Beginning inflation in an inhomogeneous universe

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work with Matthew Kleban, Andrei Linde, and Leonardo Senatore
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Motivation

- Inflation proposes to explain large scale homogeneity and flatness of the visible universe
- However most treatments of inflation assume homogeneity to begin with
- Lore: inflation can only begin if you initially assume homogeneity on Hubble scales
- Arguably no initial conditions problem if $V_{\mathrm{Inflation}} \sim M_{Pl}$, however models with $V_{\mathrm{Inflation}} \ll M_{Pl}$ popular, especially with new results from observations

Question to address: can we begin from very inhomogeneous conditions and eventually transition to exponential expansion?

Motivation

- Inflation is an attractor — need to consider large/nonlinear perturbations
- Overdensities may form black holes — strong field gravity is important
- Study in 3D (c.f. earlier work in spherical symmetry (Goldwirth & Piran, 1990))
- Use full numerical GR simulations to probe start of inflation beginning from initial conditions where gradient/kinetic energy dominates

Setup

- Einstein equations: $R_{ab} - \frac{1}{2}Rg_{ab} = T_{ab}$
- Coupled to inflaton: $\Box\phi = V'(\phi)$
- Periodic boundary conditions: either numerical convenience or universe with torodial topology
- “Flat on average"

Initial conditions

$$\phi(t=0,\mathbf{x}) = \phi_0 + \delta\phi\left[\sum_{1\leq|\mathbf{k}L/2\pi|^2\leq N}\cos(\mathbf{k}\cdot\mathbf{x}+\theta_{\mathbf{k}})\right]$$

- Start with gradient dominated energy conditions $\langle\rho\rangle \approx \langle\rho_{\rm grad}\rangle$
- Inflationary energy scale is initially much smaller: $V_{Inflation} = 10^{-3}\langle\rho\rangle$
- Start with constant K $(= -3H_0)$ set by average energy density
- Note that $H_0^2 \propto \rho(t=0) \propto k^2\delta\phi^2$ (Hamiltonian constraint)
- Hence different amplitudes corresponding to different values of H_0/k

Scale of inhomogeneities

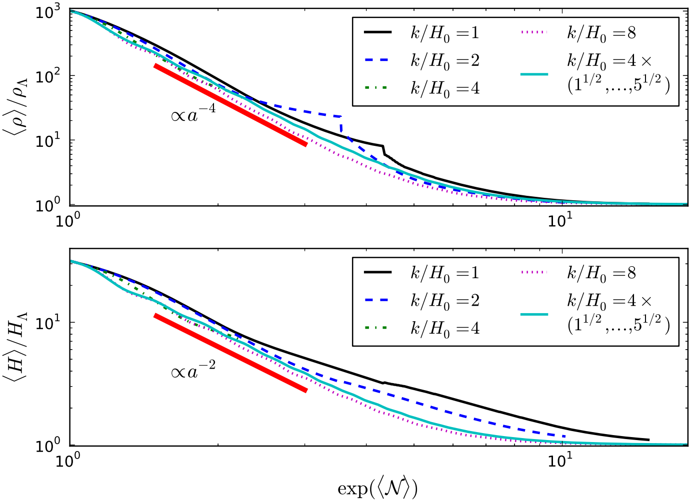
Consider different values of k/H_0

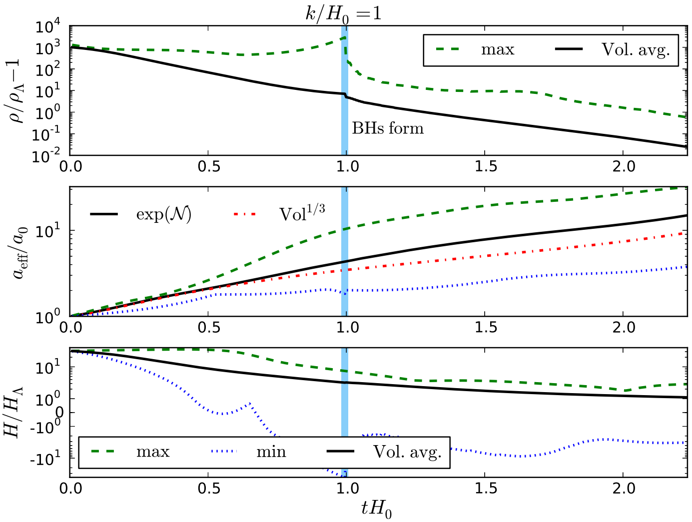
- When $k/H_0 \gg 1$ expect you should be able to average over inhomogeneities
- When $k \sim H_0$ expect black hole formation by hoop conjecture: mass in over density $\sim$ size of over density (in geometric units)
- Hence becomes a strong-field gravity problem where gravitational collapse may spoil expansion
- When $k/H_0 \ll 1$ have superhorizon modes which act like curvature (not considering).

Inflaton values are entirely contained on the flat part of the potential — inflationary “plateau":

$$V \approx \Lambda$$

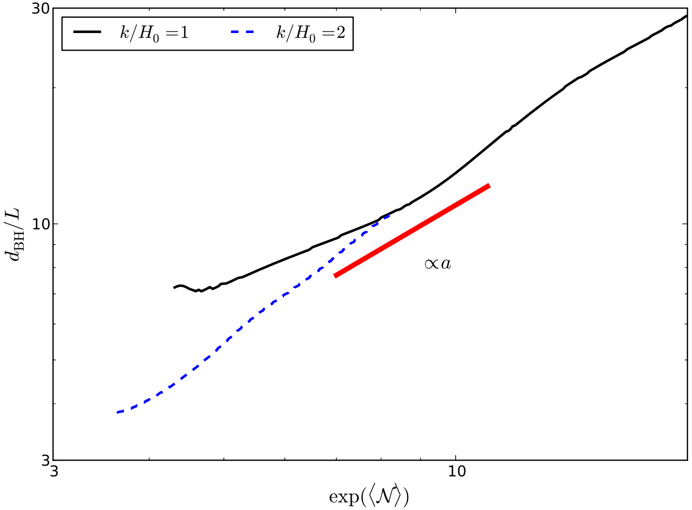
Volume-averaged energy density and expansion





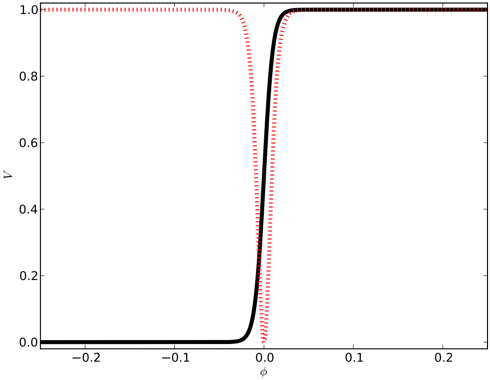
$k/H_0 = 1$
$\rho/\rho_\Lambda - 1$
max
Vol. avg.
BHs form
$a_{\rm eff}/a_0$
$\exp(\mathcal{N})$
$\mathrm{Vol}^{1/3}$
H/H_Λ
max
min
Vol. avg.
tH_0

Distance between black holes



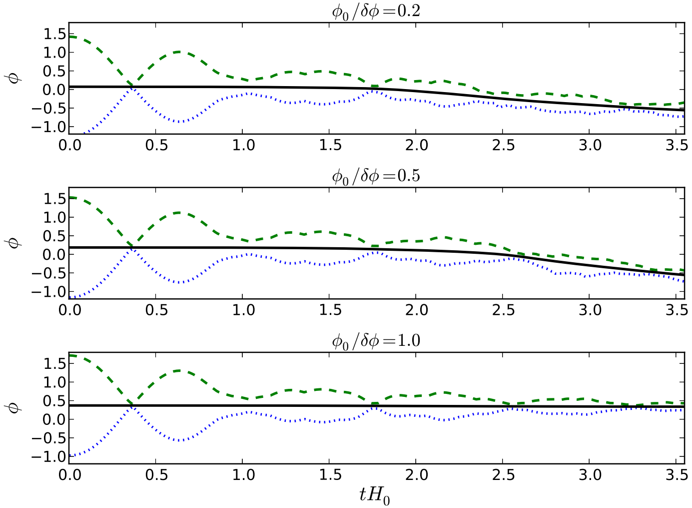
Second case

Consider cases when average field value gives slow-roll inflation, but variations can exceed inflationary plateau.

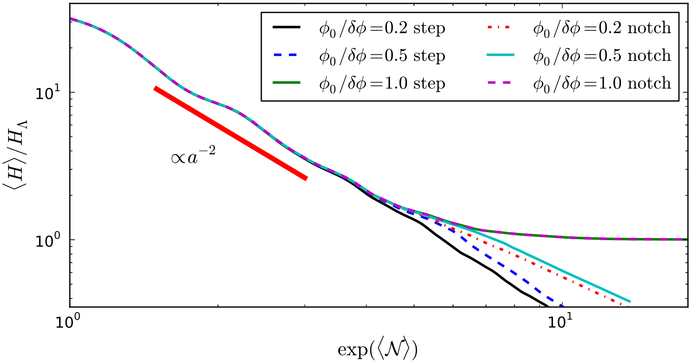


Step and notch-like potentials (e.g. cosmological attractors from Kallosh & Linde, 2013)

Step function potential



Leaving the inflationary plateau



Leaving the inflationary plateau

When $\delta\phi > \phi_0$, field "feels" bottom of potential which rapidly pulls it to lower energy/non-inflationary values. For step-like potential:

- Average "force" on scalar field is proportional to $\langle V' \rangle = V/\langle\phi\rangle$
- Slow roll condition $M_p V'/V = \epsilon < 1$
- $\langle V'(\phi) \rangle \sim V(\langle\phi\rangle)/\langle\phi\rangle = (\frac{M_p}{\langle\phi\rangle})(\frac{V'(\langle\phi\rangle)}{\epsilon}) > V'(\langle\phi\rangle)$ for $\langle\phi\rangle < M_p/\epsilon$
- Similar nonlinear effects cause potential energy decrease for notch-like potential

Conclusion

- Inflation can proceed from a large class of initial inhomogeneous initial conditions — no need to initially assume Hubble-sized homogeneous patches
- Collapsing regions with black holes form, but exponentially expanding regions eventually dominate the volume
- Holds except when distance to end of inflationary plateau $\lesssim \delta\phi$

Future work:

- Consider wider class of initial conditions/potentials
- Fate of crunching regions potentially produced during inflation by metastability of the Higgs

Many interesting applications of tools from numerical GR to addressing inhomogeneous/strong-field cosmology questions (see e.g. Eloisa Bentivegna's talk from Monday).