The variation of the fine-structure constant from disformal couplings

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Outline

1. Introduction–Is α a constant of Nature?
2. Disformal Electrodynamics
   - The Model
   - Identification of α
   - Evolution of α
   - Cosmology
     - FRW
   - Examples
     - Disformal/ Disformal & electromagnetic couplings
     - Disformal & conformal couplings
     - Disformal, conformal & electromagnetic couplings
3. Conclusion

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3 Conclusion

Introduction–Is α a constant of Nature?

- Dirac came up with the idea on the variation of the fundamental constants of Nature in his 'large numbers hypothesis'.
- Effective (3+1)–dimensional constants can vary in space and time in higher–dimensional theories.
- Current observations look for variations in the fine–structure constant:
  - Atomic Clocks [T. Rosenband *et al* '08]

$$\left.\frac{\dot{\alpha}}{\alpha}\right|_0 = (-1.6 \pm 2.3) \times 10^{-17}\ \mathrm{yr}^{-1},$$

  - Oklo natural reactor [E.D. Davis & L. Hamdan '15]

$$\frac{|\Delta\alpha|}{\alpha} < 1.1 \times 10^{-8}, \quad z \simeq 0.16,$$

  - ^{187}Re meteorites [K.A. Olive *et al* '04]

$$\frac{\Delta\alpha}{\alpha} = (-8 \pm 8) \times 10^{-7}, \quad z \simeq 0.43,$$

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  - The cosmic microwave background (CMB) radiation [Planck Coll. '15]

$$\frac{\Delta\alpha}{\alpha} = (3.6 \pm 3.7) \times 10^{-3}, \quad z \simeq 10^3,$$

  - Astrophysical data:
    - Keck/ HIRES–141 absorbers (MM method) [M.T. Murphy *et al* '04]

$$\left(\frac{\Delta\alpha}{\alpha}\right)_{\mathrm{w}} = (-0.57 \pm 0.11) \times 10^{-5}, \quad 0.2 < z < 4.2,$$

    - VLT/ UVES–154 absorbers (MM method) [J.A. King *et al* '12]

$$\left(\frac{\Delta\alpha}{\alpha}\right)_{\mathrm{w}} = (0.208 \pm 0.124) \times 10^{-5}, \quad 0.2 < z < 3.7,$$

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$$\left(\frac{\Delta\alpha}{\alpha}\right)_{\mathrm{w}} = (-0.5 \pm 1.3) \times 10^{-5}, \quad 2 < z < 3,$$

    - Comparison of HI 21–cm line with molecular rotational absorption spectra [M.T. Murphy *et al* '01]

$$\frac{\Delta\alpha}{\alpha} = (-0.10 \pm 0.22) \times 10^{-5}, \quad z = 0.25,$$

$$\frac{\Delta\alpha}{\alpha} = (-0.08 \pm 0.27) \times 10^{-5}, \quad z = 0.68,$$

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    - Recent data [P. Molaro *et al* '13, T.M. Evans *et al* '14]

Object	z	$(\Delta\alpha/\alpha) \times 10^6$	Spectrograph
Three sources	1.08	4.3 ± 3.4	HIRES
HS1549+1919	1.14	-7.5 ± 5.5	UVES/HIRES/HDS
HE0515-4414	1.15	-0.1 ± 1.8	UVES
HE0515-4414	1.15	0.5 ± 2.4	HARPS/UVES
HS1549+1919	1.34	-0.7 ± 6.6	UVES/HIRES/HDS
HE0001-2340	1.58	-1.5 ± 2.6	UVES
HE1104-1805A	1.66	-4.7 ± 5.3	HIRES
HE2217-2818	1.69	1.3 ± 2.6	UVES
HS1946+7658	1.74	-7.9 ± 6.2	HIRES
HS1549+1919	1.80	-6.4 ± 7.2	UVES/HIRES/HDS

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Disformal Electrodynamics: The Model

We consider the following action:

$$\mathcal{S} = \mathcal{S}_{\text{grav}}\left(g_{\mu\nu}, \phi\right) + \mathcal{S}_{\text{matter}}\left(\tilde{g}^{(m)}_{\mu\nu}\right) + \mathcal{S}_{\text{EM}}\left(A_\mu, \tilde{g}^{(r)}_{\mu\nu}\right) \tag{1}$$

such that,

$$\tilde{g}^{(m)}_{\mu\nu} = C_m g_{\mu\nu} + D_m \phi_{,\mu}\phi_{,\nu} \ , \tag{2}$$

$$\tilde{g}^{(r)}_{\mu\nu} = C_r g_{\mu\nu} + D_r \phi_{,\mu}\phi_{,\nu} \ , \tag{3}$$

where

$C_{r,m}$: conformal factors
$D_{r,m}$: disformal couplings

} both taken to be functions of ϕ only

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Disformal Electrodynamics: The Model

The electromagnetic sector is specified by

$$\mathcal{S}_{\mathrm{EM}} = -\frac{1}{4}\int d^4x\sqrt{-\tilde{g}^{(r)}}h(\phi)\tilde{g}^{\mu\nu}_{(r)}\tilde{g}^{\alpha\beta}_{(r)}F_{\mu\alpha}F_{\nu\beta} - \int d^4x\sqrt{-\tilde{g}^{(m)}}\tilde{g}^{\mu\nu}_{(m)}\,j_\nu A_\mu, \tag{4}$$

where

- $F_{\mu\nu}$ is the standard antisymmetric Faraday tensor,
- j^ν is the four-current,
- The function $h(\phi)$ is the direct coupling between the electromagnetic field and the scalar.

We aim to work in the Jordan frame

- The frame in which matter is decoupled from the scalar degree of freedom.

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Disformal Electrodynamics: The Model

Indeed, we know that

$$\tilde{g}^{(r)}_{\mu\nu} = \frac{C_r}{C_m}\tilde{g}^{(m)}_{\mu\nu} + \left(D_r - \frac{C_r D_m}{C_m}\right)\phi_{,\mu}\phi_{,\nu} \equiv A\tilde{g}^{(m)}_{\mu\nu} + B\phi_{,\mu}\phi_{,\nu}\ . \tag{5}$$

Then, in terms of this metric, the electromagnetic sector becomes

$$\mathcal{S}_{\rm EM} = -\frac{1}{4}\int d^4x\sqrt{-\tilde{g}^{(m)}}h(\phi)Z\left[\tilde{g}^{\mu\nu}_{(m)}\tilde{g}^{\alpha\beta}_{(m)} - 2\gamma^2\tilde{g}^{\mu\nu}_{(m)}\phi^{,\alpha}\phi^{,\beta}\right]F_{\mu\alpha}F_{\nu\beta} - \int d^4x\sqrt{-\tilde{g}^{(m)}}\tilde{g}^{\mu\nu}_{(m)}\,j_\nu A_\mu\ , \tag{6}$$

- Gauge invariance: $\tilde{\nabla}_\mu j^\mu = 0$
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where we again raise the indices with $\tilde{g}^{(m)}$

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where we raise the indices with the metric $\tilde{g}^{(m)}_{\mu\nu}$ and define

$$Z = \left(1 + \frac{B}{A}\tilde{g}^{\mu\nu}_{(m)}\partial_\mu\phi\partial_\nu\phi\right)^{1/2},$$

$$\gamma^2 = \frac{B}{A + B\tilde{g}^{\mu\nu}_{(m)}\partial_\mu\phi\partial_\nu\phi}\ .$$

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$$\tilde{\nabla}_\epsilon\left(h(\phi)ZF^{\epsilon\rho}\right) - \tilde{\nabla}_\epsilon\left(h(\phi)Z\gamma^2\phi^{,\beta}\left(\tilde{g}^{\epsilon\nu}_{(m)}\phi^{,\rho} - \tilde{g}^{\rho\nu}_{(m)}\phi^{,\epsilon}\right)F_{\nu\beta}\right) = j^\rho \qquad (7)$$

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Disformal Electrodynamics: Identification of α

- Set $\tilde{g}^{(m)}_{\mu\nu} = \eta_{\mu\nu}$ and consider ϕ to depend on time only.
- From the field equation (7), and identifying the electric field by $E^i = F^{i0}$, we find the field equation for the electric field to be given by

$$\nabla \cdot \mathbf{E} = \frac{Z\rho}{h(\phi)} \tag{8}$$

  where $\rho = j^0$ is the charge density.
- By integrating this equation over a volume $\mathcal{V}$, it is straightforward to derive the electrostatic potential

$$V(r) = \frac{ZQ}{4\pi h(\phi) r} \tag{9}$$

  where Q is the total charge contained in $\mathcal{V}$.
- Comparing this to the standard expression for the tree-level-potential from QED, one finds that α has the following dependence on Z and h:

$$\alpha \propto \frac{Z}{h(\phi)} \tag{10}$$

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$$\boxed{\alpha \propto \frac{Z}{h(\phi)}} \quad \text{(Note that } \alpha \propto h^{-1}(\phi) \text{ when } \tilde{g}^{(m)}_{\mu\nu} = \tilde{g}^{(r)}_{\mu\nu}.) \tag{10}$$

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Disformal Electrodynamics: Evolution of α

Using

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- We define the redshift evolution of α by the quantity

$$\frac{\Delta\alpha}{\alpha}(z) \equiv \frac{\alpha(z) - \alpha(z=0)}{\alpha(z=0)} = \frac{h(\phi_0)Z(z)}{h(\phi(z))Z_0} - 1, \tag{12}$$

  where ϕ_0 is the field value today and Z_0 is the value of Z evaluated today.

- In a spatially–flat FRW gravitational metric, the temporal variation of α reduces to the following

$$\frac{\dot{\alpha}}{\alpha} = \frac{1}{Z}\left(\frac{\partial Z}{\partial\phi}\dot{\phi} + \frac{\partial Z}{\partial\dot{\phi}}\ddot{\phi}\right) - \frac{1}{h}\frac{dh}{d\phi}\dot{\phi}. \tag{13}$$

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Using

$$\alpha \propto \frac{Z}{h(\phi)}, \quad Z = \left(1 + \frac{B}{A}\tilde{g}^{\mu\nu}_{(m)}\partial_\mu\phi\partial_\nu\phi\right)^{1/2} \tag{11}$$

- We define the redshift evolution of α by the quantity

$$\frac{\Delta\alpha}{\alpha}(z) \equiv \frac{\alpha(z) - \alpha(z=0)}{\alpha(z=0)} = \frac{h(\phi_0)Z(z)}{h(\phi(z))Z_0} - 1, \tag{12}$$

  where ϕ_0 is the field value today and Z_0 is the value of Z evaluated today.
- In a spatially–flat FRW gravitational metric, the temporal variation of α reduces to the following

$$\frac{\dot{\alpha}}{\alpha} = \frac{1}{Z}\left(\frac{\partial Z}{\partial\phi}\dot{\phi} + \frac{\partial Z}{\partial\dot{\phi}}\ddot{\phi}\right) - \frac{1}{h}\frac{dh}{d\phi}\dot{\phi}. \tag{13}$$

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Disformal Electrodynamics: Cosmology

We now specify our gravitational-scalar action, which leads us to the EF theory described by the following action

$$\mathcal{S} = \int d^4x\sqrt{-g}\left(\frac{1}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right) + S_{\rm matter}\left(\tilde{g}^{(m)}_{\mu\nu}\right) - \frac{1}{4}\int d^4x\sqrt{-\tilde{g}^{(r)}}h(\phi)\tilde{g}^{\mu\nu}_{(r)}\tilde{g}^{\alpha\beta}_{(r)}F_{\mu\alpha}F_{\nu\beta}\ , \tag{14}$$

where the last term in the action above describes the dynamics of the CMB photons.

- Field equations

$$G^{\mu\nu} = T^{\mu\nu}_{\phi} + T^{\mu\nu}_{(m)} + T^{\mu\nu}_{(r)}, \tag{15}$$

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Disformal Electrodynamics: Cosmology

- Klein-Gordon equation

$$\Box\phi - V' = -Q_m - Q_r, \tag{16}$$

- Conservation equations

$$\nabla_\mu T^\mu_{(m)\nu} = Q_m \phi_{,\nu}\ , \quad \nabla_\mu T^\mu_{(r)\nu} = Q_r \phi_{,\nu}\ , \tag{17}$$

where,

$$Q_m = \frac{C'_m}{2C_m} T_{(m)} + \frac{D'_m}{2C_m} \phi_{,\mu}\phi_{,\nu} T^{\mu\nu}_{(m)} - \nabla_\mu \left[\frac{D_m}{C_m} \phi_{,\nu} T^{\mu\nu}_{(m)} \right]\ , \tag{18}$$

$$Q_r = \frac{C'_r}{2C_r} T_{(r)} + \frac{D'_r}{2C_r} \phi_{,\mu}\phi_{,\nu} T^{\mu\nu}_{(r)} + \frac{h'}{h} C_r^2 \sqrt{1 + \frac{D_r}{C_r} g^{\mu\nu} \phi_{,\mu}\phi_{,\nu}}\ \tilde{\mathcal{L}}_{EM} - \nabla_\mu \left[\frac{D_r}{C_r} \phi_{,\nu} T^{\mu\nu}_{(r)} \right]\ . \tag{19}$$

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We shall now consider perfect fluid energy-momentum tensors for radiation and matter in the EF, radiation in the RF and matter in the JF as follows

$$T^{\mu\nu}_{(r)} = (\rho_r + p_r)u^\mu u^\nu + p_r g^{\mu\nu}, \quad T^{\mu\nu}_{(m)} = (\rho_m + p_m)u^\mu u^\nu + p_m g^{\mu\nu}, \quad (20)$$

$$\tilde{T}^{\mu\nu}_{(r)} = (\tilde{\rho}_r + \tilde{p}_r)\tilde{u}^\mu \tilde{u}^\nu + \tilde{p}_r \tilde{g}^{\mu\nu}_{(r)}, \quad \tilde{T}^{\mu\nu}_{(m)} = (\tilde{\rho}_m + \tilde{p}_m)\tilde{u}^\mu \tilde{u}^\nu + \tilde{p}_m \tilde{g}^{\mu\nu}_{(m)}, \quad (21)$$

Furthermore, we will now consider a zero curvature FRW EF metric, $ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$, leading to

$$\ddot{\phi} + 3H\dot{\phi} + V' = Q_m + Q_r, \quad (22)$$

$$\dot{\rho}_m + 3H(\rho_m + p_m) = -Q_m\dot{\phi}, \quad (23)$$

$$\dot{\rho}_r + 3H(\rho_r + p_r) = -Q_r\dot{\phi}, \quad (24)$$

where $H = \dot{a}/a$ is the Hubble parameter and dot represents an EF time derivative. We introduce $\eta \equiv \tilde{\mathcal{L}}_{EM}/\tilde{\rho}_r$ in what follows.

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Disformal Electrodynamics: Cosmology–FRW

$$Q_m = \frac{A_r}{A_r A_m - D_r D_m \rho_r \rho_m} \left[B_m - \frac{D_m B_r}{A_r} \rho_m \right], \tag{25}$$

$$Q_r = \frac{A_m}{A_r A_m - D_r D_m \rho_r \rho_m} \left[B_r - \frac{D_r B_m}{A_m} \rho_r \right], \tag{26}$$

where

$$A_r = C_r + D_r \left(\rho_r - \dot{\phi}^2 \right) , \quad A_m = C_m + D_m \left(\rho_m - \dot{\phi}^2 \right) , \tag{27}$$

$$\begin{aligned} B_r &= \frac{1}{2} C_r' \left(3w_r - 1 \right) \rho_r - \frac{1}{2} D_r' \dot{\phi}^2 \rho_r + \frac{h'}{h} \left(C_r - D_r \dot{\phi}^2 \right) \eta \rho_r \\ &+ D_r \rho_r \left[\frac{C_r'}{C_r} \dot{\phi}^2 + V' + 3H\dot{\phi} \left(1 + w_r \right) \right] , \\ B_m &= \frac{1}{2} C_m' \left(3w_m - 1 \right) \rho_m - \frac{1}{2} D_m' \dot{\phi}^2 \rho_m \\ &+ D_m \rho_m \left[\frac{C_m'}{C_m} \dot{\phi}^2 + V' + 3H\dot{\phi} \left(1 + w_m \right) \right] \end{aligned} \tag{28}$$

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Disformal Electrodynamics: Cosmology–FRW

- EF Friedmann equations

$$H^2 = \frac{1}{3}\left(\rho_m + \rho_r + \rho_\phi\right), \; \dot{H} = -\frac{1}{6}\left[3\left(\rho_m + \dot{\phi}^2\right) + \rho_r\left(4 - \frac{D_r}{C_r}\dot{\phi}^2\right)\right] \tag{29}$$

- The scalar field characterizing the disformal couplings is also responsible for the current acceleration of the Universe, i.e., it is the dark energy.
- Non–interacting dark sector (Type Ia supernova)

$$\underbrace{\dot{\rho}_{\mathrm{DE}}^{\mathrm{eff}} = -3H(1+w_{\mathrm{eff}})\rho_{\mathrm{DE}}^{\mathrm{eff}}, \; H^2 = \frac{1}{3}\left(a^{-4}\rho_{0,r} + a^{-3}\rho_{0,m} + \rho_{\mathrm{DE}}^{\mathrm{eff}}\right)}$$

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Disformal Electrodynamics: Examples

- We specify the following form of couplings and potential:

$$C_i(\phi) = \beta_i e^{x_i \phi} \quad , \quad D_i(\phi) = M_i^{-4} e^{y_i \phi},$$
$$V(\phi) = M_V^4 e^{-\lambda\phi}, \quad h(\phi) = 1 - \zeta(\phi - \phi_0),$$

such that the introduced parameters are tuned in order to be in agreement with the observational data on the variation of α together with the cosmological parameters.

Parameter	Estimated value
$w_{0,\phi}$	-1.006 ± 0.045
H_0	(67.8 ± 0.9) km s^{-1}Mpc^{-1}
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Ex	$M_r \neq M_m$	M_m	β_m	x_m	$\|\zeta\|$	M_V	λ
*	$\sim$ meV	$\sim$ meV	1	0	$< 5 \times 10^{-6}$	2.69 meV	0.45
**	$\sim$ meV	15 meV	8	0.14	0	2.55 meV	0.45
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Ex	$(\dot{\alpha}/\alpha)\vert_0 \times 10^{17}$	$\vert\Delta\alpha/\alpha\vert_{z_{\rm CMB}}$
*	$-2.14 \sim -1.62$	$10^{-8} \sim 10^{-6}$
**	$-2.41 \sim 0.70$	$10^{-8} \sim 10^{-7}$
***	$-2.10 \sim -1.24$	$10^{-7} \sim 10^{-6}$

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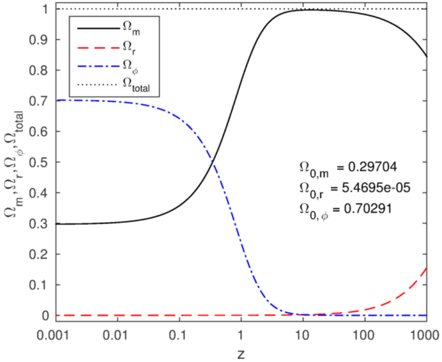
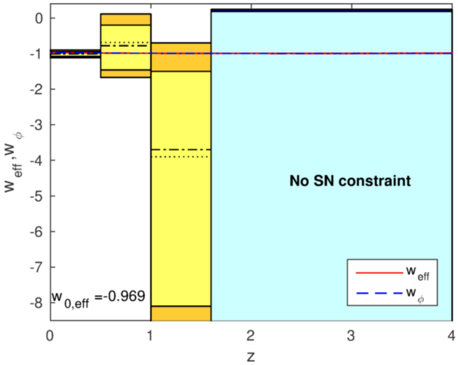
*Disformal/ Disformal & electromagnetic couplings





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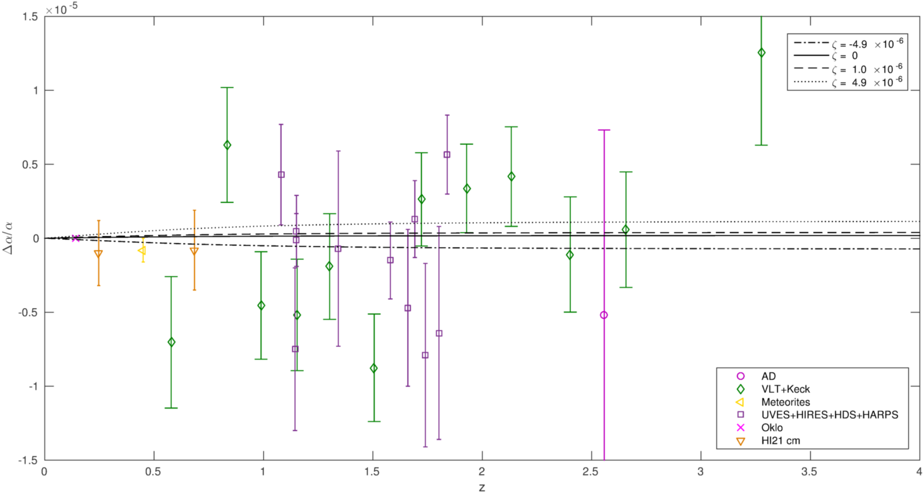

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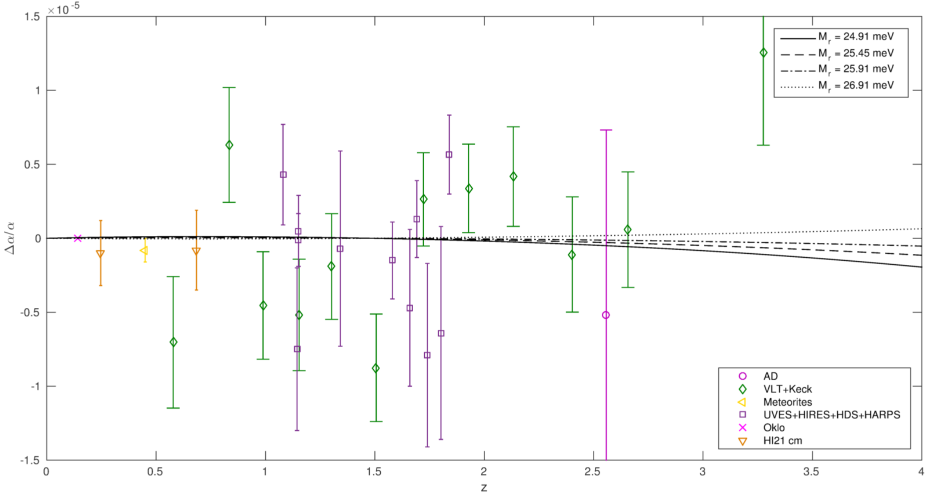
**Disformal & conformal couplings



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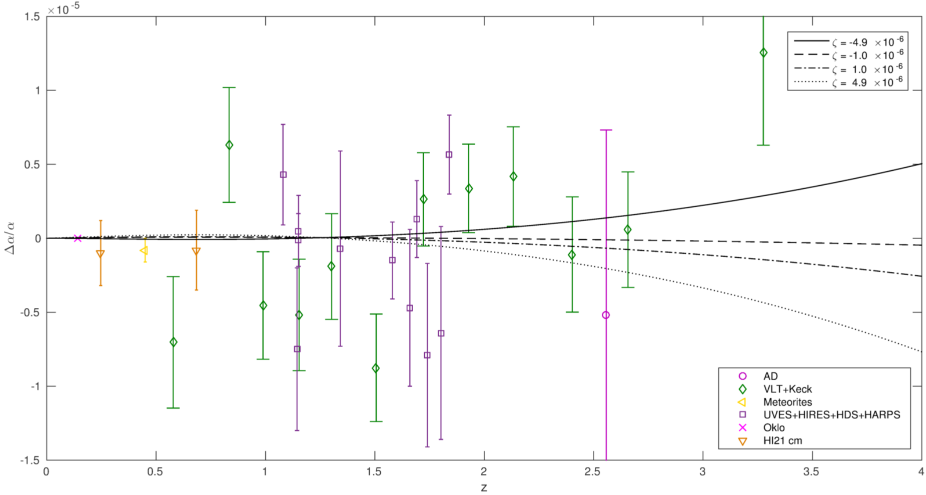

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- A variation in the fine–structure constant can be induced by disformal couplings provided that the radiation and matter disformal coupling strengths are not identical.
- Such a variation is enhanced in the presence of the usual electromagnetic coupling.
- Laboratory measurements with molecular and nuclear clocks are expected to increase their sensitivity to as high as 10^{-21} yr^{-1}.
- Better constrained data is expected from high-resolution ultra-stable spectrographs such as
  - PEPSI at the LBT
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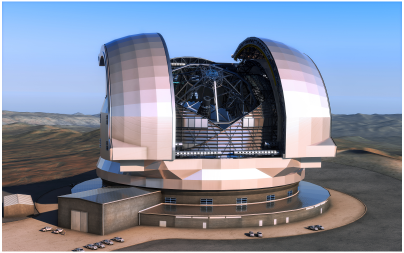
- Such a variation is enhanced in the presence of the usual electromagnetic coupling.
- Laboratory measurements with molecular and nuclear clocks are expected to increase their sensitivity to as high as 10^{-21} yr^{-1}.
- Better constrained data is expected from high-resolution ultra-stable spectrographs such as
    - PEPSI at the LBT
    - ESPRESSO at the VLT
    - ELT-Hires at the E-ELT




Thank You