Higher-order massive neutrino perturbations in large scale strcuture

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INTERNATIONAL MAX PLANCK RESEARCH SCHOOL
PT FS
FOR PRECISION TESTS OF FUNDAMENTAL SYMMETRIES

Neutrinos

Neutrinos are massive

Earth based experiments

$$0.057\,\mathrm{eV} \lesssim \sum_i m_i \lesssim 6\,\mathrm{eV}$$

Neutrino oscillations   Tritium β-decay

Cosmology

$$\sum_i m_i \lesssim O(1\,\mathrm{eV})$$

Non-observation of scale-dependent effects

Two important effects on Structure formation

Neutrino are relativistic at early times, non-relativistic at late times

→ Suppression of perturbation on small scale of all types of matter

Neutrinos contribute to non-relativistic matter $f_\nu = O(0.05)$

→ Neutrino perturbations evolve different from CDM perturbations

- Need to incorporate υ into non-linear LSS calculations
- LSS can be used to "measure" the neutrino mass

Fluid Perturbations

Non-vanishing velocity dispersion suppresses Neutrino perturbations on small scales

Add sound speed to fluid equations $c_s^2 \approx \frac{\overline{q^2}}{a^2 m^2}$ Shoji, Komtasu 2010

$$\frac{\partial\theta}{\partial s} + a\mathcal{H}\theta + a\left(\frac{3}{2}\mathcal{H}^2\Omega(\tau) - k^2 c_s^2(s)\right)\delta = 0 \qquad ds = \frac{d\tau}{a}$$

Free-Streaming scale $k_{FS}^2 = \frac{3}{2}\frac{\mathcal{H}^2\Omega}{c_s^2}$

→ $k < k_{FS}$ Suppression

→ $k > k_{FS}$ Artificial acoustic oscillations

$k \ll k_{FS}$	$k \sim k_{FS}$	$k \gg k_{FS}$
$\delta_\nu \sim \delta_{CDM}$	$\delta_\nu < \delta_{CDM}$	$\delta_\nu \ll \delta_{CDM}$
Fluid approximation	Vlasov equation	Neutrinos linear?

Fluid Perturbation Theory

$$\frac{\partial \delta}{\partial s} + a\theta = -a\left[\alpha(\mathbf{k}_1, \mathbf{k}_2)\delta(\mathbf{k}_1)\theta(\mathbf{k}_2)\right]_{\mathbf{k}} \qquad [\ldots]_{\mathbf{k}} = \int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\delta_D(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\ldots$$

$$\frac{\partial \theta}{\partial s} + a\mathcal{H}\theta + a\left(\frac{3}{2}\mathcal{H}^2\Omega(\tau) - k^2c_s^2(s)\right)\delta = -a\left[\beta(\mathbf{k}_1, \mathbf{k}_2)\theta(\mathbf{k}_1)\theta(\mathbf{k}_2)\right]_{\mathbf{k}}$$

$$-ac_s^2(s)i\mathbf{k}\cdot\mathcal{F}\left[\frac{\delta\nabla\delta}{1+\delta}\right]$$

Linear order:

$$\frac{\partial \delta^{(1)}}{\partial s} + a\theta^{(1)} = 0$$

$$\frac{\partial \theta^{(1)}}{\partial s} + a\mathcal{H}\theta^{(1)} + a\left(\frac{3}{2}\mathcal{H}^2\Omega(\tau) - k^2c_s^2(s)\right)\delta^{(1)} = 0$$

Second order:

$$\frac{\partial \delta^{(2)}}{\partial s} + a\theta^{(2)} = -a\left[\alpha(\mathbf{k}_1, \mathbf{k}_2)\delta^{(1)}(\mathbf{k}_1)\theta^{(1)}(\mathbf{k}_2)\right]_{\mathbf{k}}$$

$$\frac{\partial \theta^{(2)}}{\partial s} + a\mathcal{H}\theta^{(2)} + a\left(\frac{3}{2}\mathcal{H}^2\Omega(\tau) - k^2c_s^2(s)\right)\delta^{(2)} = -a\left[\tilde{\beta}(\mathbf{k}_1\mathbf{k}_2)\theta^{(1)}(\mathbf{k}_1)\theta^{(1)}(\mathbf{k}_2)\right]_{\mathbf{k}}$$

and so forth

Vlasov equation

A consistent treatment requires to solve the Vlasov equation

$$\frac{\partial \delta f}{\partial s} + i\frac{\mathbf{q}}{m}\cdot\mathbf{k}\delta f - iV(s)\frac{\mathbf{k}}{k^2}\cdot\frac{\partial \bar{f}}{\partial \mathbf{q}}\delta = iV(s)\left[\frac{\mathbf{k}_1}{k_1^2}\cdot\frac{\partial \delta f}{\partial \mathbf{q}}(\mathbf{k}_2)\Omega\delta(\mathbf{k}_1)\right]_{\mathbf{k}}$$

$$V(s) = a^2 m \tfrac{3}{2}\mathcal{H}^2\Omega$$

Straightforward expansion as for CDM in principle possible

See talk by Dupuy

- Much more involved due to momentum dependence
- Only interested in integral over momentum: $\delta \propto \int d^3q\,\delta f$

→ Integrate over momentum before solving

Gilbert's Equation

At linear order, distribution function determined by density contrast:

$$\delta f^{(1)} = g_{FS}(s, s_i)\delta f_i + \int_{s_i}^{s} ds' \; g_{FS}(s, s')ia^2 m \frac{3}{2}\mathcal{H}^2 \frac{\mathbf{k}}{k^2} \cdot \frac{\partial \bar{f}}{\partial \mathbf{q}} \Omega \delta^{(1)}$$

Free-streaming solution: $g_{FS}(s, s') = exp\left(-i\frac{\mathbf{q}\cdot\mathbf{k}}{m}(s - s')\right)$

Integrating over momentum yields Gilbert's equation:

$$\delta^{(1)} = I + \int_{s_i}^{s} ds' \; K(s, s')\delta^{(1)}(s') \quad \text{Solution:} \quad \delta^{(1)} = \int_{s_i}^{s} ds' \; G(s, s')I(s')$$

$$I = \int d^3q \; g_{FS}(s, s_i)\delta f_i \qquad K(s, s') = \tfrac{3}{2}\mathcal{H}^2 a^2 (s - s')\Omega \int d^3q \; g_{FS}(s, s')\bar{f}$$

e.g. Bertschinger 2007
Boyanovsky et al. 2008

Non-Linear Gilbert's Equation

Rewrite Boltzmann equation including the non-linear term:

$$\delta f = g_{FS}(s, s_i)\delta f_i + \int_{s_i}^{s} ds' \; g_{FS}(s, s')ia^2m\frac{3}{2}\mathcal{H}^2\frac{\mathbf{k}}{k^2}\cdot\frac{\partial \bar{f}}{\partial \mathbf{q}}\Omega\delta$$

$$+ \int_{s_i}^{s} ds' \; g_{FS}(s, s')ia^2m\frac{3}{2}\mathcal{H}^2\left[\frac{\mathbf{k}_1}{k_1^2}\cdot\frac{\partial \delta f}{\partial \mathbf{q}}(\mathbf{k}_2)\Omega\delta(\mathbf{k}_1)\right]_{\mathbf{k}}$$

Integrating over momentum adds a non-linear term to Gilbert's equation:

$$\delta = I + \int_{s_i}^{s} ds' \; K(s, s')\delta(s')$$

$$+ \int_{s_i}^{s} ds' \int d^3q \; g_{FS}(s, s')ia^2m\frac{3}{2}\left[\frac{\mathbf{k}_1}{k_1^2}\cdot\frac{\partial \delta f}{\partial \mathbf{q}}(\mathbf{k}_2)\Omega\delta(\mathbf{k}_1)\right]_{\mathbf{k}}$$

Non-Linear Gilbert's Equation

Formal integral equation for density contrast:

$$\delta = I + \int_{s_i}^{s} ds'\ K(s,s')\delta(s')$$

Linear terms

$$+ \int_{s_i}^{s} ds' \ \left[\tilde{\Gamma}^{(1)}\delta(s',\mathbf{k}_2)\right]_{\mathbf{k}}$$

$$+ \int_{s_i}^{s} ds_1 \int_{s_i}^{s_1} ds_2 \ \left[\Gamma^{(2)}\delta(s_1,\mathbf{k}_1)\delta(s_2,\mathbf{k}_2)\right]_{\mathbf{k}}$$

Quadratic terms

$$+ \int_{s_i}^{s} ds_1 \int_{s_i}^{s_1} ds_2 \ \left[\tilde{\Gamma}^{(2)}\delta(s_1,\mathbf{k}_1)\delta(s_2,\mathbf{k}_2)\right]_{\mathbf{k}}$$

$$+ \int_{s_i}^{s} ds_1 \int_{s_i}^{s_1} ds_2 \int_{s_i}^{s_2} ds_3 \ \left[\Gamma^{(3)}\delta(s_1,\mathbf{k}_1)\delta(s_2,\mathbf{k}_2)\delta(s_3,\mathbf{k}_3)\right]_{\mathbf{k}}$$

Third order terms

$$+ \ldots$$

Effects of $\mathbf{q}$ -dependence stored in two functions

$$\Gamma \sim \int d^3q\, e^{-i\frac{\mathbf{q}\cdot\mathbf{v}}{m}}\,\bar{f}$$

$$\mathbf{v} = \mathbf{v}(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2,\ldots;s,s_1,s_2,\ldots)$$

$$\tilde{\Gamma} \sim \int d^3q\, e^{-i\frac{\mathbf{q}\cdot\mathbf{v}}{m}}\,\delta f(s_i)$$

Correlation Functions

The linear Power Spectrum is given by

$$(2\pi)^3 P^{(1)}(k; s, s') = \int_{s_i}^{s} ds_1 \int_{s_i}^{s'} ds_2\, G(k; s, s_1) G(k; s', s_2) \langle I(\mathbf{k}, s_1) I(-\mathbf{k}, s_2) \rangle$$

The tree-level bispectrum is given by the sum of two parts

$$(2\pi)^3 B^{(2a)}(k_1,\ k_2,\ |\mathbf{k}_1 + \mathbf{k}_2|, s) = 2 \int_{s_i}^{s} ds'\, G(k_1; s, s') \int_{s_i}^{s'} ds_1 \int_{s_i}^{s_1} ds_2$$

$$\times \Gamma^{(2)}(\mathbf{k}_1 + \mathbf{k}_2, \mathbf{k}_2; s', s_1, s_2) P^{(1)}(|\mathbf{k}_1 + \mathbf{k}_2|; s_1, s) P^{(1)}(k_2; s_2, s)$$

$$(2\pi)^3 B^{(2b)}(k_1,\ k_2,\ |\mathbf{k}_1 + \mathbf{k}_2|, s) = 2 \int_{s_i}^{s} ds''\, G(k_2; s, s'') \int_{s_i}^{s} ds'\, G(k_1; s, s') \int_{s_i}^{s'} ds_1$$

$$\times \langle \tilde{\Gamma}^{(1)}(\mathbf{k}_1 + \mathbf{k}_2, -\mathbf{k}_2; s', s_1) I(\mathbf{k}_2, s'') \rangle P^{(1)}(|\mathbf{k}_1 + \mathbf{k}_2|, s_1, s)$$

Hybrid Approach

Combine exact linear theory, with non-linear term of fluid equations

- Perturbation theory as simple as for a fluid
- Captures suppression on small scales
- No artificial acoustic oscillations

Replace fluid Green's function by the "exact" ones

Use continuity equation to obtain velocity $\theta^{(1)} = -\frac{1}{a}\frac{\partial \delta^{(1)}}{\partial s}$

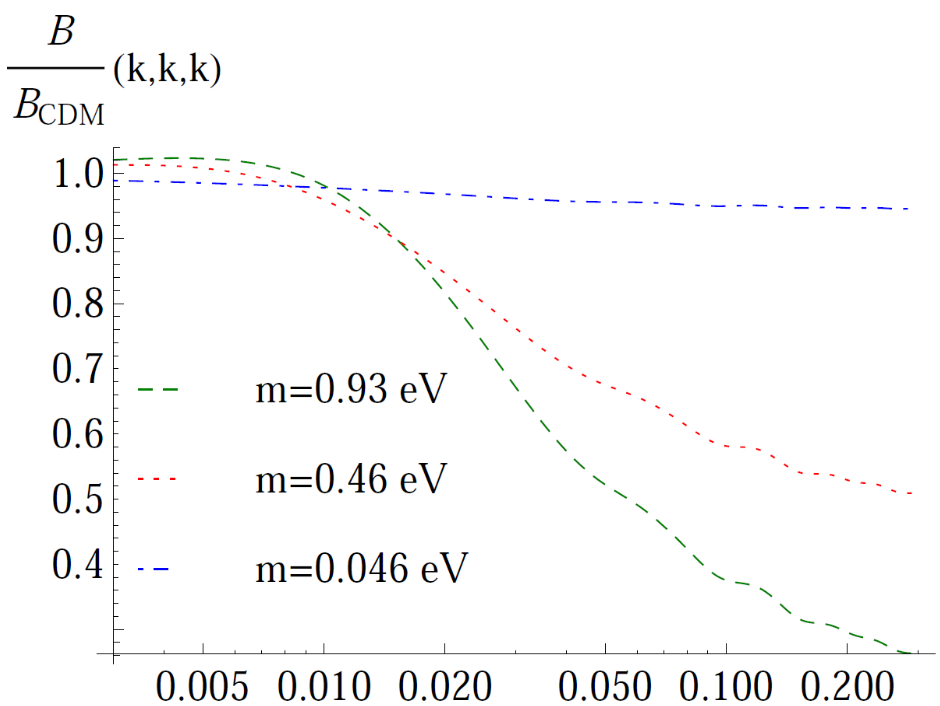
To obtain the Green's function write

$$I(s_i) = \delta(s_i) - (s - s_i)a(s_i)\theta(s_i) + \ldots = \tilde{g}_{1b}(s, s_i)\varphi_b(s_i) + \ldots$$

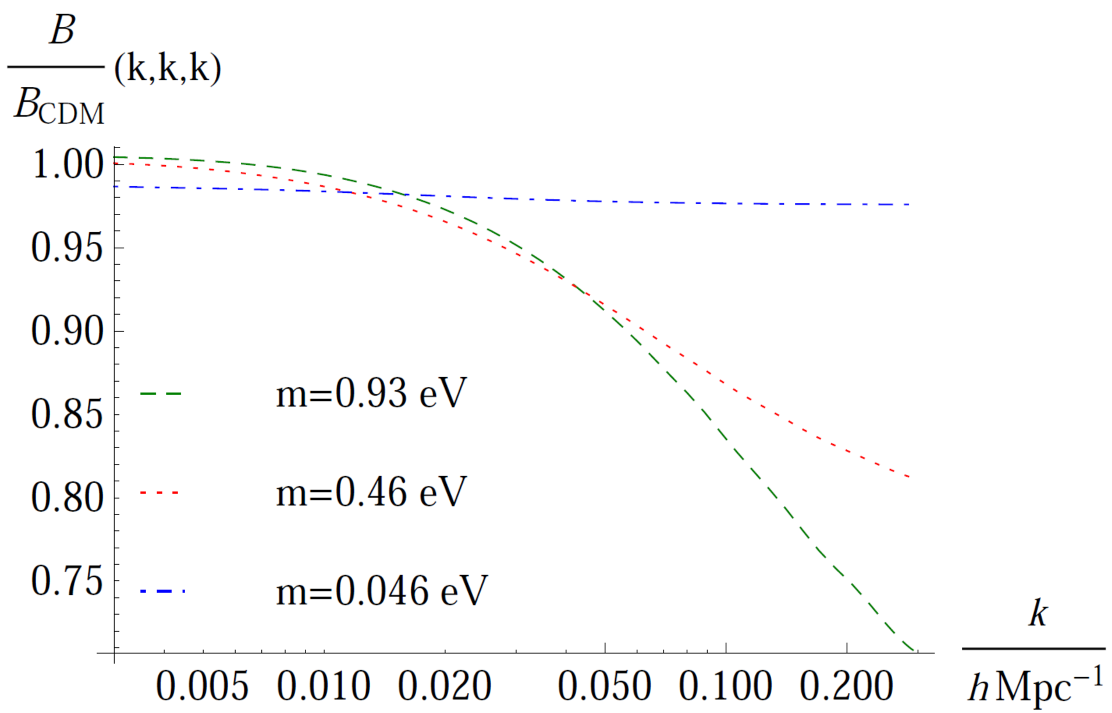
Plug into Gilbert's equation

$$\delta = \underbrace{\int_{s_i}^{s} ds'\, G(s,s')\tilde{g}_{1b}(s', s_i)}_{= g_{1b}(s, s_i)} \varphi_b(s_i) + \ldots$$

Bispectrum

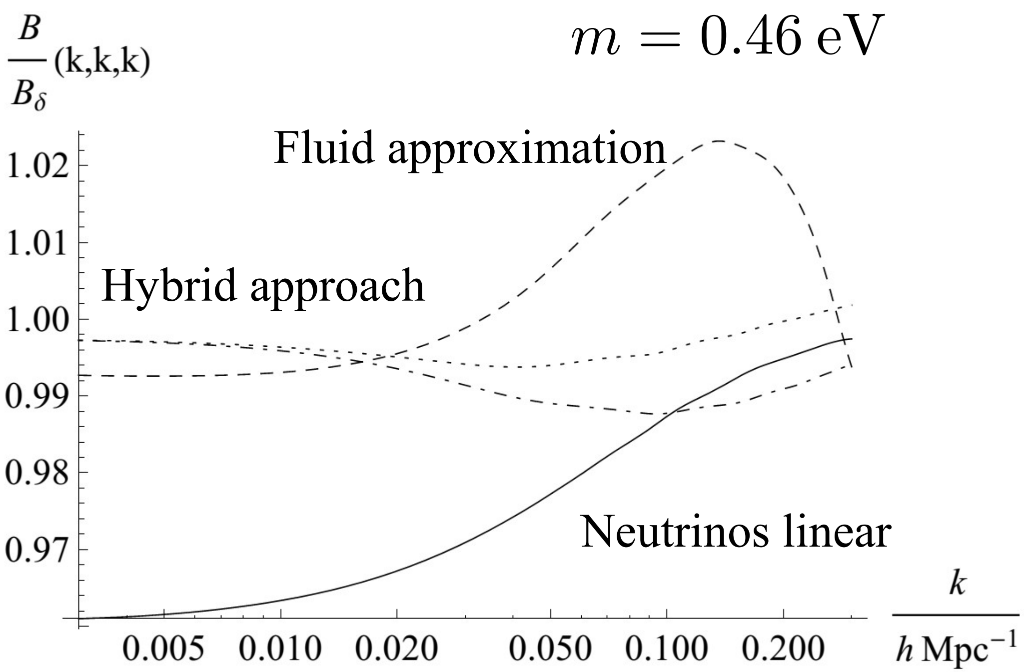


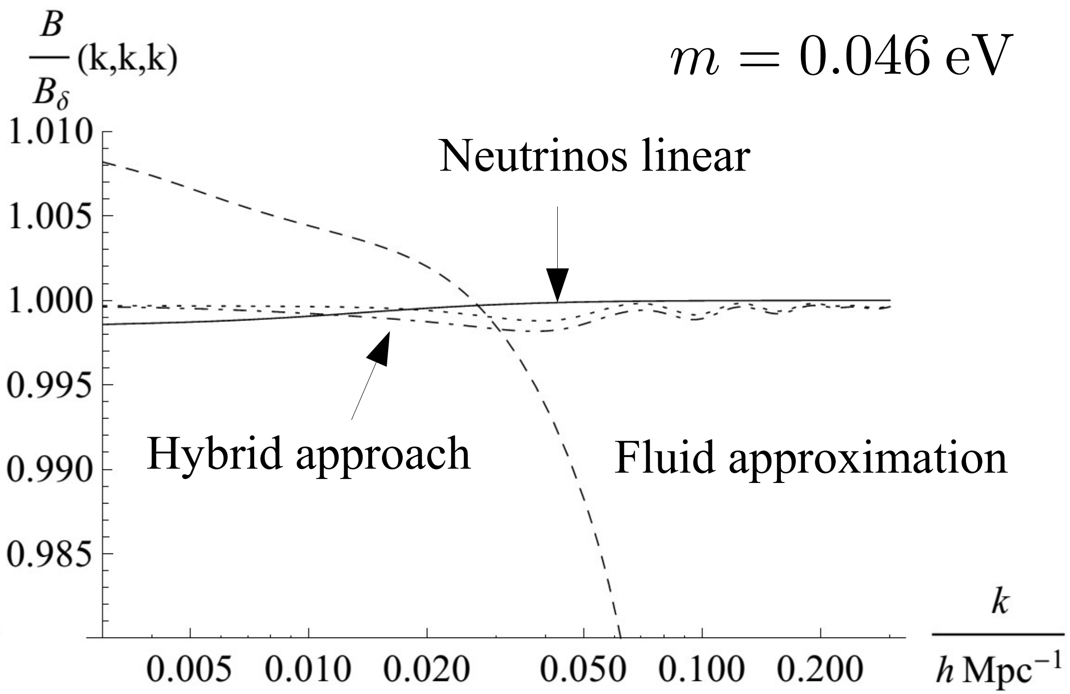
The total matter bispectrum of a neutrino+CDM cosmology dived by the bispectrum in a pure CDM cosmology



The total matter bispectrum of a neutrino+CDM cosmology dived by the CDM bispectrum in the same cosmology

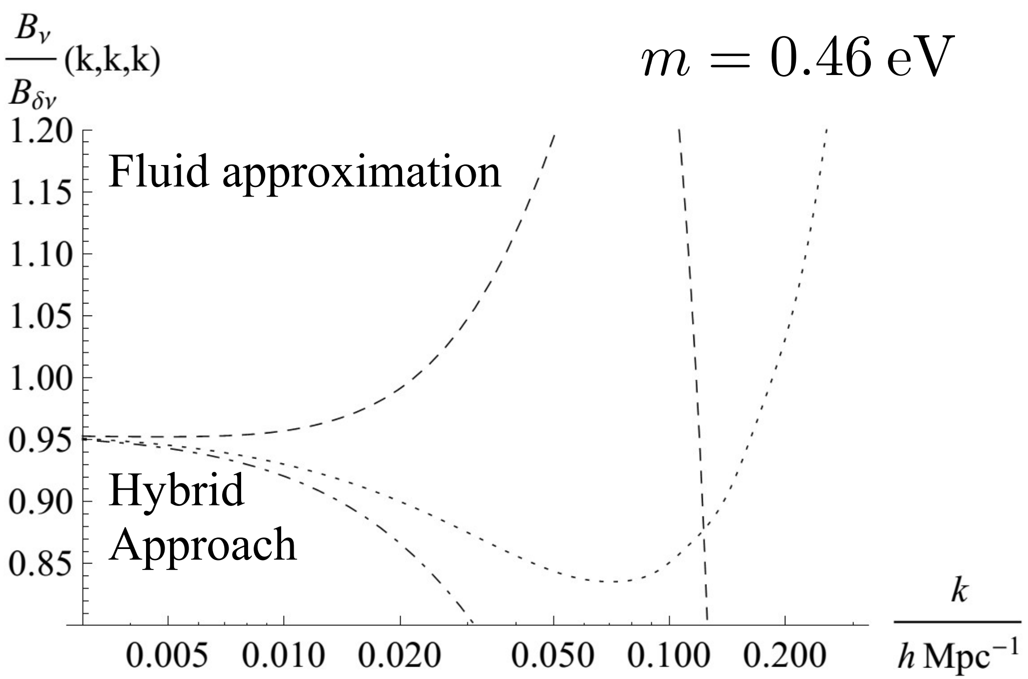
Matter Bispectrum

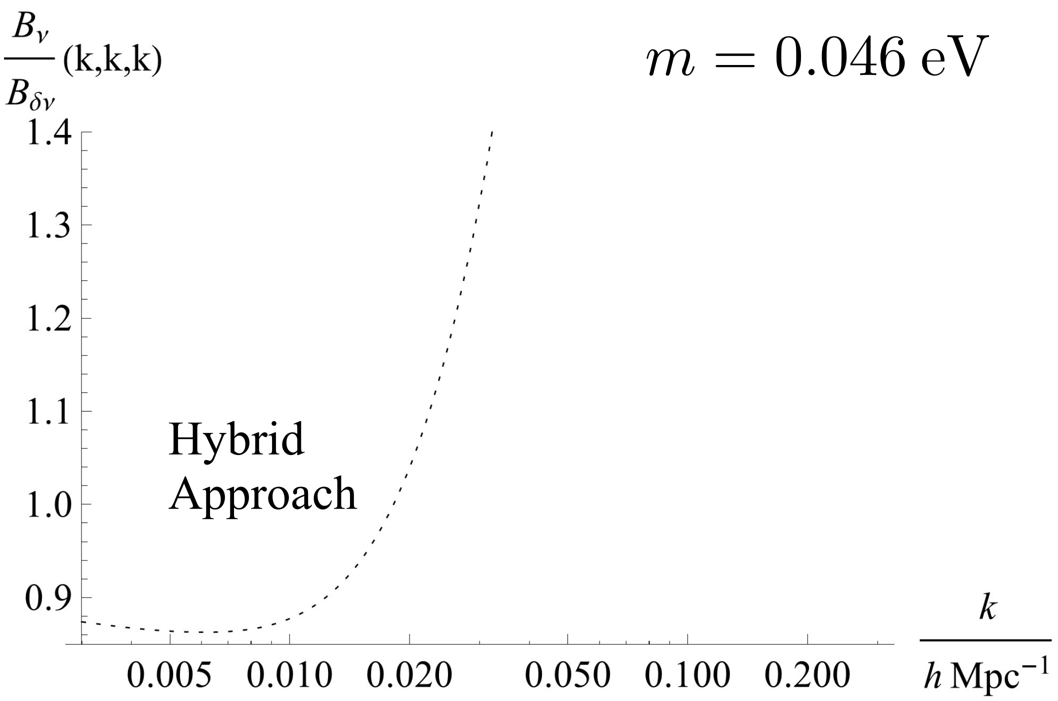




The total matter bispectrum computed using various approximations compared to the result from the non-linear Gilbert's equation

Neutrino Bispectrum





The neutrino bispectrum computed using various approximations compared to the result from the non-linear Gilbert's equation

Summary and Outlook

- First-principle approach for ν perturbations in LSS
  - Applicable to Warm Dark Matter
- Calculated the bispectrum at leading order
- Aiming at reproducing the total matter bispectrum at 1% accuracy
  - The fluid approximation fails
  - The linear approximation fails, except for a small ν mass
  - The hybrid approach succeeds
  - All tested approximations fail for the ν bispectrum
- Loop corrections to the Power Spectrum
  - Efficient numerical implementation for loops
  - Improve hybrid approach/develop new approximations