# Testing varying speed of light cosmologies in future experiments.

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## Based on:

- A. Balcerzak, MPD, PLB **728**, 15 (2014).
- V. Salzano, MPD, R. Lazkoz, PRL **114**, 101304 (2015).
- V. Salzano, MPD, R. Lazkoz, arXiv: 1511.04732.
- V. Salzano, A. Balcerzak, MPD - in progress

# Varying speed of light $c$ (VSL) theories

Attempts: Einstein (1907), Dicke (1957), J.-P. Petit (1988) (Einstein eqs remain same due to fine-tuned change of $c$ and $G$), Moffat (1992).

Albrecht & Magueijo model (1998) (AM model) (Barrow 1999; Magueijo 2003): Introduce a scalar field

$$c^4 = \psi(x^\mu) \tag{1}$$

and so the action is

$$S = \int d^4x \sqrt{-g} \left[ \frac{\psi(R + 2\Lambda)}{16\pi G} + L_m + L_\psi \right] \tag{2}$$

AM model breaks Lorentz invariance (relativity principle and light principle) so that there is a preferred frame (cosmological or CMB) in which the field is minimally coupled to gravity. The Riemann tensor is computed in such a frame for a constant $\psi = c^4$ and no additional terms $\partial_\mu \psi$ appear in this frame (though they do in other frames). Einstein eqs remain the same **except** $c$ now varies.

# VSL and varying fine structure constant $\alpha$ theories

Due to the definition of the fine structure constant

$$\alpha = \frac{e^2}{\hbar c} \quad \text{i.e.} \quad \alpha(t) = \frac{e^2}{\hbar c(t)}, \qquad \frac{\Delta\alpha}{\alpha} = -\frac{\Delta c}{c}, \tag{3}$$

VSL theories can be related with varying fine structure constant $\alpha$ (or charge $e = e_0\epsilon(x^\mu)$ theories (Webb et al. 1999, Sandvik 2002))

$$S = \int d^4x\sqrt{-g}\left(R - \frac{\omega}{2}\partial_\mu\psi\partial^\mu\psi - \frac{1}{4}f_{\mu\nu}f^{\mu\nu}e^{-2\psi} + L_m\right) \tag{4}$$

with $\psi = \ln\epsilon$ and $f_{\mu\nu} = \epsilon F_{\mu\nu}$. Constraints:
Oklo natural nuclear reactor: $\Delta\alpha/\alpha = (0.15 \pm 1.05)\cdot 10^{-7}$ at z = 0.14
VLT/UVES quasars: $\Delta\alpha/\alpha = (0.15 \pm 0.43)\cdot 10^{-5}$ at 1.59 < z < 2.92
SDSS quasars: $\Delta\alpha/\alpha = (1.2 \pm 0.7)\cdot 10^{-4}$ at 0.16 < z < 0.8
Atomic clocks (Rosenband (2008)) at $z = 0$: $\left(\frac{\dot\alpha}{\alpha}\right)_0 = (-1.6 \pm 2.3)\times 10^{-17}\text{yr}^{-1}$.

## More bounds on variation of $\alpha$.

By Webb et al. (PRL 107, 191101 (2011)) ($\alpha$-dipole $R.A.17.4 \pm 0.9h$, $\delta = -58 \pm 9$: Keck ($\Delta\alpha < 0$) and VLT) as well as other specific measurements of $\alpha$ given in the table below (in parts per million):

| Object | z | $\Delta\alpha/\alpha$ | Spectrograph | Ref. |
|---|---|---|---|---|
| HE0515$-$4414 | 1.15 | $-0.1 \pm 1.8$ | UVES | Molaro et al. (2008) |
| HE0515$-$4414 | 1.15 | $0.5 \pm 2.4$ | HARPS/UVES | Chand et al. (2006) |
| HE0001$-$2340 | 1.58 | $-1.5 \pm 2.6$ | UVES | Agafonowa et al. (2011) |
| HE2217$-$2818 | 1.69 | $1.3 \pm 2.6$ | UVES–LP | Molaro et al. (2013) |
| Q1101$-$264 | 1.84 | $5.7 \pm 2.7$ | UVES | Molaro et al. (2008) |

UVES - Ultraviolet and Visual Echelle Telescope

HARPS - High Accuracy Radial velocity Planet Searcher

LP - Large Program measurement

# VSL - simple generalization of the Einstein equations.

Einstein eqs. the same except $c$ now varies - ($\varrho$ - mass density; $\varepsilon = \varrho c^2(t)$ - energy density in $Jm^{-3} = Nm^{-2} = kgm^{-1}s^{-2}$)

$$\varrho(t) = \frac{3}{8\pi G}\left(\frac{\dot{a}^2}{a^2} + \frac{kc^2(t)}{a^2}\right), \tag{5}$$

$$p(t) = -\frac{c^2(t)}{8\pi G}\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{kc^2(t)}{a^2}\right), \tag{6}$$

but the continuity eq. contains an extra term (obtained from (5) and (6))

$$\dot{\varrho}(t) + 3\frac{\dot{a}}{a}\left(\varrho(t) + \frac{p(t)}{c^2(t)}\right) = 3\frac{kc(t)\dot{c}(t)}{4\pi G a^2}. \tag{7}$$

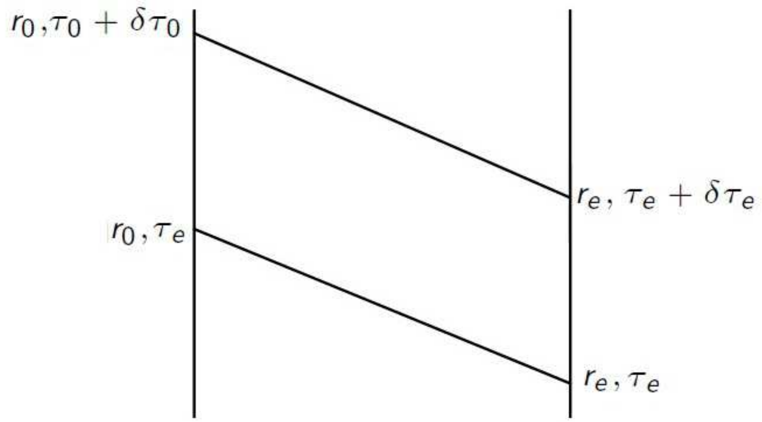
Benefits:

Can solve the horizon, flatness and singularity problems.

Can mimick dark energy (see later).

# Tests in future experiments.

1. Redshift drift (Sandage 1962, Loeb 1998) - the idea is to collect data from two light cones separated by 10-20 years to look for a change in redshift of a source as a function of time.




There is a relation between the times of emission of light by the source $\tau_e$ and $\tau_e + \Delta\tau_e$ and times of their observation at $\tau_o$ and $\tau_o + \Delta\tau_o$ (VSL adopted):

$$\int_{t_e}^{t_o} \frac{c(t)dt}{a(t)} = \int_{t_e+\Delta t_e}^{t_o+\Delta t_o} \frac{c(t)dt}{a(t)} , \qquad (8)$$

which for small $\Delta t_e$ and $\Delta t_o$ transforms into $\frac{c(t_e)\Delta t_e}{a(t_e)} = \frac{c(t_0)\Delta t_o}{a(t_o)}$ .

## Redshift drift - varying $c$

Assuming the ansatz for the variability of the speed of light

$$c(t) = c_0 a^n(t), \qquad n = \text{const.}, \tag{9}$$

and bearing in mind definitions of $\Omega$'s, assuming flat $k = 0$ model we have

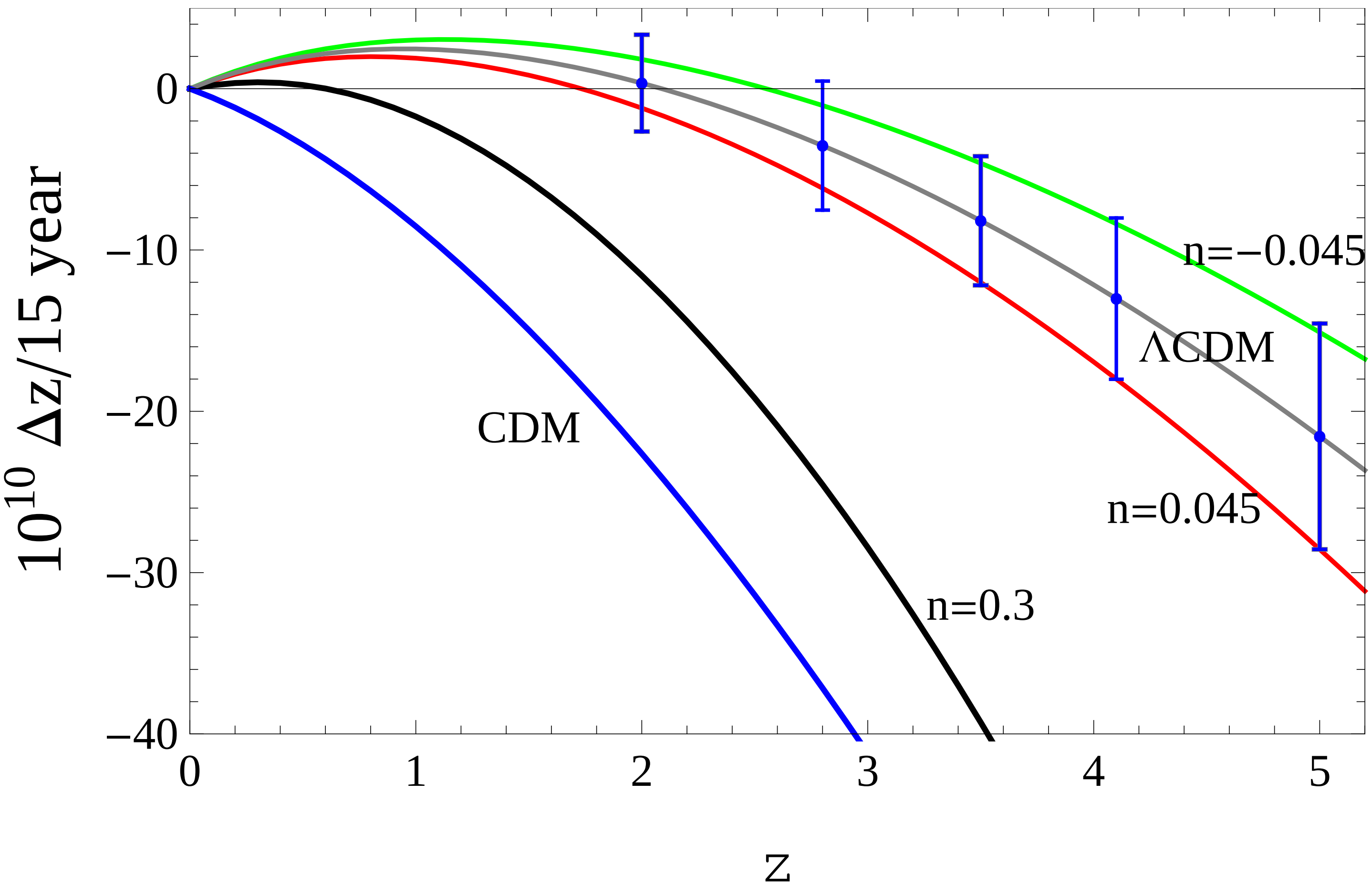
$$\frac{\Delta z}{\Delta t_0} = H_0 \left[ 1 + z - (1+z)^n \sqrt{\Omega_{m0}(1+z)^3 + \Omega_\Lambda} \right] \tag{10}$$

which can further be rewritten to define new Hubble function ($w_{eff} = w_i + \frac{2}{3}n$)

$$\tilde{H}(z) \equiv (1+z)^n H(z) = H_0 \sqrt{\sum_{i=1}^{i=k} \Omega_{wi} (1+z)^{3(w_{eff}+1)}} \ . \tag{11}$$

## Redshift drift test - varying $c$

The VSL redshift drift effect for 15 year period of observations (Balcerzak, MPD 2014).



## Redshift drift test - varying $c$

- If $n < 0$ ($c$ decreases) then dust matter becomes little negative pressure matter and the cosmological constant became phantom. **Varying $c$ mimics dark energy**.
- If $n > 0$ then (growing $c(t)$) VSL model becomes more like Cold Dark Matter (CDM) model.
- Theoretical error bars are taken from Quercellini et al. 2012 and presumably show that for $|n| < 0.045$ – **one cannot distinguish between VSL models and $\Lambda$CDM models**.
- Future observations:
- European Extremely Large Telescope (EELT) (with its spectrograph CODEX (COsmic Dynamics EXperiment)), Thirty Meter Telescope (TMT), the Giant Magellan Telescope (GMT)
- gravitational wave interferometers DECIGO/BBO (DECi-hertz Interferometer Gravitational Wave Observatory/Big Bang Observer). Detection even at $z \sim 0.2$.

# 2. Measuring $c$ with baryon acoustic oscillations (BAO)

Speed of light $c$ appears in many observational quantities.
Among them in the angular diameter distance

$$D_A = \frac{D_L}{(1+z)^2} = \frac{a_0}{1+z}\int_{t_1}^{t_2} \frac{c(t)dt}{a(t)} \quad (12)$$

where $D_L$ is the luminosity distance, $a_0$ present value of the scale factor (normalized to $a_0 = 1$ later), and we have taken the spatial curvature $k = 0$ (otherwise there would be $\sin$ or $\sinh$ in front of the integral). Using the definition of redshift and the dimensionless parameters $\Omega_i$ we have

$$D_A = \frac{1}{1+z}\int_0^z \frac{c(z)dz}{H(z)}, \quad (13)$$

where

$$H(z) = \sqrt{\Omega_{r0}(1+z)^4 + \Omega_{m0}(1+z)^3 + \Omega_\Lambda}. \quad (14)$$

## Angular diameter distance maximum.

**Due to the expansion of the universe, there is a maximum of the distance** at

$$D_A(z_m) = \frac{c(z_m)}{H(z_m)}. \tag{15}$$

which can be obtained by simple differentiating (13) with respect to $z$:

$$\frac{\partial D_A}{\partial z} = -\frac{1}{(1+z)^2}\int_0^z \frac{c(z)dz}{H(z)} + \frac{1}{1+z}\frac{c(z)}{H(z)} = 0 \tag{16}$$

In a flat $k = 0$ cold dark matter CDM model

$$z_m = 1.25 \quad \text{and} \quad D_A \approx 1230 \;\; \text{Mpc} \tag{17}$$

For standard $\Lambda$CDM model of our interest:

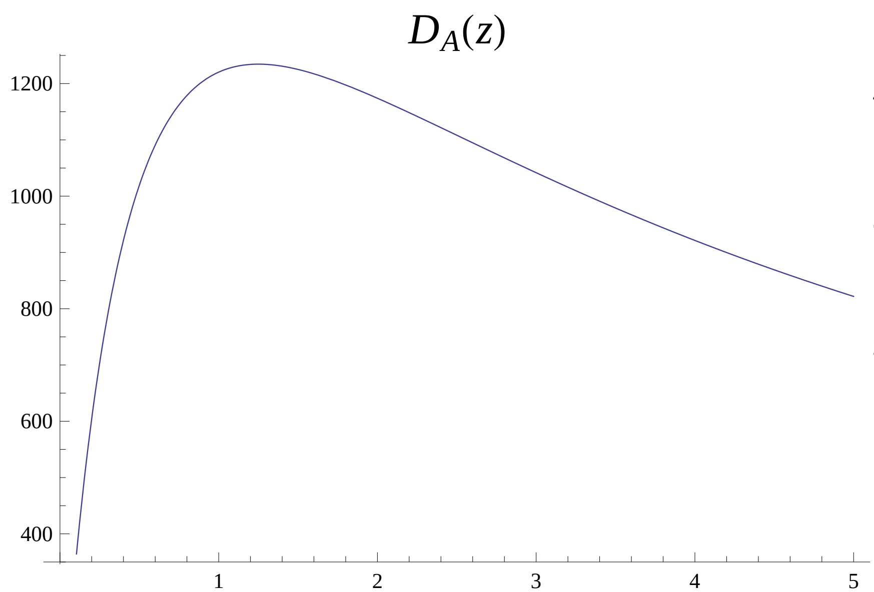
$$1.4 < z_m < 1.8. \tag{18}$$

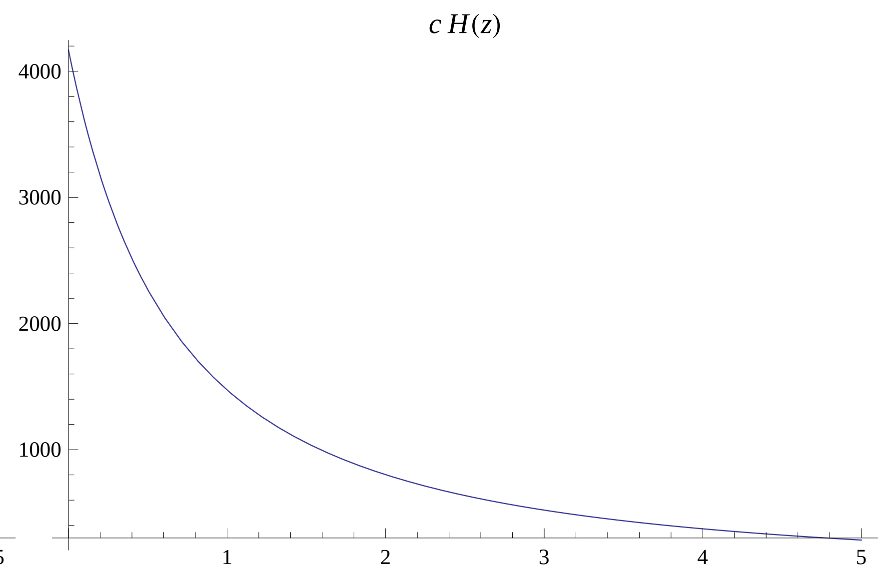
## $D_A$ versus $H(z)$

**The point:** The product of $D_A$ and $H$ gives **exactly** the speed of light $c$ at maximum (the curves intersect at $z_m$):

$$D_A(z_m)H(z_m) = c_0 \equiv 299792.458 \ \text{kms}^{-1} \tag{19}$$

if we believe it is constant! (defined officially www.bipm.org; a relative error $10^{-9}$ by Evenson et al. 1972)



## The method to measure $c$ - cosmic "rulers" and "chronometers".

In fact what we end in the relation (Salzano, MPD, Lazkoz 2015):

$$c = \frac{D_A}{\left(\frac{1}{H}\right)}. \tag{20}$$

in which $D_A$ with the dimension of length plays the role of a "cosmic ruler", and $1/H$ giving the dimension of time plays the role of a "cosmic clock/chronometer". The method is then:

- Measure independently $D_A(z)$ and $H(z)$.
- Calculate $z_m$.
- Calculate the product $D_A(z_m)H(z_m) = c(z_m)$.
- If $c(z_m)$ is not be equal to $c_0$, then one measures the deviation from $c_0$, i.e. $\Delta c = c(z_m) - c_0$.

## Measuring $z_m$

Measuring $z_m$ problematic if one uses $D_A$ only (large plateau around $z_m$ makes it difficult to avoid errors from small sample of data – besides, one has binned data, observational errors, and instrinsic dispersion).

However, one can appeal to an independent measurement of $c_0/H(z)$ which is the radial (line-of-sight) mode of the baryon acoustic oscillations surveys (BAO) for which $D_A(z)$ is the tangential mode (e.g. Nesseris et al. 2006). In other words, we have both tangential and horizontal modes as

$$y_t = \frac{D_A}{r_s} \qquad y_r = \frac{c}{Hr_s}, \tag{21}$$

where

$$r_s = \int_{z_{dec}}^{\infty} \frac{cc_s(z)dz}{H(z)} \tag{22}$$

is the sound horizon size at decoupling and $c_s$ the speed of sound.

# The scenarios.

Take background $\Lambda$CDM model with an ansatz (Magueijo 2003)

$$c(a) \propto c_0 \left(1 + \frac{a}{a_c}\right)^n \tag{23}$$

where $a_c$ is the scale factor at the transition epoch from some $c(a) \neq c_0$ (at early times) to $c(a) \to c_0$ (at late times to now).

Three scenarios (Salzano, MPD, Lazkoz 2015):

1) standard case $c = c_0$;

2) $a_c = 0.005$, $n = -0.01 \to \Delta c/c \approx 1\%$ at $z \propto 1.5$;

3) $a_c = 0.005$, $n = -0.001 \to \Delta c/c \approx 0.1\%$ at $z \propto 1.5$.

# The results - Gaussian Processes approach.

Based on $10^3$ Euclid project (Laureijs et al. 0912.0914) mock data simulations (Font-Ribeira et al. 2014):

1) $z_m = 1.592^{+0.043}_{-0.039}$ (fiducial model input $z_m = 1.596$) and $c/c_0 = 1 \pm 0.009$

2) $z_m = 1.528^{+0.038}_{-0.036}$ (fiducial $z_m = 1.532$) and $c(z_m)/c_0 = 1.00925 \pm 0.00831$

and

$$< c(z_m)/c_0 - 1\sigma_{c(z_m)/c_0} >= 1.00094^{+0.00014}_{-0.00033} \tag{24}$$

so that **a detection by Euclid of $1\%$ variation at $1\sigma$-level will be possible**.

3) $z_m = 1.584^{+0.042}_{-0.039}$ (fiducial $z_m = 1.589$) and $c(z_m)/c_0 = 1.00095 \pm 0.00852$

and

$$< c(z_m)/c_0 - 1\sigma_{c(z_m)/c_0} >= 0.99243^{+0.00016}_{-0.00013} \tag{25}$$

so that **a detection by Euclid of $0.1\%$ variation at $1\sigma$-level will not be possible**.

# The results - Matérn(9/2) function approach.

- Better estimation of errors (Seikel at al. arXiv: 1311.6678)
- To determine $H(z)$ the cosmic chronometers (passively-evolving early-type galaxies - Jimenez, Loeb Ap.J. 573, 37 (2002)) to directly measure $\Delta t$ and $\Delta z$ have been used i.e. $H(z) = -\Delta z/(1+z)\Delta t$
- Euclid will not detect at $1\sigma$ (errors larger than in GP approach)
- Possible measurement of 1% of variation of $c$ by Square Kilometer Array (SKA) even at $3\sigma$ - errors on $z_M$ about 30% smaller than from Euclid.
- Measurement of $0.1\%$ variation is possible at $1\sigma$ level, but needs reduction of errors by a factor of 10.

# The results - Euclid vs SKA.

| | *Euclid* | | | | |
|---|---|---|---|---|---|
| $\Delta c/c_0$ | $z_M$ | $c\ (p_{>1})$ | $c_{1\sigma}\ (p_{>1})$ | $c_{2\sigma}\ (p_{>1})$ | $c_{3\sigma}\ (p_{>1})$ |
| 1% | $1.559^{+0.054}_{-0.051}$ | $1.00872^{+0.00003}_{-0.00003}$ (1) | $0.99993^{+0.00013}_{-0.00024}$ (0.32) | $0.99436^{+0.00023}_{-0.00041}$ (0) | $0.98879^{+0.00032}_{-0.00056}$ (0) |
| 0.1% | $1.587^{+0.058}_{-0.052}$ | $1.000880^{+0.000006}_{-0.000006}$ (0.98) | $0.99199^{+0.00014}_{-0.00024}$ (0.001) | $0.98636^{+0.00024}_{-0.00038}$ (0) | $0.98072^{+0.00034}_{-0.00053}$ (0) |

| | SKA | | | | |
|---|---|---|---|---|---|
| $\Delta c/c_0$ | $z_M$ | $c\ (p_{>1})$ | $c_{1\sigma}\ (p_{>1})$ | $c_{2\sigma}\ (p_{>1})$ | $c_{3\sigma}\ (p_{>1})$ |
| 0% | $1.593^{+0.018}_{-0.017}$ | $1.^{+3\cdot10^{-7}}_{-4\cdot10^{-7}}$ | $0.99708^{+0.00003}_{-0.00004}$ | $0.99524^{+0.00006}_{-0.00007}$ | $0.99339^{+0.00008}_{-0.00008}$ |
| 1% | $1.561^{+0.017}_{-0.017}$ | $1.00873^{+0.00001}_{-0.00001}$ (1) | $1.00585^{+0.00003}_{-0.00003}$ (1) | $1.004036^{+0.00005}_{-0.00005}$ (1) | $1.00221^{+0.00008}_{-0.00009}$ (1) |
| 0.1% | $1.590^{+0.018}_{-0.017}$ | $1.000880^{+0.000001}_{-0.000001}$ (1) | $0.99797^{+0.00003}_{-0.00003}$ (0) | $0.99612^{+0.00006}_{-0.00006}$ (0) | $0.99428^{+0.00008}_{-0.00008}$ (0) |
| 0.1% $(err/3)$ | $1.590^{+0.006}_{-0.006}$ | $1.0008800^{+0.0000001}_{-0.0000001}$ (1) | $0.999834^{+0.000009}_{-0.000009}$ (0) | $0.99917^{+0.00001}_{-0.00001}$ (0) | $0.998510^{+0.00002}_{-0.00002}$ (0) |
| 0.1% $(err/10)$ | $1.590^{+0.003}_{-0.003}$ | $1.0008800^{+0.0000003}_{-0.0000002}$ (1) | $1.00032^{+0.00014}_{-0.00018}$ (0.94) | $0.99996^{+0.00023}_{-0.00029}$ (0.44) | $0.99961^{+0.00032}_{-0.00040}$ (0.10) |

# Observations.

- Current:
- Euclid will have $1/10$ of the errors of the current missions like WiggleZ Dark Energy Survey (e.g. Blake et al. 2011, 2012).
- Future:
- Dark Energy Spectroscopic Instrument (DESI) (Levi et al. 1308.0847)
- Square Kilometer Array (SKA) (Bull et al. 1405.1452)
- Wide-Field Infrared Survey Telescope (WFIRST) (Spergel et al. 1305.5425) (esp. having largest sensitivity at potential $z_m$ region i.e. $1.5 < z < 1.6$).

# Summary:

- Redshift drift test which give clear prediction for redshift drift effect which can potentially be measured by future telescopes (E-ELT, TMT, GMT, DECIGO/BBO).
- Baryon acoustic oscillations test to independently measure the radial $D_A$ and tangential mode $c/H$ of the volume distance at the angular diameter distance maximum $z_m$.
- In simple terms we have a "cosmic" measurement of the speed of light $c$ with $D_A$ giving the dimension of length being a "cosmic ruler" and $1/H$ giving the dimension of time being a "cosmic clock/chronometer" i.e.

$$c = \frac{D_A}{\left(\frac{1}{H}\right)}. \tag{26}$$