Inhomogeneous conformally flat models of the universe

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References

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- A. Balcerzak and MPD - in progress.

Content:

- 1. Dark energy and back-reaction of inhomogeneities.
- 2. Conformally flat inhomogeneous pressure universes.
- 3. Off-center observer position constrained by supernovae.
- 4. Full observational check (supernovae, redshift drift, BAO, CMB shift parameter) for a central observer.
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1. Dark energy and back-reaction of inhomogeneities.

- In the context of dark energy problem (Λ being 120 orders of magnitude too large) there has been more interest in the non-friedmannian models of the universe which could explain the acceleration only due to inhomogeneity (initially E. Kolb et al. astro-ph/0506354, New J. Phys. 8, 322 (2006)).
- One of the strogest claims was that **we are living in a spherically symmetric void of density** described by the Lemaître-Tolman-Bondi dust spheres model (e.g. Uzan, Clarkson, Ellis (PRL, **100**, 191303 (2008))
- Another approach was of averaging procedure of M. Buchert (GRG 32, 105 (2000); 33, 1381 (2001)).
- Recently this was challenged by S.R. Green and R.M. Wald (PRD 83, 084020 (2011); PRD 87, 124037 (2013)).
- Dispute is continued: S. Szybka et al. PRD 89, 044033 (2014); M. Buchert et al. 1507.07800.

Inhomogeneity as dark energy.

- Suppose we live in an inhomogeneous model of the Universe with the **same (small) number of parameters** as a homogeneous dark energy ΛCDM model and they both fit observations very well.
- Can inhomogeneity really mimic dark energy?
- Is one able to **differentiate** between inhomogeneity and Λ?
- Simplest inhomogeneous models are **spherically symmetric** – they violate the **Copernican Principle** (so can perhaps be toy models or describing local inhomogeneity).
- These are **two complementary models**:
- the inhomogeneous density (dust shells) Lemaître-Tolman-Bondi (LTB) models
- the conformally flat inhomogeneous pressure (gradient of pressure shells) Stephani models.

Complementary spherically symmetric universes

	pressure	density
FRW	$p = p(t)$	$\varrho = \varrho(t)$
LTB	$p = 0 \ (p(t))$	$\varrho = \varrho(t, r)$ - nonuniform
Stephani	$p = p(t, r)$ - nonuniform	$\varrho = \varrho(t)$

Lemaître-Tolman-Bondi Universe

– is the only spherically symmetric solution of Einstein equations for **pressureless matter** ($T^{ab} = \varrho u^a u^b$) and no cosmological term (G. Lemaître, Ann. Soc. Sci. Brux. **A 53**, 51 (1933); R.C. Tolman, Proc. Natl. Acad. Sci., **20**, 169 (1934); H. Bondi MNRAS **107**, 410 (1947))

$$ds^2 = -dt^2 + \frac{R'^2}{1-K} dr^2 + R^2(d\theta^2 + sin^2\theta d\phi^2) \ , \qquad (1)$$

where

$$R = R(t,r); \qquad R' = \partial R/\partial r; \qquad K = K(r) \ . \qquad (2)$$

The Einstein equations reduce to

$$\dot{R}^2 = \frac{2M(r)}{R} - K(r); \qquad 2M' = \kappa \varrho R^2 R' \ , \qquad (3)$$

and are solved by

Lemaître-Tolman-Bondi Universe

$$R(r,\eta) = \frac{M(r)}{K(r)}\Phi'(\eta); \qquad t(r,\eta) = T_0(r) + \frac{M(r)}{K^{3/2}(r)}\phi'(\eta) \ , \tag{4}$$

where for $K(r) < 0$ (hyperbolic), $K(r) = 0$ (parabolic), and $K(r) > 0$ (elliptic) appropriately ($K(r)$ is a spatially dependent "curvature index") we have

$$\Phi(\eta) = (\sinh\eta - \eta; \eta^3/6; \eta - \sin\eta) \ . \tag{5}$$

Regularity conditions:

- existence of a regular **center of symmetry** $r = 0$ – implies $R(t,0) = \dot{R}(t,0) = 0$ and $M(0) = M'(0) = K(0) = K'(0) = 0$ and $R' \to 1$.
- hypersurfaces of constant time are **orthogonal** to 4-velocity and are of topology S^3 – implies the existence of a second center of symmetry $r = r_c$ (with some 'turning value' $0 < r_{tv} < r_c$)
- a 'shell-crossing' singularity should be **avoided** – implies $R'(t,r) \neq 0$ except at turning values (though it is a weak singularity - no geodesic incompleteness)

2. Conformally flat inhomogeneous pressure universes

They are the only solutions of Einstein equations for a perfect-fluid energy-momentum tensor ($T^{ab} = (\varrho + p)u^a u^b + p g^{ab}$) which are **conformally flat**

$$g_{ab,ST} = \Omega^2_{ST,M} \eta_{ab,M} \tag{6}$$

and **embeddable** in a 5-dimensional flat space (H. Stephani Commun. Math. Phys. **4**, 167 (1967); A. Krasiński, GRG **15**, 673 (1983)). The metric for their spherically symmetric version reads as

$$ds^2_{ST} = -\frac{a^2}{\dot{a}^2}\frac{a^2}{V^2}\left[\left(\frac{V}{a}\right)^{\cdot}\right]^2 dt^2 + \frac{a^2}{V^2}\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right] , \tag{7}$$

where

$$V(t,r) = 1 + \frac{1}{4}k(t)r^2 , \tag{8}$$

and $(\ldots)^{\cdot} \equiv \partial/\partial t$. The function $a(t)$ plays the role of a **generalized scale factor**, $k(t)$ has the meaning of **a time-dependent "curvature index"**, and r is the radial coordinate.

Conformally flat Stephani universes

The energy density and pressure are given by

$$\varrho(t) = 3\left[\frac{\dot{a}^2(t)}{a^2(t)} + \frac{k(t)}{a^2(t)}\right] , \tag{9}$$

$$p(t,r) = \varrho(t)\left\{-1 + \frac{1}{3}\frac{\dot{\varrho}(t)}{\varrho(t)}\frac{\left[\frac{V(t,r)}{a(t)}\right]}{\left[\frac{V(t,r)}{a(t)}\right]^{\cdot}}\right\} \equiv w_{eff}(t,r)\varrho(t) , \tag{10}$$

and generalize the standard Einstein-Friedmann relations

$$\varrho(t) = 3\left(\frac{\dot{a}^2(t)}{a^2(t)} + \frac{k}{a^2(t)}\right) , \tag{11}$$

$$p(t) = -\left(2\frac{\ddot{a}(t)}{a(t)} + \frac{\dot{a}^2(t)}{a^2(t)} + \frac{k}{a^2(t)}\right) \tag{12}$$

to inhomogeneous models.

Topology & singularities

- Global topology still $S^3 \times R$. The models are just **specific deformations of the de Sitter** hyperboloid near the "neck circle", **but with local topology of the constant time hypersurfaces (index $k(t)$) changing in time.**
- Usually we cut hyperboloid by either $k = 1$ (S^3 topology), $k = 0$ (R^3) or $k = -1$ (H^3) – here we have "3-in-1" – the Universe may either **"open up" or "close down"**.
- standard **Big-Bang** singularities $a \to 0$, $\varrho \to \infty$, $p \to \infty$ are possible (FRW limit).
- **Finite Density (FD)** singularities of pressure appear at some particular value of a radial coordinate r – in standard FRW cosmology there exist exotic (sudden future) singularities of pressure (SFS) with finite scale factor and energy density – they differ (MPD 2005).
- There is **no global equation of state** - it changes from shell to shell and on the hypersurfaces $t =$ const.

Stephani universes - kinematics

Kinematic characteristics of the models:

$$u_{a;b} = \frac{1}{3}\Theta h_{ab} - \dot{u}_a u_b \ , \qquad \dot{u} \equiv (\dot{u}_a \dot{u}^a)^{\frac{1}{2}} \ . \tag{13}$$

where $\dot{u}$ is the acceleration scalar and the **acceleration vector**

$$\dot{u}_r = \frac{\left\{ \frac{a^2}{\dot{a}^2} \frac{a^2}{V^2} \left[\left(\frac{V}{a} \right)^{\cdot} \right] \right\}_{,r}}{\frac{a^2}{\dot{a}^2} \frac{a^2}{V^2} \left[\left(\frac{V}{a} \right)^{\cdot} \right]} \tag{14}$$

while the expansion scalar is the same as in FRW model, i.e.,

$$\Theta = 3\frac{\dot{a}}{a} \ . \tag{15}$$

Compare: LTB has non-zero expansion and shear.

Comoving observers being accelerated

- radial dependence of $w_{eff}(r,t)$ is due to the radial dependence of the fluid pressure
- this means that a comoving observer does *not* follow a geodesic.
- In fact, a comoving observer has a four-velocity with a non vanishing radial component and move in the radial direction in addition to its movement due to the expansion. Extra radial force pushes him out of a geodesic.

Null geodesics and acceleration mimicking dark energy

The four-velocity and the acceleration are

$$u_\tau = -\,c\frac{1}{V}\,, \qquad \dot{u}_r = -\,c\frac{V_{,r}}{V}\,. \tag{16}$$

The components of the vector tangent to null geodesics are

$$k^\tau = \frac{V^2}{a}\,,\;\; k^r = \pm\frac{V^2}{a^2}\sqrt{1-\frac{h^2}{r^2}}\,,\;\; k^\theta = 0\,,\;\; k^\varphi = h\frac{V^2}{a^2r^2}\,, \tag{17}$$

where h = const., and the plus sign in applies to a ray moving away from the centre, while the minus sign applies to a ray moving towards the centre. The **acceleration scalar** is

$$\dot{u} \equiv (\dot{u}_\mu \dot{u}^\mu)^{\frac{1}{2}} = \frac{V_{,r}}{a} = \frac{1}{2}\frac{k(t)}{a(t)}r \tag{18}$$

The **farther away** from the center, the **larger the acceleration (pressure)**.
Theoretically one can get an effect of dark energy!

Exact classes of inhomogeneous pressure MI and MII models.

Have been found (Dąbrowski 1993, 1995)

- Models I which fulfill the condition $(V/a)^{\cdot\cdot} = 0$
- Models II which fulfill the condition $(k/a)^{\cdot} = 0, \text{i.e.} \quad k(\tau) = -\beta a(\tau), [\beta] = Mpc^{-1}$.

A subclass of Models I is given by:

$$a(t) = \frac{1}{\gamma t + \delta}, \qquad k(t) = \frac{\alpha t + \sigma}{\gamma t + \delta} \ , \tag{19}$$

with the units of constants given by: $[\alpha]$ = Mpc s^{-1}, $[\sigma]$ = Mpc, $[\gamma]$ = Mpc^{-1} s^{-1}, and $[\delta]$ = Mpc^{-1}. It has an interesting Friedmann limit (when $\sigma = \delta = 0$) being a phantom-dominated model with $w = -5/3$ having interesting null geodesic completness features (Fernandez-Jambrina, Lazkoz 2006). In the limit $t \to 0$ one has a big-rip singularity with $a \to \infty$, $\varrho \to \infty$, and $p \to \infty$, while in the limit $t \to \infty$ one has $a \to 0$, $\varrho \to \infty$, and $p \to \infty$ (though it also depends on the radial coordinate r).

Exact classes of inhomogeneous pressure MII models.

The subclass MII has a very simple form of the metric

$$ds^2 = -\frac{1}{V^2}dt^2 + \frac{a^2}{V^2}\left(dr^2 \ + \ r^2 d\Omega^2\right) = \frac{a^2}{V^2}\left[-d\tau^2 + dr^2 \ + \ r^2 d\Omega^2\right], \quad (20)$$

of which conformal flatness is explicit (using conformal time $dt = a(\tau)d\tau$). Besides, one can nicely generalise the set of the cosmological equations using a generalised continuity equation:

$$\begin{aligned} H^2(t) &= \frac{8\pi G}{3}\,\varrho(t) - \frac{k(t)}{a^2(t)} \\ \dot{\varrho}(t) &+ 3\frac{H(t)}{V(t,r)}\left[\varrho(t) + p(t,r)\right] = 0\,. \end{aligned} \quad (21)$$

In general one can introduce an arbitrary number of comoving perfect fluids, each of them satisfying separately equation (21).

Exact inhomogeneous pressure IIA models

A subcase of model II (from now on **IIA**) was proposed by Stelmach and Jakacka (2001)) – it assumes that the **standard barotropic** equation of state

$$\frac{p(t)}{c^2} = w\varrho(t) \tag{22}$$

at the center of symmetry and **no exact form** of the scale factor. This assumption gives that

$$\frac{8\pi G}{3c^2}\varrho(t) = C^2(\tau) = \frac{A^2}{a^{3(w+1)}(t)} \quad (A = \text{const.}) \tag{23}$$

and allows to write a generalized Friedmann equation as

$$\frac{1}{c^2}\left(\frac{a_{,t}}{a(t)}\right)^2 = \frac{A^2}{a^{3(w+1)}(t)} - \frac{\beta}{a(t)} = \frac{8\pi G}{3c^2}[\rho + \rho_{inh}] \tag{24}$$

Alhough β shows up in the term of a fluid (domain walls $p_{inh} = -(2/3)\rho_{inh}$), it is **inhomogeneity parameter which gives Friedmann limit $\beta \to 0$**.

Models IIA

Similarly as in the Friedmann model, we can define critical density as

$$\varrho_{cr}(t) = \frac{3c^2}{8\pi G}\left(\frac{a_{,t}}{a(t)}\right)^2 = \frac{3c^2}{8\pi G}H^2 \tag{25}$$

and the density parameter $\Omega(t) = \varrho(t)/\varrho_{cr}(t)$ which after taking $t = t_0$ gives

$$1 = \frac{A^2}{H_0^2 a^{3(w+1)}(t_0)} - \frac{\beta c^2}{H_0^2 a_0} \equiv \Omega_0 + \Omega_{inh} \quad , \tag{26}$$

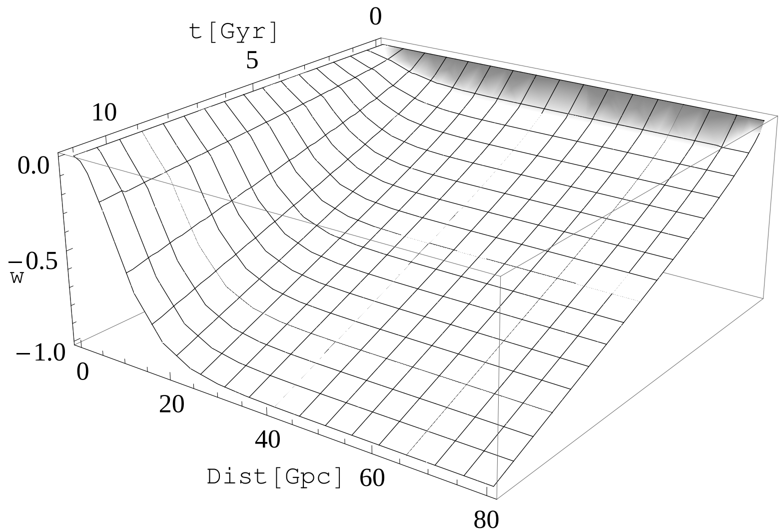
and so

$$\beta = \frac{a_0 H_0^2}{c^2}(\Omega_0 - 1) < 0 \quad . \tag{27}$$

The effective barotropic index reads as

$$w_{eff}(t,r) = w + \frac{\beta}{4}(w+1)a(t)r^2, \qquad p(t,r) = w_{eff}(t,r)\rho(t) \quad . \tag{28}$$

Model IIA - effective barotropic index



The effective barotropic index w_{eff} is getting more and more negative simulating theoretically dark energy (no observational constraints yet) for large distances away from the center (at $r = 0$) and far from the big-bang singularity (at $t = 0$).

Models IIA - the effective pressure

This is because even positive pressure fluids at $r = 0$ can give negative effective barotropic index at $r \neq 0$ ($\beta < 0$) e.g.:

$$w_{eff,\mathrm{rad}}(r,t) = \left[w_{\mathrm{rad}} + \frac{\beta}{4}(1 + w_{\mathrm{rad}})a(t)r^2 \right] = \frac{1}{3}\ \left[1 + \beta a(t) r^2 \right] \ . \tag{29}$$

and

$$p_{\mathrm{rad}}(r,t) = w_{eff,\mathrm{rad}}(r,t)\ \varrho_{\mathrm{rad}}(t) \ . \tag{30}$$

In particular,

$$w_{eff,\mathrm{rad}}(r=0) = w_{\mathrm{rad}} = \frac{1}{3} \ . \tag{31}$$

Similarly for dust:

$$w_{eff,\mathrm{dust}}(r,t) = \left[w_{\mathrm{dust}} + \frac{\beta}{4}(1 + w_{\mathrm{dust}})a(t)r^2 \right] = \frac{1}{4}\beta a(t) r^2 \leq 0 \ . \tag{32}$$

Models IIA - the speed of sound

Same refers to the velocity of sound c_S: .

$$c_S^2(r,a) = w + \left(w + \frac{2}{3}\right)\frac{\beta}{4}ar^2 = c_S^2(r=0) + \left(c_S^2(r=0) + \frac{2}{3}\right)\frac{\beta}{4}ar^2 \,. \quad (33)$$

For dust matter at $r = 0$, we get $c^2_{S,\mathrm{m}}(r,a) = \frac{\beta}{6}ar^2$ which means not only the pressure, but also the velocity of sound squared becomes negative for $ar^2 \neq 0$ (but it is equal to standard fluid $C_S^2 = w$ both at the Big-Bang $a = 0$, and center $r = 0$).

- Same problem in standard cosmology for a dark energy component with constant negative w.
- Here we face it even for dust and similarly as for the dark energy the problem should be addressed when considering the formation of structure.
- It further affects the CMB sound horizon and acoustic oscillations.
- So this effect should be extremely small (β sufficiently small).

Models IIA - departure from standard speed of sound

The departure Δc_S^2 from the standard sound speed for a barotropic perfect fluid with constant w is

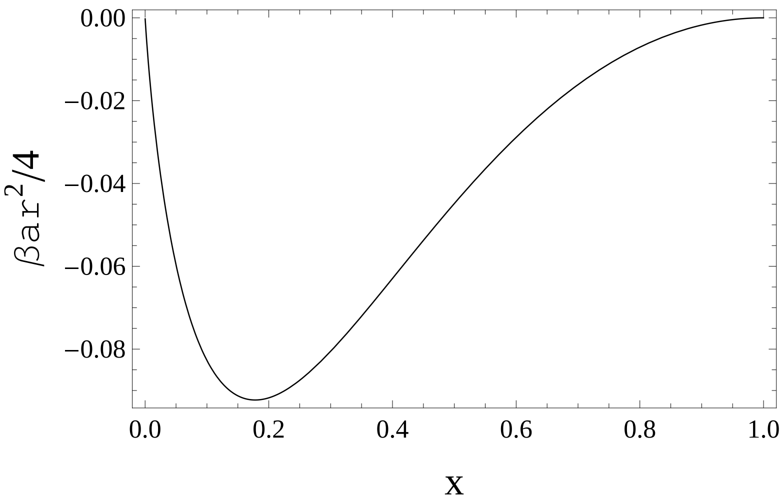
$$\Delta c_S^2(r,a) \propto \beta a r^2 c^2 \propto -\Omega_{\beta,0}\frac{a}{a_0}H_0^2(a_0 r)^2 \, . \tag{34}$$

The quantity $H_0 a_0 r$ has unit of velocity and can be estimated from

$$H_0 a_0 r = 100h\frac{a_0 r}{\mathrm{Mpc}}\mathrm{km/s} \quad h \equiv H_0/(100\mathrm{km/s/Mpc}). \tag{35}$$

It does not become too large for the observational data which can be seen if one plots the value of $\beta a r^2/4$ below ($x = a/a_0 = 1/(1+z)$):

Models IIA - departure from standard speed of sound



- Vanishes both at the Big Bang $a = 0$ and today at $r = 0$.
- Has a modification of about -9% at $z \sim 4$.
- At very small values of $x \lesssim 10^{-3}$, we have $\frac{1}{4}|\beta|ar^2 \lesssim 10^{-3}$
- Plot for $\Omega_{inh,0} = 0.68$ and $w \equiv w(r = 0) = -0.08$.

Exact inhomogeneous pressure model IIB

In Model **IIB** (Dąbrowski 1993, 1995) the scale factor is of the **dust-like type**

$$a(t) = \sigma t^{2/3},\ k(t) = -\alpha\sigma a(t), \quad , \tag{36}$$

($[\alpha] = (s/km)^{2/3} Mpc^{-4/3}$, $[\sigma] = (km/s)^{2/3} Mpc^{1/3}$, $[t] = sMpc/km$) but the equation of state at the center of symmetry is no longer barotropic:

$$\rho = p\left(\frac{32\pi^2 G^2}{3\alpha^3 c^8} p^2 - \frac{3}{2}\right) \quad . \tag{37}$$

In the limit of the **inhomogeneity parameter** $\alpha \to 0$ one obtains the Friedmann universe. FD singularity of pressure is at $r \to \infty$.

3. Off-center observer position constrained by supernovae.

The luminosity distance is given by

$$d_L = \frac{a_0(1+z)\hat{r}'}{1+\frac{\beta}{4}a_0 r_0^2} \,, \tag{38}$$

with an **off-center** observer placed at r_0, θ_0, ϕ_0 as meant in the coordinate system $\{t, r, \theta, \varphi\}$ of the Stephani metric. More precisely we have

$$d_L = \frac{(1+z)}{1-\frac{a_0 H_0^2 \Omega_{inh}}{4} r_0^2} \, \hat{r}'(\Omega_{inh}, w, r_0, \theta_0, \varphi_0, H_0, \hat{\theta}', \hat{\varphi}', z) \,, \tag{39}$$

where

$$\hat{r}' = \hat{r}'(a) = \frac{1}{H_0}\int_{a_e}^{1} \frac{dx}{\sqrt{(1-\Omega_{inh})x^{1-3w} + \Omega_{inh}x^3}} \,, \tag{40}$$

and a_e is the value of the scale factor at the moment of an emission of the light ray.

Off-center observers

For the redshift one takes

$$1 + z = \frac{a_0 (4 - a_e H_0^2 \Omega_{inh} r_e^2)}{a_e (4 - a_0 H_0^2 \Omega_{inh} r_0^2)} \;, \tag{41}$$

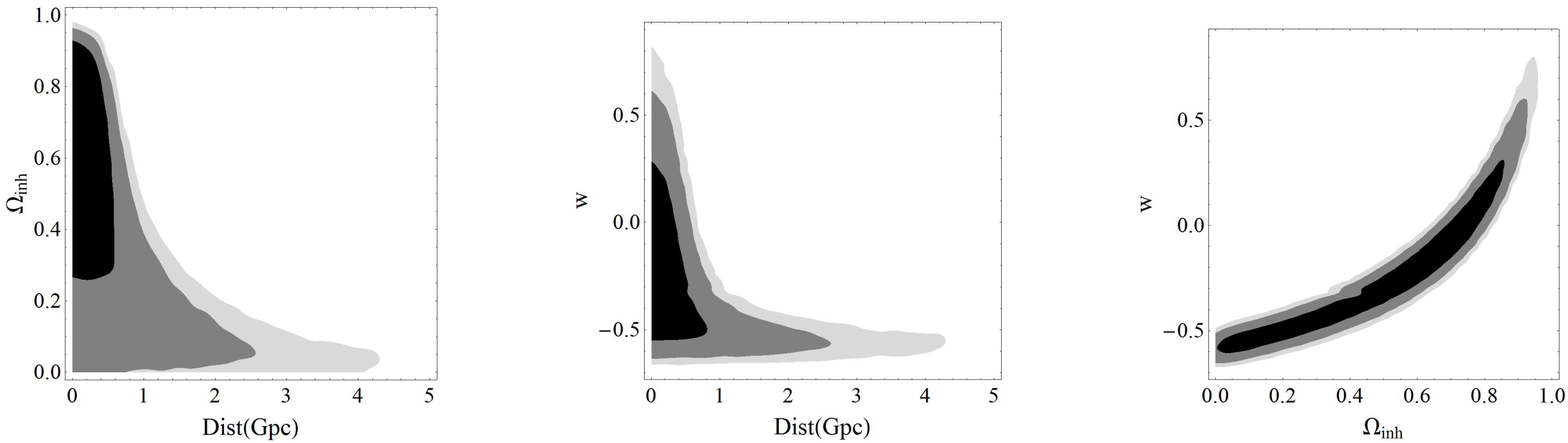
where

$$\begin{aligned} r_e^2 &= (r_0 \sin\theta_0 \cos\varphi_0 + \hat{r}'(a) \sin\hat{\theta}' \cos\hat{\varphi}')^2 \\ &+ (r_0 \sin\theta_0 \sin\varphi_0 + \hat{r}'(a) \sin\hat{\theta}' \sin\hat{\varphi}')^2 \\ &+ (r_0 \cos\theta_0 + \hat{r}'(a) \cos\hat{\theta}' \sin\hat{\varphi}')^2 \end{aligned} \tag{42}$$

and $\hat{\theta}'$ and $\hat{\varphi}'$ are the coordinates of a supernova as seen by an off-center observer in the sky (Balcerzak, MPD, Denkiewicz 2014).

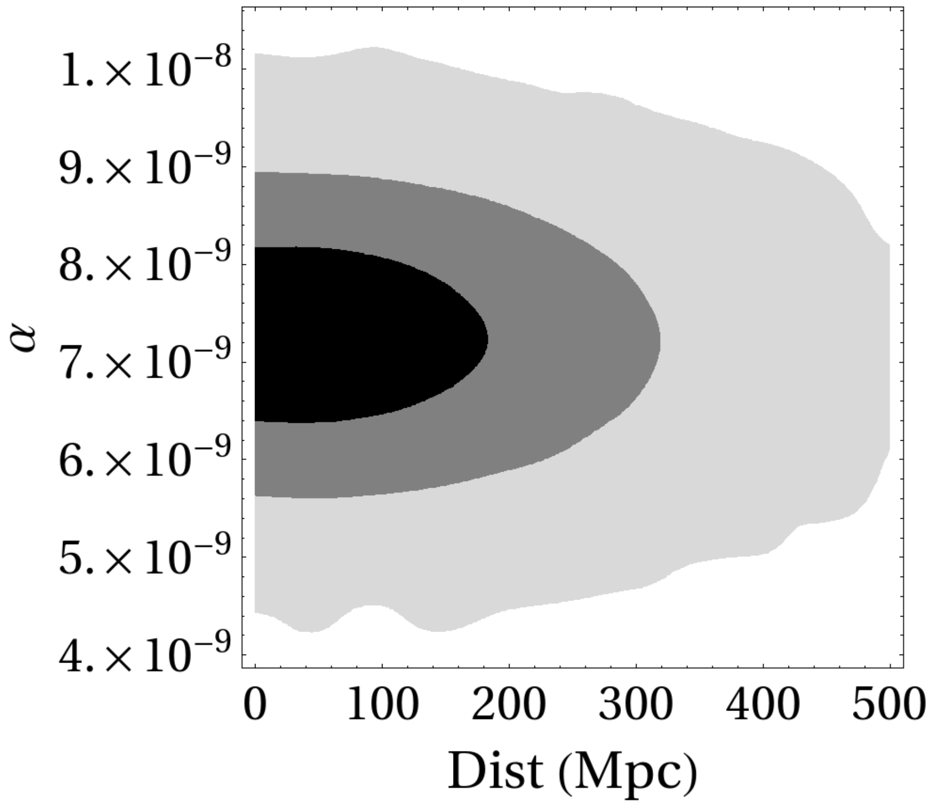
We applied **Union2 557 supernovae data** of Amanullah et al. (2010, ApJ, 716, 712) - we note the courtesy of M. Kowalski and U. Feindt to consult the sample.

Off-center observers - model IIA



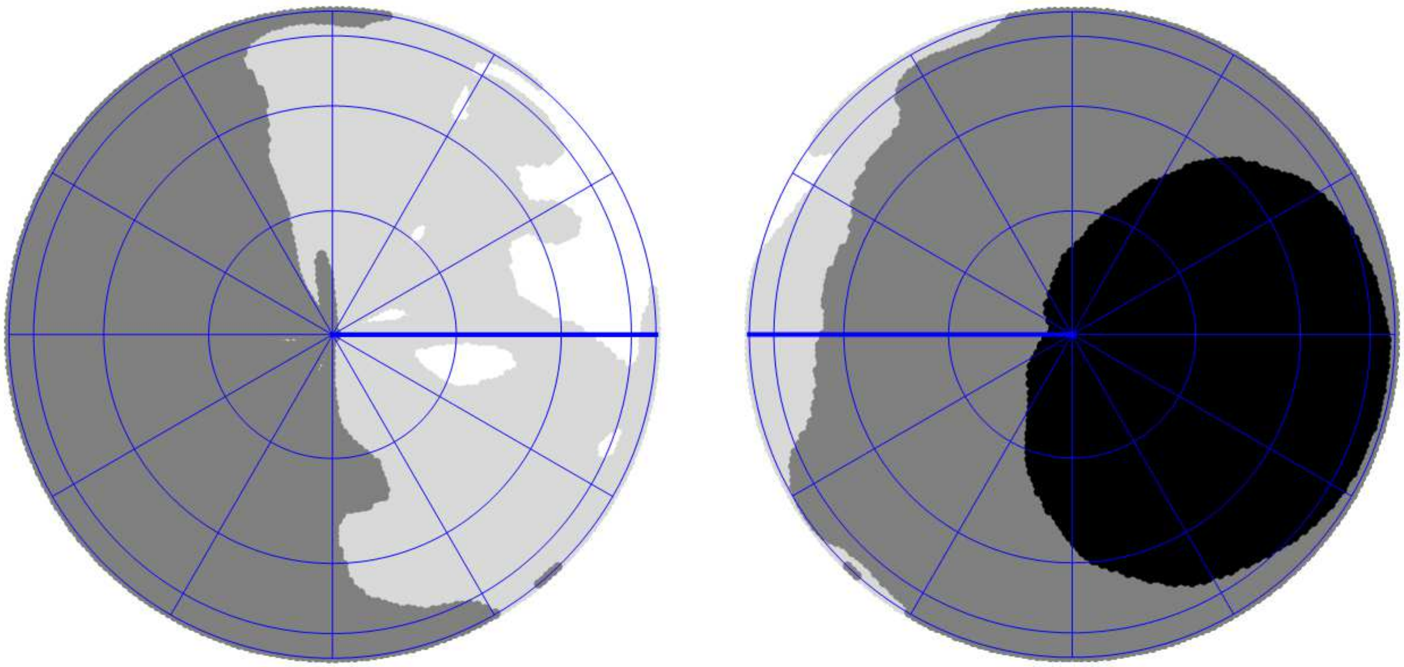
Best-fit values: inhomogeneity density $\Omega_{inh} \sim 0.77$, center of symmetry equation of state barotropic index $w \sim 0.093$, off-center observer position $Dist = 341$ Mpc $(\chi^2 = 526)$.

Off-center observers - model IIB



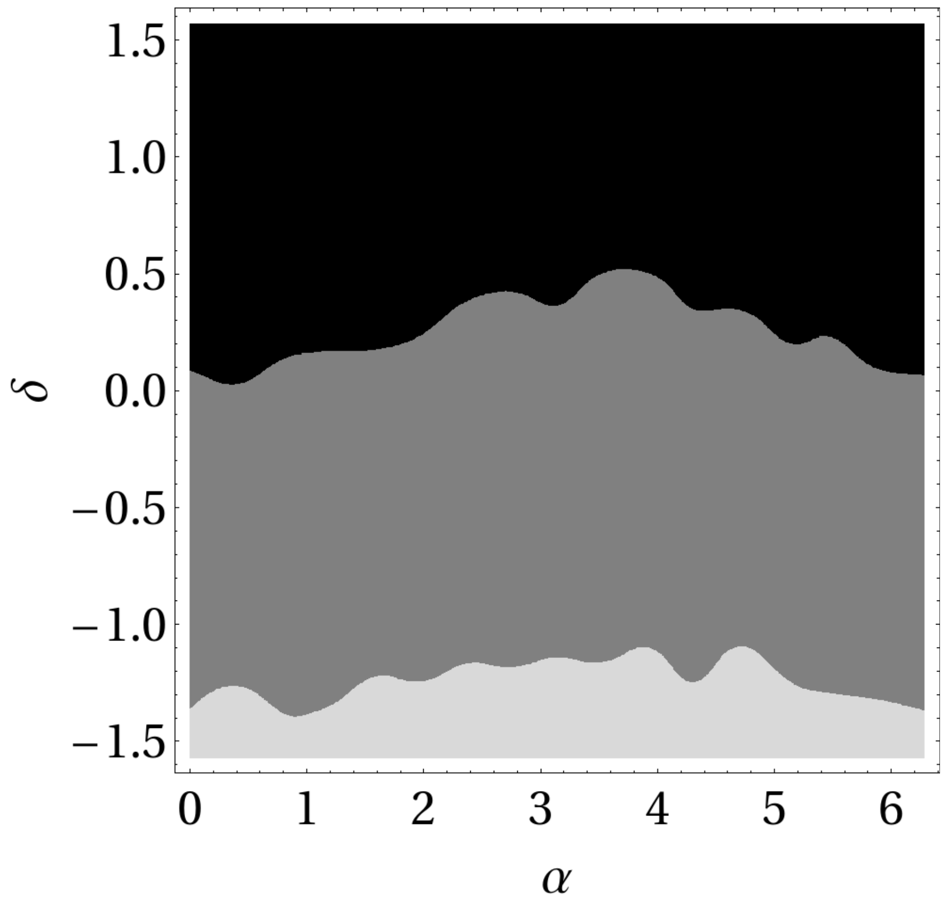
Non-barotropic EOS **limits stronger** the position of an observer. **Best-fit values**: inhomogeneity parameter $\alpha = 7.31 \cdot 10^{-9}\ (s/km)^{2/3} Mpc^{-4/3}$, off-center observer position $Dist = 68$ Mpc $(\chi^2 = 557)$ $(\alpha = 0$ case (dust) excluded).

Position of the center of symmetry (inhomogeneity)- model IIA



Best fit position: declination $\delta = -65.75^\circ$ and R.A. is $a = 187.33^\circ$. North Celestial Hemisphere (left), South Celestial Hemisphere (right), Meridian line ($\delta = 0$) in bold. In galactic coordinates: $(l, b) = (300.66^\circ, -2.98^\circ)$.

Position of the center of symmetry (inhomogeneity)- model IIB



Best fit position: $\delta = 69.35°$, $a = 8.39°$. In galactic coordinates: $(l, b) = (121.35°, 6.53°)$.

Dipole puzzles:

- Dark flow direction (Watkins et al. 2009) at $(l, b) = (287 \pm 9, 8 \pm 6)$
- Dark energy dipole (Mariano et al. 2012) at $(l.b) = (309, -15)$
- Fine structure α dipole (Webb et al. 2011) at $(l, b) = (320, -11)$
- kSZ effect on CMB (Kashlinsky et al. 2010) at $(l, b) = (296 \pm 13, 140 \pm 13)$
- Dark flow direction (Turnball et al. 2012) at $(l, b) = (319 \pm 18, 70 \pm 14)$
- Model IIA ($(l, b) = (300.66^\circ, -2.98^\circ)$) seems to be compatible with the above results (Balcerzak, MPD, Denkiewicz 2014).

4. Full observational check (supernovae, redshift drift, BAO, CMB shift parameter) for a central observer.

The luminosity distance for a central observer $r_0 = 0$ is (same as Friedmann)

$$d_L = (1+z)a_0 r \ , \tag{43}$$

and the distance modulus is

$$\mu(z) = 5\log_{10} d_L(z) + 25. \tag{44}$$

From the null geodesic equations we have (model IIA)

$$r = c\int_a^{a_0} \frac{da}{\sqrt{c^2A^2a^{1-3w} - \beta c^2 a^3}} = r = \frac{c}{H_0 a_0}\int_{a/a_0}^{1} \frac{dx}{\sqrt{\Omega_0 x^{1-3w} + (1-\Omega_0)x^3}} \ , \tag{45}$$

where $x \equiv a/a_0$.

Luminosity distance, apparent magnitude

Using the definition of redshift (57) one can rewrite (45) as

$$z(x) = \frac{1}{x} - 1 + \frac{\Omega_0 - 1}{4} \left[\int_{a/a_0}^{1} \frac{dx}{\sqrt{\Omega_0 x^{1-3w} + (1-\Omega_0) x^3}} \right]^2 , \qquad (46)$$

and so the luminosity distance (43) reads as

$$d_L(x) = \frac{c(1+z)}{H_0} \sqrt{\frac{4[z(x) + 1 - 1/x]}{\Omega_0 - 1}} . \qquad (47)$$

CMB shift parameter.

The shift parameter is defined as:

$$\mathcal{R} = \frac{l_1'^{TT}}{l_1^{TT}} \ , \tag{48}$$

where l_1^{TT} – the temperature perturbation CMB spectrum multipole of the first acoustic peak in inh. pressure model
$l_1'^{TT}$ – the multipole of a reference flat standard Cold Dark Matter model. The multipole number is related to an angular scale of the sound horizon r_s at decoupling by

$$\theta_1 = \frac{r_s}{d_A} \propto \frac{1}{l_1} \ . \tag{49}$$

For our Stephani model the angular diameter distance is given by

$$d_A = \frac{a_{\rm dec}}{V(t_{dec}, r_{dec})} r_{\rm dec} \tag{50}$$

with $r_{\rm dec}$ given by (45) taken at decoupling.

CMB shift parameter.

Using the above, we may write that for our Stephani models the shift parameter is

$$\mathcal{R} = \frac{2cV(t_{dec}, r_{dec})}{H_0\sqrt{\Omega_0} r_{\rm dec}} \ . \tag{51}$$

Finally, the rescaled shift parameter is

$$\bar{\mathcal{R}} \quad = \quad \frac{H_0\sqrt{\Omega_0} r_{\rm dec}}{cV(t_{dec}, r_{dec})} \ . \tag{52}$$

The WMAP data gives $\bar{\mathcal{R}} = 1.70 \pm 0.03$ (Wang, Mukherjee 2006).

Baryon acoustic oscillations.

One calculates the distortion of a spherical object in the sky without knowing its true size by measuring its transverse extent using the angular diameter distance, r

$$r = \frac{l}{\Delta\theta} \ , \tag{53}$$

where l and $\Delta\theta$ are the linear and angular size of an object, and its line-of-sight extent, Δr, using the redshift distance

$$\Delta r = \frac{c\Delta t}{a(t)} \tag{54}$$

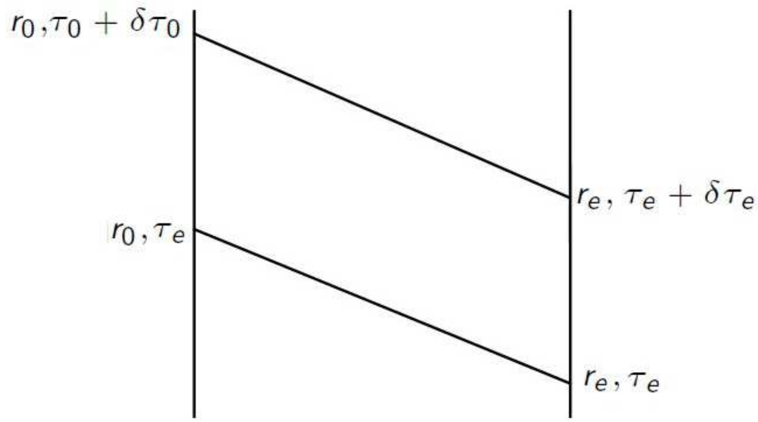
(see e.g. Nesseris (2006)). As a result one can define the volume distance, D_V, as

$$D_V^3 = r^2 \Delta r \quad . \tag{55}$$

Eisenstein et al. (2005) gave $D_V(\Delta z = z_{BAO} = 0.35) = 1370 \pm 64$ Mpc (an acoustic peak for 46748 luminous red galaxies (LRG) selected from the SDSS (Sloan Digital Sky Survey).

Redshift drift (RD) test.

Redshift drift (Sandage 1962) test is an idea to collect data from two light cones separated by 10-20 years to look for a change in redshift of a source as a function of time.



There is a relation between the times of emission of light by the source τ_e and $\tau_e + \delta\tau_e$ and times of their observation at τ_o and $\tau_o + \delta\tau_o$:

$$\int_{\tau_e}^{\tau_o} \frac{d\tau}{a(\tau)} = \int_{\tau_e+\delta\tau_e}^{\tau_o+\delta\tau_o} \frac{d\tau}{a(\tau)} , \qquad (56)$$

which for small $\delta\tau_e$ and $\delta\tau_o$ reads as $\frac{\delta\tau_e}{a(\tau_e)} = \frac{\delta\tau_o}{a(\tau_o)}$.

Redshift drift in inhomogeneous pressure models.

For small $\delta\tau_e$ and $\delta\tau_o$ we expand in Taylor series

$$
\begin{aligned}
(u_a k^a)_o &= (u_a k^a)(r_0, \tau_0 + \delta\tau_0) = (u_a k^a)(r_0, \tau_0) + \left[\frac{\partial(u_a k^a)}{\partial\tau}\right]_{(r_0,\tau_0)} \delta\tau_0 \\
(u_a k^a)_e &= (u_a k^a)(r_e, \tau_e + \delta\tau_e) = (u_a k^a)(r_e, \tau_e) + \left[\frac{\partial(u_a k^a)}{\partial\tau}\right]_{(r_e,\tau_e)} \delta\tau_e \ ,
\end{aligned}
$$

where for inhomogeneous pressure models the readshift reads as

$$
1 \ + \ z \ = \frac{(u_a k^a)_e}{(u_a k^a)_O} = \frac{\frac{V(t_e,r_e)}{R(t_e)}}{\frac{V(t_0,r_0)}{R(t_0)}} \tag{57}
$$

From the definition of the redshift drift by Sandage (1962):

$$
\delta z = z_e - z_0 = \frac{(u_a k^a)(r_e, \tau_e + \delta\tau_e)}{(u_a k^a)(r_0, \tau_0 + \delta\tau_0)} - \frac{(u_a k^a)(r_e, \tau_e)}{(u_a k^a)(r_0, \tau_0)} \ , \tag{58}
$$

Redshift drift in inhomogeneous pressure models.

For a general spherically symmetric Stephani metric we obtain

$$\frac{\partial}{\partial \tau}\left(u_a k^a\right) = -\left(\frac{1}{a}\right)^{\cdot} - \frac{1}{4}\left(\frac{k}{a}\right)^{\cdot} r^2 \, , \tag{59}$$

and

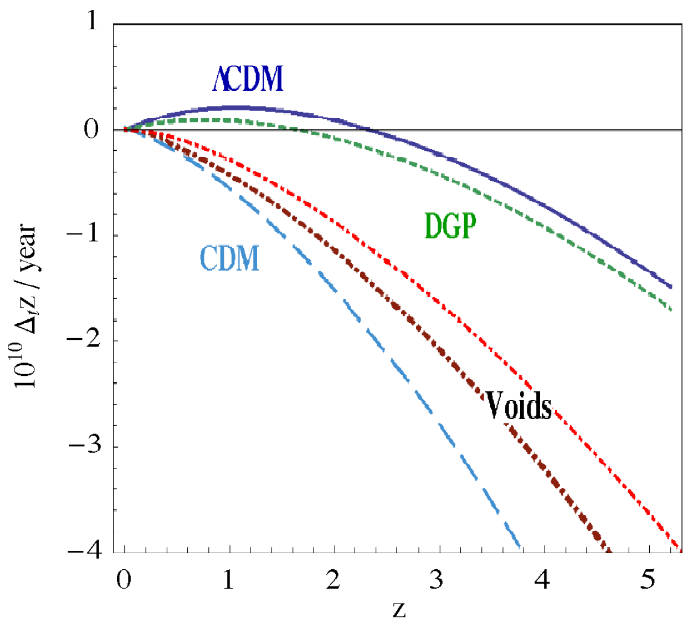
$$\frac{\delta z}{\delta \tau_0} = \frac{\left[\left(\frac{1}{a}\right)^{\cdot} - \frac{1}{4}\left(\frac{k}{a}\right)^{\cdot} r^2\right]_e}{\left[1+\frac{1}{4}kr^2\right]_e} a(\tau_e) - \frac{\left[\left(\frac{1}{a}\right)^{\cdot} + \frac{1}{4}\left(\frac{k}{a}\right)^{\cdot} r^2\right]_o}{\left[1+\frac{1}{4}kr^2\right]_o} a(\tau_0)(1+z) \tag{60}$$

For the model with $(k/a)^{\cdot} = 0$ we have

$$\frac{\delta z}{\delta \tau} = -\frac{H_0}{1+\frac{1}{4}k(\tau_0)r_0^2}\left[\frac{H_e(z)}{H_0} - (1+z)\right] \, . \tag{61}$$

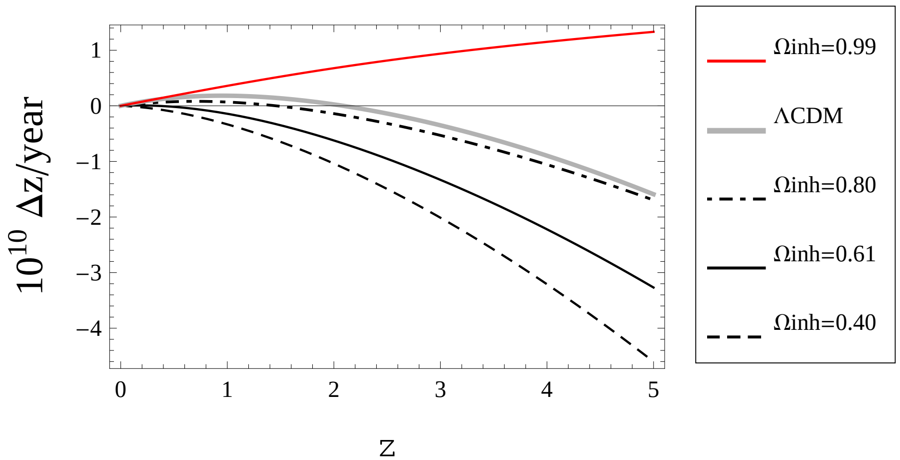
Sandage-Loeb CDM formula for $\Omega_{inh} \to 0$; $H_e(z) = H_0(1+z)^{3/2}$, $r_0 \to 0$.

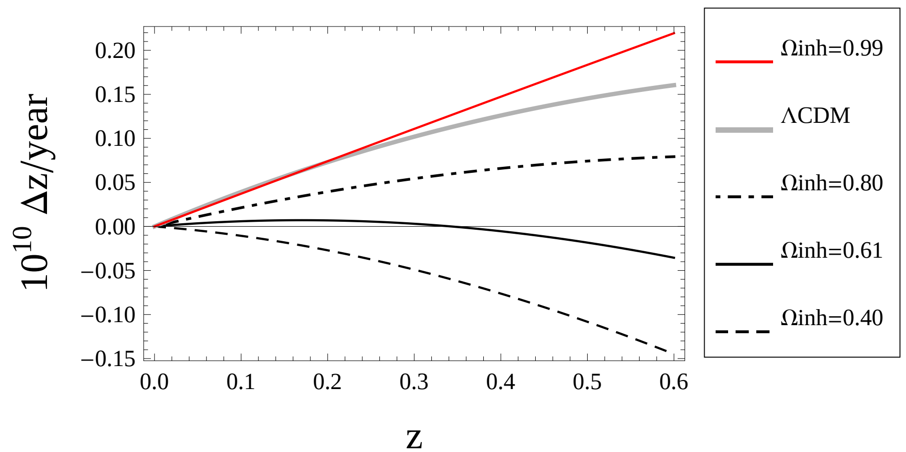
Redshift drift - LTB voids.



- Plots for **3 different LTB** void models, ΛCDM, brane DGP, Cold Dark Matter (CMD) (Quercellini et. al, 2012).
- ΛCDM – the drift is **positive at small redshift**, but becomes negative for $z \gtrsim 2$.
- Giant void (LTB) model mimicking dark energy - the drift is **always negative**.

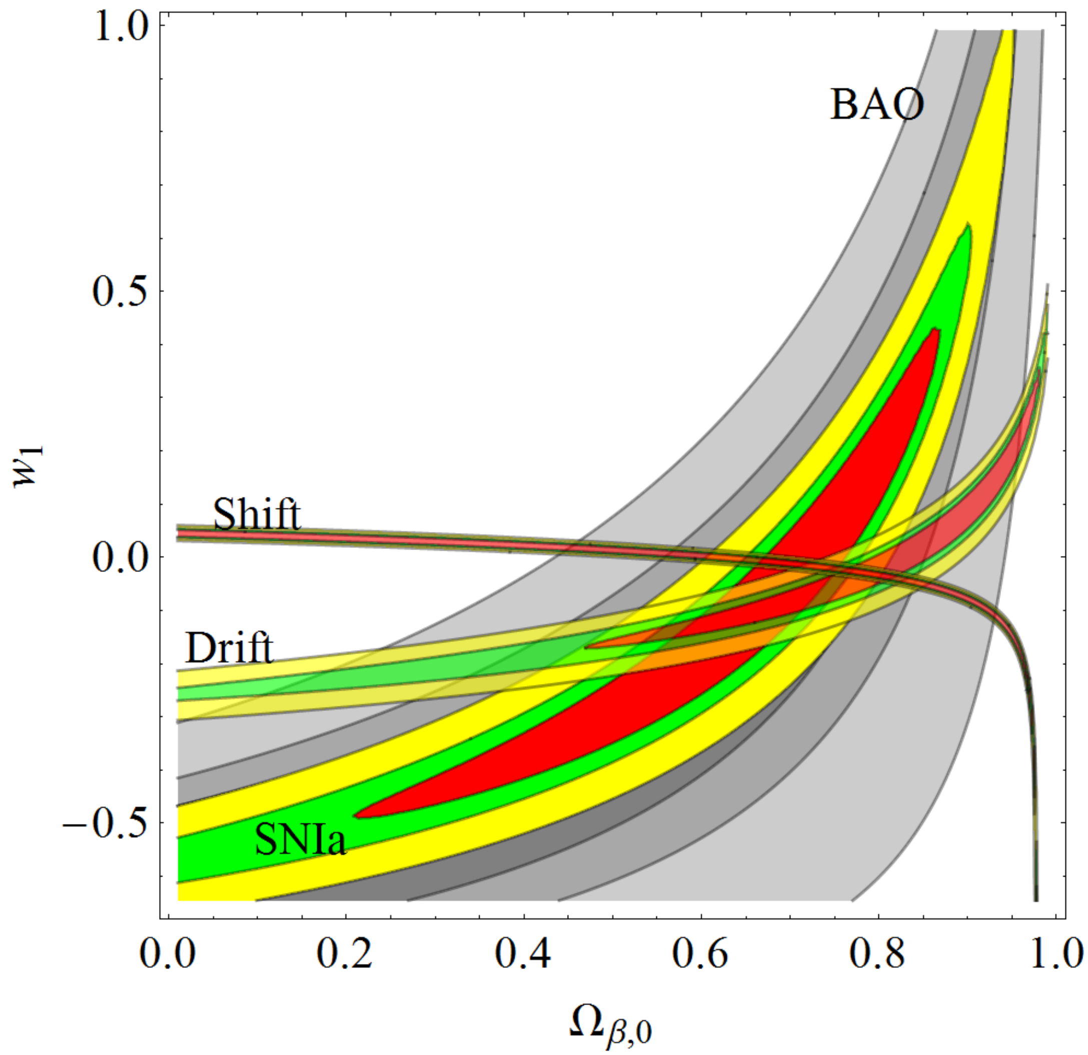
Redshift drift - inhomogeneous pressure models ($r_0 = 0$, $w = 0$).





- Ω_{inh} **small** - drift as in LTB and CDM models
- Ω_{inh} **larger** - drift as in ΛCDM models (first positive, then negative), e.g. for $\Omega_{inh} = 0.61$ drift is positive for $z \in (0, 0.34)$.
- Ω_{inh} **very large** - drift positive ($\Omega_{inh} = 0.99$ up to $z = 17$; $\Omega_{inh} = 1$ (inhomogeneity-domination) $z > 0$) and $\frac{\delta z}{\delta t} = H_0 \frac{z}{2}$ which means that the drift **grows linearly** with redshift.

Inhomogeneous pressure - combined tests (SNIa, RD, BAO, shift parameter)



Inhomogeneous pressure - combined tests: results and improvements

- Stephani model with $w = w_{eff}(r=0) \equiv w_1$ fits well the data for the SNIa, redshift drift, and BAO (contours overlap at 1σ CL).
- However, it does not fit the data for the shift parameter features of ΛCDM model.
- Possible solution: replace constant barotropic index w by $w(a)$ and assume that $w(a)$ suddenly changes somewhere between $z = 5$ and z_{dec}, and then remains constant.

Now we have

$$H^2(a) = H_0^2 \left[\Omega_0 \, f(a) + \Omega_{inh,0} \, \frac{a_0}{a} \right] \, , \tag{62}$$

where

$$\varrho(a) = \varrho_0 \, \exp\left[-3 \int_{a_0}^{a} da' \, \frac{1 + w(a')}{a'} \right] \equiv \varrho_0 \, f(a) \, , \tag{63}$$

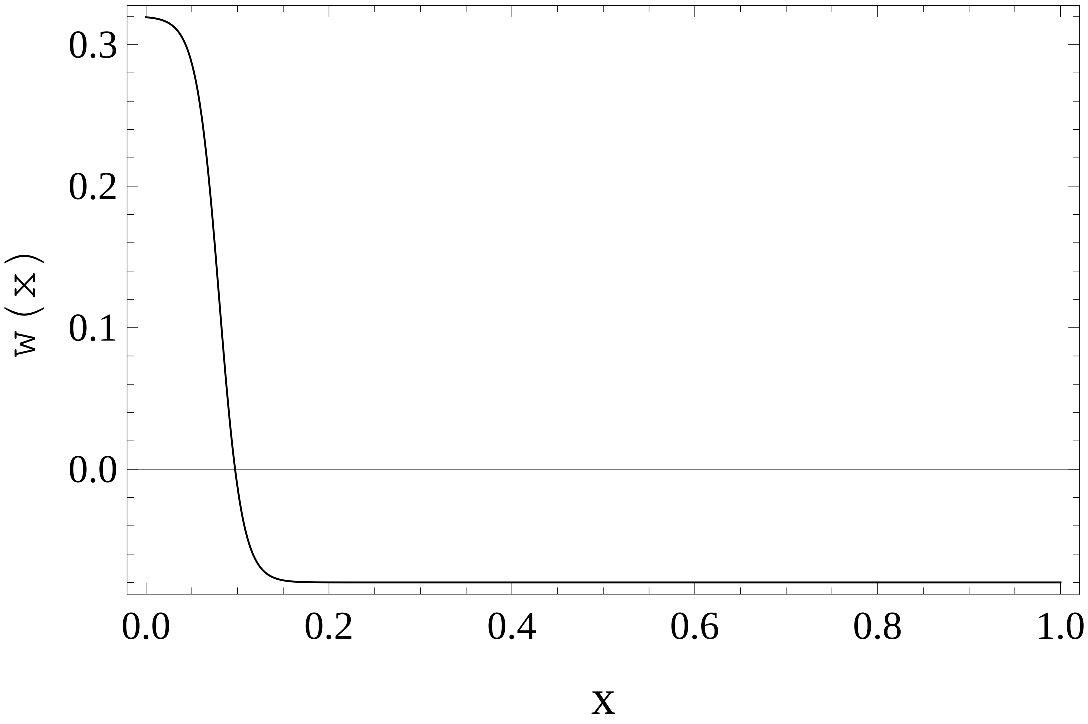
wch for $w(a) = w$const. gives standard $\rho = \rho_0 (a/a_0)^{-3(w+1)}$.

Inhomogeneous pressure models - $w(a)$ parametrization

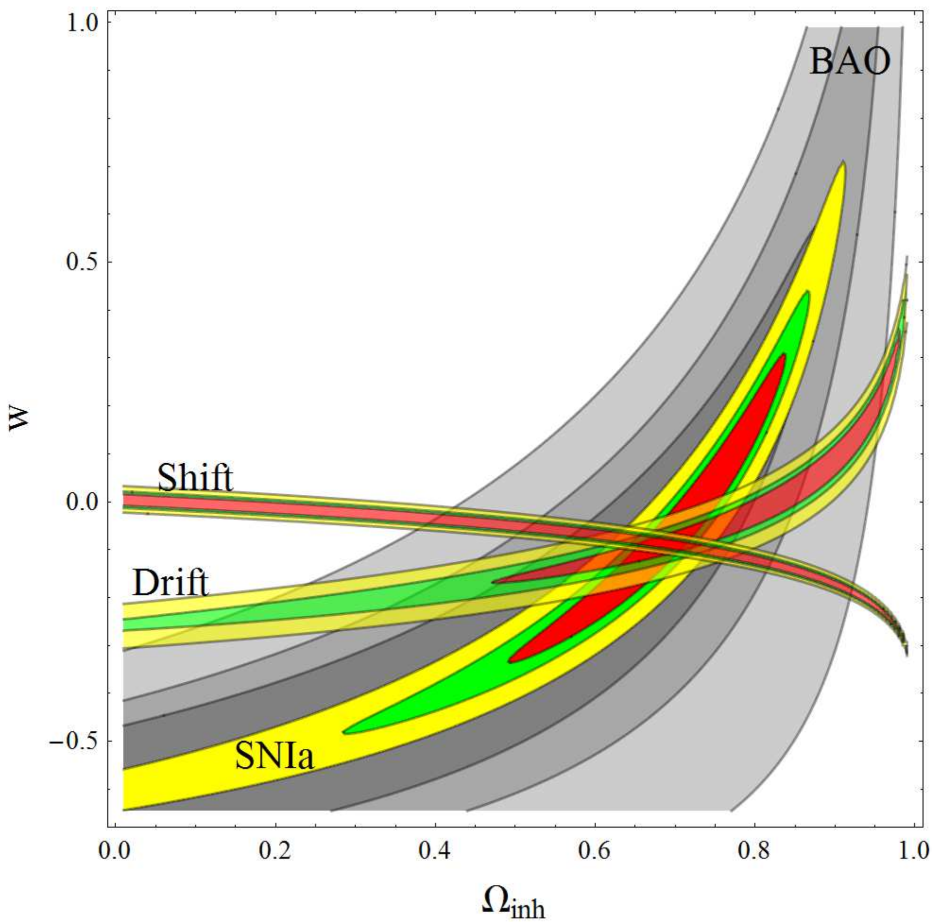
An example of a barotropic index parametrization $w(a)$ which can fit the data is:

$$w(a) = w_1 + \frac{w_2}{2}\left(1 + \tanh[\lambda(a_{tr} - a)]\right) \ . \tag{64}$$

where w, w_0, λ, and a_{tr} are constants. Here: $\lambda = 40$, $a_{tr} = 0.08$, $w_1 = -0.08$, $w_2 = 0.4$ and $\Omega_{inh} = 0.68$, $z_{tr} \sim 10.49$.



Inhomogeneous pressure models - combined tests for $w(a)$



This model fits very well the data for the SNIa, redshift drift, shift parameter, and BAO (contours overlap at 1σ CL).

Advantages for future studies

The general inhomogeneous pressure metric

$$
\begin{aligned}
ds^2 &= -\frac{a^2}{\dot{a}^2}\frac{a^2}{V^2}\left[\left(\frac{V}{a}\right)^{\cdot}\right]^2 dt^2 + \frac{a^2}{V^2}\left[dx^2 + dy^2 + dz^2\right] , \quad (65)\\
V(t,x,y,z) &= 1 + \frac{1}{4}k(t)\left\{[x - x_0(t)]^2 + [y - y_0(t)]^2 + [z - z_0(t)]^2\right\} ,
\end{aligned}
$$

with x_0, y_0, z_0 being arbitrary functions of time is just a generalization of both the FRW and spherically symmetric Stephani (7) metrics in isotropic coordinates.

- It is fully inhomogeneous - **no symmetries** acting on spacetime.
- It is still **conformally flat** (Weyl tensor $C_{abcd} = 0$).
- Can be made **"density inhomogeneized"** taking $x_0(t) \to x_0(t, x, y, z)$ etc.
- **Energy conditions** studied by MPD (PRD 71, 103505 (2005)) and can be fulfilled according to Green and Wald assumptions (PRD 87, 124037, 2013) - MPD, A. Balcerzak, J. Ostrowski - in progress.

5. Conclusions

- Complementary to LTB cosmologies which **can theoretically drive acceleration**.
- Sole application of 557 Union2 supernovae data **restricts the position of non-centrally placed observers**. Best fit values are: $Dist = 341Mpc$ (model IIA) with inhomogeneity density $\Omega_{inh} = 0.77$ (model IIA) and $Dist = 68Mpc$ with inhomogeneity parameter $\alpha = 7.31 \cdot 10^{-9}$ $(s/km)^{2/3}Mpc^{-4/3}$ (model IIB).
- Gives a dipole which is directed at $(l, b) = (300.66°, -2.98°)$ (model IIA) and $(l, b) = (121.35°, 6.53°)$ (model IIB) and can be compared (if aligned) with other dipoles (dark energy, dark flow, varying-α dipole, etc.)
- Stephani model fits well the data for SNIa, redshift drift, shift parameter, and BAO provided a specific parametrization for $w = w(a)$ is applied and the inhomogeneity is small $\frac{1}{4}|\beta|ar^2 \lesssim 10^{-3}$ for large redshift $z \lesssim 1000$.
- Due to its conformal flatness and possible modelling of a **full spacetime inhomogeneity** it can perhaps be applied to test Green and Wald theorem.

Thank You!

Physical interpretation of inhomogeneous pressure models

- A fluid with spatially varying equation of state (spatially varying vacuum energy like Λ) is assumed which gives a nongravitational force in the Universe (which manifests as non-zero acceleration of comoving observers).
- Inhomogeneous pressure models can be considered as a kind of interior of a TOV exotic star filled with matter like generalized (anti)-Chaplygin gas $p = \pm A^2/\varrho^\alpha$ ($A =$ const.) (e.g. Kamenschchik et al. 2004, 2008).
- In these models there exists a static spherically symmetric configuration in which the central pressure at $r = 0$ was constant, while on some shell of constant radius r_s it became minus infinity (which is an analogue of a FD singularity). Everywhere between $r = 0$ and $r = r_s$, the pressure is lower than at the center, so that the particles are accelerated away which is exactly the effect which is present in the inhomogeneous pressure model.
- There is also an ideal gas interpretation of these models (Sussmann 2000).