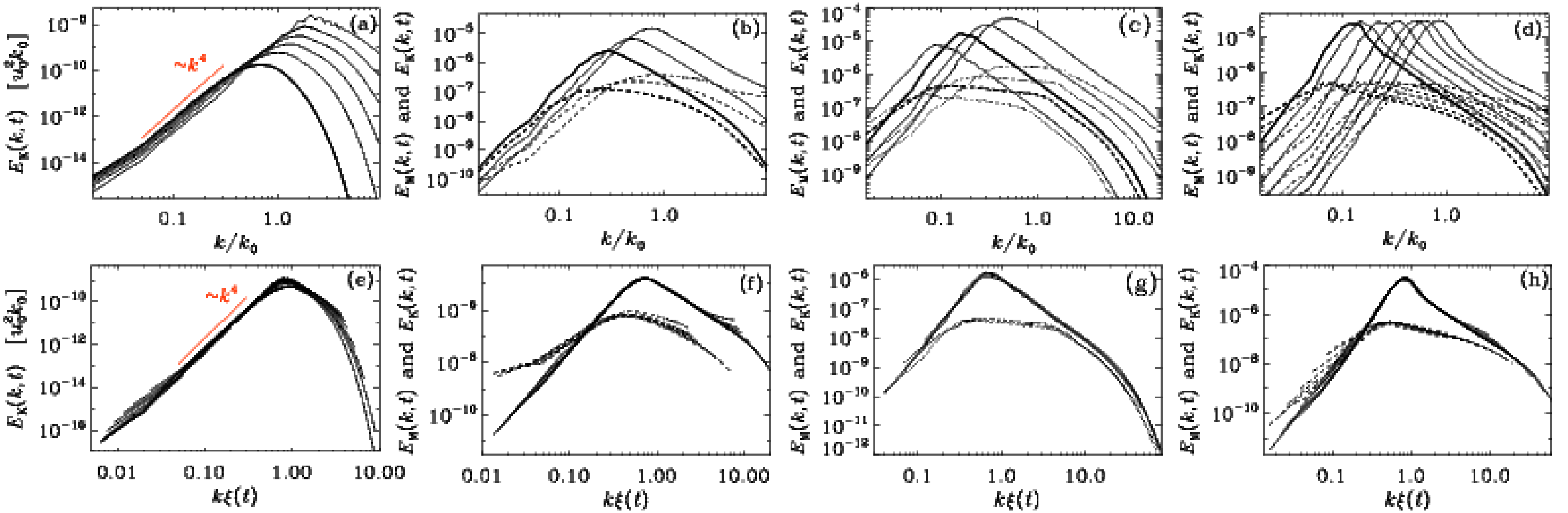
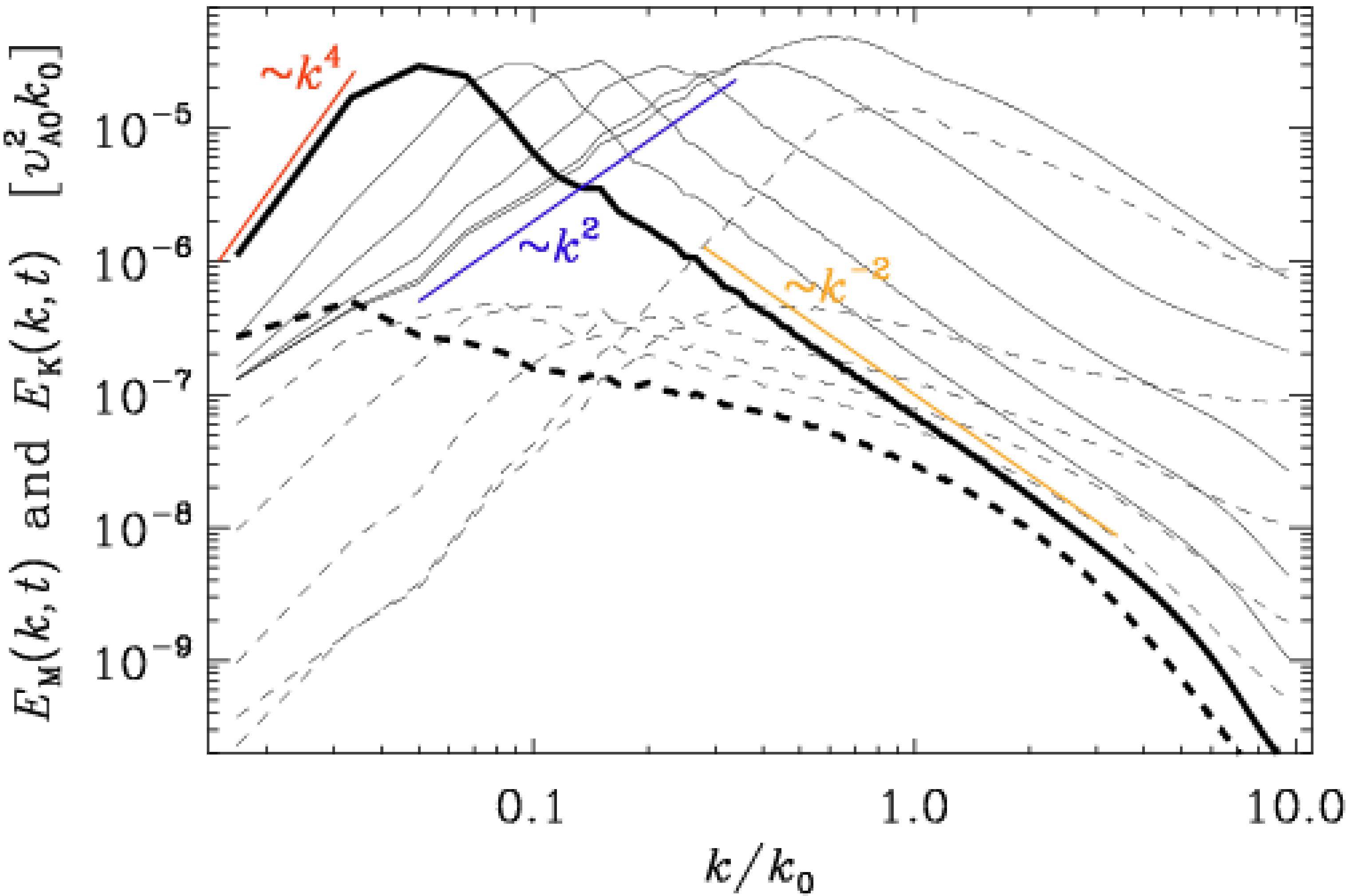


FIG. 1: Kinetic energy spectra in a hydrodynamic simulation (a), compared with magnetic (solid) and kinetic (dashed) energy spectra in a hydromagnetic simulation without helicity (b) and (c), and with (d). Panels (e)–(h) show the corresponding collapsed spectra obtained by using $\beta_{\rm M}=3$ (e), $\beta_{\rm M}=2$ (f), $\beta=1$ (g), and $\beta=0$ (h). In (f) we used $\beta_{\rm K}=1\neq\beta_{\rm M}$.

Prevalance of k^4 spectrum



Helical decay: collapsed

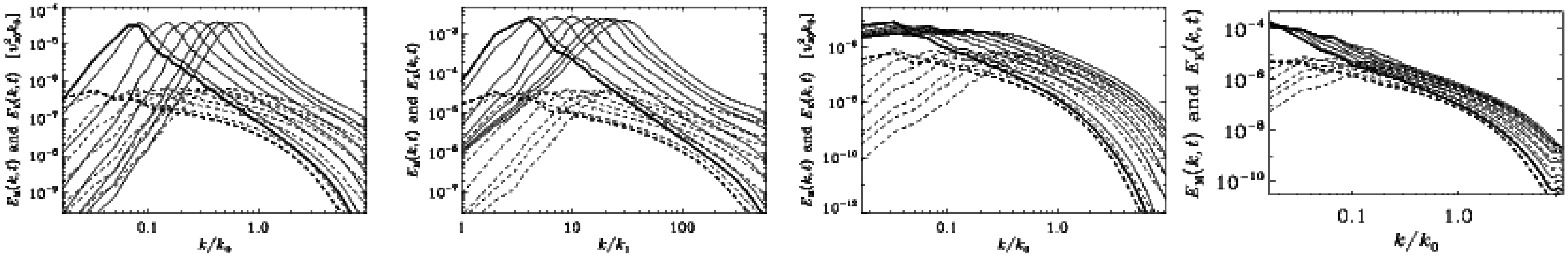


Figure 1: Magnetic and kinetic energy spectra for initial spectra with $E_{\rm M} \sim k^{\alpha}$, with $\alpha = 4$ (left), 2, 0, and -1, using 1152^3 meshpoints, $\sigma = 1$, and $\nu = \eta = 1 \times 10^{-5}$, 5×10^{-6}, 2×10^{-5}, and again 2×10^{-5}.

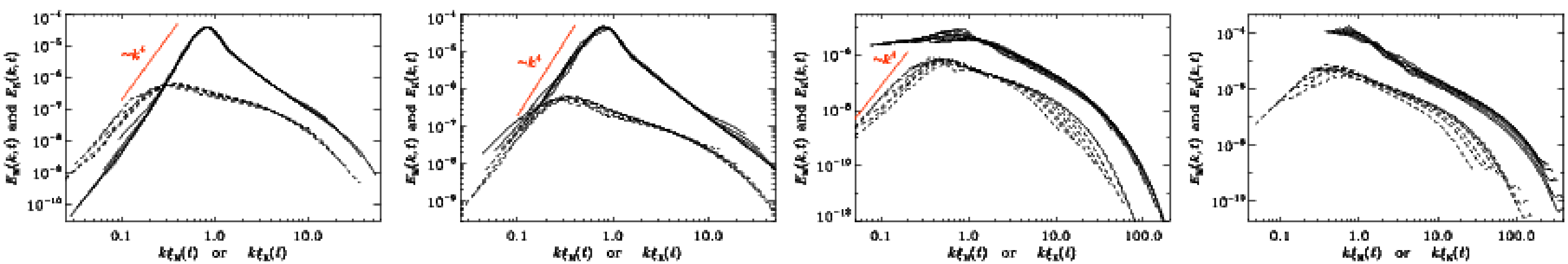
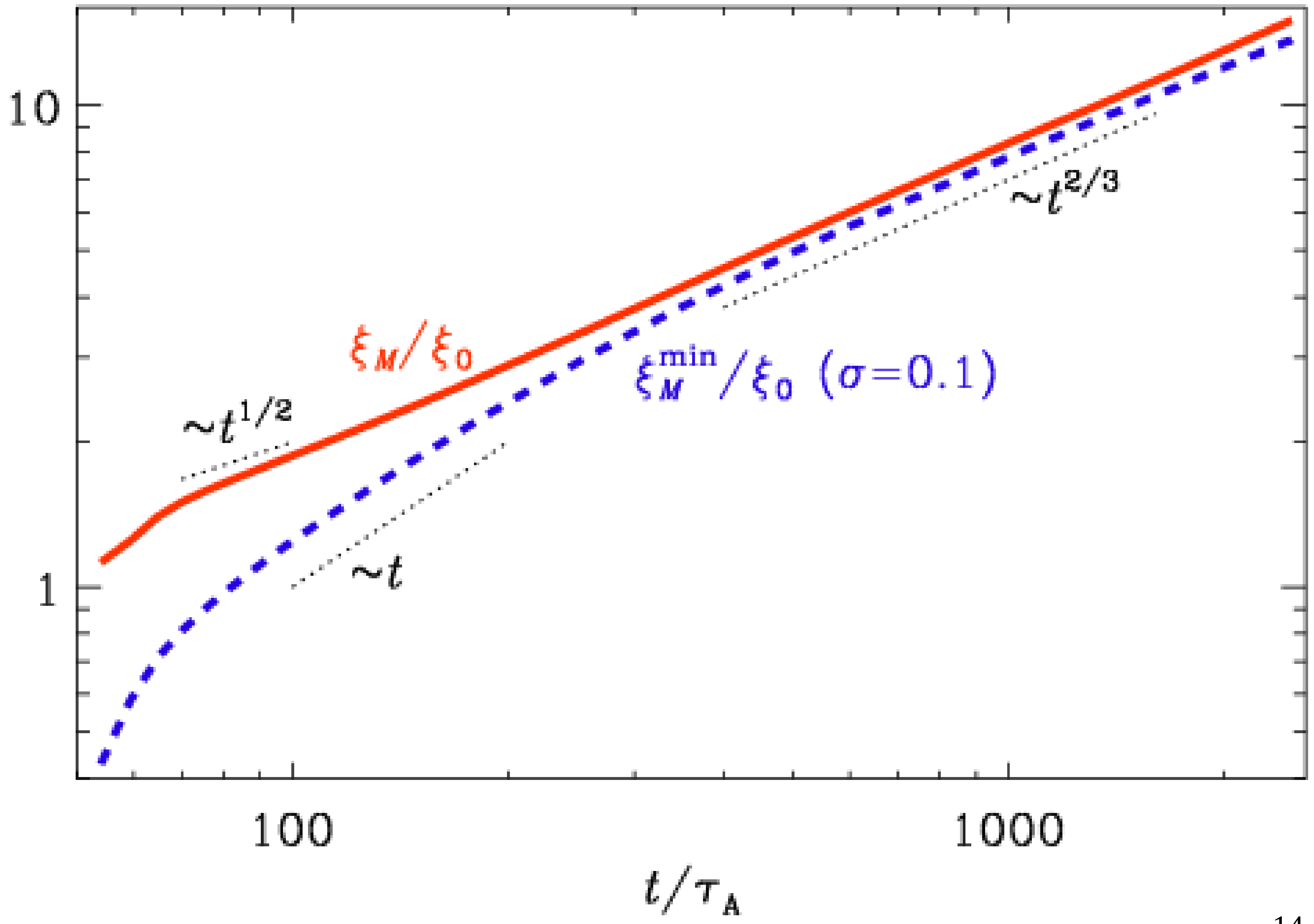


Figure 2: Magnetic and kinetic energy spectra for initial spectra with $E_{\rm M} \sim k^{\alpha}$, with $\alpha = 4$ (left), 2, 0, and -1, using 1152^3 meshpoints, $\sigma = 1$, and 1152^3 meshpoints, $\sigma = 1$, collapsed with $\beta = 0$. $\nu = \eta = 1 \times 10^{-5}$, 5×10^{-6}, 2×10^{-5}, and again 2×10^{-5}.

Scaling relations

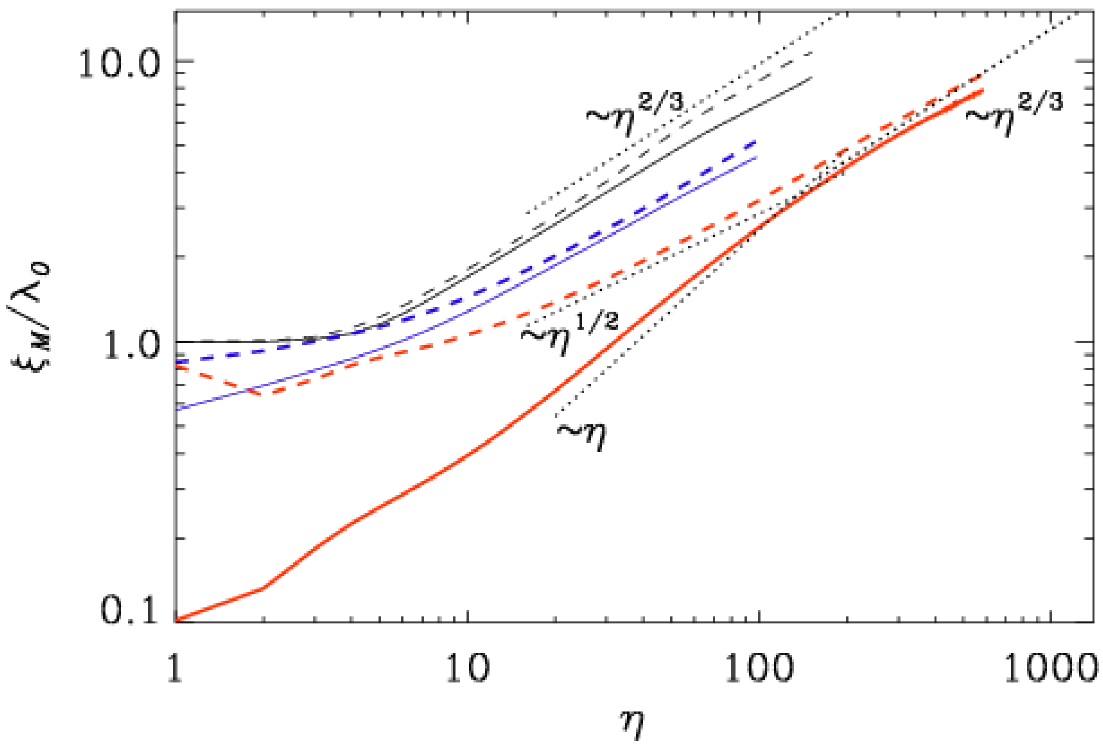


Fractional initial helicity

$$k\,H(k) \leq 2E(k)$$

$$\int H(k)\mathrm{d}k \leq 2\int k^{-1}E(k)\mathrm{d}k$$

$$\xi_M \equiv \frac{\int k^{-1}E(k)\mathrm{d}k}{\int E(k)\mathrm{d}k} \geq \frac{\int H(k)\mathrm{d}k}{2\int E(k)\mathrm{d}k} \equiv \xi_M^{\min}$$



Tevzadze, Kisslinger, Brandenburg, Kahniashvili (2012, ApJ, in press)

Conclusions

Vetenskapsrådet

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- Helicity slows down decay
- Large scale energy increases
- Nonhelical inverse transfer, <A^2>?
- Revised interpretation to Olesen
- Self-similar spectra
- β determined by physics,
- not initial conditions