Parity odd CMB power spectrum via helical magnetic field

Hector Javier Hortua, Departamento de Física,
Universidad Nacional de Colombia, Bogotá, Colombia
Leonardo Castañeda, Grupo de Gravitación y Cosmología, O.A.N,
Universidad Nacional de Colombia, Bogotá, Colombia

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Abstract

In this work we compute the temperature-polarization correlation C_l^{TB} and C_l^{EB} in the cosmic microwave background (CMB) generated by the presence of causal magnetic fields with helical contribution. We analize the effect of an infrared cutoff in the power spectrum of causal fields on the cross-correlation and we compare our results with previous work.

1 The model and statistics for a stochastic PMF

We consider a causal stochastic PMF generated after inflation, thus the maximum coherence lenght for the fields must be not less than the Hubble horizon and besides the region of spectra is realized when $n_s \geq 2$ (blue spectral) [1]. Now, the PMF power spectrum which is defined as the Fourier transform of the two point correlation can be written as

$$\langle B_i(\mathbf{k})B_j^*(\mathbf{k}')\rangle = (2\pi)^3\delta^3(\mathbf{k}-\mathbf{k}')\Big(P_{ij}P_B(k) + i\epsilon_{ijk}\hat{k}^k P_H(k)\Big), \tag{1}$$

where $P_{ij} = \delta_{ij} - \frac{\mathbf{k}_i\mathbf{k}_j}{k^2}$ is a projector onto the transverse plane, $P_B(k)$, $P_H(k)$ are the symmetric and helical PMF power spectrum, and ϵ_{ijk} is the antisymmetric symbol. We focus our attention to the evolution of a causally-generated PMF parametrized by a power law with index $n_s \geq 2$, with an ultraviolet cut-off k_D and the dependence of an infrared cutoff, k_m. In this framework, we consider for $k_m \leq k \leq k_D$, the power spectrum can be defined as

$$P_B(k) = A_s k^{n_s}, \quad \text{and} \quad A_s = \frac{B_\lambda^2 2\pi^2\lambda^{n_s+3}}{\Gamma(\frac{n_s+3}{2})}, \tag{2}$$

$$P_H(k) = A_H k^{n_H}, \quad \text{and} \quad A_H = \frac{H_\lambda^2 2\pi^2\lambda^{n_H+3}}{\Gamma(\frac{n_H+4}{2})}, \tag{3}$$

where B_λ, H_λ are the comoving PMF strength and magnetic helicity smoothing over a Gaussian sphere of comoving radius λ. The equation for anisotropic trace-free part in the Fourier space is

$$\Pi_{ij} = \int \frac{d^3k'}{2(2\pi)^4}\left[B_i(k')B_j(k-k') - \frac{\delta_{ij}}{3}B_l(k')B^l(k-k')\right], \tag{4}$$

Now, we define the two point correlation function as [3]

$$\langle \Pi_i(k)\Pi_j^*(k')\rangle = (2\pi)^3\delta^3(k-k')\Big(P_{ij}\left|\Pi(k)\right|_S^2 + i\epsilon_{ijl}\hat{k}^l\left|\Pi(k)\right|_A^2\Big), \tag{5}$$

Now, to calculate the power spectrum, we use the eqs. (1), (4), (5) and the Wick's theorem to evaluate the 4-point correlator of the PMF. The power spectrum for anisotropic stress is

$$\left|\Pi^{(V)}(k)\right|_S^2 = \frac{1}{128\pi^5}\int d^3p'\Big([(1-\gamma^2)(1+\beta^2)-\gamma^2\beta^2$$
$$+ \quad \mu\gamma\beta]P_B(p')P_B(|\mathbf{k}-\mathbf{p}'|)-(\gamma\beta-\mu)P_H(p')P_H(|\mathbf{k}-\mathbf{p}'|)\Big),$$

$$\left|\Pi^{(V)}(k)\right|_A^2 = \frac{1}{128\pi^5}\int d^3p'\Big([\beta+\gamma\mu-2\gamma^2\beta]P_B(p')\cdot$$
$$\cdot \quad P_B(|\mathbf{k}-\mathbf{p}'|)+(\gamma+\beta\mu-2\beta^2\gamma)P_H(p')P_H(|\mathbf{k}-\mathbf{p}'|)\Big),$$

where S, A means symmetric and antisymmetric parts. Finally, the anisotropic stress tensor part becomes

$$\left|\Pi^{(T)}(k)\right|_S^2 = \frac{1}{512\pi^5}\int d^3p'\Big([(1+\gamma^2)(1+\beta^2)]\cdot$$
$$\cdot \quad P_B(p')P_B(|\mathbf{k}-\mathbf{p}'|)+4(\gamma\beta)P_H(p')P_H(|\mathbf{k}-\mathbf{p}'|)\Big),$$

$$\left|\Pi^{(T)}(k)\right|_A^2 = \frac{1}{512\pi^5}\int d^3p'\Big((\beta+\gamma^2\beta)P_B(p')P_B(|\mathbf{k}-\mathbf{p}'|)$$
$$+ \quad (\gamma+\beta^2\gamma)P_H(p')P_H(|\mathbf{k}-\mathbf{p}'|)\Big).$$

being the angular functions defined as

$$\beta = \frac{\mathbf{k}\cdot(\mathbf{k}-\mathbf{k}')}{k\,|\mathbf{k}-\mathbf{k}'|}, \quad \mu = \frac{\mathbf{k}'\cdot(\mathbf{k}-\mathbf{k}')}{k'\,|\mathbf{k}-\mathbf{k}'|}, \quad \gamma = \frac{\mathbf{k}\cdot\mathbf{k}'}{kk'}. \tag{6}$$

We found these results using xAct [2] package and these results are in agreement with [1] and [3].

2 The cutoff dependence and the concern scale

We work with an upper cutoff k_D corresponds to the damping scale due to magnetic field energy is dissipated into heat through the damping of magnetohydrodynamics MHD waves found by [1] as

$$k_D \approx \left(1.7\times 10^2\right)^{\frac{2}{n+5}}\left(\frac{B_\lambda}{10^{-9}nG}\right)^{\frac{-2}{n+5}}\left(\frac{k_\lambda}{1Mpc^{-1}}\right)^{\frac{n+3}{n+5}}\frac{h^{\frac{1}{n+5}}}{Mpc}. \tag{7}$$

The most general scenario at PMFs takes into account a infrared cutoff k_m for low values of k and depends on the generation model of PMF [4],[5]. We define this infrared cut off as

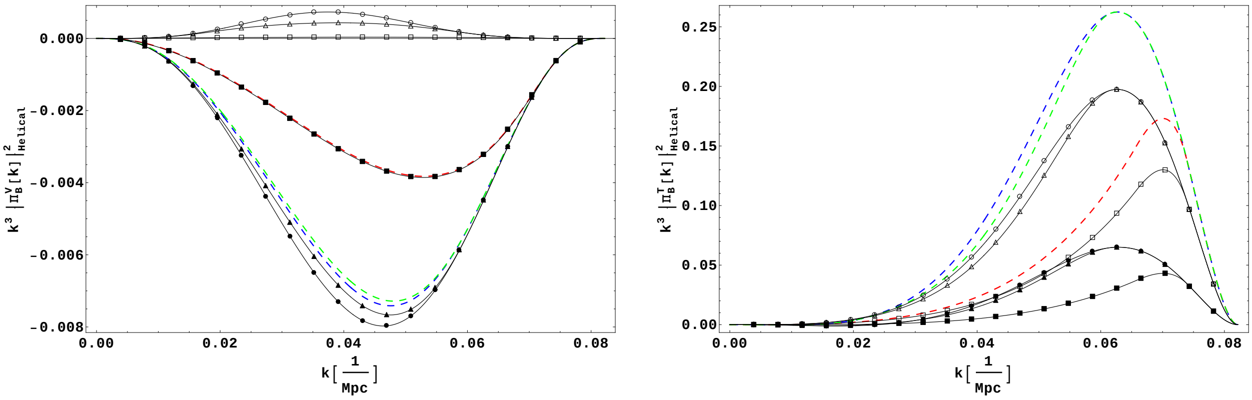
$$k_m = \alpha k_D \tag{8}$$

where $0 < \alpha < 1$.

3 PMF power spectra

Here we show the power spectrum for vector fig. 1a and tensor fig. 1b (at a scale of $\lambda = 1$Mpc, $n_s = n_H = 2$ and $B = H = 1nG$, maximal helicity for all plots). In our previous work [5] we show the change of the spectrum for different values of n and we also have studied mathematically the implications when an infrared cutoff is introduced, even the case a physical scenario for such methodology is still under debate.

The open(full) symbols denote antisymmetric(symmetric) helical part while the circle, triangle and square refer to $\alpha = 0.01$, $\alpha = 0.5$, and $\alpha = 0.8$ respectively. The colors describe the total contribution, Blue ($\alpha = 0.01$), Green ($\alpha = 0.5$), Red ($\alpha = 0.8$). The integration domain and the effects of the upper and lower cutoff are described in [5].



(a) Plot of vector part power spectrum $|\Pi_B^{(V)}|^2$ with differents lower cutoff.

(b) Plot of tensor part power spectrum $|\Pi_B^{(T)}|^2$ with differents lower cutoff.

Figure 1: The vector and tensor anisotropic stress power spectrum.

4 CMB power spectra

Using the total angular momentum formalism [6], the Cross-Correlation power spectrum of the CMB is given by

$$(2l+1)^2 C_l^{\Theta\,B\,(X)} = \frac{4}{\pi}\int dk k^2 \Theta_l^{(X)\,*}(\tau_0,k) B_l^{(X)}(\tau_0,k), \tag{9}$$

$$(2l+1)^2 C_l^{E\,B\,(X)} = \frac{4}{\pi}\int dk k^2 E_l^{(X)\,*}(\tau_0,k) B_l^{(X)}(\tau_0,k), \tag{10}$$

where $X = \{V,\ T\}$. Now, we use the approximate values for vector and tensor contribution of the E-type, B-type and temperature moments obtained in [7]

$$\frac{\Theta_l^{(V)}(\tau_0,k)}{2l+1} \approx \sqrt{\frac{l(l+1)}{2}}\frac{5\Pi_B^{(V)}(k)}{kL_\gamma(\rho_{\gamma 0}+P_{\gamma 0})}\frac{j_l(k\tau_0)}{k\tau_0}, \tag{11}$$

$$\frac{B_l^{(V)}(\tau_0,k)}{2l+1} \approx -\sqrt{\frac{(l-1)(l+2)}{18}}\frac{5\Pi_B^{(V)}(k)}{(\rho_{\gamma 0}+P_{\gamma 0})}\frac{j_l(k\tau_0)}{k\tau_0}, \tag{12}$$

$$\frac{E_l^{(V)}(\tau_0,k)}{2l+1} \approx -\sqrt{\frac{(l-1)(l+2)}{18}}\frac{5\Pi_B^{(V)}(k)}{(\rho_{\gamma 0}+P_{\gamma 0})}\left((l+1)\frac{j_l(k\tau_0)}{(k\tau_0)^2} - \frac{j_{l+1}(k\tau_0)}{k\tau_0}\right), \tag{13}$$

$$\frac{\Theta_l^{(T)}(\tau_0,k)}{2l+1} \approx -\frac{7(2\pi)^2}{50}\sqrt{\frac{l(l+2)!}{(l-2)!}}\left(Gz_{eq}\tau_0^2\ln\left(\frac{z_{in}}{z_{eq}}\right)\right)\Pi_B^{(T)}(k)\frac{J_{l+3}(k\tau_0)}{(k\tau_0)^3}, \tag{14}$$

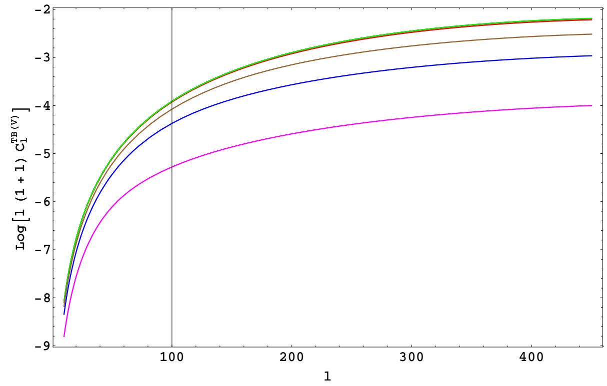
$$\frac{B_l^{(T)}(\tau_0,k)}{2l+1} \approx \frac{7(2\pi)^2}{100}\sqrt{l}\left(Gz_{eq}\tau_0\ln\left(\frac{z_{in}}{z_{eq}}\right)\right)\frac{\Pi_B^{(T)}(k)}{k}\left(l\frac{J_{l+3}(k\tau_0)}{k\tau_0} - J_{l+4}(k\tau_0)\right), \tag{15}$$

$$\frac{E_l^{(T)}(\tau_0,k)}{2l+1} \approx -\frac{7(2\pi)^2}{100}\sqrt{l}\left(Gz_{eq}\tau_0\ln\left(\frac{z_{in}}{z_{eq}}\right)\right)\frac{\Pi_B^{(T)}(k)}{k}\left(J_{l+3}(k\tau_0)\left(1-\frac{l^2}{2(k\tau_0)^2}\right)+\frac{J_{l+4}(k\tau_0)}{k\tau_0}\right), \tag{16}$$

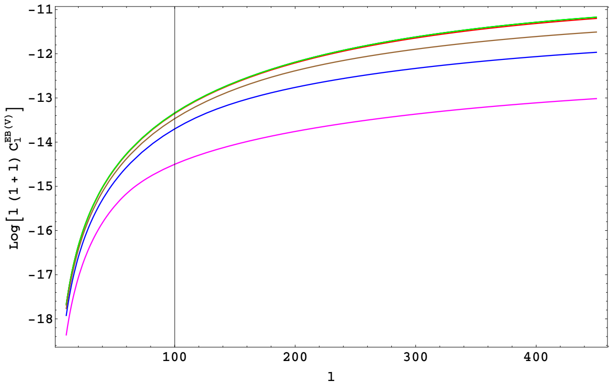
being j_l and J_l the spherical Bessel and Bessel first kind functions respectively. Using the last expressions and eqns. (9)-(10) we get the CMB Cross-Correlations power spectrum, the solutions are shown in figs. 2a-2b-2c-2d

Where the integrals for vector and tensor temperature-polarization correlations are given by

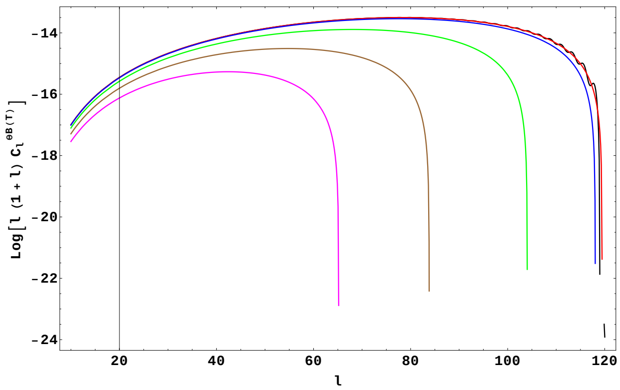
$$l(l+1)C_l^{\Theta\,B\,(V)} = -\frac{100l(l+1)}{\pi L_\gamma(\rho_{\gamma 0}+P_{\gamma 0})^2}\sqrt{\frac{l(l-1)(l+2)(l+1)}{32}}\int k\left|\Pi_B^{(V)}(k)\right|^2\left(\frac{j_l(k\tau_0)}{(k\tau_0)}\right)^2 dk, \tag{17}$$



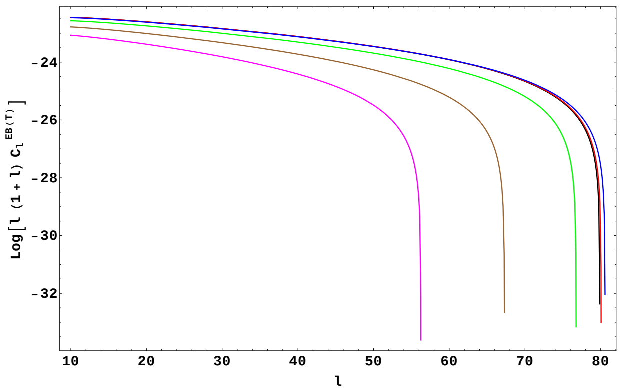
(a) CMB parity-odd correlator $l(l+1)C_l^{\theta\, B}$ induced by vector contribution of PMFs.



(b) CMB parity-odd correlator $l(l+1)C_l^{E\, B}$ induced by vector contribution of PMFs.



(c) CMB parity-odd correlator $l(l+1)C_l^{\theta\, B}$ induced by tensor contribution of PMFs.



(d) CMB parity-odd correlator $l(l+1)C_l^{E\, B}$ induced by tensor contribution of PMFs.

Figure 2: Vector and Tensor temperature-polarization correlations $C_l^{\theta\, B}$ and $C_l^{E\, B}$ in the cosmic microwave background.

$$l(l+1)C_l^{E\,B\,(V)} = -\frac{100l(l^2-1)(l+2)}{18\pi(\rho_{\gamma_0}+P_{\gamma_0})^2}\int k^2\left|\Pi_B^{(V)}(k)\right|^2\frac{j_l(k\tau_0)}{k\tau_0}\left((l+1)\frac{j_l(k\tau_0)}{(k\tau_0)^2}-\frac{j_{l+1}(k\tau_0)}{k\tau_0}\right)dk, \quad (18)$$

$$\begin{aligned} l(l+1)C_l^{\Theta\,B\,(T)} &= -\frac{196(2\pi)^4}{5000\pi}l\sqrt{\frac{(l+2)!}{(l-2)!}}\left(Gz_{eq}\ln\left(\frac{z_{in}}{z_{eq}}\right)\right)^2\cdot \\ &\cdot \int k^{-2}\left|\Pi_B^{(T)}(k)\right|^2 J_{l+3}(k\tau_0)\left(l\frac{J_{l+3}(k\tau_0)}{k\tau_0}-J_{l+4}(k\tau_0)\right)dk, \end{aligned} \quad (19)$$

$$\begin{aligned} l(l+1)C_l^{E\,B\,(T)} &= -\frac{196(2\pi)^4}{10000\pi}l\left(Gz_{eq}\tau_0\ln\left(\frac{z_{in}}{z_{eq}}\right)\right)^2\int\left|\Pi_B^{(T)}(k)\right|^2\cdot \\ &\cdot\left(J_{l+3}(k\tau_0)\left(1-\frac{l^2}{2(k\tau_0)^2}\right)+\frac{J_{l+4}(k\tau_0)}{k\tau_0}\right)\left(l\frac{J_{l+3}(k\tau_0)}{k\tau_0}-J_{l+4}(k\tau_0)\right)dk, \end{aligned} \quad (20)$$

where for our case, the integration is only up to $2k_D$, since it is the range where power spectrum is different from zero.

Here, the colours denote different values of infrared cutoff of the PMF: Black($\alpha = 0.01$), Red($\alpha = 0.3$), Blue($\alpha = 0.5$), Green($\alpha = 0.7$), Brown($\alpha = 0.8$) and Magenta($\alpha = 0.9$).
We can observe that helical(antisymmetric) contribution generates nonzero values of the parity-odd correlators $C_l^{\theta\, B}$ and $C_l^{E\, B}$ as also pointed out in[1] [3]. We also observe the effects of IR cutoff on the parity odd spectra.

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