The Chiral Magnetic Effects and Its Role in Astrophysics and Cosmology

1. A general approach to the chiral magnetic effect
2. Application to hot supernova cores
3. Role for primordial magnetic fields
4. Conclusions and outlook

partly based on Sigl, Leite, arXiv:1507:04983 (to appear in JCAP)

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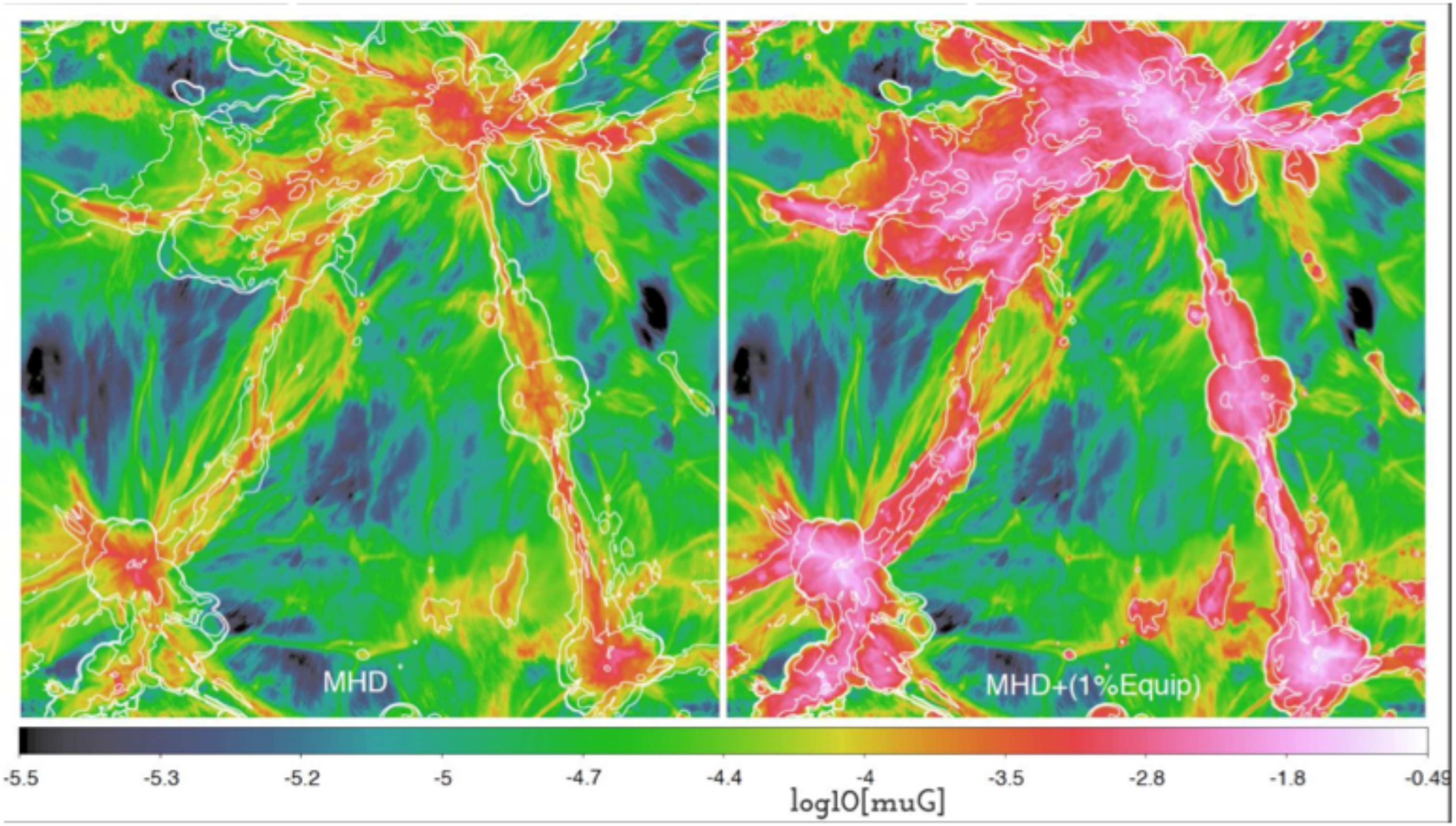
The Magnetic Universe:
Understanding the origin and evolution of B fields



(Vazza et al. 2014)

- Determine the role of magnetism in regulating galaxy evolution
- Detection and characterization of the magnetic cosmic web
- Magnetic evolution of AGN over cosmic time

Primordial Magnetic fields - Basic MHD

Magnetohydrodynamics (MHD)

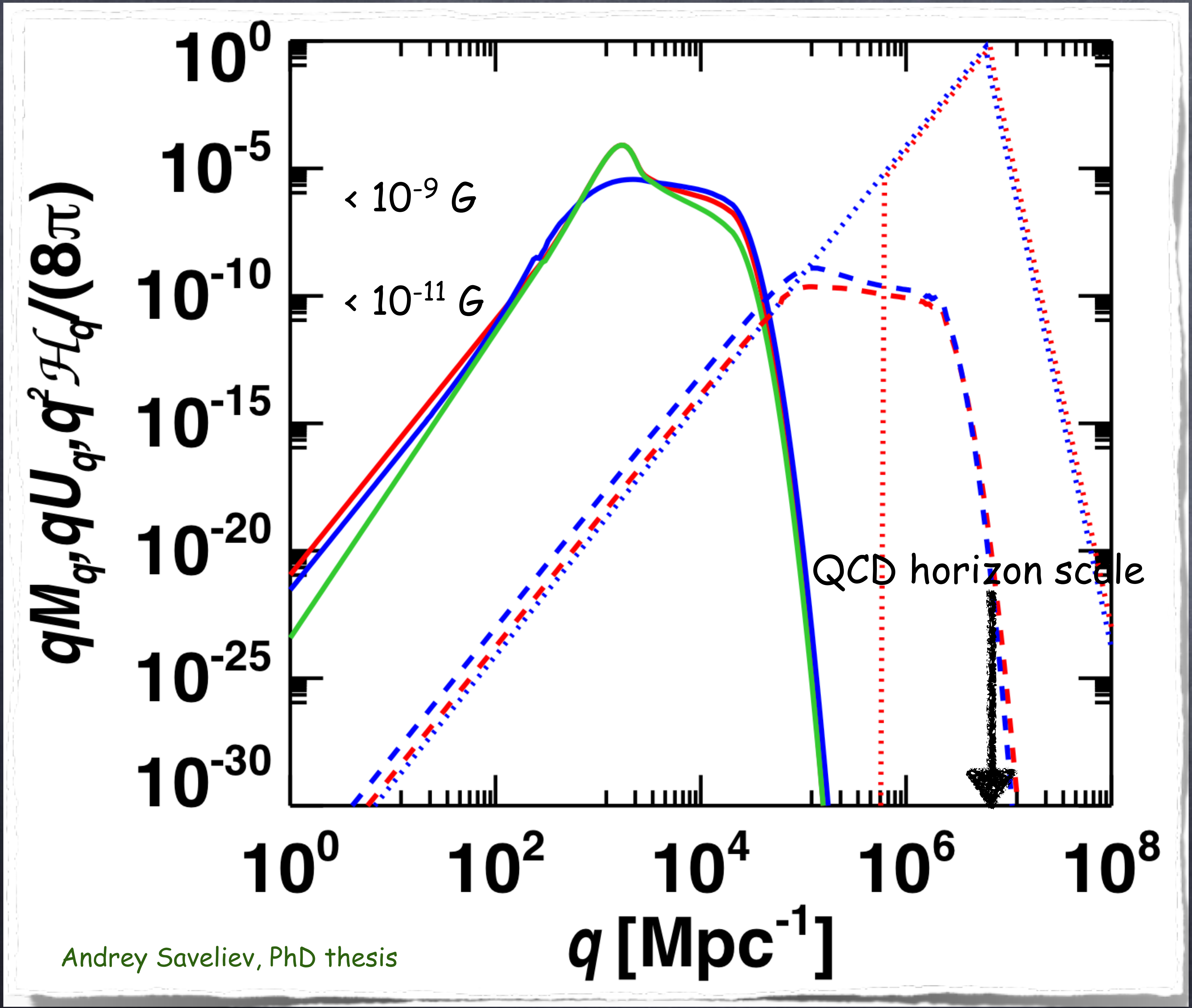
- Maxwell's equations:
  $\nabla \cdot \mathbf{B} = 0,\ \nabla \times \mathbf{E} = -\partial_t \mathbf{B},\ \nabla \times \mathbf{B} = 4\pi \mathbf{j}$
- Continuity equation for mass density ρ: $\partial_t \rho + \nabla(\rho \mathbf{v}) = 0$
- Navier-Stokes equations:
  $\rho\left(\partial_t \mathbf{v} + (\mathbf{v}\nabla)\,\mathbf{v}\right) = -\nabla p + \mu \Delta \mathbf{v} + (\lambda + \mu)\nabla\left(\nabla \mathbf{v}\right) + \mathbf{f}$

For the magnetic field and the turbulent fluid it follows therefore

$$\partial_t \mathbf{B} = \frac{1}{4\pi\sigma}\Delta \mathbf{B} + \nabla \times (\mathbf{v} \times \mathbf{B})$$

$$\partial_t \mathbf{v} = -\left(\mathbf{v}\nabla\right)\mathbf{v} + \frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{4\pi\rho} + \mathbf{f}_v .$$

Primordial Magnetic Fields: Full-Blown Numerical MHD Simulations versus semi-analytical methods based on transport equations



normalized to turbulence energy, < 10^{-6} G

magnetic fields

turbulent velocity

magnetic helicity

dotted = initial condition

dashed = final state without helicity

solid = final state with maximal helicity

A General Approach to the Chiral Magnetic Effect

For the electron chiral asymmetry $N_5 \equiv N_L - N_R$ and the magnetic helicity $\mathcal{H} \equiv \int d^3\mathbf{r}\, \mathbf{B} \cdot \mathbf{A}$ the electromagnetic chiral anomaly gives

$$\frac{d}{dt}\left(N_5 - \frac{e^2}{4\pi^2}\mathcal{H}\right) = 0\,, \tag{1}$$

and $e^2\mathcal{H}/(4\pi^2)$ is just the Chern-Simons number of the electromagnetic field. The generalized Maxwell-Ampère law

$$\boldsymbol{\nabla} \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \left(\mathbf{j}_{\rm em} + \mathbf{j}_{cB}\right)\,, \quad \text{with} \quad \mathbf{j}_{cB} = -\frac{e^2}{2\pi^2}\mu_5 \mathbf{B}\,, \tag{2}$$

and Ohm's law for $\mathbf{j}_{\rm em}$ in the absence of external currents gives

$$\mathbf{E} \simeq -\mathbf{v} \times \mathbf{B} + \eta \left(\boldsymbol{\nabla} \times \mathbf{B} + \frac{2e^2}{\pi}\mu_5 \mathbf{B}\right)\,, \tag{3}$$

where η is the resistivity and the effective chemical potential is given by

$$\mu_5 = \frac{\mu_L - \mu_R}{2} + V_5 = \frac{\mu_L + V_L - \mu_R - V_R}{2}\,, \tag{4}$$

where V_5 is a possible effective potential due to a different forward scattering amplitude for left- and right-chiral electrons. Inserting this into the induction equation the MHD is modified to

$$\partial_t \mathbf{B} = \boldsymbol{\nabla} \times (\mathbf{v} \times \mathbf{B}) + \eta \Delta \mathbf{B} - \frac{2e^2}{\pi} \eta \mu_5 \boldsymbol{\nabla} \times \mathbf{B}\,. \tag{5}$$

This equation is similar to the mean field dynamo equation which also has growing solutions. Neglecting the velocity term the evolution equations for the power spectra M_k and H_k [note $U_B = \int d\ln k M_k$ and $\mathcal{H} = \int d\ln k H_k$] now become

$$\begin{aligned} \partial_t M_k &= -\eta k^2 \left(2M_k + \frac{e^2}{2\pi^2} \mu_5 H_k \right) \\ \partial_t H_k &= -\eta \left(2k^2 H_k + 32 e^2 \mu_5 M_k \right)\,. \end{aligned} \tag{6}$$

Integrating over $\ln k$ gives

$$\partial_t \mathcal{H} = -\eta \int d\ln k \left(2k^2 H_k + 32 e^2 \mu_5 M_k \right)\,. \tag{7}$$

In an FLRW metric these are comoving quantities and conformal time.

Now express N_5 in terms of μ_5,

$$N_5 = c(T, \mu_e) V \mu_5\,, \quad \text{with} \quad c(T, \mu_e) = \frac{\mu_e^2}{\pi^2} + \frac{T^2}{3} \quad \text{for } \mu_e^2 + T^2 \gg m_e^2\,, \tag{8}$$

where the second expression holds for relativistic electrons. Applying this to Eq. (1) we get

$$d\mathcal{H} = \frac{4\pi^2}{e^2} dN_5 = \frac{4\pi^2 V c(T, \mu_e)}{e^2} d\mu_5 \,. \tag{9}$$

We now also have to include the *chirality-flip rate*

$$R_f \simeq \left(\frac{m_e}{T}\right)^2 R \sim \left(\frac{m_e}{T}\right)^2 \frac{e^2}{T^2} 100\, T^3 \sim \frac{m_e^2}{T} = \frac{T_5}{T} H(T_5) \,, \tag{10}$$

where we have used $\sim e^2/T^2$ for the cross section and $\sim 100T^3$ for the relativistic target number density. This becomes comparable to the Hubble rate for $T = T_5 \simeq 80\,\mathrm{TeV}$.

Inserting Eq. (7) into Eq. (9) then yields

$$\partial_t \mu_5 = -\frac{e^2\eta}{2\pi^2 V c(T, \mu_e)} \int d\ln k \left(k^2 H_k + 16 e^2 \mu_5 M_k\right) - 2R_f \left(\mu_5 - \mu_{5,b}\right) \,. \tag{11}$$

Here was added a damping term R_f due to the chirality-flips and $\mu_{5,b} = V_5 + \mu_s$ is the equilibrium value of the effective chemical potential μ_5 in the absence of resistivity. Both a possible effective potential V_5 and a possible source term $2R_f\mu_s$ due to other processes such as electroweak interactions with other species as for example neutrinos can contribute to $\mu_{5,b}$.

From Eq. (6) growing solutions exist for wavenumbers

$$k < k_5 \equiv k_5(\mu_5) \equiv \frac{2e^2}{\pi}|\mu_5|\,. \tag{12}$$

This follows from using helicity modes in Eq. (5) which gives

$$\partial_t b^{\pm}_{\mathbf{k}} = \eta k \left(\mp \frac{2e^2}{\pi}\mu_5 - k \right) b^{\pm}_{\mathbf{k}}\,, \tag{13}$$

Thus if the condition Eq. (12) is fulfilled, the helicity with the opposite sign as μ_5 will grow whereas the same sign helicity will decay and the absolute value of the helicity will be close to the maximal value given by

$$|H_k| \leq \frac{8\pi M_k}{k}\,. \tag{14}$$

In contrast, for $k \gtrsim k_5$ both helicities will decay with roughly the resistive rate. For the helicity with opposite sign to μ_5 the first term in Eq. (13) corresponds to a growth rate

$$R_c(k) = \frac{2e^2}{\pi}\eta k|\mu_5| \simeq 2\times 10^{10}\left(\frac{\mathrm{TeV}}{T}\right)\left(\frac{k}{k_5}\right)\left(\frac{\mu_5}{T}\right)^2 H(T)\,, \tag{15}$$

The total rate $R_c - R_r$ reaches its maximum value $R_{\rm max} = \eta k_5^2/4$ at $k = k_5/2$ which for

$$\frac{\mu_5}{T} \gtrsim 10^{-5} \left(\frac{T}{\rm TeV}\right)^{1/2} \tag{16}$$

is larger than the Hubble rate. Furthermore, Eq. (11) shows that for growing modes $|\mu_5|$ shrinks for either sign of μ_5. Therefore, the **chiral magnetic instability transforms energy in the electron asymmetry N_5 into magnetic energy**. This is because by definition of the chemical potential μ_5 the energy U_5 associated with the chiral lepton asymmetry is given by

$$dE_5 = \mu_5 dN_5 = V c(T, \mu_e)\mu_5 d\mu_5\,, \quad U_5 = \frac{V c(T, \mu_e)\mu_5^2}{2}\,. \tag{17}$$

Imagine now an initial chiral asymmetry $\mu_{5,i}$ and no magnetic field. Since the sign of $d\mu_5$ is opposite to the sign of $\mu_{5,i}$, Eq. (9) also confirms that the magnetic helicity will have the opposite sign as $\mu_{5,i}$. The growth rate peaks at wavenumber $k = k_5/2$ given by Eq. (12) and for a given mode k growth stops once $|\mu_5|$ has decreased to the point that Eq. (12) is violated. Since the instability produces maximally helical fields saturating Eq. (14), with Eq. (9) we obtain

$$\begin{aligned} dU_B &\simeq dM_{k_5} \simeq k_5 |dH_{k_5}|/(8\pi) \simeq k_5 |d\mathcal{H}|/(8\pi) = Vc(T,\mu_e)\mu_5 d\mu_5\,, \\ \Delta E_m &\simeq \frac{Vc(T,\mu_e)(\mu_{5,i}^2-\mu_5^2)}{2}\,. \end{aligned} \tag{18}$$

Adding Eqs. (17) and (18) gives a total energy $U_{\rm tot} = U_5 + U_B \simeq Vc(T,\mu_e)\mu_{5,i}^2/2$ which only depends on the initial asymmetry $\mu_{5,i}$. The maximal magnetic energy density is then given by

$$\frac{\Delta U_B}{V} \lesssim \frac{c(T,\mu_e)\mu_{5,i}^2}{2} \simeq \frac{\mu_{5,i}^2 T^2}{6}\,, \tag{19}$$

where the last expression follows from Eq. (8). Eq. (11) also implies that $\partial_t \mu_5 = 0$ if

$$\tilde{\mu}_5 = \frac{R_f \mu_{5,b} - \frac{2e^2\eta}{\pi c(T,\mu_e)} \int d\ln k\, k \frac{M_k}{V} \left(\frac{H_k}{8\pi M_k/k}\right)}{R_f + \frac{4e^4\eta}{\pi^2 c(T,\mu_e)} \frac{U_B}{V}}\,, \tag{20}$$

where H_k has again be normalized to its maximal value given by Eq. (14). For negligible magnetic fields $\tilde{\mu}_5 \simeq \mu_{5,b}$, as expected and magnetic field modes with $k < k_5(\mu_{5,b})$ are growing exponentially with rate $R_c(k) - R_r$ given by Eq. (15).

The magnetic field terms start to dominate for

$$\frac{U_B}{V} \gtrsim \frac{c(T,\mu_e)R_f}{4e^4\eta} \simeq \frac{10\pi}{3e^4}T^2m_e^2 \simeq 2\times 10^5 T^2 m_e^2\,, \tag{21}$$

In this case Eq. (20) gives

$$\tilde{\mu}_5 \simeq -\frac{\pi}{2e^2U_B}\int d\ln k\, kM_k\left(\frac{H_k}{8\pi M_k/k}\right)\,. \tag{22}$$

This is what Ruchayskiy et al call *tracking solution.* Note that $\tilde{\mu}_5$ from Eq. (20) varies with rates in general much slower than R_f and R_c. Also, since in general $\tilde{\mu}_5 \neq \mu_{5,b}$, the two terms in Eq. (11) do not vanish separately but only tend to compensate each other and are both roughly constant since μ_5 is approximately constant. Due to Eq. (9) the magnetic helicity changes linearly in time with a rate

$$\partial_t\mathcal{H} \simeq \frac{8\pi^2 V c(T,\mu_e)}{e^2}R_f(\mu_5-\mu_{5,b})\,. \tag{23}$$

Since helicity is nearly maximal this also implies that the magnetic energy also roughly grows or decreases linearly with time, depending on the sign of $(\mu_5 - \mu_{5,b})/\mathcal{H}$.

Combining Eqs. (6), (11) and (17) the rate of change of the total energy is

$$\begin{aligned}\partial_t U_{\rm tot} &= \partial_t U_B + \partial_t U_5 = \qquad (24)\\ &= -2\eta \int d\ln k M_k \left\{ (k-k_5)^2 + 2k_5 k \left[\left(\frac{H_k}{8\pi M_k/k} \right) {\rm sign}(\mu_5) + 1 \right] \right\} \\ &\quad -2R_f V c(T,\mu_e)\mu_5 \left(\mu_5 - \mu_{5,b}\right) ,\end{aligned}$$

where $k_5 = k_5(\mu_5)$ is given by Eq. (12). Since the expression in large braces in the integrand in Eq. (24) is non-negative due to Eq. (14) this shows that, apart from the term proportional to $\mu_{5,b}$ which describes a possible energy exchange with external particles, the total energy can only decrease due to the finite resistivity and the chirality-flip rate. The only equilibrium state in which the total energy is exactly conserved is given by $\mu_5 = \mu_{5,b}$ and a magnetic energy which is concentrated in the mode $k = k_0 = k_5(\mu_{5,b})$ and has maximal magnetic helicity with the opposite sign as $\mu_{5,b}$, $H_{k_0} = {\rm sign}(\mu_{5,b}) 8\pi M_{k_0}/k_0$.

The evolution of $\mu_{5,b}$ due to energy exchange with the background matter can be modeled as follows: In absence of magnetic fields multiplying Eq. (11) with $c(T,\mu_e)$ and using Eq. (8) gives

$$\partial_t n_5 = -2R_f[n_5 - c(T,\mu_e)\mu_{5,b}] = R_w n_b - 2R_f n_5 , \qquad (25)$$

where the gain term was written as a parity breaking electroweak rate R_w times the number density n_b of the background lepton species. This implies

$$n_b = 2c(T,\mu_e)\frac{R_f}{R_w}\mu_{5,b}\,, \qquad \frac{\mu_{5,b}}{T} \simeq 0.1 g_b \frac{R_w}{R_f}\,, \tag{26}$$

where the second expression holds for g_b non-degenerate relativistic fermionic degrees of freedom. The energy U_b associated with these background particles is thus given by

$$\frac{U_b}{V} = \int_0^{\mu_{5,b}} \mu'_{5,b} dn_b = \frac{R_f}{R_w} c(T,\mu_e)\mu_{5,b}^2 \sim 3\times 10^{-3} g_b^2 \frac{R_w}{R_f} T^4\,, \tag{27}$$

where the last expression again holds in the non-degenerate relativistic case. Note that for $\mu_{5,i} \sim \mu_{5,b} \sim (R_w/R_f)T$ Eq. (27) is of order $(R_w/R_f)T^4$ whereas U_5 from Eq. (17) is of order $(R_w/R_f)^2T^4$. Both energies vanish in the limit of parity conservation, $R_w \to 0$, as it should be. In terms of initial equilibrium chiral potential $\mu_{5,bi}$ and for $R_w \lesssim R_f$ the maximal magnetic energy is then

$$\frac{\Delta U_B}{V} \lesssim \frac{R_f}{R_w} c(T,\mu_e)\mu_{5,bi}^2 \sim 3\times 10^{-3} g_b^2 \frac{R_w}{R_f} T^4\,. \tag{28}$$

Setting $\partial_t U_b = -\partial_t U_5$ to ensure that the interactions conserve energy and using the last term in Eq. (24) for the contribution of the interactions to $\partial_t U_5$ yields an equation for the evolution of $\mu_{5,b}$,

$$\partial_t \mu_{5,b} = R_w \frac{\mu_5}{\mu_{5,b}} (\mu_5 - \mu_{5,b}) \,. \tag{29}$$

When U_b is included in $U_{\rm tot}$ the second term in Eq. (24) is absent in the time derivative of $U_{\rm tot}$. The total energy is then dissipated exclusively through resistive magnetic field damping. Eq. (29) indicates that $\mu_{5,b}$ typically changes with the rate R_w. An stationary state is reached if $\mu_{5,b} = \mu_5$ and the magnetic field concentrates in $k_5 = k_5(\mu_{5,b})$ with maximal helicity of sign opposite to $\mu_{5,b}$. This requires magnetic field growth rates larger than the Hubble rate, see Eq. (16).

μ_5 is continuously recreated with a rate $R_f \mu_{5,b} \sim 0.1 g_b R_w T$, see Eq. (26), and a time-independent or slowly varying μ_5 can be established which is given by Eq. (20). This can be the case, for example, in a supernova or a neutron star due to URCA processes which absorb left-chiral electrons with a rate R_w and turn them into neutrinos that subsequently escape the star.

Due to Eq. (6) amplification stops and resistive damping sets in when $2\eta k_5^2 t \sim 1$, thus

$$k_5 \sim k_5^0 \left(\frac{t_0}{t}\right)^{1/2} \,, \quad \mu_5 \sim \mu_5^0 \left(\frac{t_0}{t}\right)^{1/2} \,. \tag{30}$$

The Chiral Magnetic Effect in Hot Supernova Cores

The spin flip rate is dominated by the modified URCA rate

$$\epsilon_{\rm URCA} = \frac{457\pi}{10080}(1+3g_A^2)\cos^2\theta_C G_F^2 m_n m_p \mu_e T^6 \,,$$

The resistivity $\eta=1/(4\pi\sigma)$ is given by the conductivity

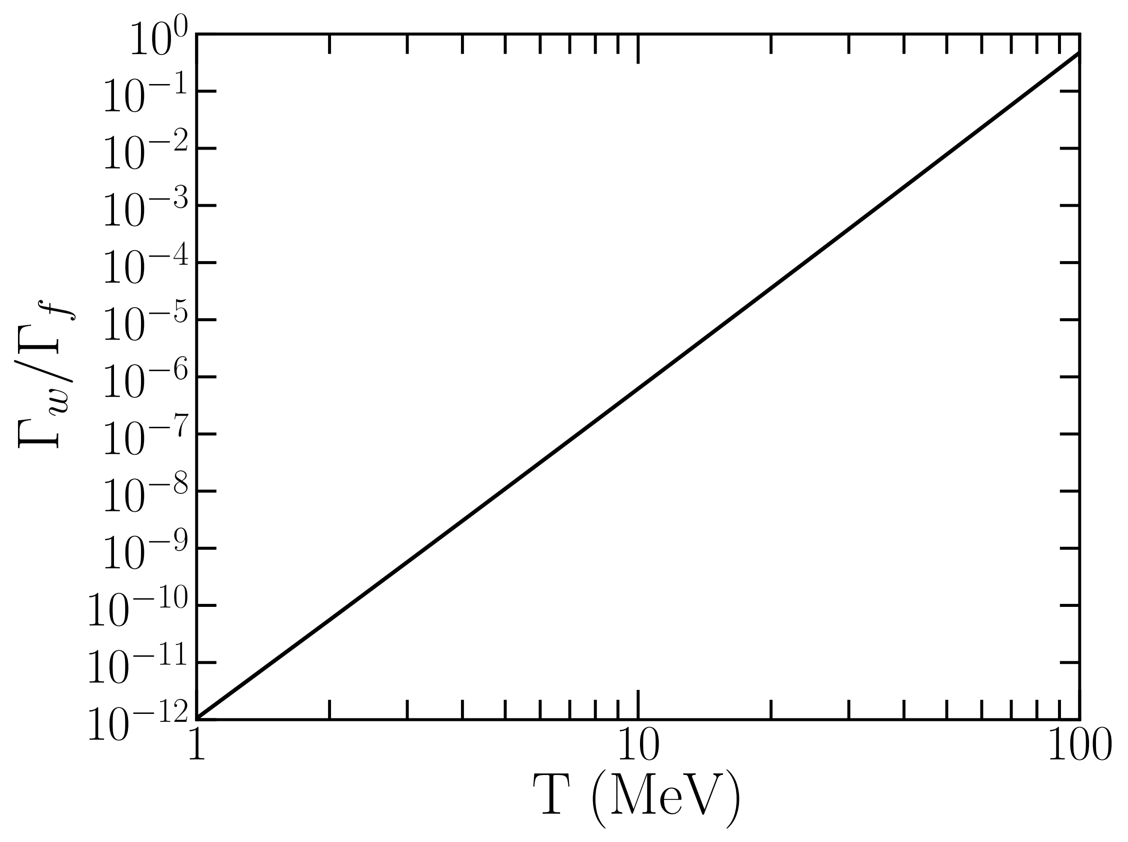
$$\sigma \simeq 1.5\times10^{45}\left(\frac{\rm K}{T}\right)^2\left(\frac{\rho_p}{10^{13}\,{\rm g\,cm^{-3}}}\right)^{3/2}\,{\rm s}^{-1}\,,$$

Comparing the velocity and chiral magnetic term for a velocity spectrum $v(l)\sim(l/L)^{n/2}$ for integral scale L at the length scale of maximal growth $l=2\pi/k_5=(\pi/e)^2/|\mu_5|$ gives

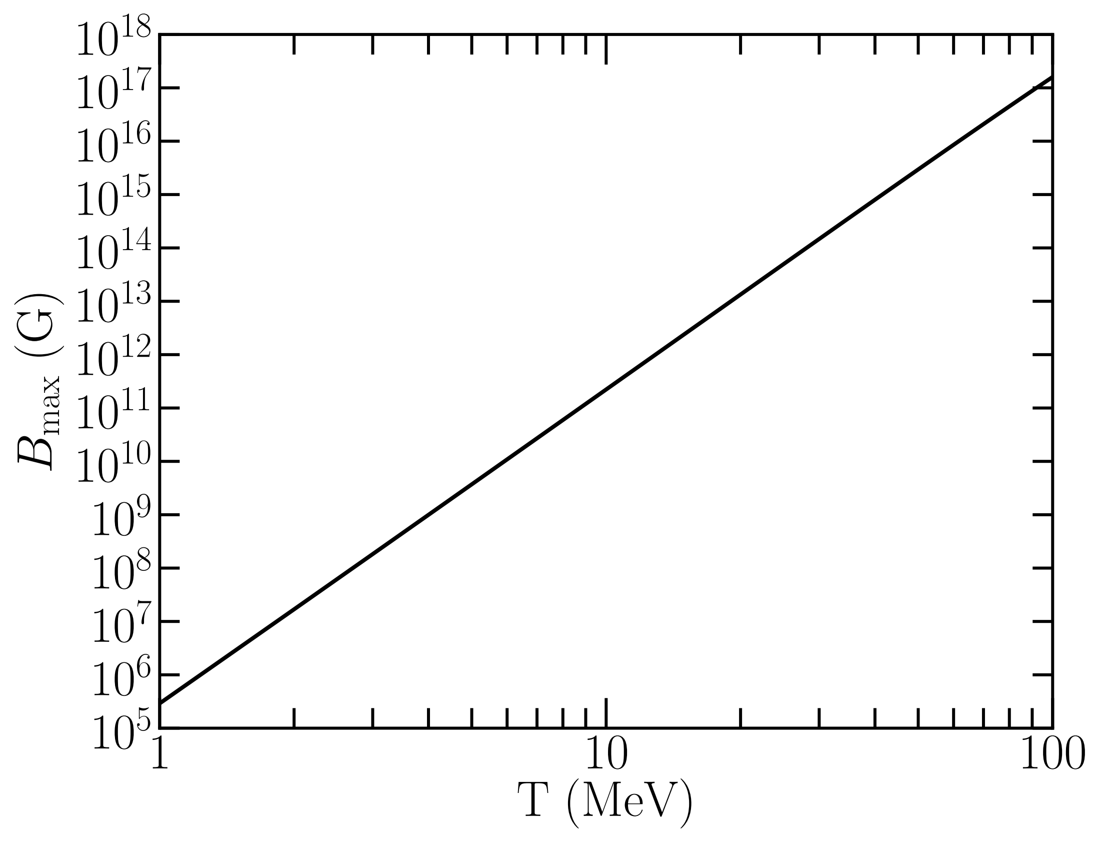
$$\frac{\boldsymbol{\nabla}\times(\boldsymbol{v}\times\mathbf{B})}{e^2/(2\pi^2\sigma)\mu_5\boldsymbol{\nabla}\times\mathbf{B}} \sim 2\sigma L v_{\rm rms}\left[\left(\frac{e}{\pi}\right)^2 L\mu_5\right]^{-(n/2+1)}\,,$$

For $v_{rms}\sim10^{-2}$ in a supernova this is $\lesssim 1$ if $n \gtrsim 4/3$.

URCA to spin flip rate



Resulting maximal field in hot neutron star within our formalism



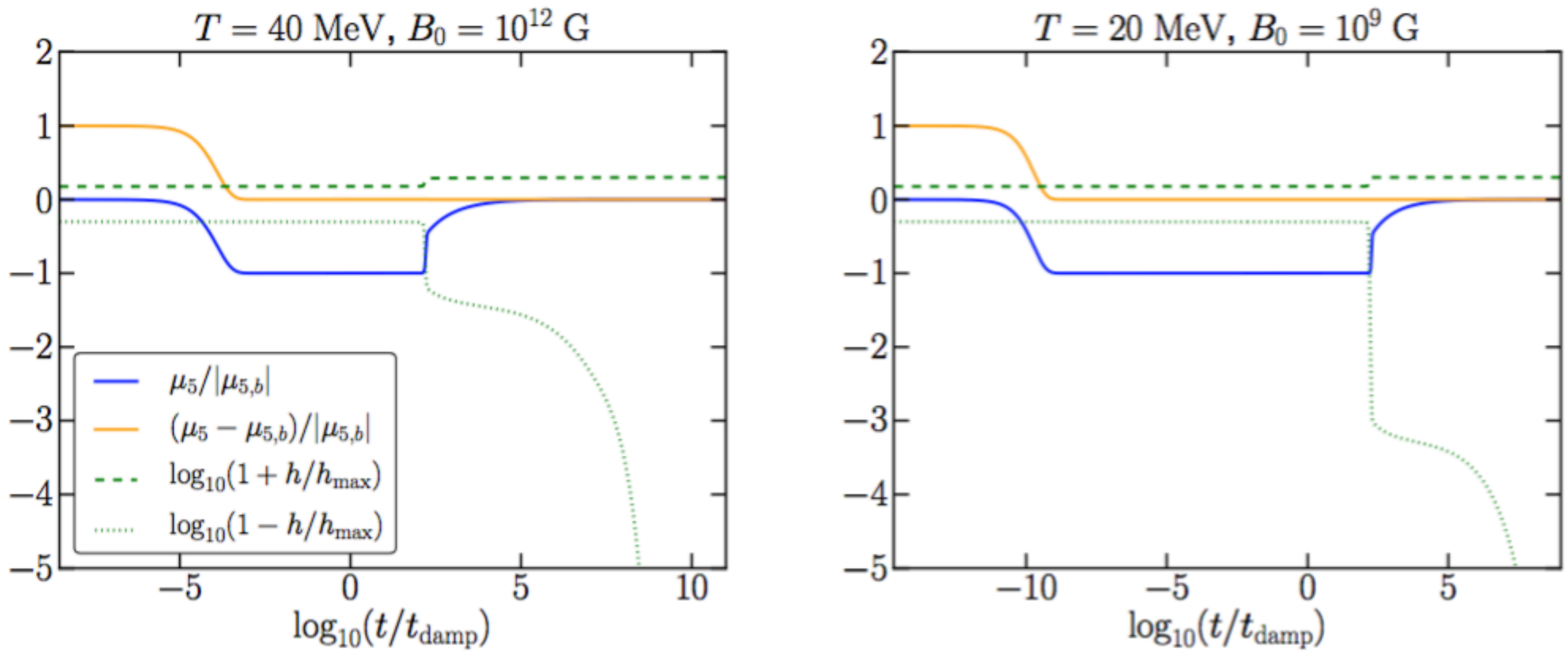
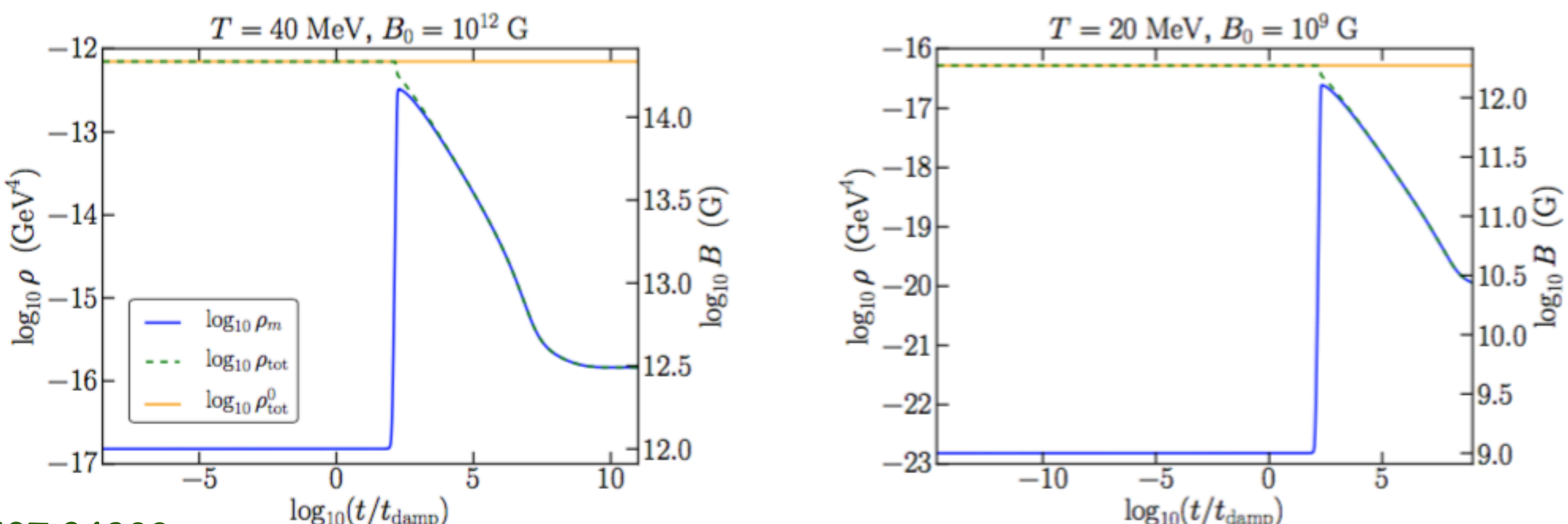


Figure 2. Time evolution of the chiral chemical potential normalized to the equilibrium value, $\mu_5/|\mu_{5,b}|$, relative difference of the chiral chemical potential to the equilibrium value, $(\mu_5 - \mu_{5,b})/|\mu_{5,b}|$ and, in logarithmic units, relative deviation of the helicity density from its maximal and minimal value, $1 \pm h/h_{\rm max}$. The left panel is for a temperature of $T = 40$ MeV and seed field $B_0 = 10^{12}$ G, and the right panel is for $T = 20$ MeV and a seed field of $B_0 = 10^9$ G.



Sigl, Leite, arXiv:1507:04983

Figure 3. Time evolution of the magnetic energy density ρ_m and total energy density $\rho_{\rm tot}$. Also shown is the initial total energy density which limits the maximal magnetic energy density that can be reached by the instability. In the left panel $T = 40$ MeV and in the right panel $T = 20$ MeV.

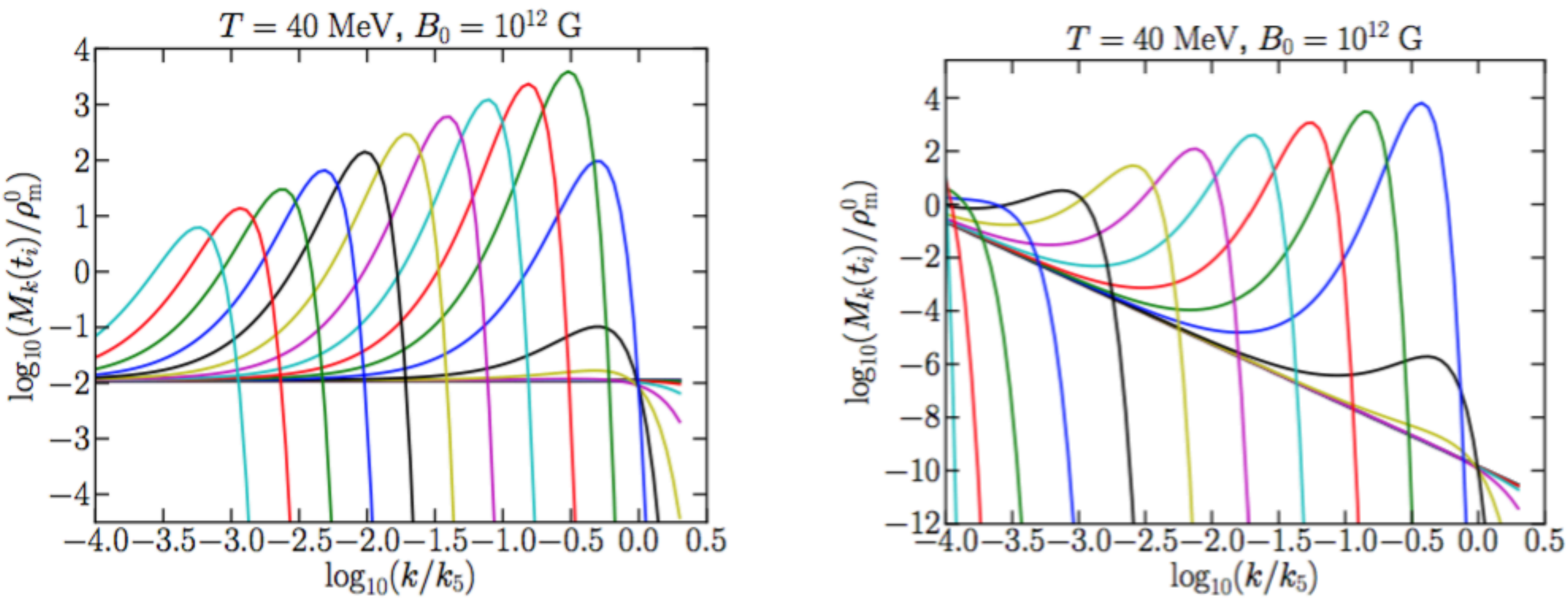
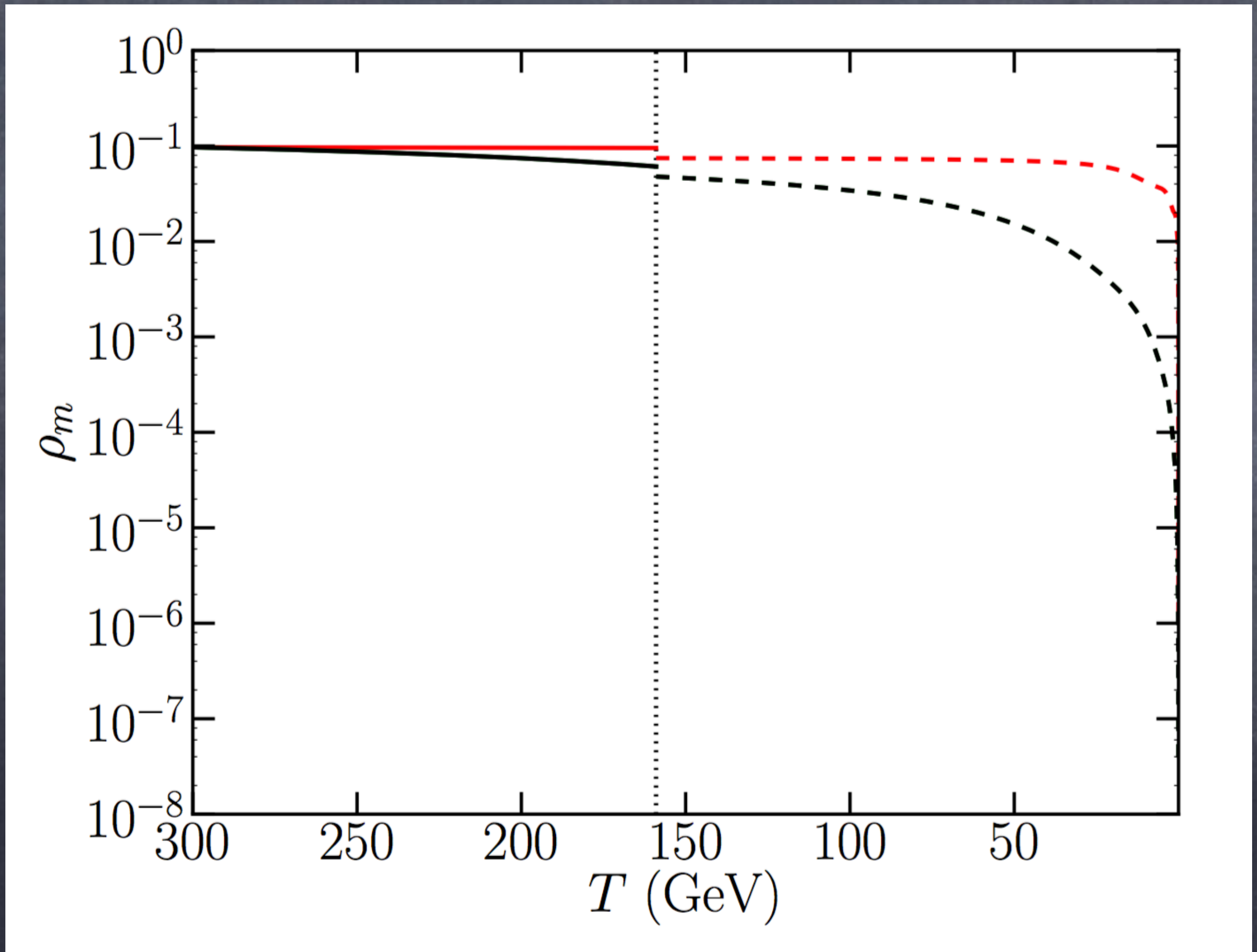


Figure 4. Time evolution of the magnetic field power spectrum normalized to the initial magnetic energy density, M_k/ρ_m^0, as a function of wavenumber k normalized to k_5. The power spectra are shown for equally spaced intervals in the logarithm of time between $t = t_{\rm damp}$ and $t = 10^8 t_{\rm damp}$, for $T = 40$ MeV. Left panel: Initially flat power spectrum. Right panel: Initial power spectrum has a Kolmogorov distribution.

The Chiral Magnetic Effect around the Electroweak Transition

Conductivity σ ~ 70 T

preliminary result: No significant magnetic field enhancement under realistic conditions, only resistive damping rate is somewhat slowed down



also, turbulence would play a more important role than in hot neutron stars

Pavlovic, Leite, Sigl, in preparation

Conclusions

1.) The chiral magnetic effect can lead to growing, helical magnetic fields in the presence of a chiral asymmetry in the lepton sector.

2.) However, spin flip interactions can damp the chiral asymmetry faster than the magnetic field growth rate.

3.) In hot supernova cores the chiral magnetic effect could play a significant role. This is less likely in the early Universe.

Outlook

1.) Role of turbulence unclear: If velocity term > chiral term the chiral magnetic effect could be considerably modified. Suppression if magnetic fields transported toward smaller scale ? Enhancement if transported toward larger scales (inverse cascade) ?

2.) Spatially varying chiral potential should be discussed quantitatively