# Dualities and Symmetries of Galileons

Based on work in collaboration with: James Scargill, Vishagan Sivanesan, Mikael von Strauss
arXiv: 1503.02700, 1506.03446

**Johannes Noller**, University of Oxford

**Texas 2015**, Geneva, 16/12/15

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# Symmetries of Scalar Field Theories

**We will consider single real scalar field theories on flat space:**

$$\mathcal{L} \equiv \mathcal{L}[\pi, \partial\pi, \partial\partial\pi, \ldots]$$

**with a** *global, continuous and infinitesimal (Wess-Zumino)* **symmetry**

$$\pi \to \pi + \delta\pi$$

**where**

$$\delta\pi = \delta\pi[x, \pi, \partial\pi, \partial\partial\pi, \ldots]$$

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\delta\pi &= c + b_a x^a + c^{(2)}\pi + b_a^{(2)}\pi^a \\
&+ c^{(3)}\pi^2 + \pi b_a^{(3)}\pi^a + \pi b_a^{(4)} x^a + r_{ab}x^a\pi^b \\
&+ s_{ab}x^a x^b + s_{ab}^{(2)} x^a \pi^b + s_{ab}^{(3)}\pi^a\pi^b \\
&+ \ldots
\end{aligned}$$

*JN, Sivanesan, von Strauss '15*

# Why care?

- **Model-building in an *EFT* sense**

$$\pi \to \pi + c, \qquad \Longrightarrow \qquad \pi, \cancel{\pi^2}, \cancel{\pi^3}, \ldots, \partial^a \pi \partial_a \pi, \Box\pi, \ldots$$

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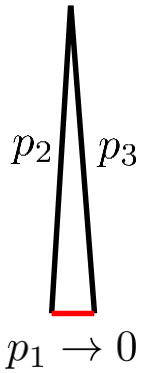
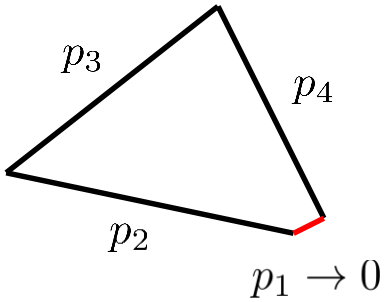
- **Soft-pion theorems/phenomenology**



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$$\pi \to \pi + \sum_{N=0}^{\infty} c_{a_1,\ldots,a_n} x^{a_1} \ldots x^{a_n}, \qquad \Longrightarrow \qquad \mathcal{L} = -\frac{1}{2}\partial_a\pi\partial^a\pi$$

- **DBI symmetry**:

$$\text{Dilatations} + \text{SCTs} \qquad \Longrightarrow \qquad \mathcal{L} \sim \sqrt{1 + \partial_a\pi\partial^a\pi}$$

- **Galileon**:

$$\pi \to \pi + c + b_a x^a \qquad \Longrightarrow \qquad \mathcal{L} = \sum_{n=0}^{D} c_{n+1}\pi\epsilon_{a_1\ldots a_n}\epsilon^{b_1\ldots b_n}\pi^{a_1}_{b_1}\ldots\pi^{a_n}_{b_n}$$

*Nicolis, Rattazzi, Trincherini '09, Hinterbichler, Joyce '14*

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- **???** **Quartic Galileon**

*Cheung, Kampf, Novotny, Trnka '14*

# Enhanced Galileon Symmetry

**The symmetry ansatz:**

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\delta\pi^{(2)} &= c + b_a x^a + c^{(2)}\pi + b_a^{(2)}\pi^a \\
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**Quadratic order enhanced symmetry:**

$$\begin{aligned}\delta\pi &= c + b_a x^a \\ &+ b_a^{(2)}\pi^a + r_{ab}x^a\pi^b \\ &+ s_{ab}\left(x^a x^b + d_{(1,1)}x^a\pi^b + d_{(0,2)}\pi^a\pi^b\right),\end{aligned}$$

**where**

$$d_{(1,1)} = \frac{c_3}{c_2}, \qquad d_{(0,2)} = \frac{c_3^2 - c_2 c_4}{c_2^2},$$

$$c_5 = \frac{2c_3c_4}{c_2} - \frac{c_3^3}{c_2^2}.$$

*Hinterbichler, Joyce '15, JN, Sivanesan, von Strauss '15*

# More enhancement?

**Higher order symmetries?:**

$$\begin{aligned}\delta\pi^{(3)} = & c + Q^{(1)}_a x^a + Q^{(2)}_{ab} x^a x^b + 2\alpha\, Q^{(2)}_{ab} x^a \pi^b \\ & + \alpha^2\, Q^{(2)}_{ab} \pi^a \pi^b + Q^{(3)}_{abc} x^a x^b x^c + 3\alpha\, Q^{(3)}_{abc} x^a x^b \pi^c \\ & + 3\alpha^2\, Q^{(3)}_{abc} x^a \pi^b \pi^c + \alpha^3\, Q^{(3)}_{abc} \pi^a \pi^b \pi^c\end{aligned}$$

**where**

$$\alpha = \frac{c_3}{2c_2}, \qquad c_4 = \frac{3c_3^2}{4c_2}, \qquad c_5 = \frac{c_3^3}{2c_2^2}$$

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**Galileon Dualities:**

$$\sigma = -\pi + \tfrac{s}{2\Lambda}\pi_a\pi^a - \tfrac{s^2}{2\Lambda^2}\pi^a\pi^b\pi_{ab} + \tfrac{s^3}{2\Lambda^3}\pi^a\pi^b\pi_a{}^c\pi_{bc} + \tfrac{s^3}{6\Lambda^3}\pi^a\pi^b\pi^c\pi_{abc} + \mathcal{O}(\pi^5)$$

**There exists an invertible, non-linear, non-local field re-definition $\pi \to \sigma$, that leaves the Galileon form invariant.**

**We can use the duality to always set $c_3 = 0$ and completely fix the redundancy associated with the duality in this way.**

*de Rham, Keltner, Tolley '14, Kampf, Novotny '14, JN, Scargill '15, JN, Sivanesan, von Strauss '15*

# Enhanced Galileon Symmetries II

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# Conclusions

- **Quartic Galileons (modulo duality) have an enhanced symmetry**

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- **There is strong evidence no further enhanced symmetries exist for Galileons, except for those of the kinetic term**

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**Thank you!**