Beyond General Relativity: the geometric deformation and new black hole solutions

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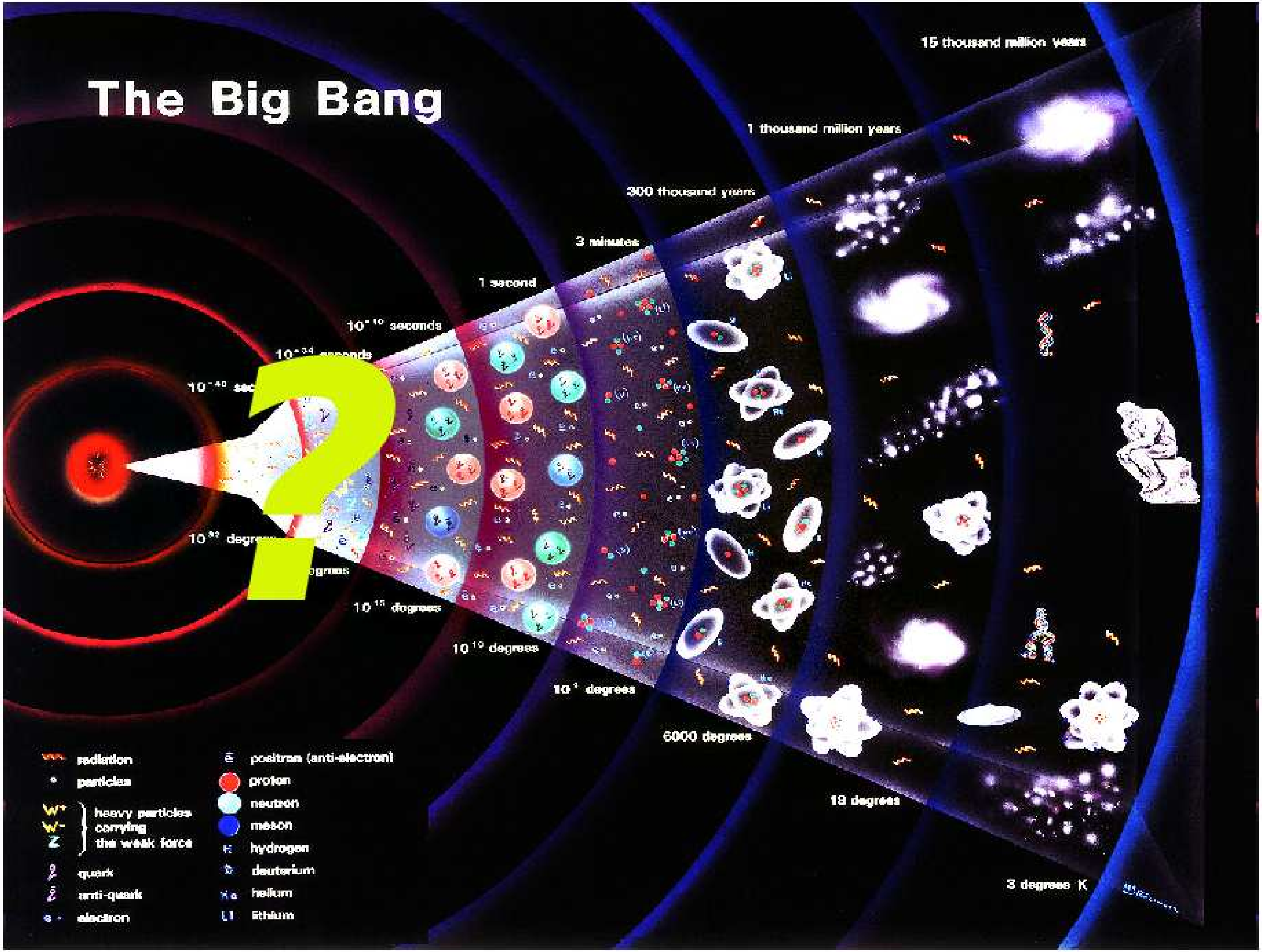
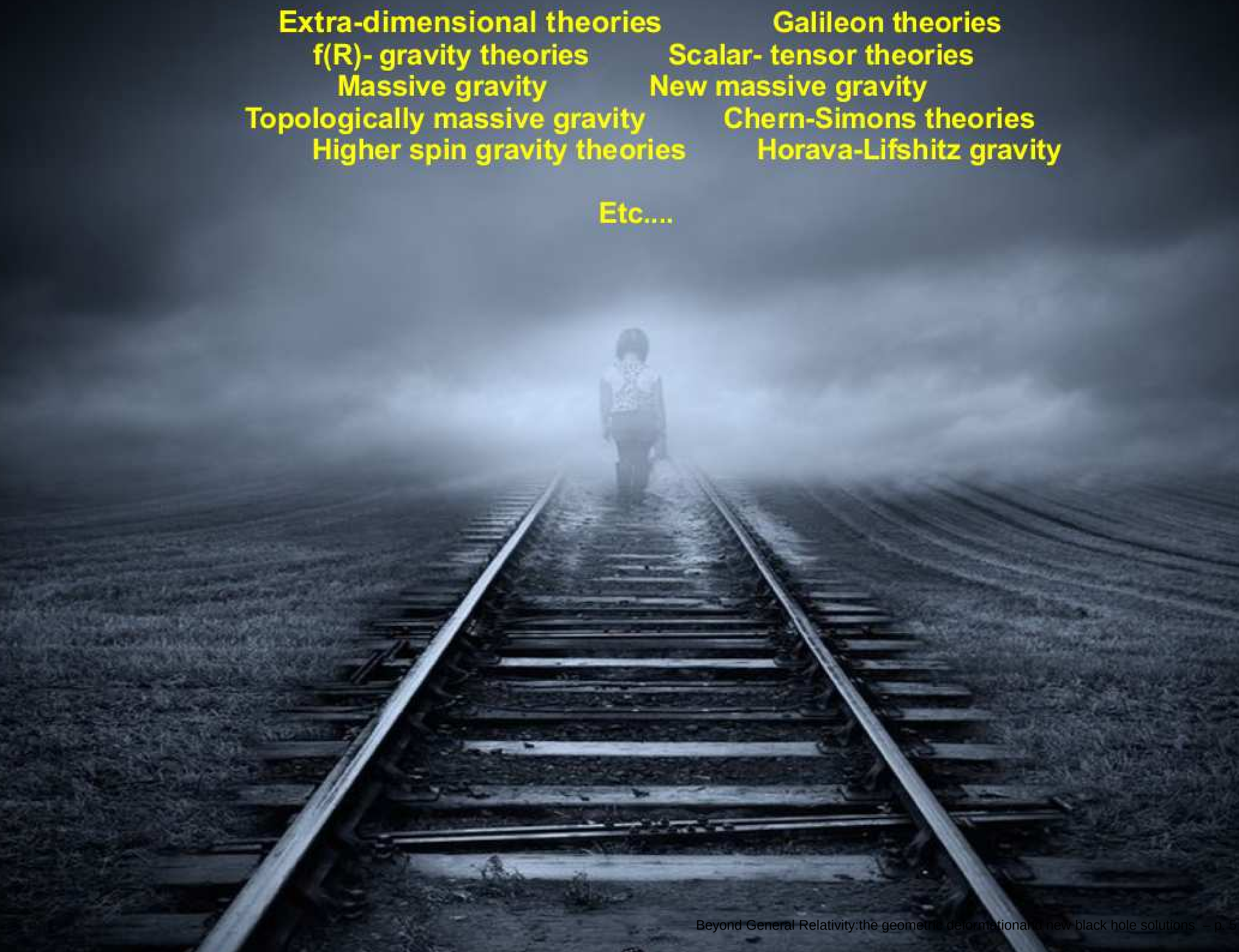
Beyond Einstein...

- Motivation
  - The standard model and the general relativity represents the two great theories in fundamental physics. The success of general relativity is beyond any doubt, however due to its inconsistency with quantum mechanics, it is not possible to ensure that this theory keeps its original structure at high energies.
  - One of the goals of the current study is to see what features of theories beyond Einstein could lead to an answer to any of the open problems in astrophysics (dark matter) or cosmology (dark energy)
  - In this talk: Geometric Deformation (GD), new BH solutions, GD on f(R) (preliminary results) .

Black holes, neutron stars, quark stars



The Big Bang
15 thousand million years
1 thousand million years
300 thousand years
3 minutes
1 second
6000 degrees
18 degrees
3 degrees K
radiation
particles
W+
W-
Z
heavy particles carrying the weak force
quark
anti-quark
electron
positron (anti-electron)
proton
neutron
meson
hydrogen
deuterium
helium
lithium


Extra-dimensional theories
Galileon theories
f(R)- gravity theories
Scalar- tensor theories
Massive gravity
New massive gravity
Topologically massive gravity
Chern-Simons theories
Higher spin gravity theories
Horava-Lifshitz gravity
Etc....

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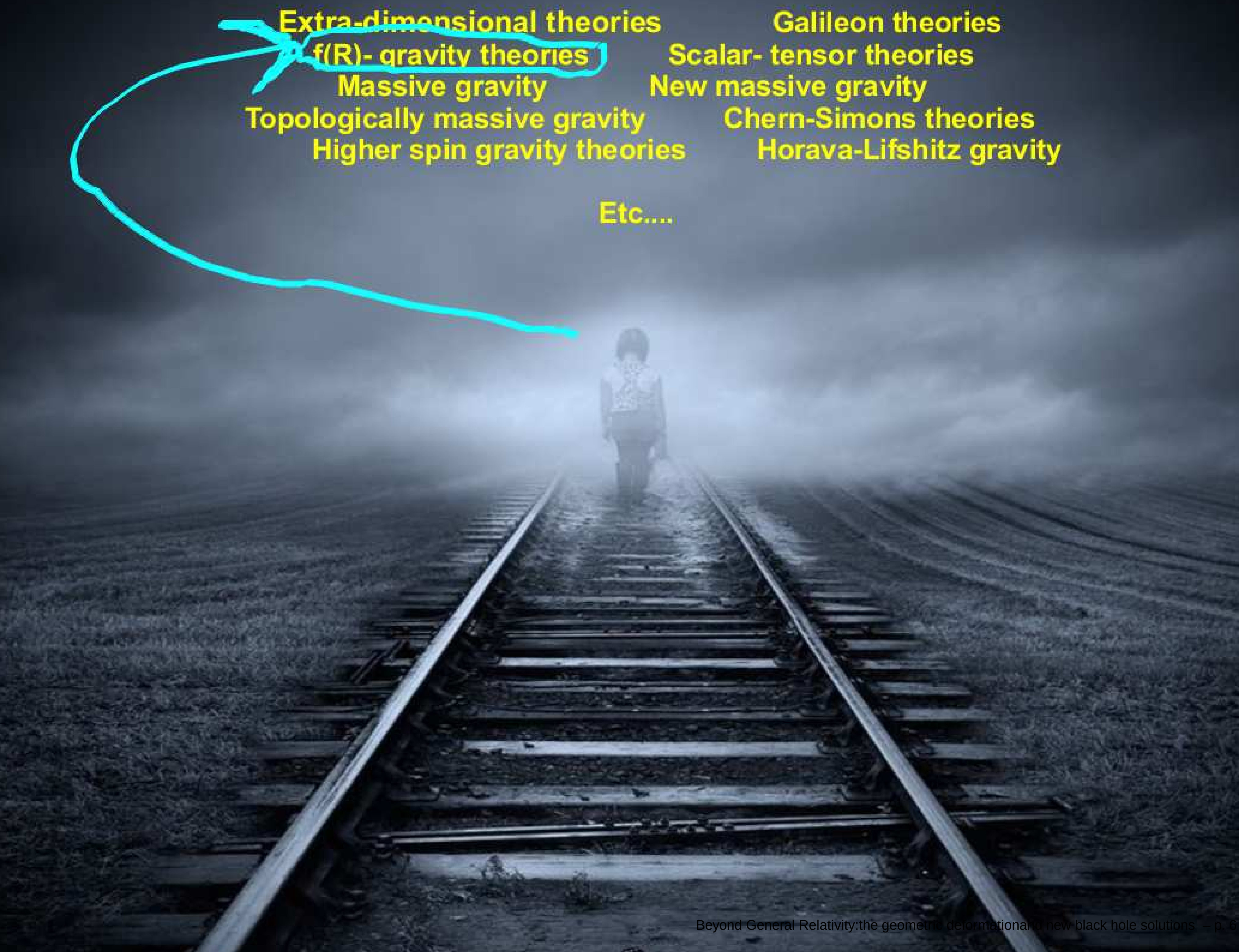
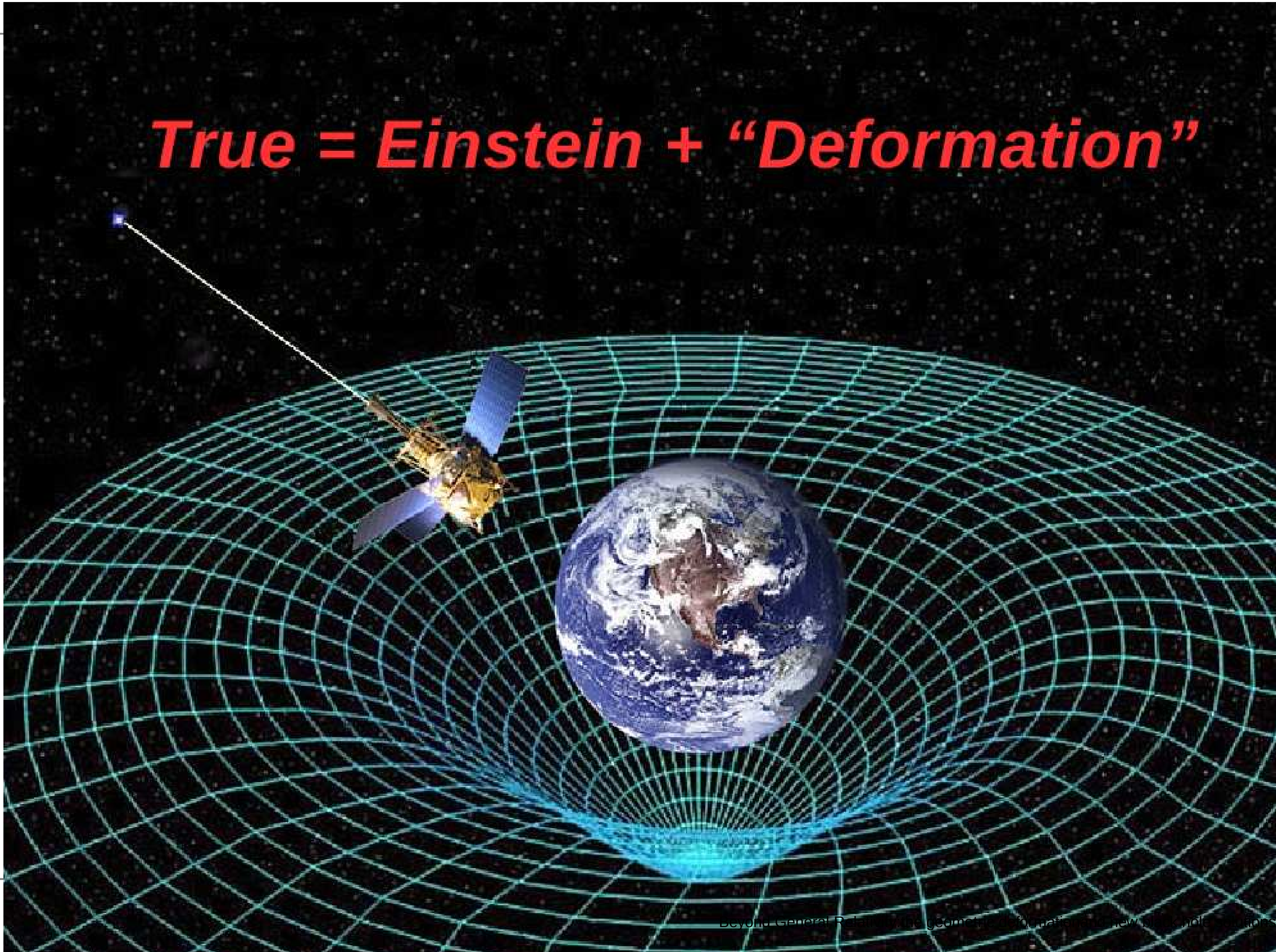


The Minimal Geometric Deformation



New theory

New theory = GR + "Corrections"

Corrections $\longrightarrow$ effects on T_{ab}

$T_{ab} \longrightarrow T_{ab} + x\,\mathcal{T}_{ab}$

$x = 0 \longrightarrow$ GR

$1/g_{rr} = 1 - 2m(r)/r$ + Corrections

Corrections = **Anisotropic consequences** + χ(something)

$\chi = 0$ we cannot ragain GR !!!

CONSISTENCE PROBLEM!!!

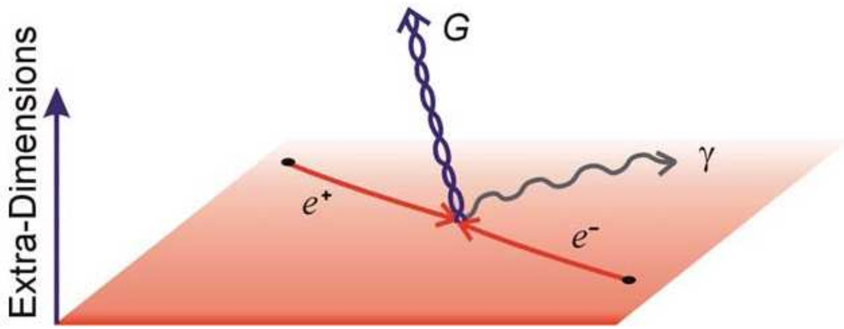
However, keeping under control anisotropic consequences on GR coming from the extended theory, we are able to obtain

Corrections = [zero] + χ (something)

This happen when we force a GR solution keep being a solution in the extended theory

$\longrightarrow$ ***MINIMAL GEOMETRIC DEFORMATION***

Extra dimension



- **Large Extra Dimension** (ADD theory) Arkani-Hamed, Dimopoulos, Dvali (1998)
- **Braneworld** (RS theory) L. Randall and R. Sundrum (1999)
  - Both models explain the hierarchy problem
  - ADD: **Many flat** extra dimensions
  - Braneworld: **Only one** extra dimension with a **warped geometry**.
- **No experimental evidence for extra dimensions so far:**
  - **LEP:** LEP Exotica Working Group, LEP Exotica WG 2004-03;
  - **Tevatron:** CDF Collaboration, Phys. Rev. Lett. 101 (2008) 181602; D0 Collaboration, Phys. Rev. Lett. 101 (2008) 011601.
  - **LHC:** ATLAS Collaboration, Phys. Lett. B 705 (2011) 294; Phys. Lett. B 709 (2012) 322.
  - **LHC:** CMS Collaboration, Phys. Rev. Lett. 107 (2011) 201804.
  - Recently: **LHC:** ATLAS collaboration, arXiv:1204.4646v2[hep-ex] Sep.2012.

Einstein field equations on the brane

The Einstein field equations on the brane may be written as a modification of the standard field equations [Shiromizu et al 2002]

5D Einstein equations:

$$G_{ab} + \Lambda_5 g_{ab} = \kappa_5^2 T_{ab}; \quad \kappa_5 = 8\pi G_5 \quad a = 0, ...4 \quad (Bulk)$$

$$G_{\mu\nu} = -8\pi T_{\mu\nu}^{T} - \Lambda g_{\mu\nu}, \quad \mu = 0, ...3 \quad (Brane)$$

where the energy-momentum tensor has new terms carrying bulk effects onto the brane:

$$T_{\mu\nu} \to T_{\mu\nu}{}^{T} = T_{\mu\nu} + \frac{6}{\sigma} S_{\mu\nu} + \frac{1}{8\pi} \mathcal{E}_{\mu\nu}$$

Here σ is the brane tension

High energy corrections + bulk gravitons

The new terms and are the high-energy corrections $S_{\mu\nu}$ and the projection of the bulk Weyl tensor on the brane $\mathcal{E}_{\mu\nu}$

$$S_{\mu\nu} = \frac{1}{12}T_{\alpha}^{\ \alpha}T_{\mu\nu} - \frac{1}{4}T_{\mu\alpha}T^{\alpha}_{\ \nu} + \frac{1}{24}g_{\mu\nu}\left[3T_{\alpha\beta}T^{\alpha\beta} - (T_{\alpha}^{\ \alpha})^2\right]$$

$$-8\pi\mathcal{E}_{\mu\nu} = -\frac{6}{\sigma}\left[\mathcal{U}(u_\mu u_\nu + \frac{1}{3}h_{\mu\nu}) + \mathcal{P}_{\mu\nu} + \mathcal{Q}_{(\mu}u_{\nu)}\right]$$

$$\begin{array}{rcl} \mathcal{U} & \rightarrow & Dark\ \ radiation \\ \mathcal{P}_{\mu\nu} & \rightarrow & Anisotropic\ \ stress \\ \mathcal{Q}_\mu & \rightarrow & Energy\ \ flux \end{array}$$

Spherically symmetric static distribution

Schwarzschild-like coordinates

$$ds^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right)$$

A perfect fluid (General Relativity)+high energy corrections+Weyl functions

$$-8\pi \left(\rho + \frac{1}{\sigma} \left(\frac{\rho^2}{2} + 6\mathcal{U} \right) \right) = -\frac{1}{r^2} + e^{-\lambda} \left(\frac{1}{r^2} - \frac{\lambda'}{r} \right),$$

$$-8\pi \left(-p - \frac{1}{\sigma} \left(\frac{\rho^2}{2} + \rho p + 2\mathcal{U} \right) + \frac{\mathcal{P}}{\sigma} \right) = -\frac{1}{r^2} + e^{-\lambda} \left(\frac{1}{r^2} + \frac{\nu'}{r} \right),$$

$$-8\pi \left(-p - \frac{1}{\sigma} \left(\frac{\rho^2}{2} + \rho p + 2\mathcal{U} \right) - \frac{\mathcal{P}}{2\sigma} \right) = \frac{1}{4} e^{-\lambda} \left[2\nu'' + \nu'^2 - \lambda'\nu' + 2\frac{(\nu' - \lambda')}{r} \right],$$

$$p' = -\frac{\nu'}{2}(\rho + p).$$

Minimal geometric deformation

Let us see the "solution" for the geometric function

$$e^{-\lambda} = \underbrace{1 - \frac{k^2}{r}\int_0^r x^2\,\rho\,dx}_{\mathrm{GR-solution}} + \underbrace{e^{-I}\int_0^r \frac{e^I}{\frac{\nu'}{2}+\frac{2}{x}}\left[H(p,\rho,\nu) + \frac{k^2}{\sigma}\left(\rho^2 + 3\,\rho\,p\right)\right]dx + \beta(\sigma)\,e^{-I}}_{\mathrm{Geometric\ deformation}},$$

It can be written as

$$e^{-\lambda} = \underbrace{1 - \frac{k^2}{r}\int_0^r r^2\rho dr}_{General\ Relativity}$$ **+ "DEFORMATIONS"**

$$e^{-\lambda} = 1 - \frac{k^2}{r}\int_0^r r^2\left[\rho + \frac{1}{\sigma}\left(\frac{\rho^2}{2} + \frac{6}{k^4}\mathcal{U}\right)\right]dr,$$

JO Mod. Phys. Lett. A **23**, 3247 (2008).

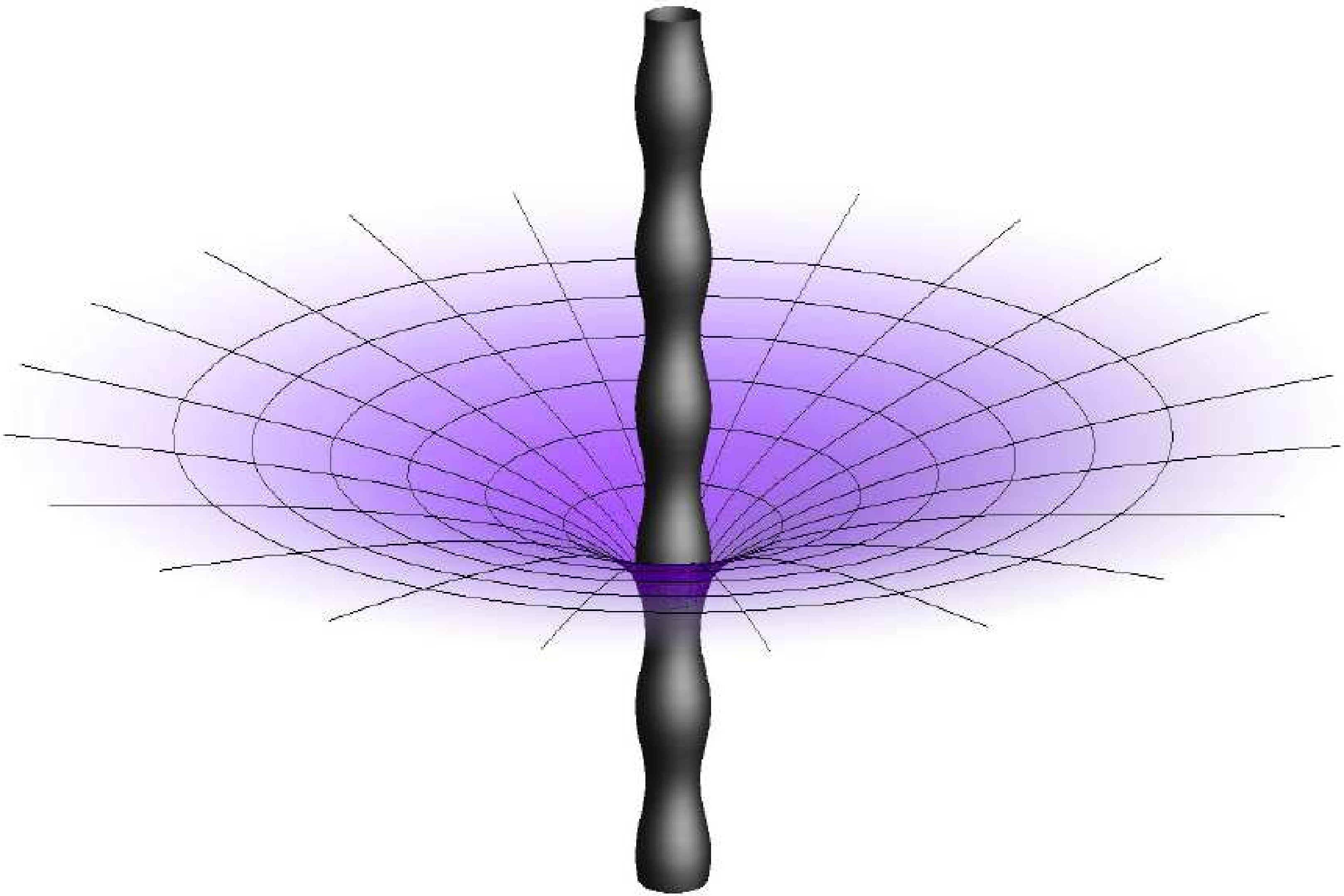
JO *Braneworld stars: anisotropy minimally projected onto the brane*, in Gravitation and Astrophysics (ICGA9), Ed. J. Luo, World Scientific, Singapore, 173-182, (2010).

JO, Int. J. Mod. Phys. D **18**, 837 (2009).

Minimal geometric deformation

- THE MGD WORKS!
- When a solution of the four-dimensional Einstein equations is considered as a possible solution of the BW system, the geometric deformation produced by extra-dimensional effects is minimized, and the open system of effective BW equations is automatically satisfied JO, F. Linares, A. Pascua, A. Sotomayor Class. Quant. Grav. **30** 175019 (2013).
- This approach was successfully used to generate physically acceptable interior solutions for stellar systems and even exact solutions were found:
  - JO, Int. J. Mod. Phys. D **18**, 837 (2009);
  - JO, F. Linares Phys. Rev. D **88**, 104026 (2013).

Black Strings

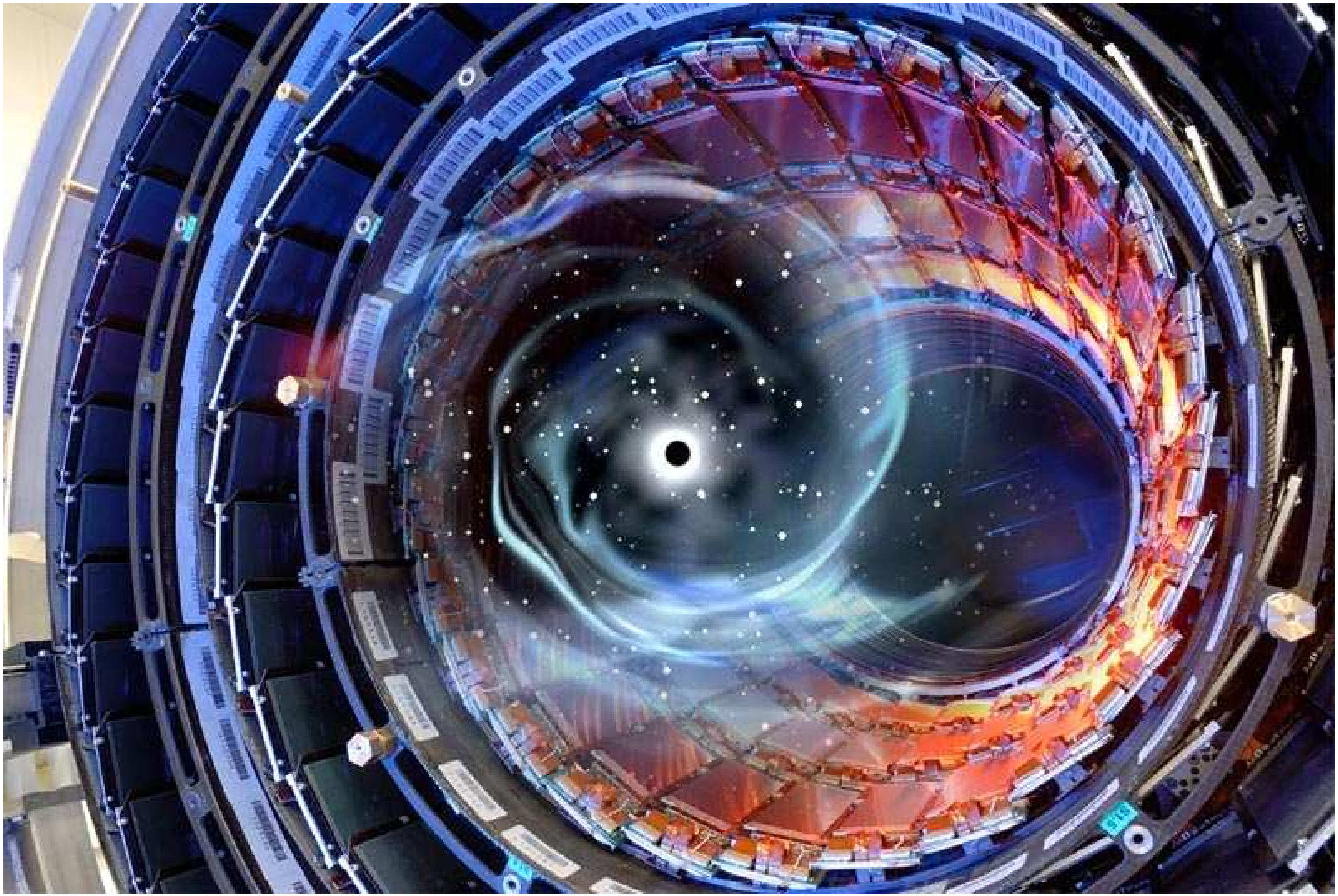


Black Strings

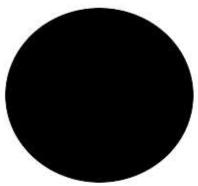
$$
\begin{aligned}
\mathsf{g}_{\mu\nu}(x,y) =\; & g_{\mu\nu}(x,0) - \kappa_5^2\left[T_{\mu\nu} + \tfrac{1}{3}(\sigma - T)g_{\mu\nu}\right]\,|y| \\
& + \left[-\mathcal{E}_{\mu\nu} + \tfrac{1}{4}\kappa_5^4\left(T_{\mu\alpha}T^{\alpha}{}_{\nu} + \tfrac{2}{3}(\sigma - T)T_{\mu\nu}\right) + \tfrac{1}{6}\left(\tfrac{1}{6}\kappa_5^4(\sigma - T)^2 - \Lambda_5\right)g_{\mu\nu}\right]\,y^2 + \\
& + \Bigg[2K_{\mu\beta}K^{\beta}{}_{\alpha}K^{\alpha}{}_{\nu} - (\mathcal{E}_{\mu\alpha}K^{\alpha}{}_{\nu} + K_{\mu\alpha}\mathcal{E}^{\alpha}{}_{\nu}) - \tfrac{1}{3}\Lambda_5 K_{\mu\nu} - \nabla^{\alpha}\mathcal{B}_{\alpha(\mu\nu)} + \tfrac{1}{6}\Lambda_5\left(K_{\mu\nu} - g_{\mu\nu}K\right) \\
& + K^{\alpha\beta}R_{\mu\alpha\nu\beta} + 3K^{\alpha}{}_{(\mu}\mathcal{E}_{\nu)\alpha} - K\mathcal{E}_{\mu\nu} + \left(K_{\mu\alpha}K_{\nu\beta} - K_{\alpha\beta}K_{\mu\nu}\right)K^{\alpha\beta} - \tfrac{\Lambda_5}{3}K_{\mu\nu}\Bigg]\,\frac{|y|^3}{3!} + \\
& + \Bigg[\tfrac{\Lambda_5}{6}\left(R - \tfrac{\Lambda_5}{3} + K^2\right)g_{\mu\nu} + \left(\tfrac{K^2}{3} - \Lambda_5\right)K_{\mu\alpha}K^{\alpha}{}_{\nu} + (R - \Lambda_5 + 2K^2)\mathcal{E}_{\mu\nu} \\
& + \left(K^{\alpha}{}_{\sigma}K^{\sigma\beta} + \mathcal{E}^{\alpha\beta} + KK^{\alpha\beta}\right)\,R_{\mu\alpha\nu\beta} - \tfrac{1}{6}\Lambda_5 R_{\mu\nu} + 2K_{\mu\beta}K^{\beta}{}_{\sigma}K^{\sigma}{}_{\alpha}K^{\alpha}{}_{\nu} + K_{\sigma\rho}K^{\sigma\rho}K\,K_{\mu\nu} \\
& + \mathcal{E}_{\mu\alpha}\left(K_{\nu\beta}K^{\alpha\beta} - 3K^{\alpha}{}_{\sigma}K^{\sigma}{}_{\nu} + \tfrac{1}{2}KK^{\alpha}{}_{\nu}\right) + \left(\tfrac{7}{2}KK^{\alpha}{}_{\mu} - 3K^{\alpha}{}_{\sigma}K^{\sigma}{}_{\mu}\right)\mathcal{E}_{\nu\alpha} - \tfrac{13}{2}K_{\mu\beta}\mathcal{E}^{\beta}{}_{\alpha}K^{\alpha}{}_{\nu} \\
& + \left(3\,K^{\alpha}{}_{\mu}K^{\beta}{}_{\alpha} - K_{\mu\alpha}K^{\alpha\beta}\right)\mathcal{E}_{\nu\beta} - K_{\mu\alpha}K_{\nu\beta}\mathcal{E}^{\alpha\beta} - 4K^{\alpha\beta}R_{\mu\nu\gamma\alpha}K^{\gamma}{}_{\beta} - \tfrac{7}{6}K^{\sigma\beta}K_{\mu}{}^{\alpha}R_{\nu\sigma\alpha\beta}\Bigg]\,\frac{y^4}{4!} + \cdots
\end{aligned}
$$

R. Casadio, JO, R. da Rocha, Class. Quantum Grav. **30** 175019 (2014).

Black Holes in the Braneworld



Black Holes in the Braneworld



$$ds^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right)$$

- Dadhich,Maartens,Papadopoulos and Rezania (DMPR Solution):

$$e^{\nu^+} = e^{-\lambda^+} = 1 - \frac{2\mathcal{M}}{r} + \frac{q}{r^2}, \quad \mathcal{U}^+ = -\frac{\mathcal{P}^+}{2} = \frac{4}{3}\pi q \sigma \frac{1}{r^4},$$

- Casadio, Fabbri and Mazzacurati (CFM Solution)

$$e^{\nu^+} = \left[\frac{\eta + \sqrt{1 - \frac{2\mathcal{M}}{r}(1+\eta)}}{1+\eta} \right]^2, \quad e^{\lambda^+} = \left[1 - \frac{2\mathcal{M}}{r}(1+\eta) \right]^{-1},$$

$$\frac{16\pi \mathcal{P}^+}{k^4 \sigma} = -\frac{\mathcal{M}(1+\eta)\eta}{\eta + \sqrt{1 - \frac{2\mathcal{M}}{r}(1+\eta)}} \frac{1}{r^3}, \quad \mathcal{U}^+ = 0,$$

Finding the tidal charge q

Taking $p_R = 0$ and imposing the boundary constraint

$$R\,\nu'_R = -\frac{(M-\mathcal{M}) - \frac{2\,\mathcal{M}\,K\,M_{\rm P}}{\sigma\,R^2\,\ell_{\rm P}}}{(M-\mathcal{M}) - \frac{\mathcal{M}\,K\,M_{\rm P}}{\sigma\,R^2\,\ell_{\rm P}}}$$

where K is a (dimensionful) constant we can fix later, we obtain a simple relation between q and $\mathcal{M}$ given by (R. Casadio, JO, Phys. Lett. B, 715, 251-255 (2012)).

$$q = \frac{2\,K\,\mathcal{M}}{\sigma\,R} \qquad (*)$$

- it vanishes for $\mathcal{M} \to 0$ and for $\sigma^{-1} \to 0$, and
- it vanishes for very small star density, that is for $R \to \infty$ at fixed $\mathcal{M}$ and σ.

As the pressure does not need to vanish at the surface in the BW, we can get the same simple $q = q(\mathcal{M}, \sigma)$ solution by

$$4\,\pi\,R^3\,p_R = \frac{M_{\rm P}\,\mathcal{M}\,K}{\ell_{\rm P}\,\sigma\,R^2}\left(2 + R\,\nu'_R\right) - (M-\mathcal{M})\left(1 + R\,\nu'_R\right)$$

In our solution $(*)$ R is still a free parameter. We need an interior solution to fix it!

Micro black holes and critical mass

We obtain the horizon radius

$$h = \frac{\ell_{\rm P}}{M_{\rm P}} \left(\mathcal{M} + \sqrt{\mathcal{M}^2 + q\, \frac{M_{\rm P}^2}{\ell_{\rm P}^2}} \right)$$

and the classicality condition $h \gtrsim \lambda_M$ reads

$$\frac{M}{M_{\rm P}^2} \left(\mathcal{M} + \sqrt{\mathcal{M}^2 + q\, \frac{M_{\rm P}^2}{\ell_{\rm P}^2}} \right) \gtrsim 1$$

We expand for $M \sim \mathcal{M} \simeq M_{\rm G} \ll M_{\rm P}$, thus obtaining

$$\frac{h^2}{\lambda_C^2} \simeq \frac{M^2}{M_{\rm P}^2}\, \frac{q}{\ell_{\rm P}^2} \simeq \frac{M_{\rm G}^2}{M_{\rm P}^2}\, \bar{M}^2\, \bar{q}\, \frac{\ell_{\rm G}^2}{\ell_{\rm P}^2} \simeq \bar{M}^2\, \bar{q} \simeq 1$$

or $\bar{M}^4 \simeq n\left(n_1 + \bar{M}^2\right)$, which yields

$$M_c \simeq 1.3\, M_{\rm G}$$

Always a critical mass M_c above M_G

What we knew until January 2015

It is *impossible* to have Schwarzschild exterior for a spherically symmetric selfgravitating system made of regular matter.

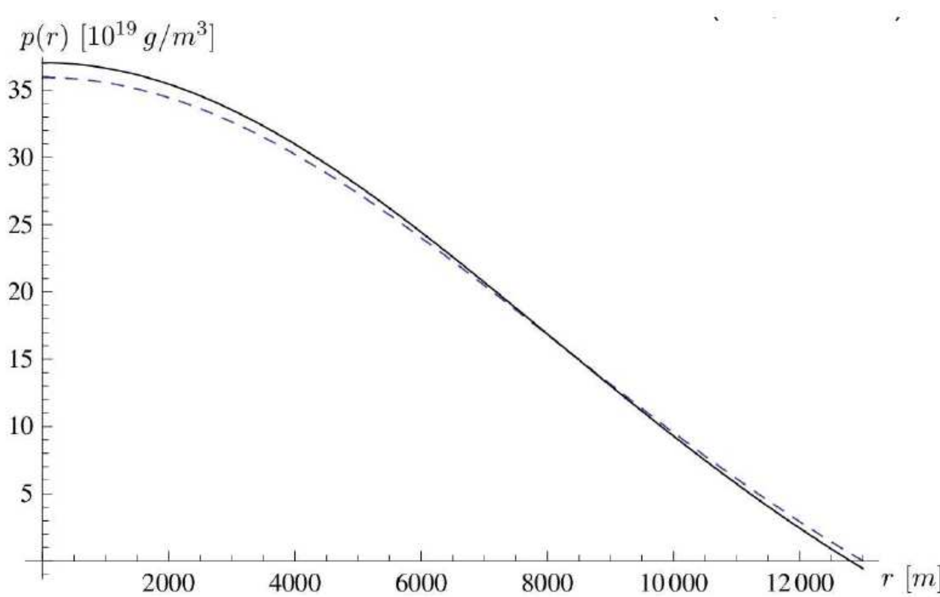
$ds^2 = Schwarzschild$

ONLY NEGATIVE PRESSURE IS ALLOWED!

Dark Stars

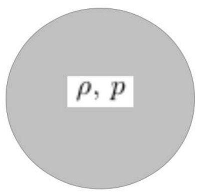
Contrary to what was previously believed, it is possible to have a selfgravitating system made of regular matter and with a Schwarzschild exterior. In order to accomplish the above, we consider the exact interior brane-world solution found in JO, Int. J. Mod. Phys. D **18**, 837 (2009).

$$e^{\nu} = A(1 + Cr^2)^4 \; ; \qquad e^{-\lambda} = 1 - \frac{8}{7}\, C\, r^2 \, \frac{(3 + C\, r^2)}{(1 + C\, r^2)^2} + f^*(r) \; ,$$

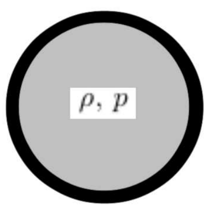


Dark Stars = Dark Matter...?

Even if severe constraints from lensing or other tests are derived for brane-world stars, their existence cannot yet be ruled out, if their exterior is the same as in GR. JO, László Gergely and Roberto Casadio, Class. Quantum Grav. 32 045015 (2015) .



$$ds^2 = Schwarzschild + \mathcal{O}(\sigma^{-1})$$



$$ds^2 = Schwarzschild$$

There is a solid crust which thickness $\Delta \sim 1/R\,\sigma$.

A POSSIBLE DARK MATTER SOURCE!

f(R) theory (in progress)

The general action for $f(R)$ gravity is given by [See, for instance, Capozziello, S. and De Laurentis, M. (2011). Extended Theories of Gravity. Physics Reports 509: 167]

$$S = \frac{c^4}{16\pi G}\int d^4x\sqrt{-g}f(R) + S_{\rm matter}$$

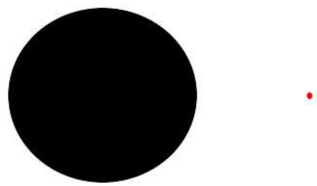
The function $f(R)$ can be written as

$$f(R) = R + \alpha h(R)\ ,$$

- The vacuum:
  - The standard MGD apprach (deformation only in the radial metric component) produces no consequences on the Schwarzschild solution.
  - A complete deformation is necessary, hence a temporal deformation must be considered!!!

The MGD must be extended in f(R) theories

The vacuum: beyond Schwarzschild



$$ds^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

$$R_\mu^{\ \mu} = e^{-\lambda}\left(\nu'' + \frac{\nu'^2}{2} + 2\,\frac{\nu'}{r} + \frac{2}{r^2}\right) - \lambda'\, e^{-\lambda}\left(\frac{\nu'}{2} + \frac{2}{r}\right) - \frac{2}{r^2} = 0$$

e$^{-\lambda} = 1 - \frac{2\,M}{r} + \beta(\sigma)\, e^{-I}$

The deformed exterior metric components read

$$e^{\nu} = 1 - \frac{2\,M}{r}$$

$$e^{-\lambda} = \left(1 - \frac{2\,M}{r}\right)\left(1 - \frac{\beta(\sigma)}{r - \frac{3\,M}{2}}\right)$$

$$\beta(\sigma) = R^3\left(\frac{1 - \frac{3\,M}{2\,R}}{1 - \frac{2\,M}{R}}\right)\left[\left(\frac{\nu'_R}{R} + \frac{1}{R^2}\right)\frac{f_R^*}{8\pi} + p_R\right]$$

Extended Geometric Deformation

In its original representation, only deformations in the radial metric component is allowed.
Next logical step is to consider (if possible) a deformation in the temporal metric component
R Casadio, JO, R da Rocha, Class. Quantum Grav. 32, 215020 (2015)

$$\nu(r) = \nu_s + h(r)$$

$$R = 0 \, ,$$

$$\left(\frac{\nu'}{2} + \frac{2}{r}\right) f' + \left(\nu'' + \frac{\nu'^2}{2} + \frac{2\nu'}{r} + \frac{2}{r^2}\right) f + F(h) = 0 \, ,$$

$$F(h) = \mu' \, \frac{h'}{2} + \mu \left(h'' + \nu_s' \, h' + \frac{h'^2}{2} + 2 \, \frac{h'}{r}\right) \, .$$

$$e^{-\lambda(r)} = 1 - \frac{2\,M}{r} + \underbrace{e^{-I(r,R)} \left(\beta - \int_R^r \frac{e^{I(x,R)}\, F(h)}{\frac{\nu'}{2} + \frac{2}{x}} \, dx\right)}_{\text{Geometric deformation}} \, .$$

The simplest solution

Let us consider $F(h) = 0$ whose solution is given by the simple expression

$$e^{h/2} = a + \frac{b}{2\,M}\,\frac{1}{\sqrt{1-2\,M/r}}$$

$$r \to \infty \quad \Rightarrow \quad e^{\nu} \to 1 \quad \Rightarrow \quad h \to 0\,,$$

$$a = 1 - \frac{b}{2\,M}$$

$$e^{\nu} = \left(1 - \frac{2\,M}{r}\right)\left[1 + \frac{b(\sigma)}{2\,M}\left(\frac{1}{\sqrt{1-\frac{2\,M}{r}}} - 1\right)\right]^2 \;;\;\; r > 2M$$

$$e^{-\lambda} = 1 - \frac{2\,M}{r} + \beta\,e^{-I}$$

Black hole solution...?

A particularly simple case is given when $\beta = 0$

$$e^{-\lambda} = 1 - \frac{2\,M}{r}$$

which will produce no geometric deformation in the radial metric component, so it is exactly the Schwarzschild form and diverges for $r \to 2\,M$. However, due to the modified time component this is now a real singularity: $R_{\mu\nu\rho\sigma}\,R^{\mu\nu\rho\sigma}$ diverges at $r = 2\,M$.
The above solution could therefore only represent the exterior geometry of a self-gravitating system with radius $R > 2\,M$. For large r we have

$$e^{\nu} \simeq 1 - \frac{2\,M - b}{r} - \frac{b\,(2\,M - b)}{4\,r^2}$$

from which one can read off the ADM mass

$$\mathcal{M} = M - \frac{b}{2}$$

and the tidal charge

$$Q = \frac{b\,(2\,M - b)}{4}$$

New solutions (in progress)

By considering vacuum solutions around Schwarzschild in the form

$$e^{\nu} = (1 - 2\,M/r)^{1+k}\ ; \quad \mathcal{M} = (1+k)\,M\ ,$$

we obtain a “master” exact solution yielding to

$k = 0 \rightarrow$ Standard geometric deformation

$k = 1 \rightarrow$ Tidally charged solution

$$e^{\nu}\ =\ e^{-\lambda} = 1 - \frac{2\,\mathcal{M}}{r} + \frac{Q}{r^2}\ ,$$

$k = 2 \rightarrow$ Extension of the tidally charged solution

$$e^{\nu}\ =\ 1 - \frac{2\,\mathcal{M}}{r} + \frac{Q}{r^2} - \frac{2}{9}\,\frac{\mathcal{M}\,Q}{r^3}\ ,$$

$$e^{-\lambda} = 1 - \frac{2\,\mathcal{M}}{3\,r} + \left(1 - \frac{2\,\mathcal{M}}{3\,r}\right)^{-1}\left[\frac{128\beta\ell_2}{r}\left(1 - \frac{\mathcal{M}}{6r}\right)^7 + \frac{20\mathcal{M}}{21\,r}\left(1 - \frac{7\mathcal{M}}{15\,r}\right)\right]$$

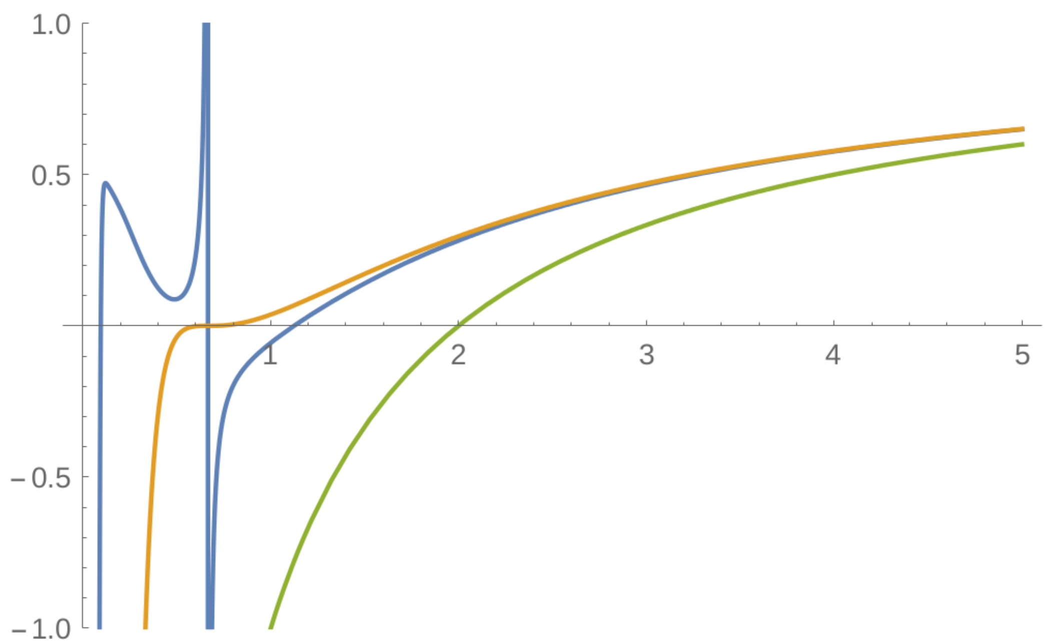
$k > 2$ High order extensions of the tidally charged solution (exacts)

New BH solutions

This solution displays two zeros of g_{rr}^{-1}, namely $r = r_i$ and h, and a surface $r = r_c$ where g_{rr}^{-1} diverges (and $g_{tt} = 0$). These surfaces separate the space-time in four regions, namely

- $0 < r < r_i$
- $r_i < r < r_c$
- $r_c < r < h$
- $r > h$.

An exterior observer at $r > h > r_c$ will never see this singularity, as it is hidden behind the horizon h.



Conclusions and outlook

- The MGD, in its original form, is incompatible with $f(R)$ theories.
- The MGD deformation was consistently extended to the case when both gravitational potentials, namely the radial and time metric components, are affected by bulk gravitons.
- We showed that the deformation for the time metric component induces part of the deformation in the radial metric component.
- The MGD in the context of $f(R)$ theories: both gravitational potentials must be deformed (In progress)

Thanks!