Black hole mimickers

Daniela Pérez, Gustavo E. Romero
Instituto Argentino de Radioastronomía,
C.C. 5 (1894) Villa Elisa, Buenos Aires, Argentina
Eric Gourgoulhon, Jerome Novak
Observatoire de Paris, 5 Place Jules Janssen, 92190 Meudon, France

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Abstract

We show that a spherically symmetric gravitational collapse of a star can result in a bounce if the equation of state behaves with sufficient rigidity just before the formation of an event horizon. The relativistic time dilation produced by the strong gravity makes the whole process to be indistinguishable from a black hole on timescales shorter than the Hubble time for a distant observer. We solve the Misner-Sharp equations for stellar collapse with a suitable equation of state that reduces to a polytropic at low densities and mimicks an effective cosmological constant during the collapse, finally inverting the direction of the velocity field of the fluid. We also present some preliminary results of numerical simulations of these objects. We conclude that evolving gravitational systems might mimick, in this way, most of the properties attributed to static black holes.

1 Introduction

Though there is strong astronomical evidence that supports the existence of black holes ([1], [2], [3]), until now, it has not been proved incontrovertibly that the compact objects associated with extreme astrophysical events are black holes. The basic property that defines a black hole is the existence of an event horizon. Since the event horizon is a surface of infinite redshift, it is impossible the empirical validation of its existence through electromagnetic observations [4].

From the theoretical point of view, the existence of event horizons seems to violate the unitary evolution required by Quantum Mechanics. There are several possible solutions to this problem: it has been suggested that General Relativity is no longer valid at the event horizon; some propose that there is a quantum field on the horizon with huge vacuum fluctuations that destroy anything that falls onto the black hole, doing work on the quantum system and destroying the unitariety. This chaotic surface of quantum states of very high energy is called *firewall* [5]. One of the most serious shortcomings of this proposal is the violation of the Equivalence Principle which has been probed at the level of the curvature of the event horizon. On the other hand, in the case of supermassive black holes, the region of the event horizon is not even one of strong gravitational field.

One possible solution to the problem is to suppose that event horizons do not exist, which in turn implies that black holes do not exist. In this way, both General Relativity and Quantum Mechanics remain untouched. Black holes, then, might be replaced by astrophysical objects that behave like black holes for any possible astrophysical observation...but ultimately they are not black holes. We shall call these type of objects *mimickers*.

Some proposals for mimickers are Boson stars: the particles in these objects do not obey Paulis exclusion principle, so they can be very compact [6]; *superspinars*: the angular momentum is larger than for Kerr black holes, and some kind of matter replaces the interior ring singularity [7]; *quark stars* or strange stars are hypothetical objects with an equation of state that allows matter to be in a hyperdense state. *Gravitational Vacuum Stars* or *gravastars* [9] are mathematically constructed as compact objects whose interior is de Sitter and its exterior geometry is Schwarzschild's for an arbitrary total mass M. These regions are separated by small layer of material of finite width which has an equation of state of the form $P = +\rho c^2$ that replaces both de Sitter and Schwarzschild horizons. The interior region has an equation of state of the form $P = -\rho c^2$ in such a way that the energy conditions are violated and singularities theorems can not be applied. The solution, then, has neither singularities nor event horizons. Recent theoretical works, however, have shown that *gravastars* and other alternative models for black holes are not stable if they rotate [8]. Furthermore, as shown by Cattoen and collaborators [10], in order to have static compact stars it is required an anisotropic energy-momentum tensor.

A dynamical model for a mimicker was proposed by Barceló and co-authors [11]: *dark stars* are ever collapsing objects that, nonetheless, never develops an horizon. To achieve this, quantum effects are usually invoked.

Another possibility is to explore gravitational collapse of matter with an equation of state such that a smooth transition is allowed from a polytropic state to a state of infinite rigidity in order to enforce a bounce.

An equation of state of this type has been recently proposed by Mbonye and Kazanas [12] and used to describe a regular black hole interior. This interior, however, is dynamically and thermodynamically unstable [13].

In this work, we propose a model for a mimicker that consists of a bouncing system whose equation of state for the matter is given by Mbonye and Kazanas [12]. Since the bounce will take place very close to the corresponding Schwarzschild radius of the system, where the redshift is very high, it is impossible to observe.

In the following section we describe the model of the proposed mimicker.

2 Model of a mimicker

In what follows, we only consider the dynamical evolution of the core of the system in the last stages of the gravitational collapse.

We assume that the equation of state of the fluid has the form [12]:

$$p(\rho) = \left[\alpha - (\alpha+1)\left(\frac{\rho}{\rho_{\mathsf{max}}}\right)^2\right]\left(\frac{\rho}{\rho_{\mathsf{max}}}\right)\rho, \tag{1}$$

where p is the pressure, $\alpha = 2.2135$, and ρ_{max} stands for the maximum density. The equation of state 1 describes the behaviour of matter that changes smoothly from normal to a core of "exotic fluid" in such a way that $p_r = -\rho$ when $r \to 0$. Outside the central region, for low densities and where matter is normal, the equation of state reduces to that of a polytropic. We adopt here a value of $\rho_{\mathsf{max}} = 0.2 \times 10^{18}$ g/cm^3, higher than the highest densities probed at the Large Hadron Collider. It is conceivable that at such densities matter might undergo a change to some repulsive state as the one described in [12]. In Figure 1 we show a plot of Eq. 1.

We study the Misner-Sharp equations [14] for a fluid described by [12].

The metric has the diagonal form:

$$ds^2 = -e^{2\varphi}dt^2 + e^{\lambda}dr^2 + R^2 d\Omega^2, \tag{2}$$
$$d\Omega^2 = d\theta^2 + \sin^2\theta d\chi^2. \tag{3}$$

Here φ, λ, and R are functions of the radial coordinate r and the coordinate t. The components of the four-velocity in comoving coordinates are:

$$u^t = e^{-\varphi}, \qquad u^i = 0; \quad i = r, \theta, \chi. \tag{4}$$

Misner and Sharp introduce the quantity U which gives the relative velocity $Ud\theta$ of adjacent particles on the same sphere of constant r:

$$U = D_t R \equiv e^{-\varphi}\dot{R}, \tag{5}$$

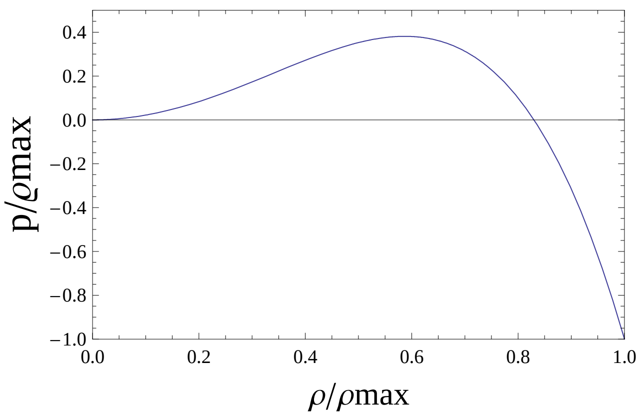

Figure 1: Plot of Eq. 1 that gives the relation between the radial pressure and the density.

and D_t is the comoving proper-time derivative:

$$D_t = u^\mu \frac{\partial}{\partial x^\mu} = e^{-\varphi}\left(\frac{\partial}{\partial t}\right)_r . \tag{6}$$

Instead of operating with the function $\lambda(r,t)$, we can use the function $m(r,t)$ defined as:

$$e^{\lambda(r,t)} = g_{rr} = \left[1 + U^2 - \frac{2m(r,t)}{R}\right]^{-1}\left(\frac{\partial R}{\partial r}\right)^2 . \tag{7}$$

The complete set of dynamical equations for the spherically symmetric gravitational collapse are:

$$\begin{aligned} D_t R &= U, && (8)\\ D_t m &= -4\pi R^2 p U, && (9)\\ D_t U &= -\left[\frac{1+U^2-2mR^{-1}}{\epsilon+p}\right]\left(\frac{\partial p}{\partial r}\right)_t \\ &- \frac{m+4\pi R^3 p}{R^2}, && (10)\\ \left(\frac{\partial m}{\partial R}\right)_t &= 4\pi R^2 \epsilon, && (11)\\ e^{\varphi} &= (-g_{00})^{1/2} = 1/h, && (12)\\ \left(\frac{dA}{dr}\right)_{t=0} &= \frac{4\pi R^2 n}{(1+U^2-2mR^{-1})^{1/2}}\frac{\partial R}{\partial r}. && (13) \end{aligned}$$

In Eq. 12, $h = u + pv = (\epsilon + p)/n$ is the specific enthalpy, $n(r,t)$ is the baryon density, $v = 1/n$ is the specific volume, $u = \epsilon v$ is the specific internal energy, and dA represents an amount of matter defined by a fixed coordinate range dr, which is independent of time.

The set of equations 8-13 together with the equation of state of the fluid determine the dynamical evolution of the spherically symmetric collapse of the core of the system.

In order to solve the Misner and Sharp equations, it is sufficient to specify the equation of state $p(n)$, and the energy density of the matter $\epsilon(n)$; from these two quantities, we can calculate the enthalpy $h = d\epsilon/dn$. From Eq. 1:

$$\begin{aligned} p(n) &= \left[\alpha - (\alpha+1)\left(\frac{n}{n_{\max}}\right)^2\right]\left(\frac{n^2}{n_{\max}}\right), && (14)\\ \epsilon(n) &= \left[\alpha - \frac{1}{3}(\alpha+1)\left(\frac{n}{n_{\max}}\right)^2\right]\left(\frac{n^2}{n_{\max}}\right), && (15)\\ h(n) &= 2\left[\alpha - \frac{2}{3}(\alpha+1)\left(\frac{n}{n_{\max}}\right)^2\right]\left(\frac{n}{n_{\max}}\right). && (16) \end{aligned}$$

The Misner and Sharp equations can only be solved numerically; in order to do so, we make use of the set of classes of $C++$ LORENE[1], and the numerical codes LORENE-rotstar-dirac [15] and CoCoNuT[2] .

We generate the initial data using the numerical code LORENE-rotstar-dirac. The input we have provided to run this code is:

1. Angular velocity $\Omega = 0$ (we suppose the collapsing object is not rotating).

2. The value of the central enthalpy, defined as:

$$H = \int \frac{dp}{e+p}, \tag{17}$$

where e is the energy density, and p is the pressure. The chosen value for the central enthalpy is $H = 0.5c^2$.

3. Range of variation for the density: $\rho_{\rm inf} = 1.7 \times 10^9$ g/cm$^3 \leq \rho \leq \rho_{\rm inf} = 10^{17}$ g/cm^3. Notice that for this range of densities, the equation of state 1 is polytropic (polytropic index $n = 2$).

4. Total energy density $\epsilon(n)$, given by Eq. 15.

5. The equation of state, given by Eq. 1.

For the set of values specified above, the gravitational mass and circunferential equatorial radius are $M_{\rm G} = 0.42M_\odot$ and $R_{\rm eq} = 2.92$ km.

Once we have the initial data, we proceed to run the code CoCoNuT. In the following section, we describe the results obtained.

3 Results

We show in Figs. 2 and 3 the results of two successive runs of the code. In Fig. 2 we plot the central density as a function of time (time for an observer at infinity). In the first run (red line) the central density grows up to a maximum. It is at this point that the code stops (the pressure as a function of the density diminishes, the sound speed becomes complex: there is no propagation of sound waves because of the exotic behaviour of the matter). At the beginning of the second run (black line), the central density decreases. For $t > 0.15$ ms, the central density increases quickly till it reaches the value where the sound speed becomes complex, and the code stops.

The plot of the conformal factor[3] as a function of time is shown in Fig. 3. At the beginning of the first run ($t = 0$), the conformal factor is approximately 1.35 ($\phi = 1$ corresponds to Minkowski spacetime). As time increases, the conformal factor grows until the code stops ($\phi \approx 2$ and $t = 0.04$). During the second run, the conformal factor decreases to a minimum value, then it starts to grow till the code stops. The values of the conformal factor that correspond to the points where the first and second run stop are different. This indicates an asymmetry in the bounce due to the existence of a preferential direction for the action of the gravitational field on large scales.

The code CoCoNuT also computes whether there are apparent horizons. In all the runs we made, there was no formation of apparent horizons nor trapped surfaces.

[1]LORENE is a free software under the GNU General Public License. It is developed in the Meudon section of Paris Observatory, at LUTH laboratory, mostly by Eric Gourgoulhon, Philippe Grandclément, Jean-Alain Marck, Jérôme Novak and Keisuke Taniguchi. For more information about LORENE, see http://www.lorene.obspm.fr.

[2]CoCoNuT is a general relativistic hydrodynamics code with dynamical space-time evolution developed by Harald Dimmelmeier at the Max-Planck Institute for Astrophysics (Garching bei München, Germany) and Jérôme Novak from the LUTH laboratory at the Observatoire de Paris (Meudon, France). The code is not of public domain. For more information about CoCoNut see http://www.uv.es/coconut/.

[3]For a definition of the conformal factor and its properties see [16] and [17].

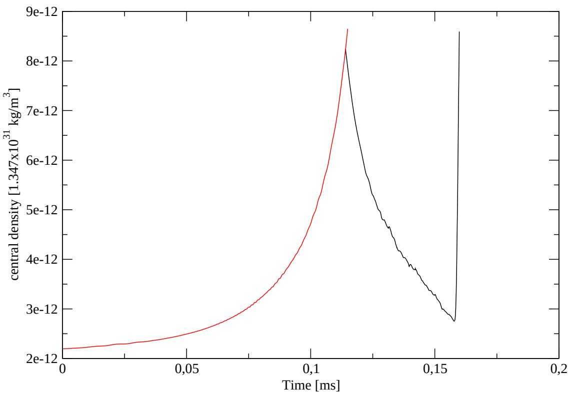

Figure 2: Central density as a function of time. The red line corresponds to the results of the first run, and the black line to the results of the second run.

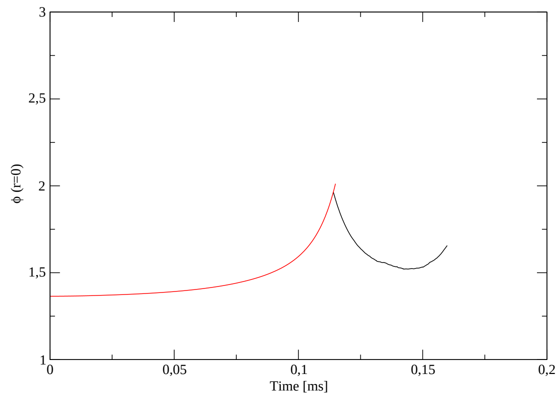

Figure 3: Conformal factor as a function of time. The red line corresponds to the results of the first run, and the black line to the results of the second run.

4 Conclusions

The results of the last section allows us to infer the behaviour and possible evolution of the system.

During the first run the system collapses under the action of the gravitational field; the central density grows, and the values of the conformal factor point out that the spacetime curvature is becoming more important. For a fix value of the density, the fluid reaches an infinite rigidity and there is no propagation of sound waves. The code stops and we proceed to make a second run taking as initial data the output of the first run and reversing the sign of the radial velocity of the fluid.

The system, now, begins to expand (the central density decreases, and the curvature is less strong). This expansion, however, is not ever lasting. Before the central density starts to grow again, the lapse function has already changed its behaviour. Then, the action of the gravitational field collapses the system until the maximum value of the density is reached and the bounce occurs again.

We have, then, an oscillating system: it collapses, bounces, expands, recollapses, and this cycle continues indefinitely. We expect, however, that the amplitudes of the system diminish because of the energy losses (in Figs 3 the value of the conformal factor is different at the bounce).

Because of the very high gravitational redshift, a distant observer will never detect the oscillations of the system. We see, thus, that all the phenomenology associated with black holes can be reproduced through the model of mimicker proposed here.

Future work includes additional runs of the code in order to compute the amplitude of oscillation of the system, as well as the exact location of the bounces. All these new calculations will allow us to obtain a better characterization of the properties of the mimicker.

References

[1] Camenzind, M. (2007), Compact objects in astrophysics : white dwarfs, neutron stars, and black holes, Springer-Verlag

[2] Paredes, J. M. (2009), Black Holes in the Galaxy, In Compact Objects and their Emission, Argentinian Astronomical Society Book Series, eds. G. E. Romero & P. Benaglia

[3] Romero, G. E. and Vila, G. S. (2014), Introduction to Black Hole Astrophysics, Lecture Notes in Physics, Berlin Springer Verlag

[4] Abramowicz, M. A. and Kluźniak, W. and Lasota, J.-P. (2002), No observational proof of the black-hole event-horizon, Astronnomy & Astrophysics 396, L31-L34

[5] Almheiri, A., Marolf, D., Polchinski, J. and Sully, J. (2013), Black holes: complementarity or firewalls?, Journal of High Energy Physics 2, 62

[6] Liebling, S. L. and Palenzuela, C. (2012), Dynamical Boson Stars, Living Reviews in Relativity 15, 6

[7] Gimon, E. G. and Hořava, P. (2009), Astrophysical violations of the Kerr bound as a possible signature of string theory, Physics Letters B 672, 299-302

[8] Cardoso, V., Pani, P., Cadoni, M., Cavaglià, M. (2008), Ergoregion instability of ultracompact astrophysical objects, Physical Review D 77, 124044

[9] Mazur, P. O. and Mottola, E. (2001), Gravitational Condensate Stars: An Alternative to Black Holes, gr-qc/0109035

[10] Cattoen, C., Faber, T. and Visser, M. (2005), Gravastars must have anisotropic pressures, Classical and Quantum Gravity 22, 4189-4202

[11] Barceló, C., Liberati, S., Sonego, S. and Visser, M. (2008), Physical Review D, Fate of gravitational collapse in semiclassical gravity 77, 044032

[12] Mbonye, M. R. and Kazanas, D. (2005), Nonsingular black hole model as a possible end product of gravitational collapse, Physical Review D 72, 024016

[13] Pérez, D., Romero, G. E. and Perez Bergliaffa, S. E. (2013), An Analysis of a Regular Black Hole Interior Model, International Journal of Theoretical Physics 53, 734-753

[14] Misner, C. W. and Sharp, D. H. (1964), Relativistic Equations for Adiabatic, Spherically Symmetric Gravitational Collapse, Physical Review 136, B571- B576.

[15] Lin, L.-M. and Novak, J. (2006), Rotating star initial data for a constrained scheme in numerical relativity, Classical and Quantum Gravity 23, 4545-4561

[16] Gourgoulhon, E. (2012), 3+1 Formalism in General Relativity, Lecture Notes in Physics, Berlin Springer Verlag

[17] Alcubierre, M. (2008), Introduction to 3+1 Numerical Relativity, Oxford University Press, Oxford, UK