ON THE POSSIBILITY OF ATTENUATION OF GRAVITATIONAL WAVES IN THE EARLY UNIVERSE

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Abstract

It has been previously shown that a cloud of charged particles could absorb gravitational waves (GWs) incident upon it. Here we calculate the degree of possible attenuation of GWs emanating from the very early universe when they travel through the plasma before the end decoupling era of the universe. We attempt to show that significant absorption of waves is possible and the detection of primordial GWs could be much harder than previously anticipated.

Background

GWs emanating from the inflationary era of the cosmos en route to us must have also traveled through the plasma before the decoupling era. We calculate the degree of wave attenuation. The strain satisfies

$$\Box^2 h^{\mu\nu} = -2\kappa T^{\mu\nu} = 0; \quad h \ll 1 \qquad (1)$$

with plane wave solutions in *empty space* given by

$$h^{\mu\nu} = A^{\mu\nu} exp(i\kappa_\rho x^\rho) \quad \text{with} \quad A^{\mu\nu}\kappa_\nu = 0$$

These waves are commonly thought of as having little or no interaction with matter, reaching us unattenuated, making GWs indispensable in detecting events during and after the inflationary era.

We use the TT gauge. On Earth, the TT gauge is approximately the same as the proper detector frame since $\Gamma^i_{00} \approx 0$ and $h \sim 10^{-21}$ (expected) in our frame. The motion of free particles in the proper detector frame can be described by a *Newtonian type of force*

$$F_i = \frac{1}{2} m \, \ddot{h}^{TT}_{ij} \xi^j$$

Therefore, we have the ability to describe the effects of GWs on particles in the proper detector frame purely in a Newtonian way, without referring to GR, remembering that h and ξ are the values measured in the TT gauge [1]!

For an astrophysical source, the GW strain at a distance r is $h \sim 2GMv^2/rc^4$. To understand how such a wave would interact with a cloud of charged particles, we need to look at the Einstein field equations

$$R_{\mu\nu} - \frac{1}{2}R = -\kappa(T_{\mu\nu} + F_{\mu\nu}[EM]) \qquad (2)$$

with

$$g^{\mu\nu}\nabla_\mu\nabla_\mu A_\alpha = \mu_0 j_\alpha \qquad (3)$$

In addition, the following Lorenz gauge condition should also be met.

$$\nabla_\alpha A^\alpha = 0 \quad \text{and} \quad [A^\mu] = \left(\frac{\phi}{c}, \vec{A}\right)$$

Thus, the first order solution to the inhomogeneous wave equation becomes

$$A_0^{(1)} = \frac{1}{4\pi\epsilon_0 c}\int d^3y \frac{\rho}{|\vec{x}-\vec{y}|} + \frac{1}{4\pi}\int d^3y \frac{h^{\mu\nu}\partial_\mu\partial_\nu A_0}{|\vec{x}-\vec{y}|} \qquad (4)$$

While the first term of the foregoing gives the electrostatic potential due to charges, the second term emerging from the derivatives of the charge density within the cloud results in a time-varying vector potential giving rise to an electromagnetic (EM) field at the observation point P. Since this field is produced due to the GWs ($h^{\mu\nu}$) some amount of energy of the incident waves must be lost in transit via the cloud.

Method

Consider the following simple, specific GW form propagating along the $\hat{r}$ direction at a distance r from a source, as an example.

$$h^{\mu\nu} = \delta^\mu_\nu \frac{\lambda}{r}\cos\omega\left(t - \frac{r}{c}\right) \quad \text{with} \quad h^{00} = h^{11} = 0$$

Then, Eq (4) becomes

$$A_0^{(1)} = \frac{1}{4\pi\epsilon_0 c}\int d^3y \frac{\rho}{|\vec{x}-\vec{y}|} + \frac{\lambda}{4\pi r}\cos(\omega(t - r/c))\int d^3y \frac{(\partial_3\partial_3 A_0 - \partial_2\partial_2 A_0)}{|\vec{x}-\vec{y}|} \qquad (5)$$

Now, we see that the GWs incident upon the cloud changes its original charge density ρ to an effective value ρ_{eff}. As the waves do not physically impose any motions, at least in principle, we make the reasonable assumption here that any resulting magnetic fields could be neglected.
The EM fields may be calculated from ρ_{eff}. Via Jefimenko equations, we obtain the electric fields resulting due to ρ_{eff} at an observation point r. The resultant field becomes

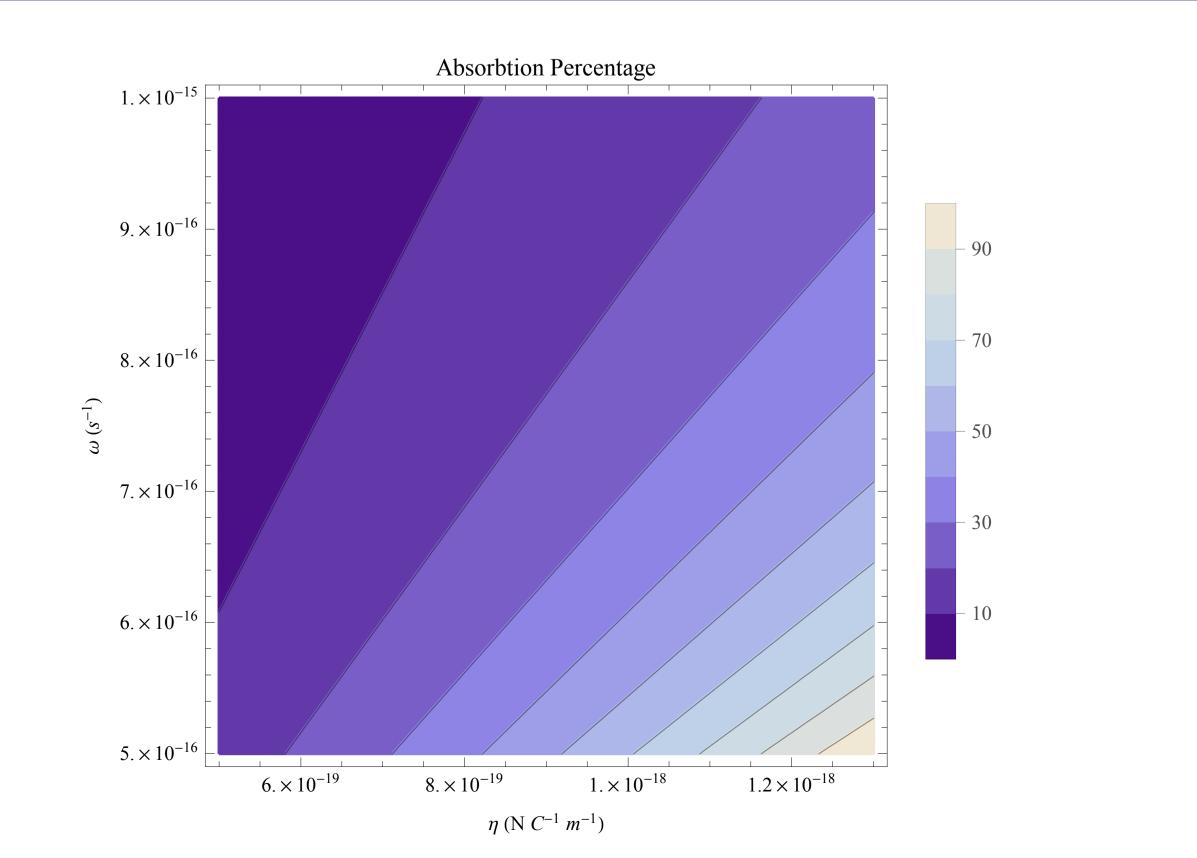$$E = \frac{1}{4\pi\epsilon_0}\int d^3y \frac{\rho}{|\vec{x}-\vec{y}|} + \left(\frac{\alpha}{r} + \frac{\beta}{r}\right) \qquad (6)$$

where

$$\alpha = \frac{\lambda c}{8\pi}f(2) \quad \text{and} \quad \beta = \frac{\lambda\omega}{8\pi}f(1)$$

and $f(n) = 1/(4\pi\epsilon_0 c)\int d^3y/(|\vec{x}-\vec{y}|)^n(\partial_3^2 - \partial_2^2)\int d^3y\rho/(\vec{x}-\vec{y})$ The first term in Eq (6) is just the electric field at P due to the charged cloud while the other two terms at the observation point r or $\vec{x}$ arise due the presence of the GW sweeping through the cloud.
Denoting U_{gw} and U_{em} as the energy densities at P due to GWs and due to the EM fields respectively, we obtain

$$\frac{U_{\text{em}}}{U_{\text{gw}}} = \frac{32\pi G\epsilon_0(\alpha^2 + \beta^2)}{c^2\omega^2\lambda^2} \qquad (7)$$

This should be the fraction of energy of the GWs absorbed by the cloud.
We are interested here at the amount of such wave attenuation at the end of the decoupling era. If we denote the size of the universe at the end of this epoch by R_{LSS}, which is the radius of the last scattering surface, we obtain

$$\frac{U_{\text{em}}}{U_{\text{gw}}} = 8\pi G\epsilon_0\left(\frac{\eta\, R_{\text{LSS}}}{c\omega}\right)^2\left[\frac{1}{9} + \left(\frac{\omega\, R_{\text{LSS}}}{2c}\right)^2\right] \qquad (8)$$

where $\eta = 1/(4\pi\epsilon_0)(\partial_3^2 - \partial_2^2)\int d^3y\,\rho/(|\vec{x}-\vec{y}|)$

Absorbtion Percentage

Results and Discussion

We see from Eq (5) that in the asymptotic region far away from the charge distribution the Poynting flux is proportional to $1/|\vec{x}-\vec{y}|^2$. Thus, the cloud of charges absorbs a *finite amount* of energy provided that those gradients of the charge density are nonzero. In addition, we see that the amount of absorbed energy depends on the inhomogeneity of the charge cloud and how far it is from the source of GWs.

For an EM wave, a charged particle oscillating in the transverse directions stays at rest even after the wave passes showing that the particle does not absorb energy. This is not the case in the direction of propagation [4]. The displacement of the charged particle in the direction of propagation of the wave is proportional to $1/2(B_0^2 - A_0^2)$. Then it is clear that unless the incident radiation is circularly polarized, a charged particle will oscillate in the direction of propagation, absorbing energy.

But for a GW, longitudinal proper oscillations in the direction of propagation of the wave will still not vanish even if the incoming GW is circularly polarized ($h_+ = \pm h_\times$) [4]. Thus as time goes on, GWs sweeping through a charged cloud will modify the proper charge density and the gradients in Eq (5) will be established.

We use the foregoing Eq (6) to analyse the wave attenuation in the plasma of the universe before decoupling. The physical meaning of η is that it represents the rate of change or the gradient of the resultant electric field on the plane of the sky (perpendicular to $\hat{r}$). Thus, the value of η depends heavily on the inhomogeneity observed in the CMB. Letting $\omega \sim 10^{-15}\,\text{s}^{-1}$ and the radius of the last scattering surface to be the proper distance to the particle horizon when the universe decoupled matter from radiation so that $R_{\text{LSS}} \sim ct \sim 10^{21}$ m, we get $U_{\text{em}}/U_{\text{gw}} \sim 10^{35}\eta^2$.

Therefore we see that for a tiny inhomogeneity of the gradient of the tangential electric field on the plane of the sky (amply supported by the observation of CMB) at a level of $\eta \sim 10^{-18}$ N C$^{-1}m^{-1}$, total absorption of GWs is possible. These findings suggest that detection of primordial GWs is slim.

Acknowledgement and References

References

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