MHD Turbulence and particles

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Basic properties of MHD turbulence

$$\partial_t \mathbf{w}^{\pm} + \hat{S}(\mathbf{w}^{\mp} \cdot \nabla)\mathbf{w}^{\pm} = 0$$

Elsasser variables: $\mathbf{w}^{\pm} = \mathbf{v} \pm \mathbf{B}/\sqrt{4\pi\rho}$ *Solenoidal projection:* $\hat{S}$

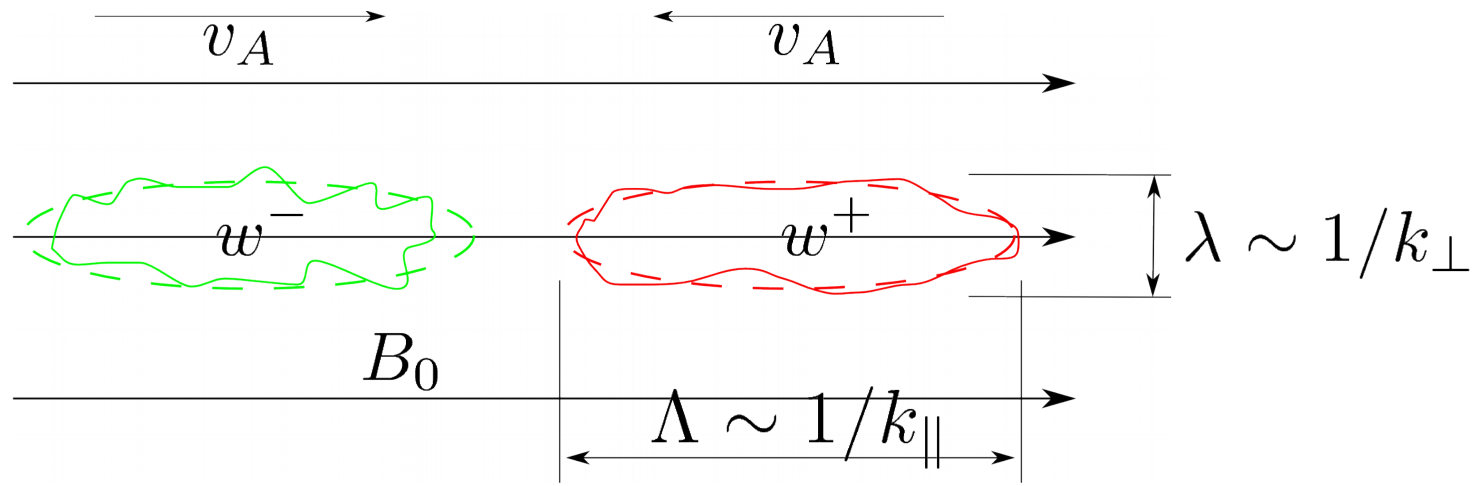
Dynamics is different from hydro, because there is a mean field.

$$\partial_t \delta\mathbf{w}^{\pm} \mp (\mathbf{v_A} \cdot \nabla)\delta\mathbf{w}^{\pm} + \hat{S}(\delta\mathbf{w}^{\mp} \cdot \nabla)\delta\mathbf{w}^{\pm} = 0$$

Mean field (Kraichnan 1965, Iroshnikov 1963)

If universality exists, it is different from hydro.

Basic properties of MHD turbulence

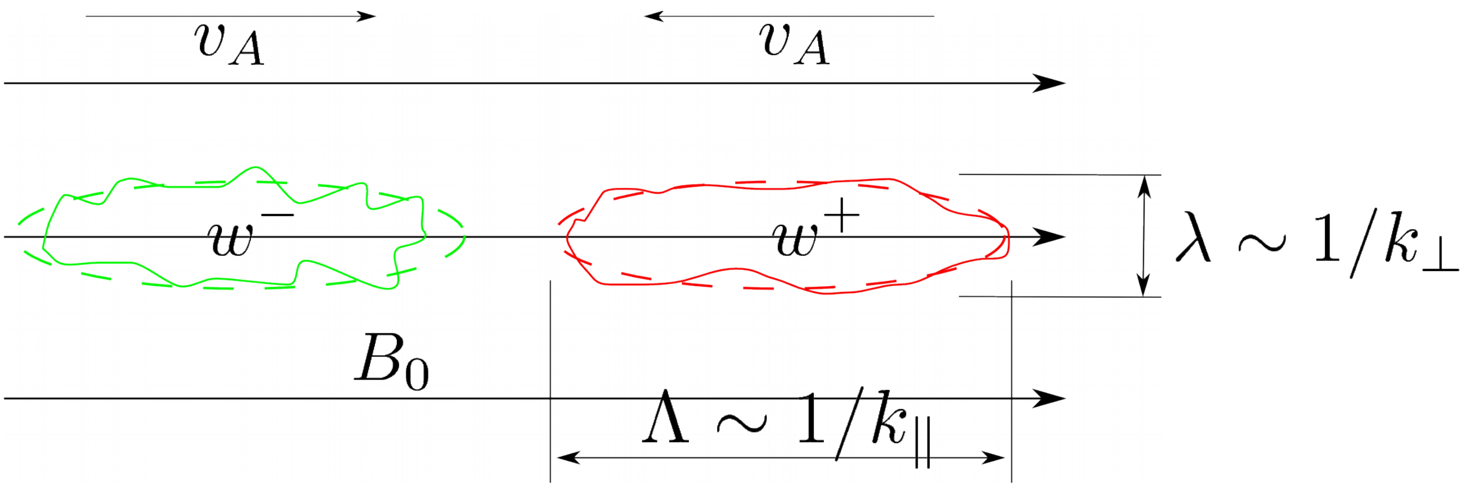


$$\mathbf{k}_{\perp\mathbf{1}} + \mathbf{k}_{\perp\mathbf{2}} = \mathbf{k}_{\perp\mathbf{3}}$$

$$\omega_1 = -\omega_2;\ \omega_3 = 0$$

$k_{||}$ is conserved, $k_\perp$ is increasing

Goldreich-Sridhar (1995) model:



Contribution of nonlinear term has a tendency to increase, thus leading to "strong turbulence", despite a strong mean field. It was then argued that turbulence can not be "too strong", because of the uncertainty relation between cascade time and the wave frequency $\omega\tau_{\rm cas} \sim 1$. This has been criticized as arbitrary, but confirmed by measurements.

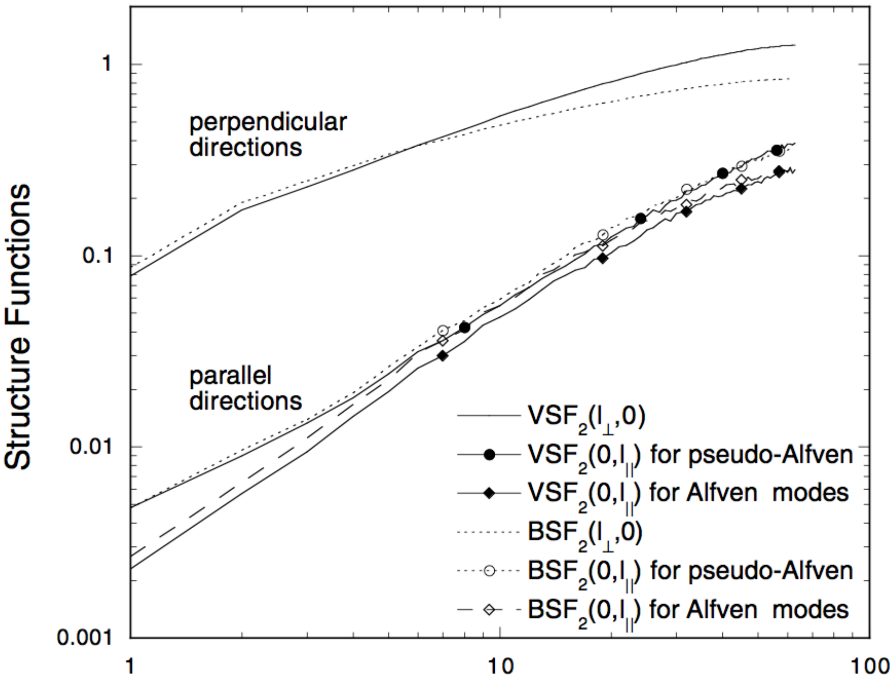
Strong cascade, $k^{-5/3}$ spectra in perpendicular direction and critical balance

$$v_A k_{||}/\delta w k_\perp \sim 1$$

leads to a k^{-2} spectrum in parallel direction

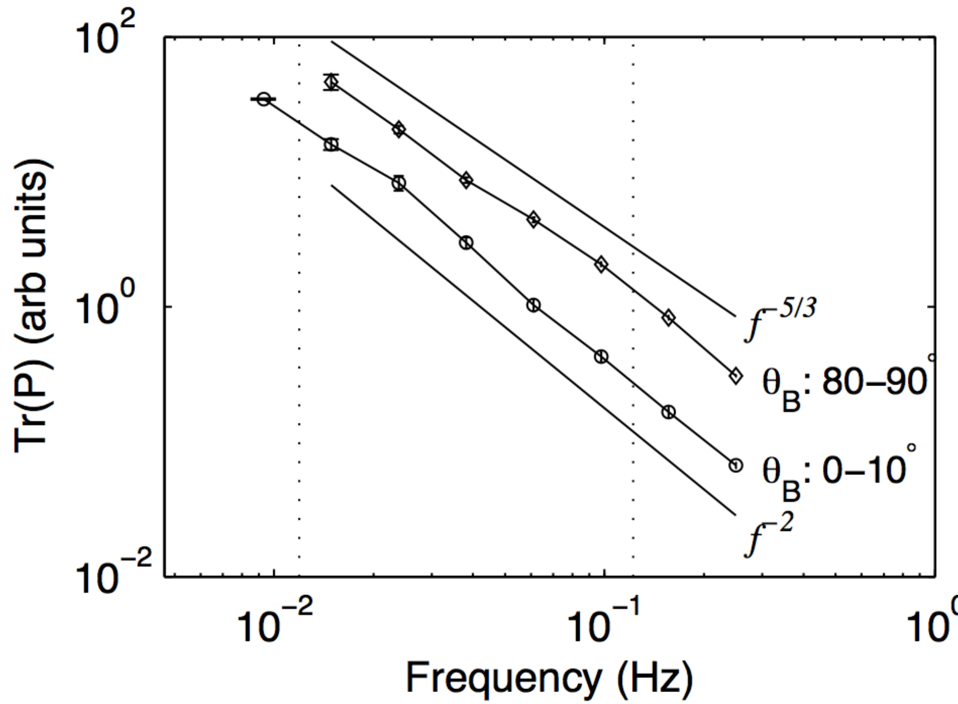
Parallel spectrum in measurements

simulations



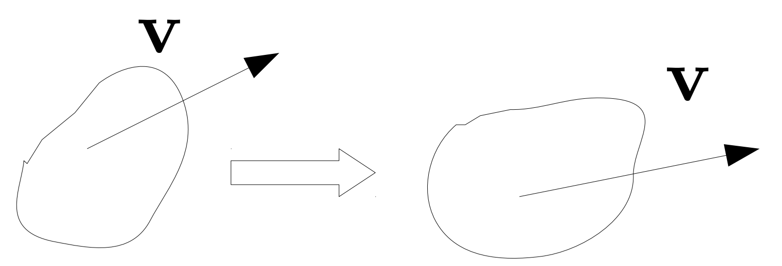
Cho et al 2001

Solar wind



Horbury et al 2008

Lagrangian frequency spectrum



$$\delta \mathbf{v}_\tau \cdot \delta \mathbf{v}_\tau / \tau = \epsilon$$

$$SF(\tau) = \langle (\mathbf{v}(t+\tau) - \mathbf{v}(t))^2 \rangle = \epsilon \tau$$

$$E(\omega) = \epsilon \omega^{-2}$$ (Landau 1941)

Cutoff: $\tau_\eta = (\nu/\epsilon)^{1/2}$ (Kolmogorov timescale)

Parallel spectrum

In MHD turbulence Alfven waves propagate **exactly** along magnetic field lines. By following magnetic field line *in space* we follow evolution of the wave *in time,* so that $t \sim s/v_A$

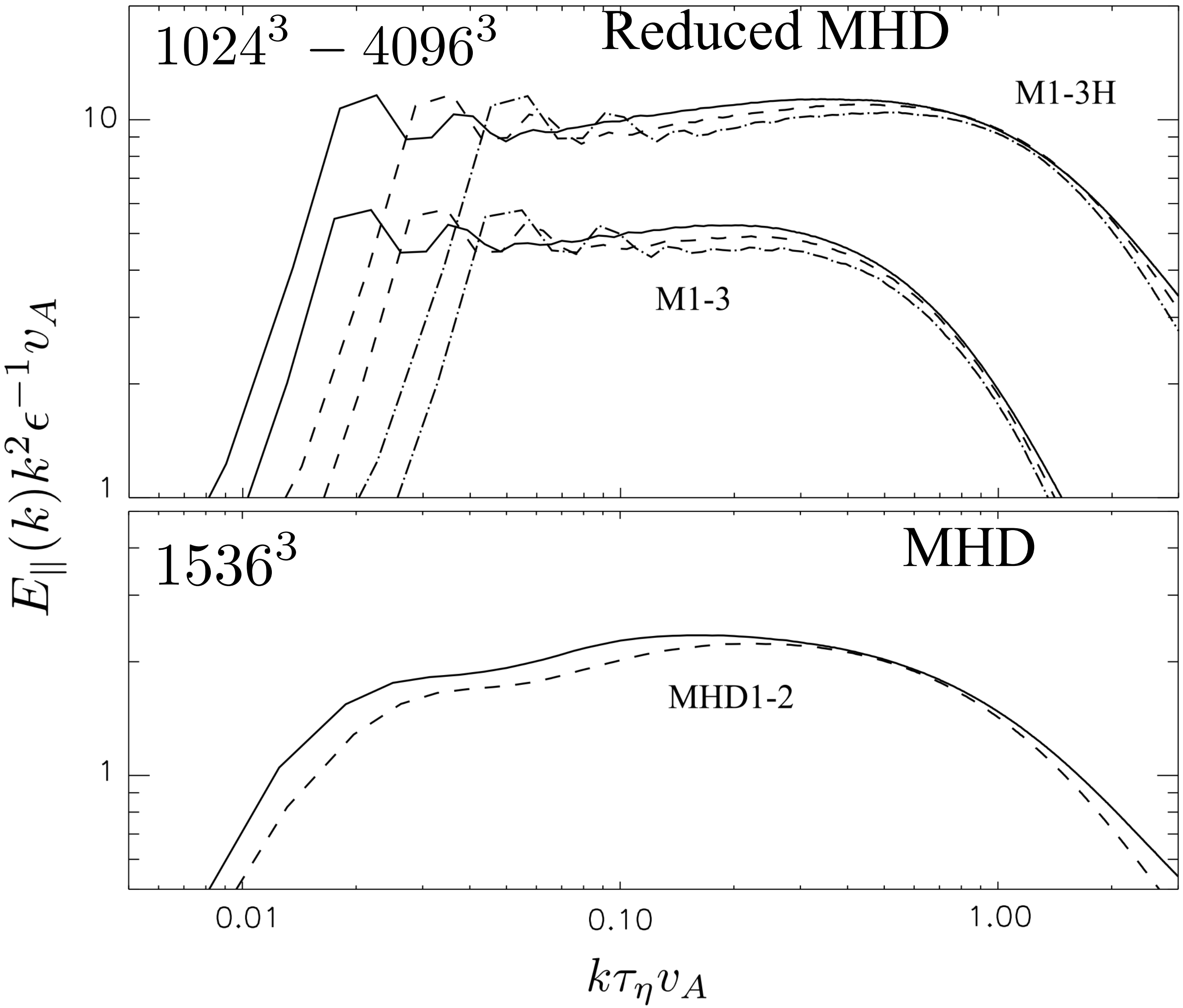
Therefore, perturbation spectrum along the field line is simply the Lagrangian frequency spectrum! This also could be obtained with units and using Alfven symmetry of reduced MHD.

$$E(k_{||})dk_{||} = const \text{ under } k_{||}v_A = const$$

The only dimensionally correct expression:

$$E(k_{||}) = C_{||}\epsilon v_A^{-1} k_{||}^{-2}$$

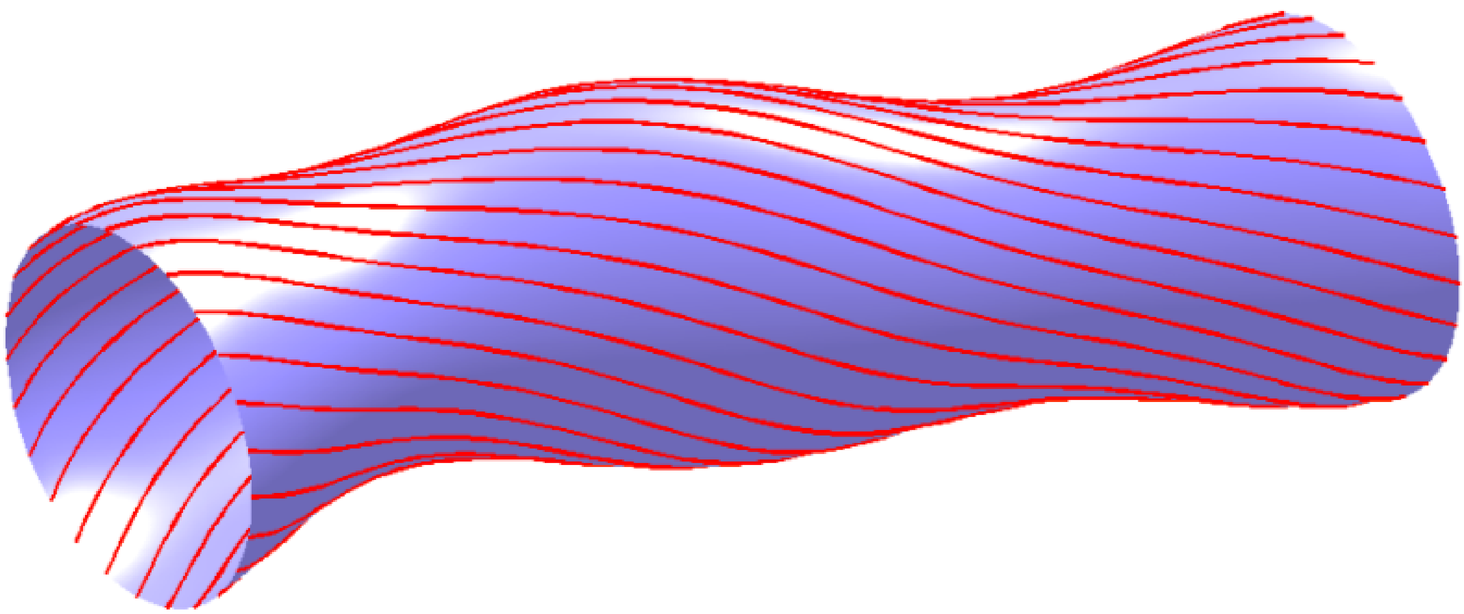
Parallel spectra in simulations



$$E(k_\parallel) = C_\parallel \epsilon v_A^{-1} k_\parallel^{-2}$$

Magnetic flux surfaces (“flux ropes”, etc)

A concept born out of flux conservation $\nabla \cdot \mathbf{B} = 0$. These were expected to magnetically isolate plasmas in tokamaks, stellarators and other plasma devices.



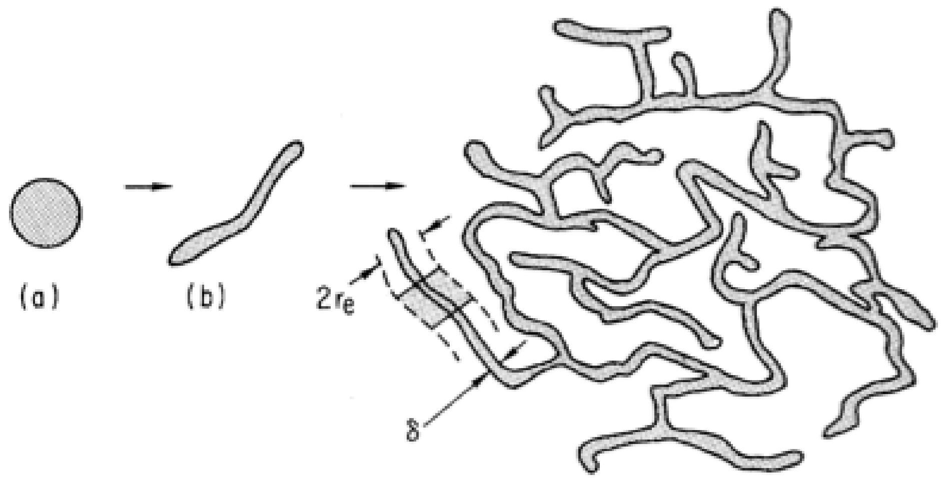
Theory of “classic transport”: relies on collisions, predicts that magnetic isolation is extremely efficient.

Practice: transport is much larger than predicted.

“Destroyed magnetic surfaces”

Rechester & Rosenbluth, 1978

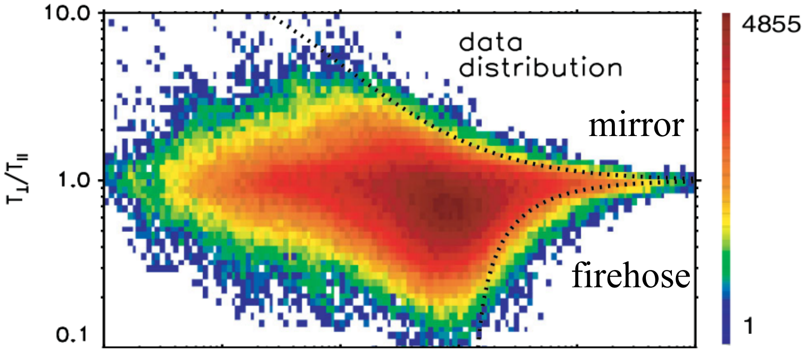
0) Analogy with classical mechanics: how conservative systems increase their entropy?



1) separation of field lines is <u>not restricted</u> by flux conservation.
2) this separation causes mixing that <u>does not depend on collisions or gyro-radius</u> so it is very important in highly magnetized cases

Some open questions in solar wind plasmas

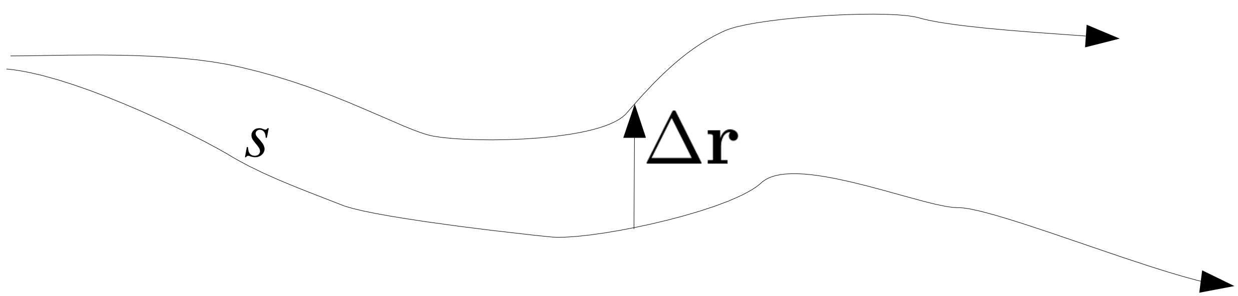
Temperature anisotropy in the solar wind at 1 AU:
Bale et al (2009)



Maybe this is due to the solar wind expansion, but the expansion timescale is much larger than the Larmor period. There must be some very rapid local processes that make plasma non-thermal.

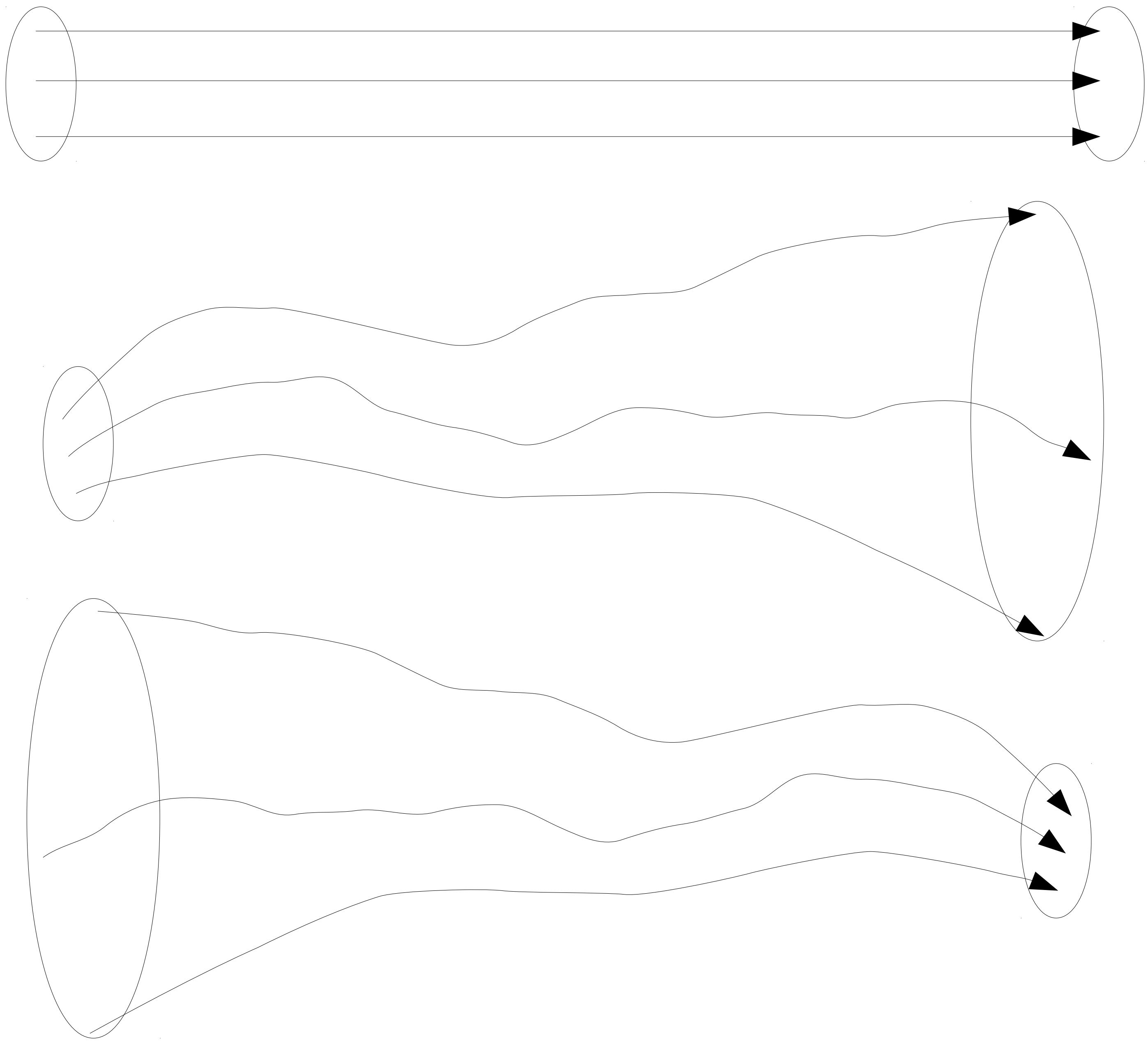
How magnetic field lines separate from each other?

We will study statistically of volumetrically averaged separation of field lines and call this “diffusion”.

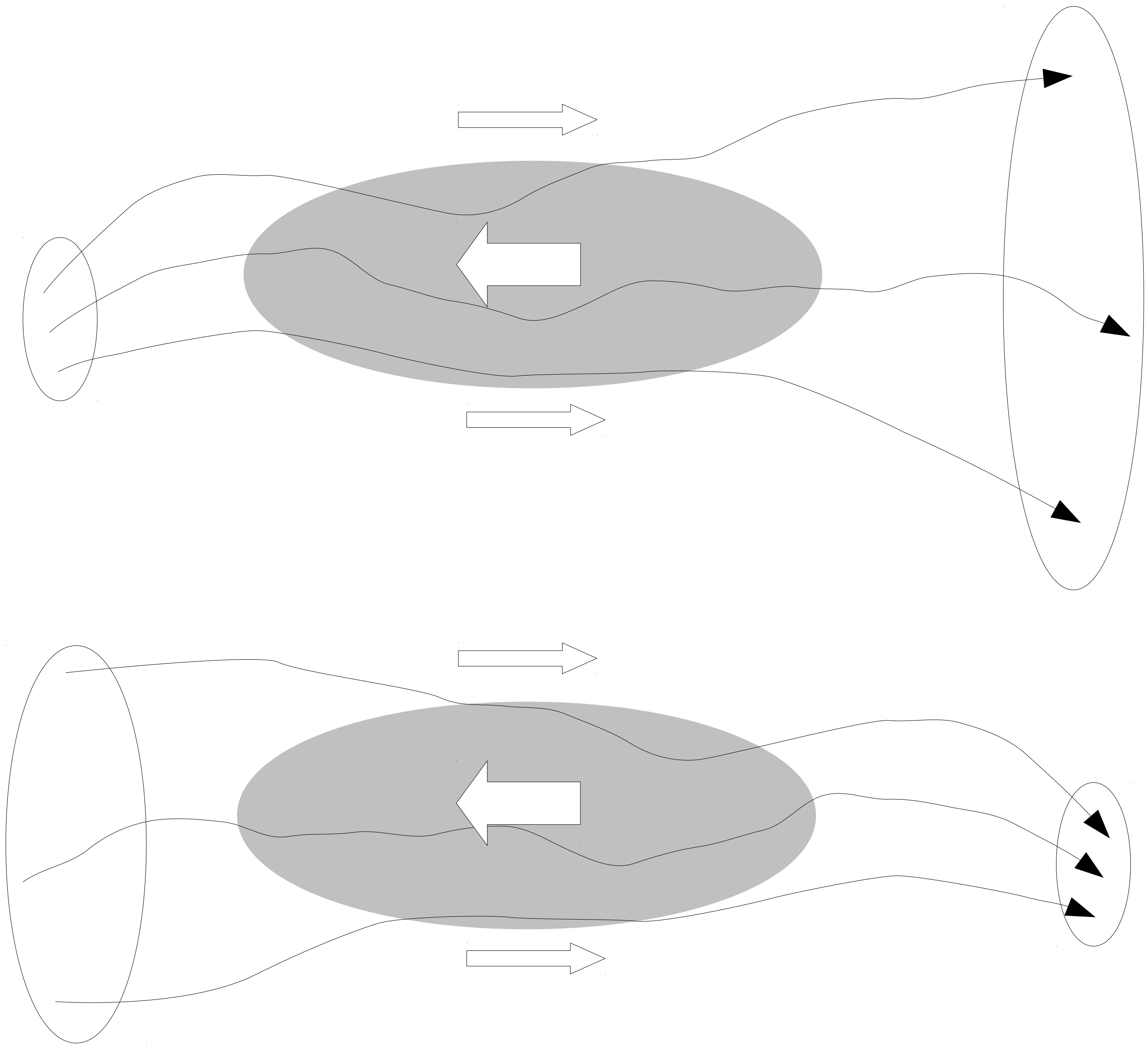


$$\langle (\Delta \mathbf{r})^2 \rangle = f(s)$$

Diffusion along the **B** direction and backward



Why would you expect diffusion to be symmetric?



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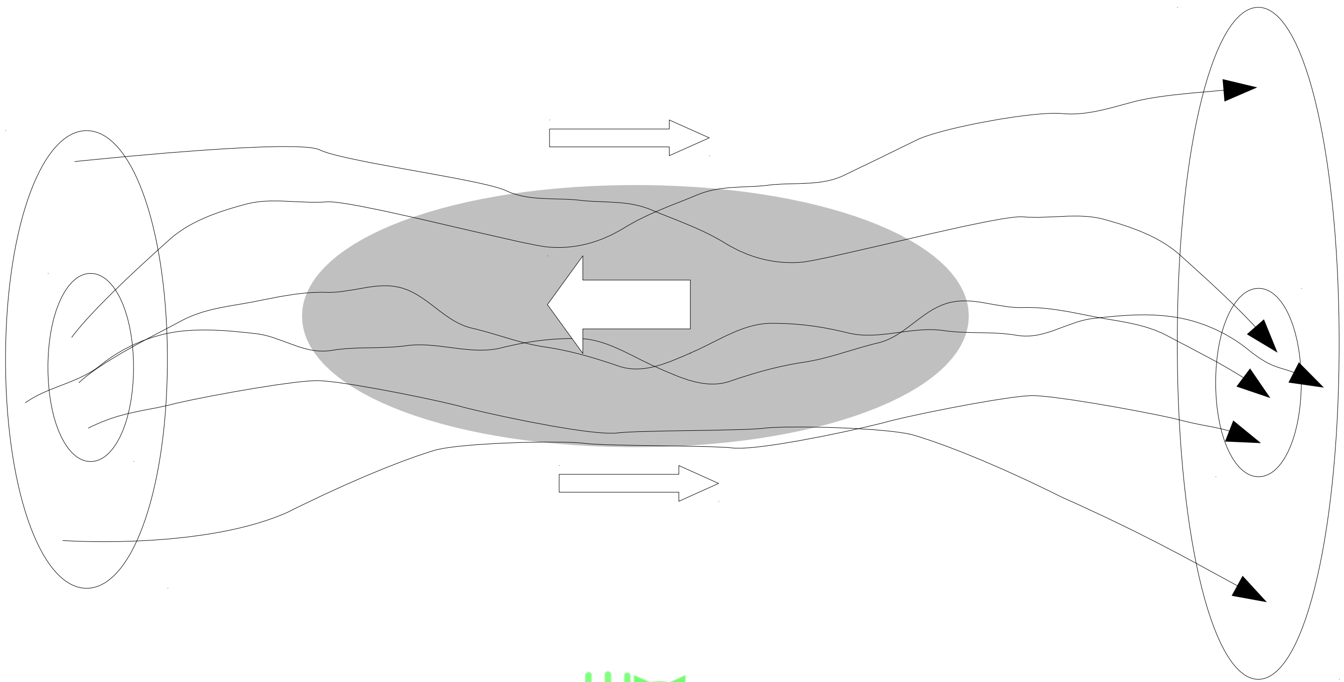
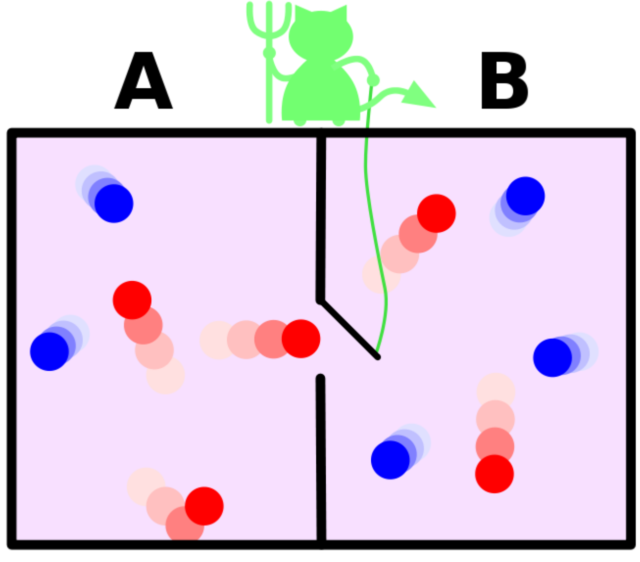




Case 3: inertial range separations

$$d\Delta\mathbf{r}/ds = \mathbf{B}_r/B_0 \qquad B_r = Cr^{1/3}$$

Let's imagine it's Markovian process

$$\langle \Delta r^2 \rangle = \frac{8}{27} C^3 \frac{|s|^3}{B_0^3}$$

Diffusion of particles in turbulence

Richardson's diffusion (1926)

Two-particle separation

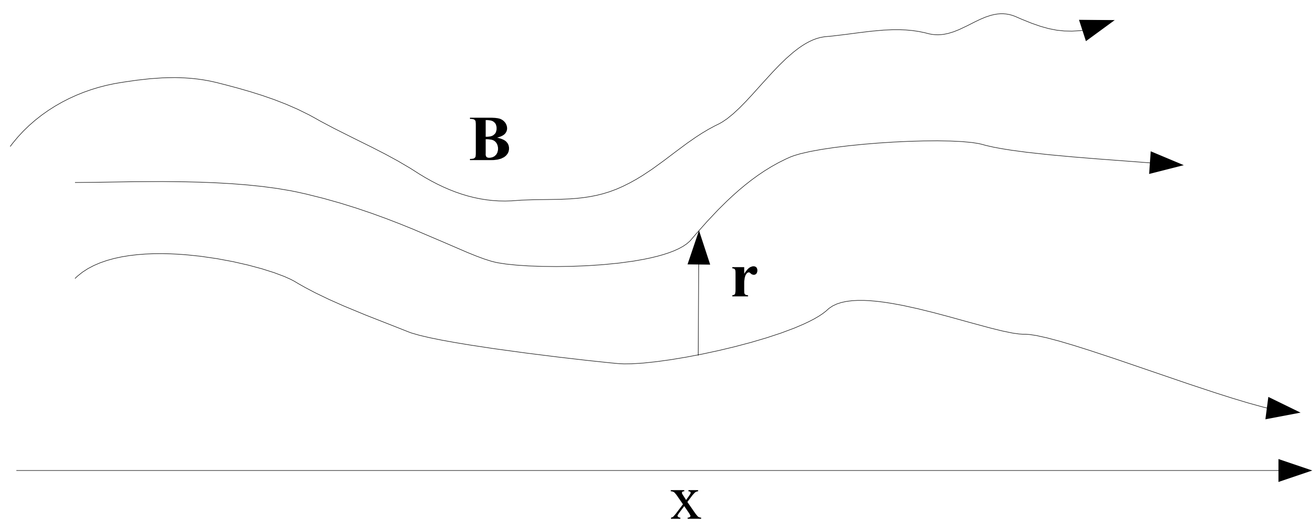
$$D \sim l^{4/3}$$

$$\langle (\Delta \mathbf{r})^2 \rangle = g_0 \epsilon t^3$$

ϵ – dissipation rate per unit mass – cm^2/s^3

g_0 – dimensionless number, “Richardson constant”

“Diffusion” of magnetic field lines

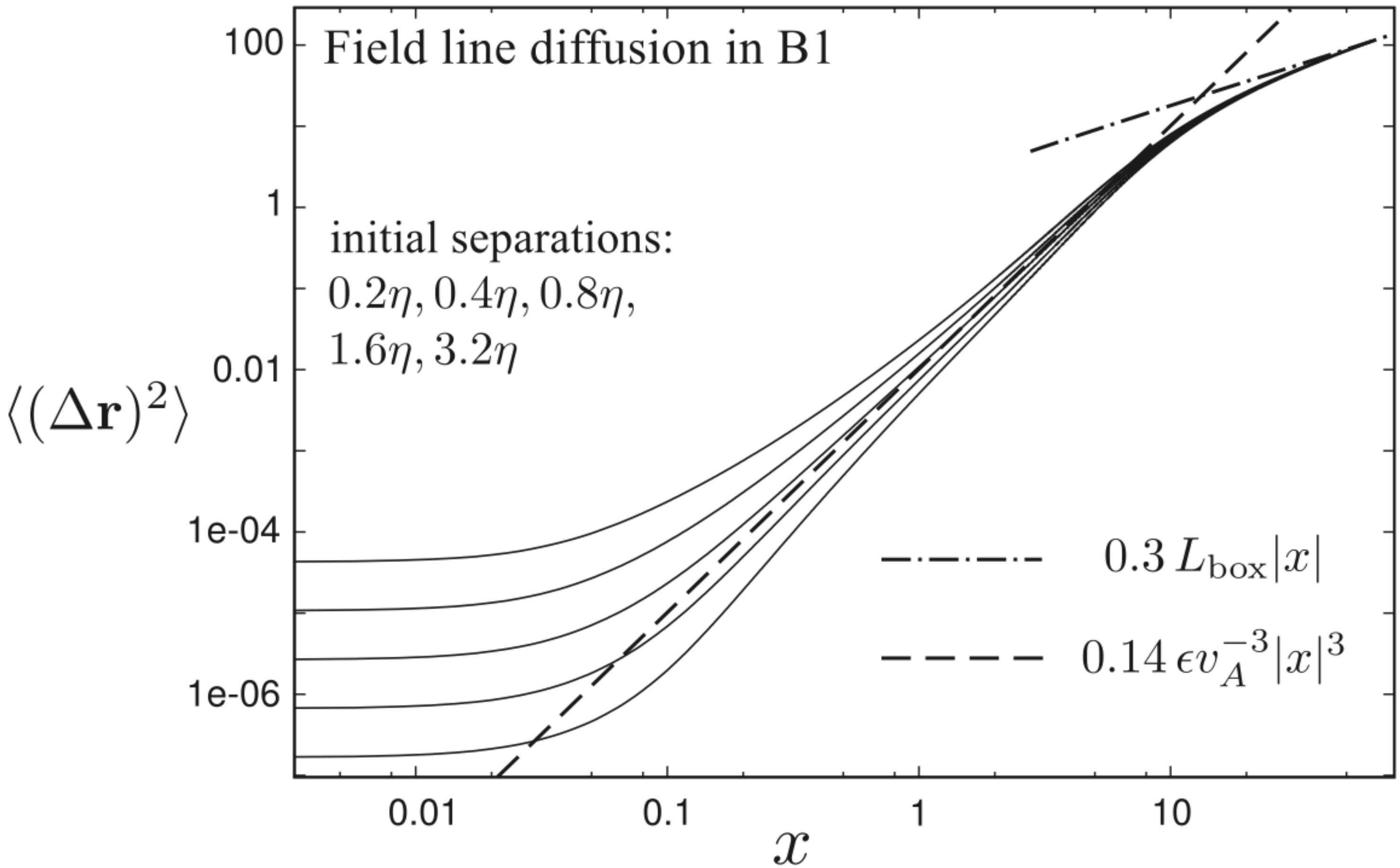


$$\langle (\Delta \mathbf{r})^2 \rangle = g_m \epsilon v_A^{-3} |x|^3$$

“Richardson-Alfven” diffusion

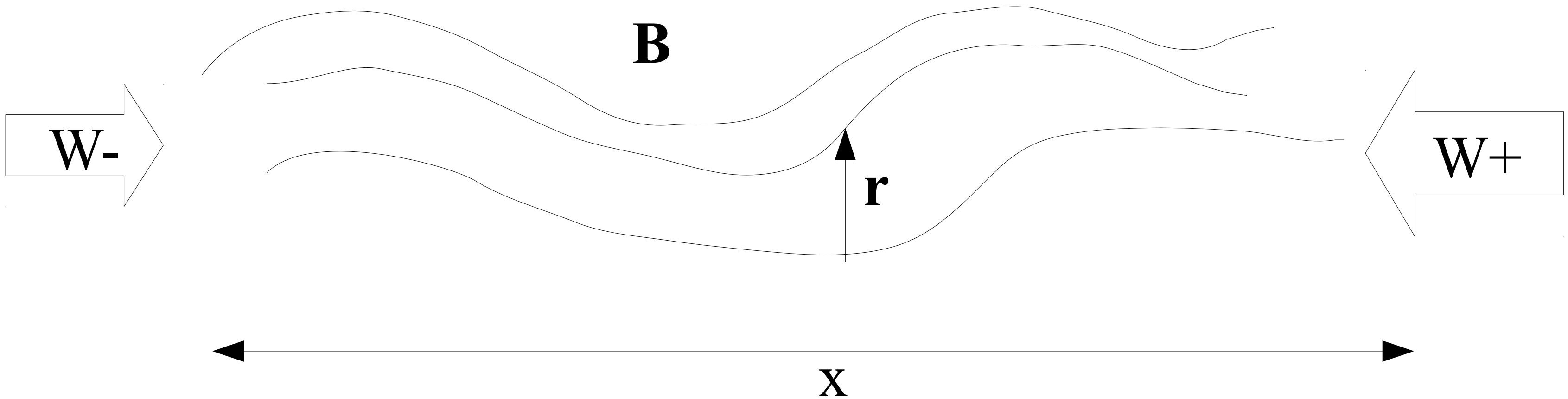
(note: must satisfy **exact** Alfven symmetry, i.e. *x* only enters as x/v_A)

Diffusion in simulated turbulence

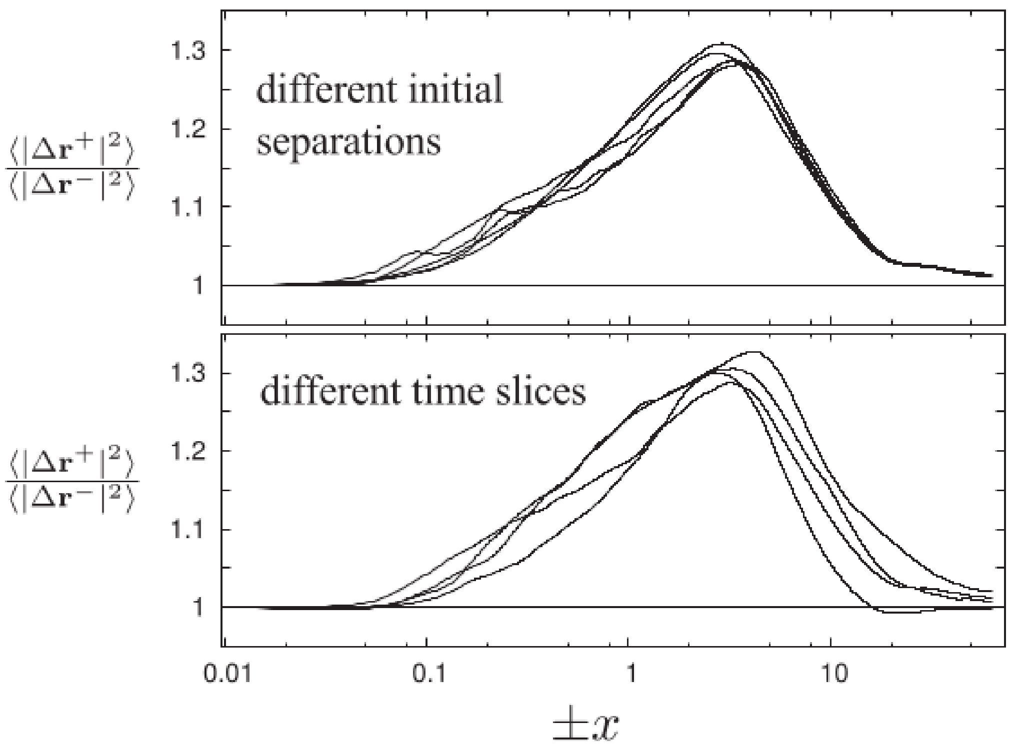


$$\langle(\Delta\mathbf{r})^2\rangle = g_m \epsilon v_A^{-3}|x|^3$$

Diffusion of magnetic field lines



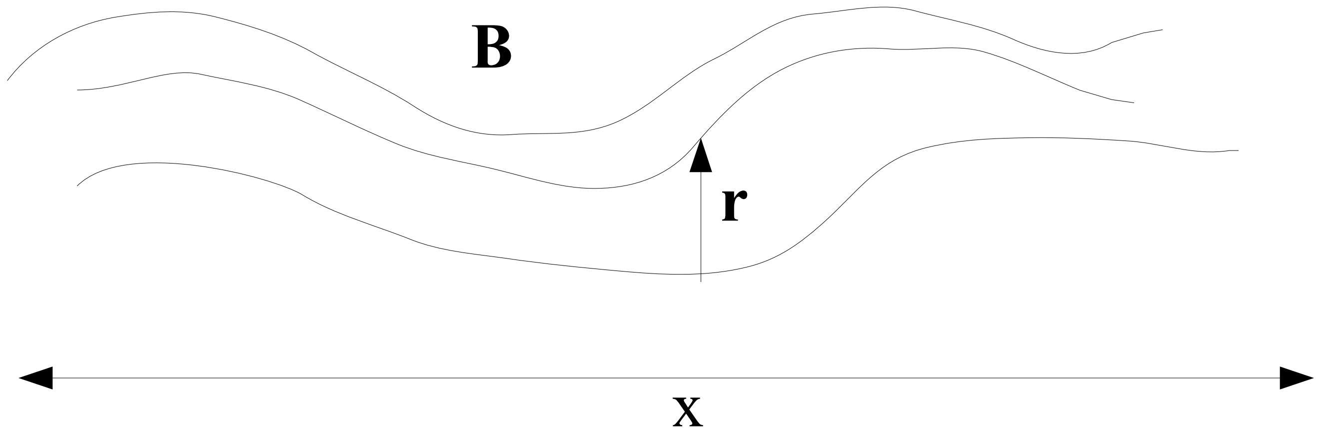
Forward and backward:



Imbalanced simulation with ratio of fluxes

$$\epsilon^+/\epsilon^- = 2$$

Diffusion of magnetic field lines



Forward and backward are different, why?

a) **B** consists of two components, $\mathbf{b}^+$ and $\mathbf{b}^-$

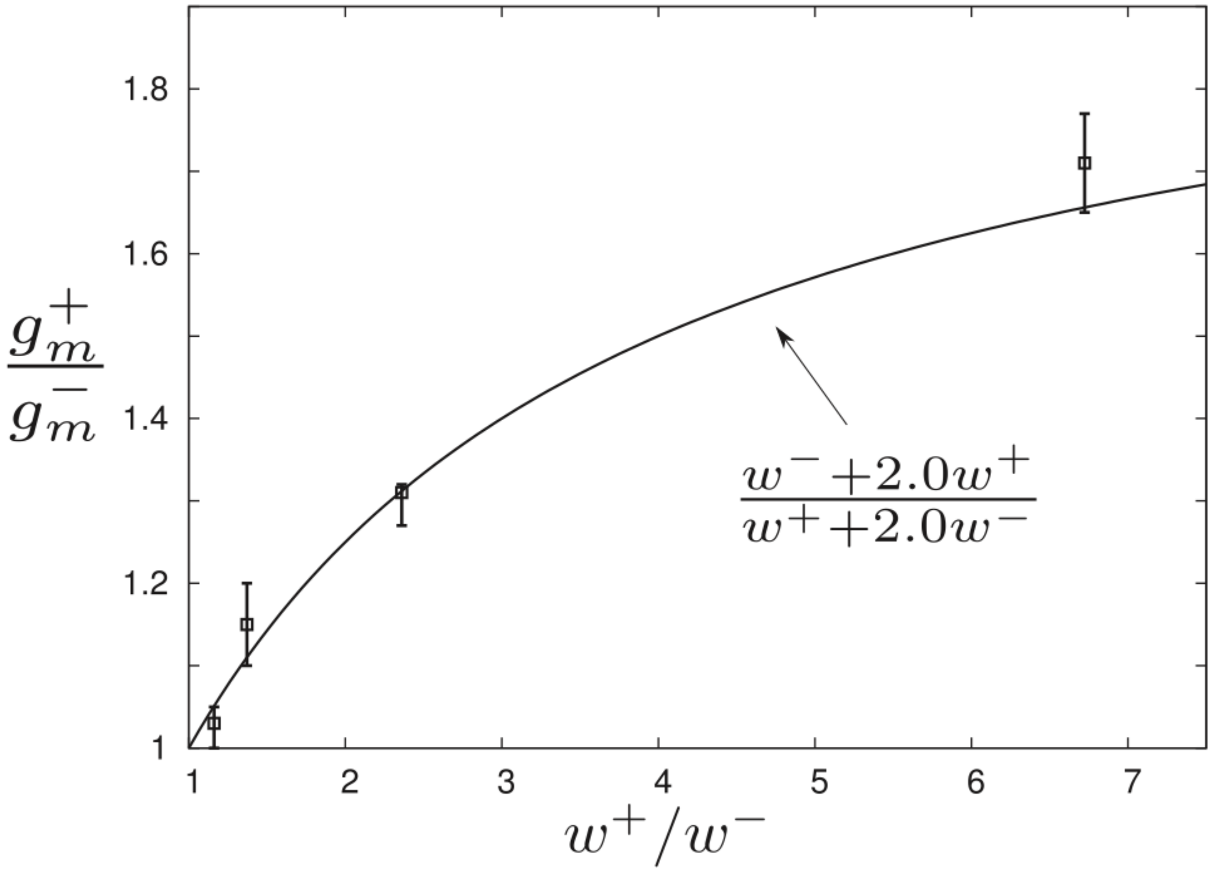
b) hydrodynamic Richardson's diffusion is known to be time-asymmetric.

c) b^- propagates along **B** and b^+ and against **B.** When we go along B, we follow evolution of b^- forward in time and b^+ backward in time.

$$\frac{g_m^+}{g_m^-} = \frac{w^- + a_m w^+}{a_m w^- + w^+}$$

Diffusion of magnetic field lines

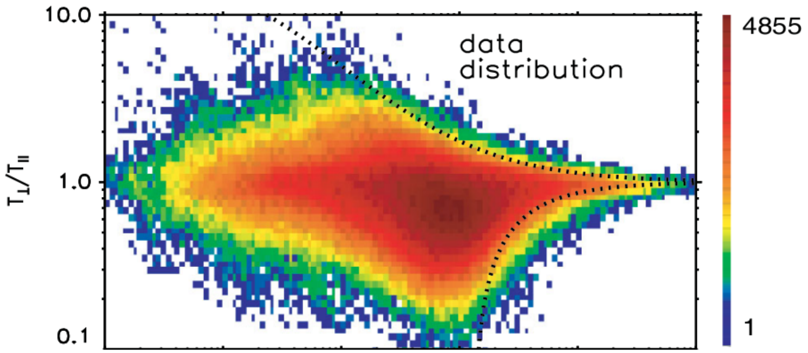
Asymmetry of diffusion vs imbalance:



$$\frac{g_m^+}{g_m^-} = \frac{w^- + a_m w^+}{a_m w^- + w^+}$$

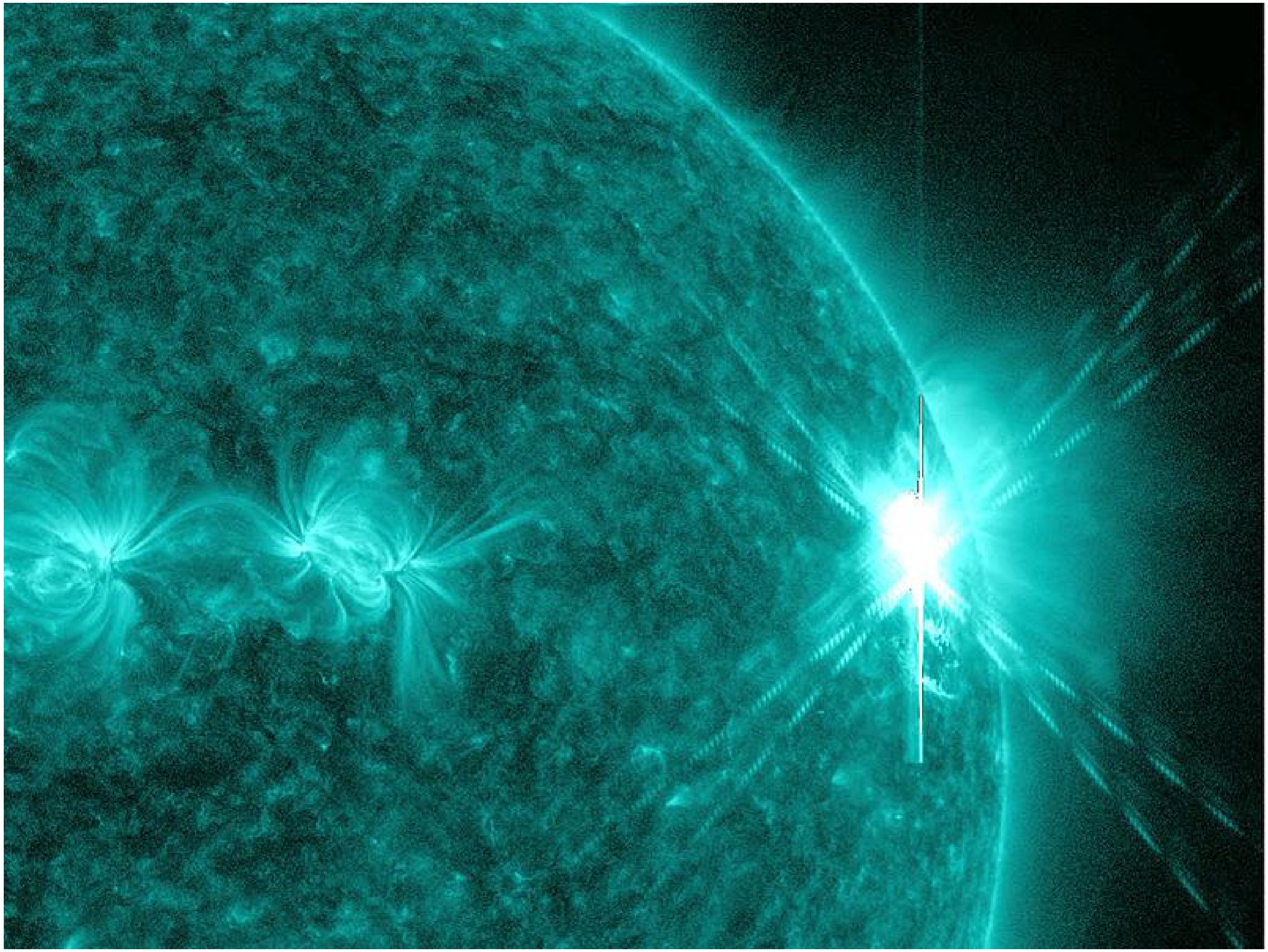
Asymmetric diffusion leads to streaming:

Streaming particle distributions are unstable, e.g. the solar wind has imbalanced turbulence and strong perpendicular gradients. This could explain why there's plenty of free energy in particles even at large distances from the Sun.



This can also excite small-scale turbulence and enhance particle scattering.

Solar X-ray flares

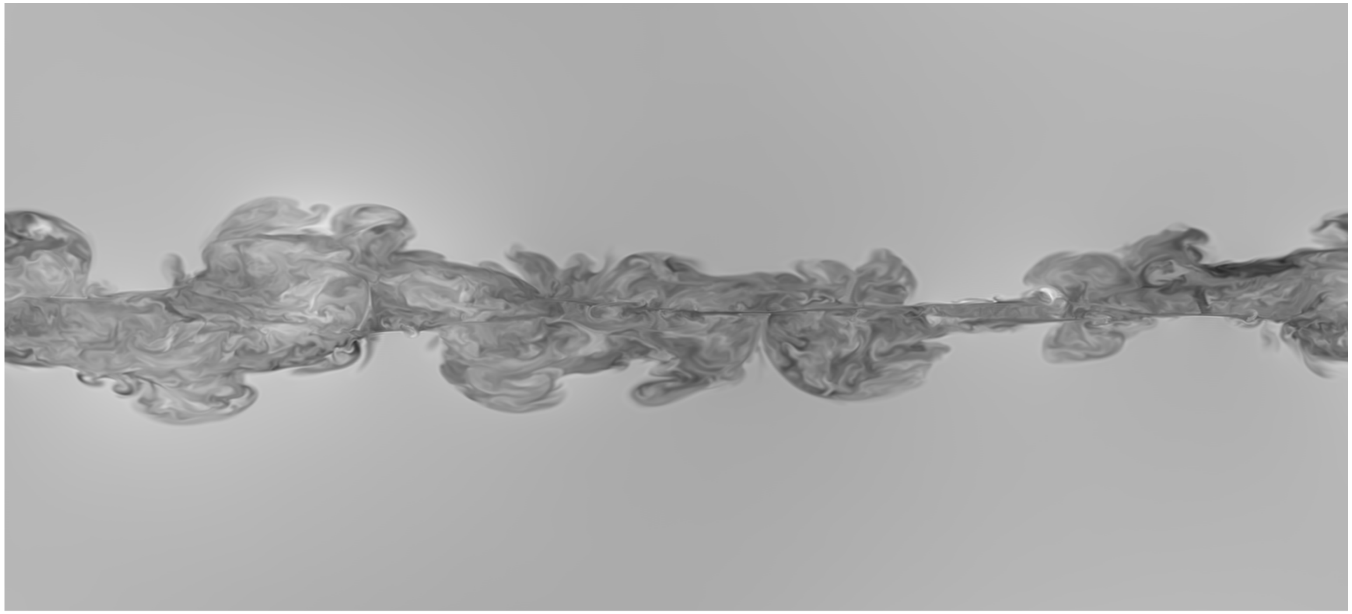


X-class flare: $6 \times 10^{32}\mathrm{erg} \approx 10^{10}\mathrm{MT}$ of energy released

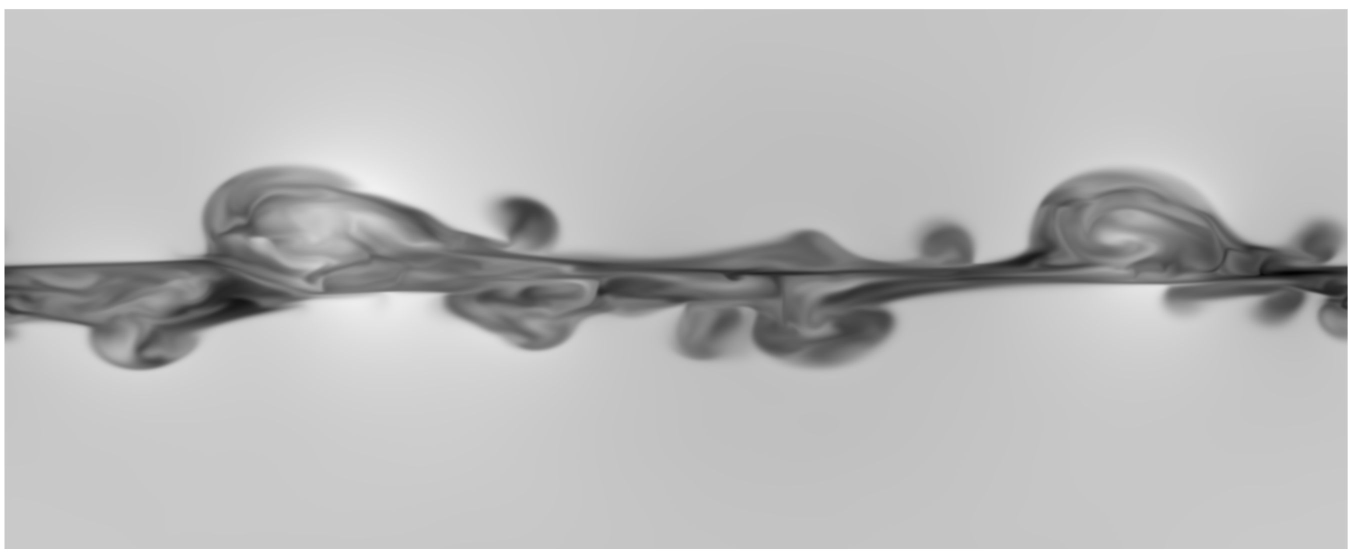
(note: this is 100% magnetic energy)

3D: initial conditions are erased!

hyper

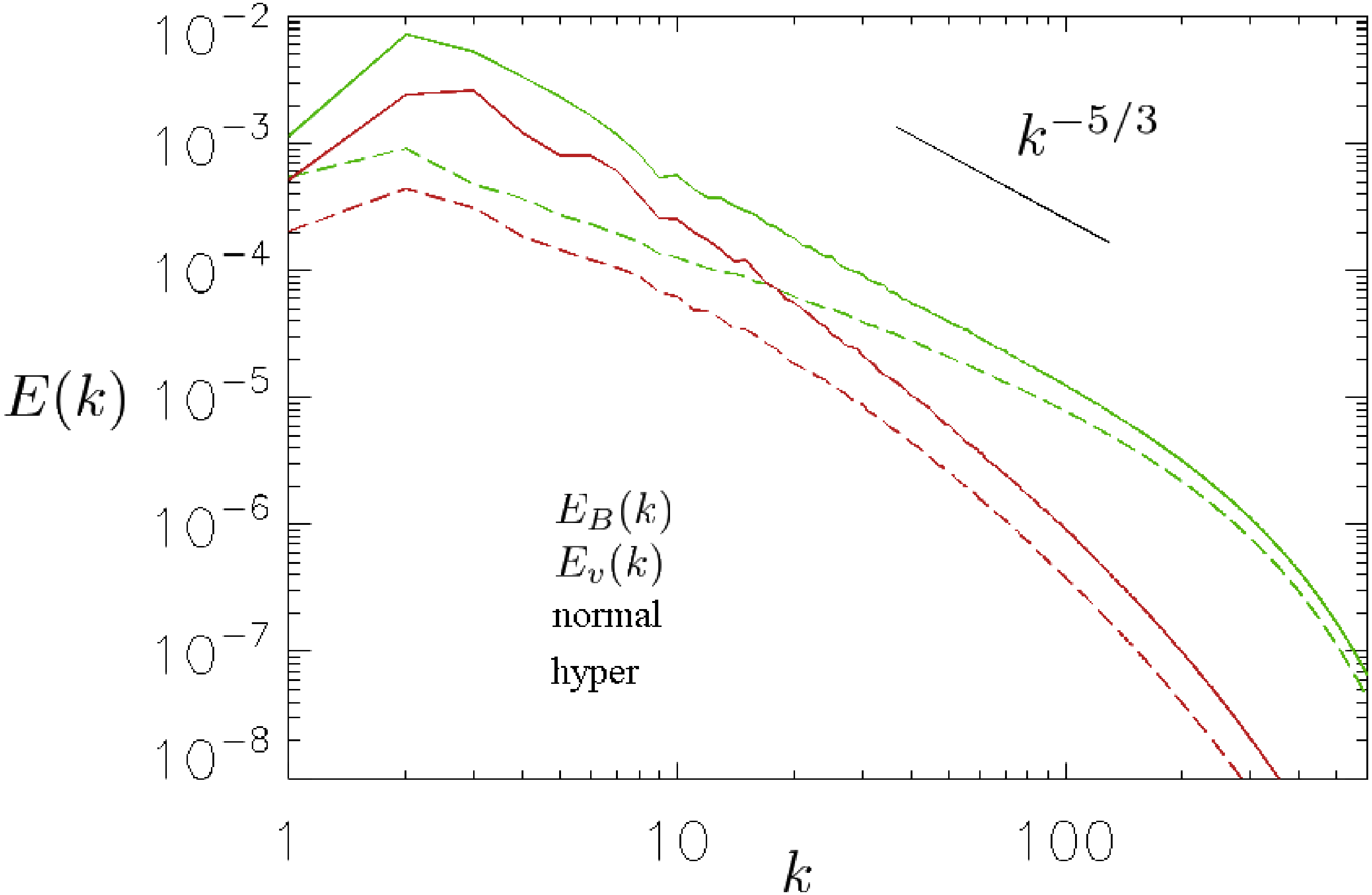


normal



|B| grayscale

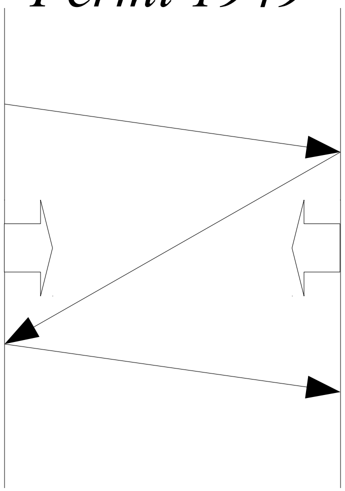
Spectra



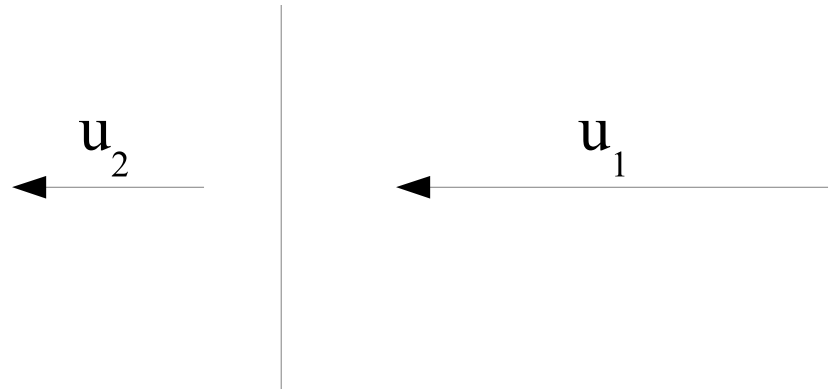
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
$E(k)$
$k^{-5/3}$
$E_B(k)$
$E_v(k)$
normal
hyper
1
10
100
k

How particles are accelerated?

Fermi 1949



shock

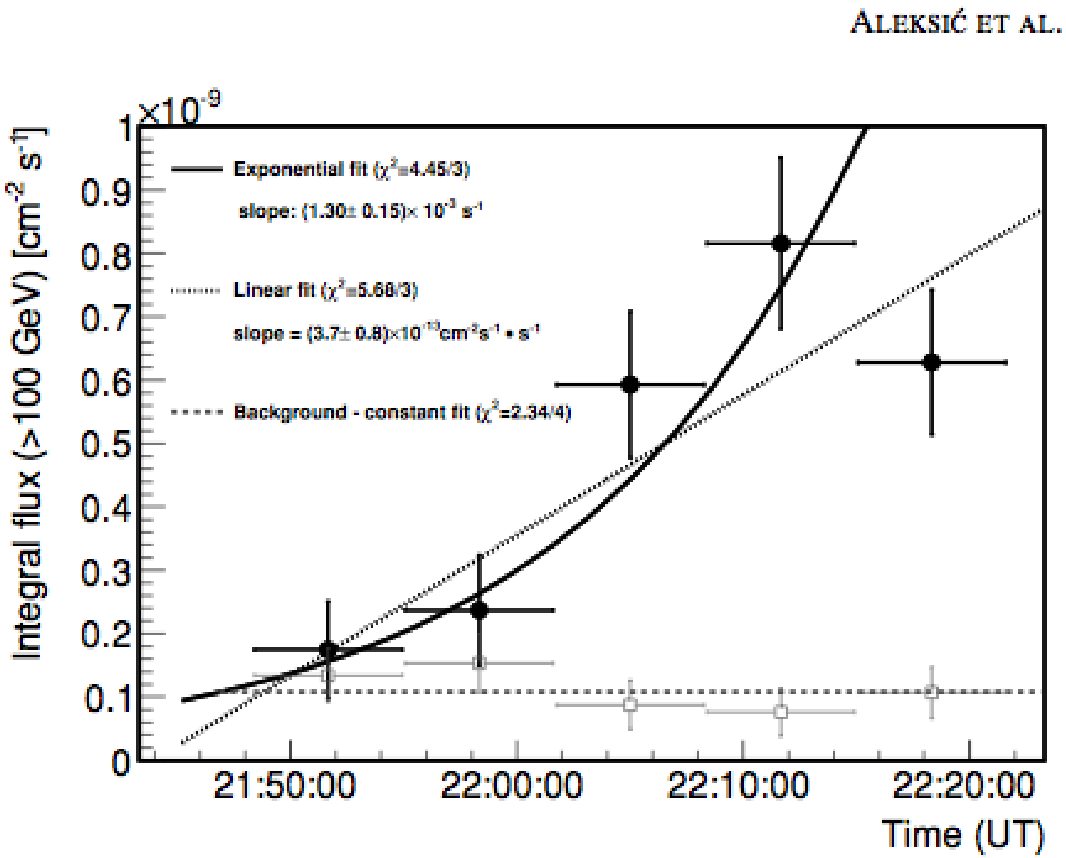


$$\tau \sim \frac{D}{U_s^2}$$

Let's say $D \sim E$ (Bohm diffusion)

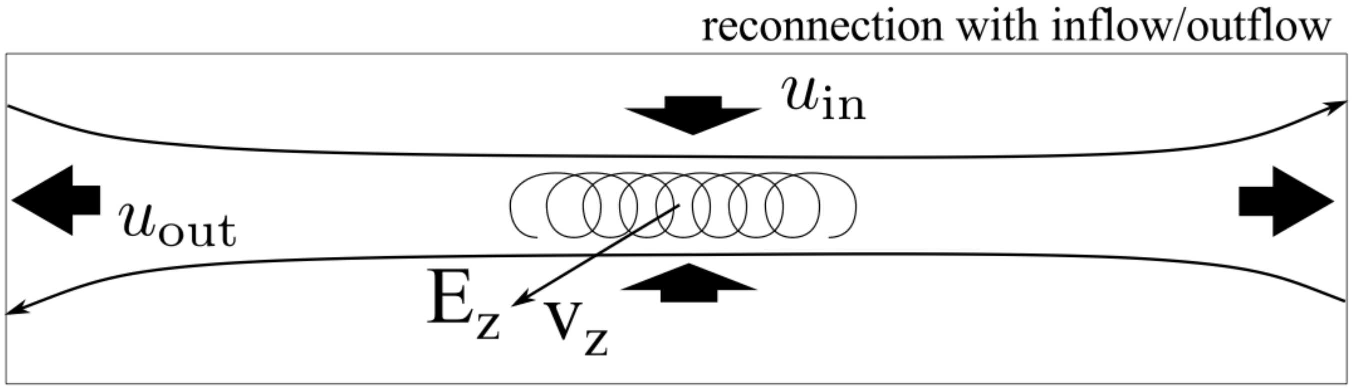
It takes a long time to accelerate high energy particles.

Nevertheless...

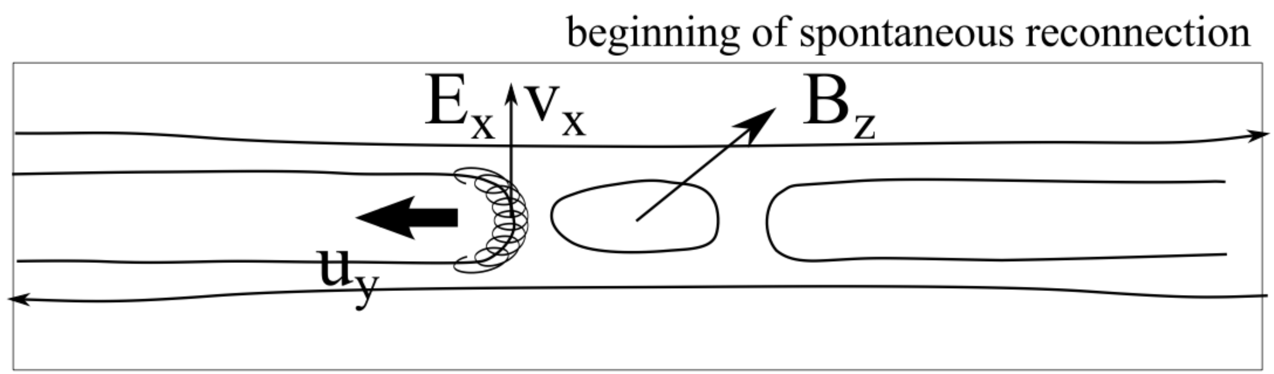


Very short variability in high energy

$$\tau \sim \frac{D}{U_s^2}$$



Particles are compressed between converging mirrors



Particles are accelerated by contracting magnetic field lines
e.g. Drake 2006

A surprising analytical connection between fluid dynamics energy transfer term and the curvature drift acceleration

V<->B energy transfer $\mathbf{u} \cdot [\mathbf{j} \times \mathbf{B}]/c =$

$$-(\mathbf{u} \cdot \nabla) B^2/8\pi + \mathbf{u} \cdot (\mathbf{B} \cdot \nabla)\mathbf{B}/4\pi$$

$$\frac{1}{4\pi}\mathbf{u} \cdot (\mathbf{B} \cdot \nabla)\mathbf{B} = \frac{1}{4\pi}(\mathbf{u} \cdot \mathbf{B})(\mathbf{b} \cdot \nabla)B + \frac{B}{4\pi}\mathbf{u} \cdot (\mathbf{B} \cdot \nabla)\mathbf{b}$$

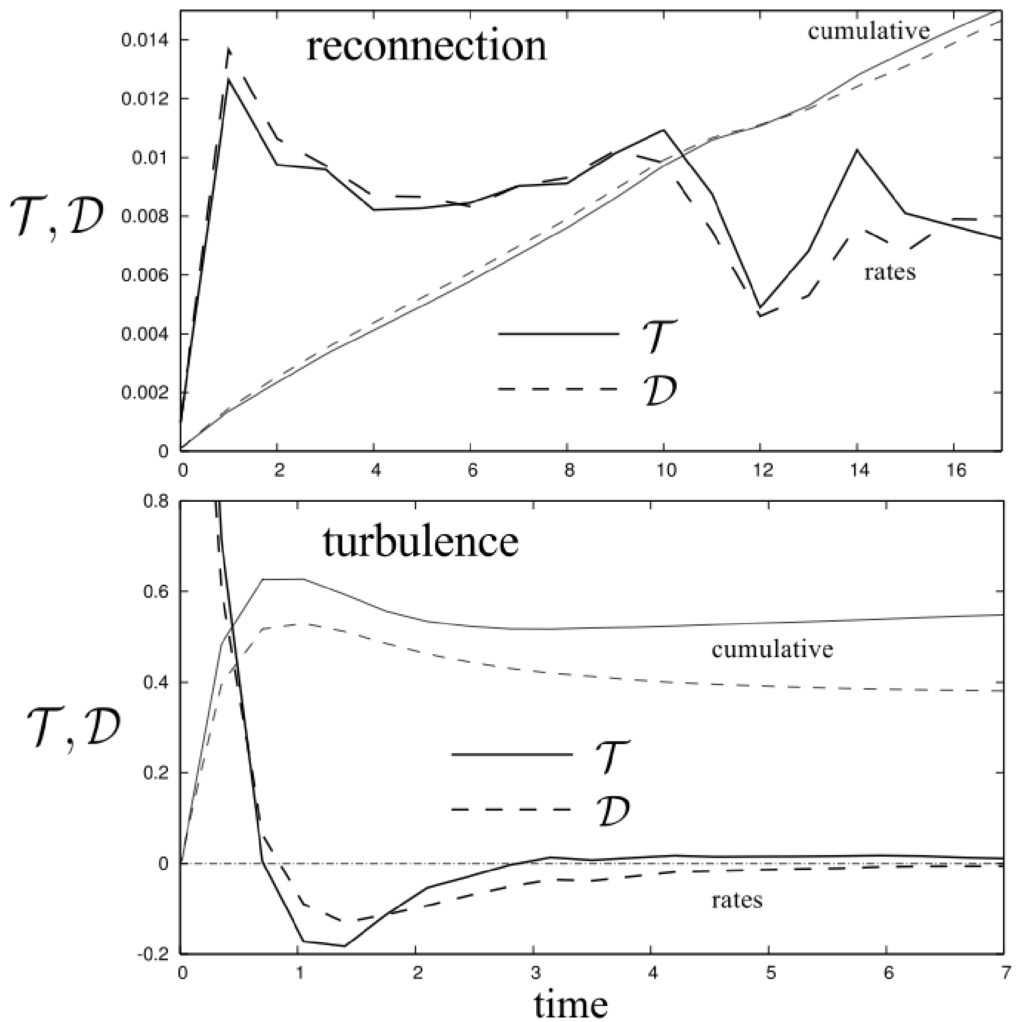
$$\mathcal{X} + \qquad \mathcal{D}$$

A surprising analytical connection between fluid dynamics energy transfer term and the curvature drift acceleration

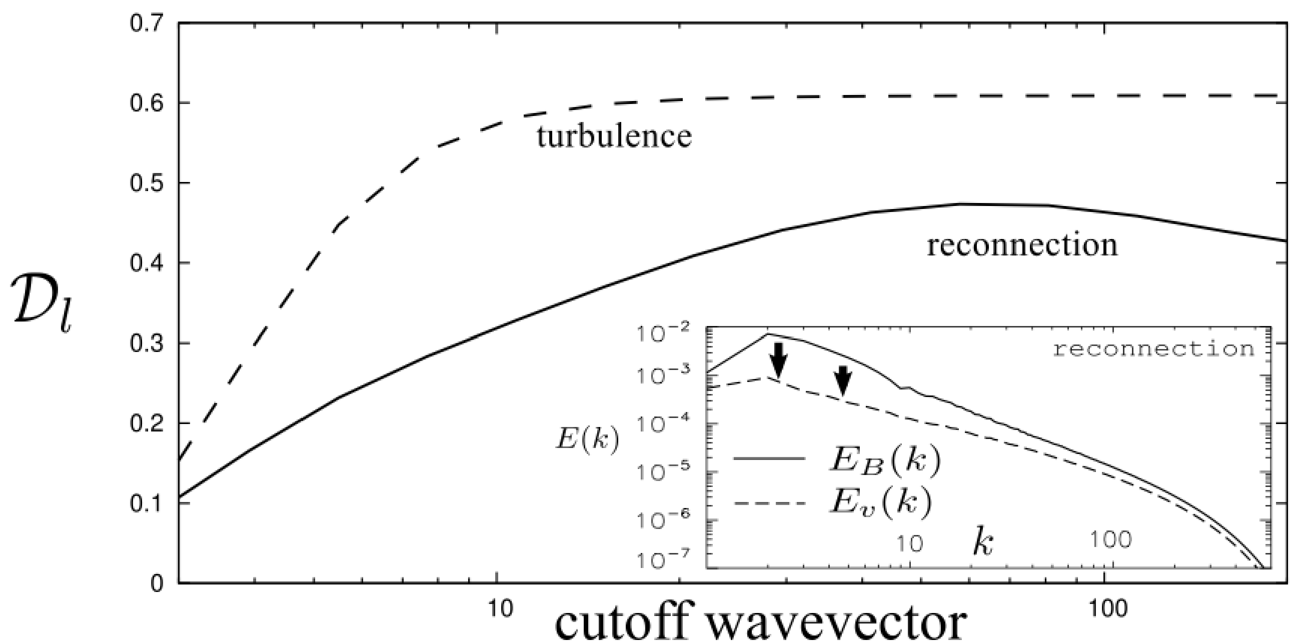
$$d\mathcal{E}/dt = -2(\mathcal{E}_{\|}/B)[\mathbf{u} \times \mathbf{B}] \cdot [\mathbf{b} \times (\mathbf{b} \cdot \nabla)\mathbf{b}] = \mathcal{E}_{\|} \frac{8\pi}{B^2} \mathcal{D}$$

$$\mathcal{D} = \frac{B}{4\pi} \mathbf{u} \cdot (\mathbf{B} \cdot \nabla)\mathbf{b}$$

The case study of spontaneous reconnection and decaying MHD turbulence

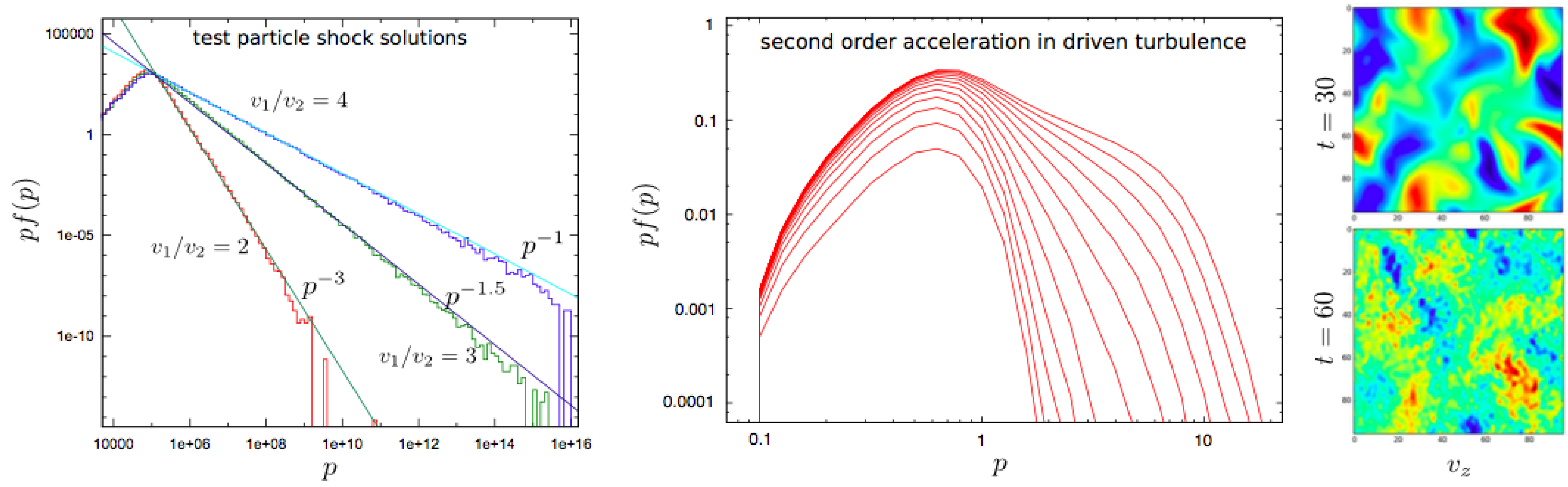


Acceleration rate as a function of energy



New directions

New PIC-MHD code (very flexible)



a) Lorentz force+ad hoc diffusion (pseudoparticles);
b) particle splitting/merging;
c) flexible precision-controlled solver,
d) designed with parallelization in mind.