Lorentz violation in gravity

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O. Pujolàs, S. Sibiryakov, K. Yagi, N. Yunes

Review (w/ E. Lim) 1412.4828 [gr-qc]

Modified gravity checklist

- Learning something fundamental about gravity/Nature
- Improve the short distance properties of GR (QG, BH)
- New ideas for cosmic acceleration/dark matter
- Interesting (testable) phenomenology

…the Lorentz violating (LV) case

- ✓ Is there a 'fundamental' preferred frame in the universe?
- ✓ Hořava gravity as a proposal for QG — Barvinski, DB, et al 15
- ✓ Natural dark energy and possible 5th forces/MOND
- ✓ Consequences at all scales (massless extra polarization)

Modified gravity checklist

- Learning something fundamental about gravity/Nature
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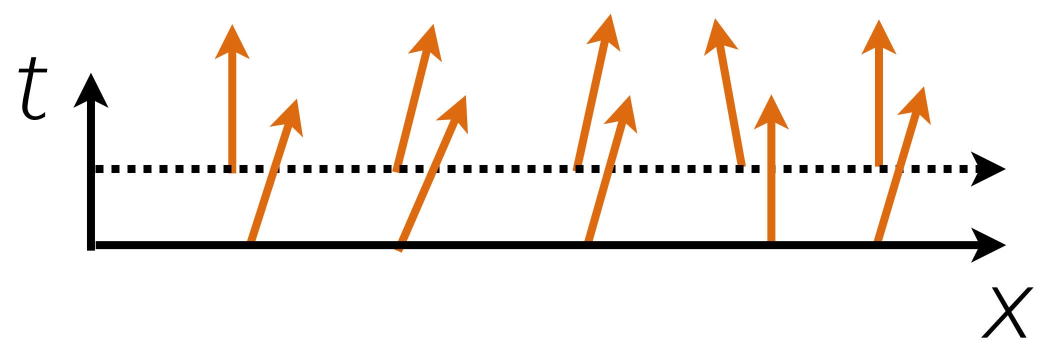
...the Lorentz violating (LV) case

sweet spot in modified gravity

✓
✓
✓
✓

Working with a preferred frame

Space-time filled by a preferred **time** direction associated to a time-like unit vector u_μ



Generic: **Einstein-æther**	Hypersurface orthogonal: **Khronometric**
Jacobson, Mattingly 01	DB, Pujolas, Sibiryakov 09

$$u_\mu u^\mu = 1$$

$$u_\mu \equiv \frac{\partial_\mu \varphi}{\sqrt{\partial_\alpha \varphi \partial^\alpha \varphi}}$$

Scalar-vector

Scalar: khronon $\chi\rho\acute{o}\nu o\varsigma$

Lagrangian ('low' energies)

Ingredients: $u_\mu \ , \ g_{\mu\nu}$

Khronometric $u_\mu \equiv \dfrac{\partial_\mu \varphi}{\sqrt{\partial_\alpha \varphi \partial^\alpha \varphi}}$

DB, Pujolas, Sibiryakov 09
Horava 09

$$\mathcal{L}_{\chi GR} = \mathcal{L}_{EH} + M_P^2 \sqrt{-g}\left(\lambda \left(\nabla^\mu u_\mu\right)^2 + \alpha \left(u^\nu \nabla_\nu u_\mu\right)^2 + \beta \nabla_\mu u_\nu \nabla^\nu u^\mu\right)$$

- **massless** spin 2 graviton: $\omega^2 = c_t^2\, \mathrm{k}^2 ,\qquad c_t^2 = \dfrac{1}{1-\beta}$
- massless **scalar** $\varphi = t + \chi$: $\omega^2 = c_\chi^2\, \mathrm{k}^2 ,\qquad c_\chi^2 = \dfrac{\beta+\lambda}{\alpha}$

Stable Minkowski & no gravitational Cherenkov:

$$0 < \alpha < 2 \quad , \quad c_t^2 \geq 1, \ c_\chi^2 \geq 1$$

Einstein-æther: extra term $\gamma \nabla_\mu u_\nu \nabla^\mu u^\nu$

Jacobson, Mattingly 01

- extra **vector** $u_\mu u^\mu = 1$

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DB, Pujolas, Sibiryakov 09
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Jacobson, Mattingly 01

- extra **vector** $u_\mu u^\mu = 1$

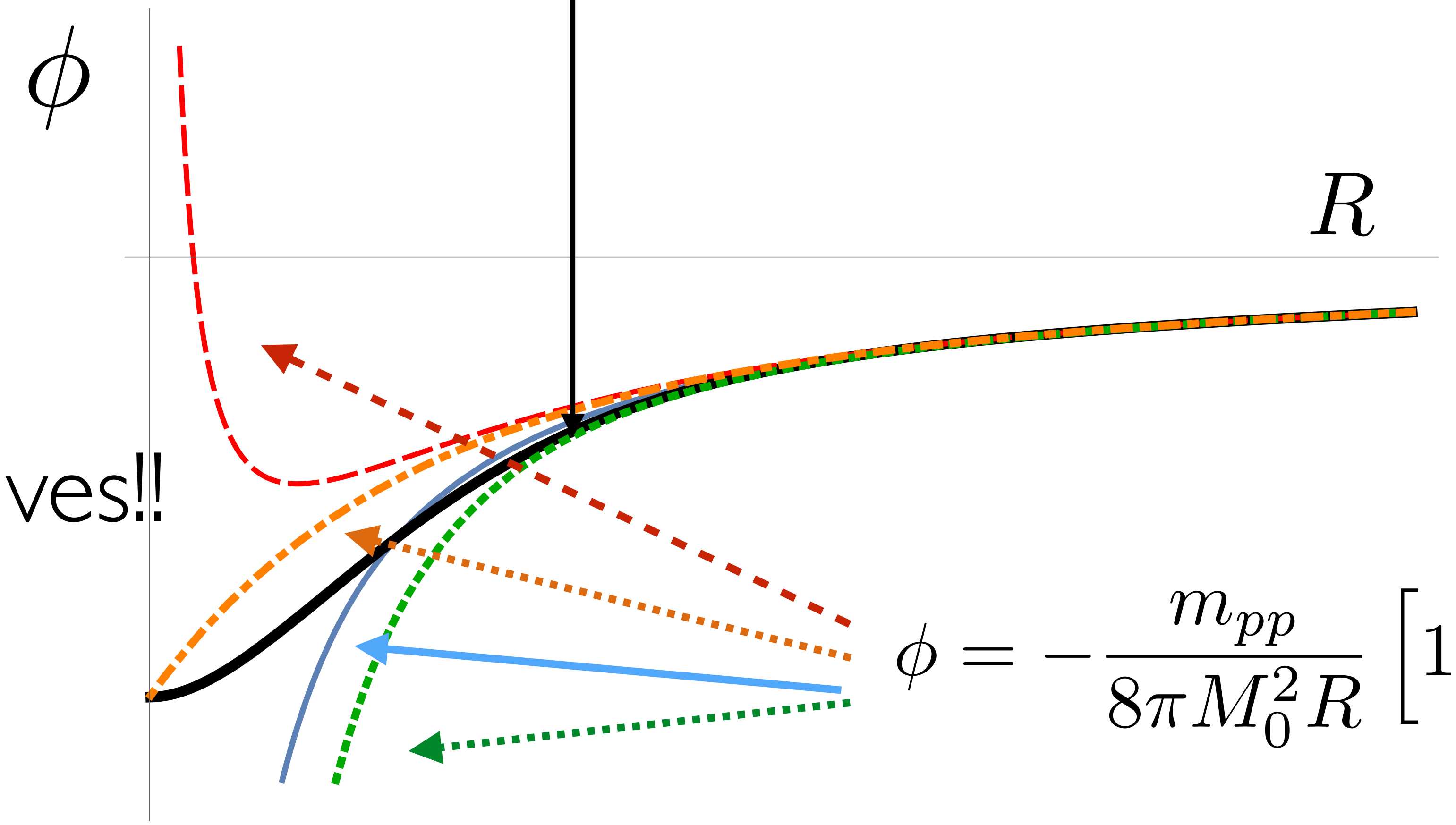
Kh can be renormalized! Barvinski, DB, et al 15

+ higher derivatives: $\dfrac{1}{{M_\star}^{d-4}} O_{d>4}(u_\mu, g_{\mu\nu}, \nabla_\mu)$

(Toy) potential at short distances

$$\phi = -\frac{m_{pp}}{8\pi M_P^2 R}\left[1 - e^{-M_\star R}\cos\left(M_\star R\right)\right]$$

DB, Lim 14

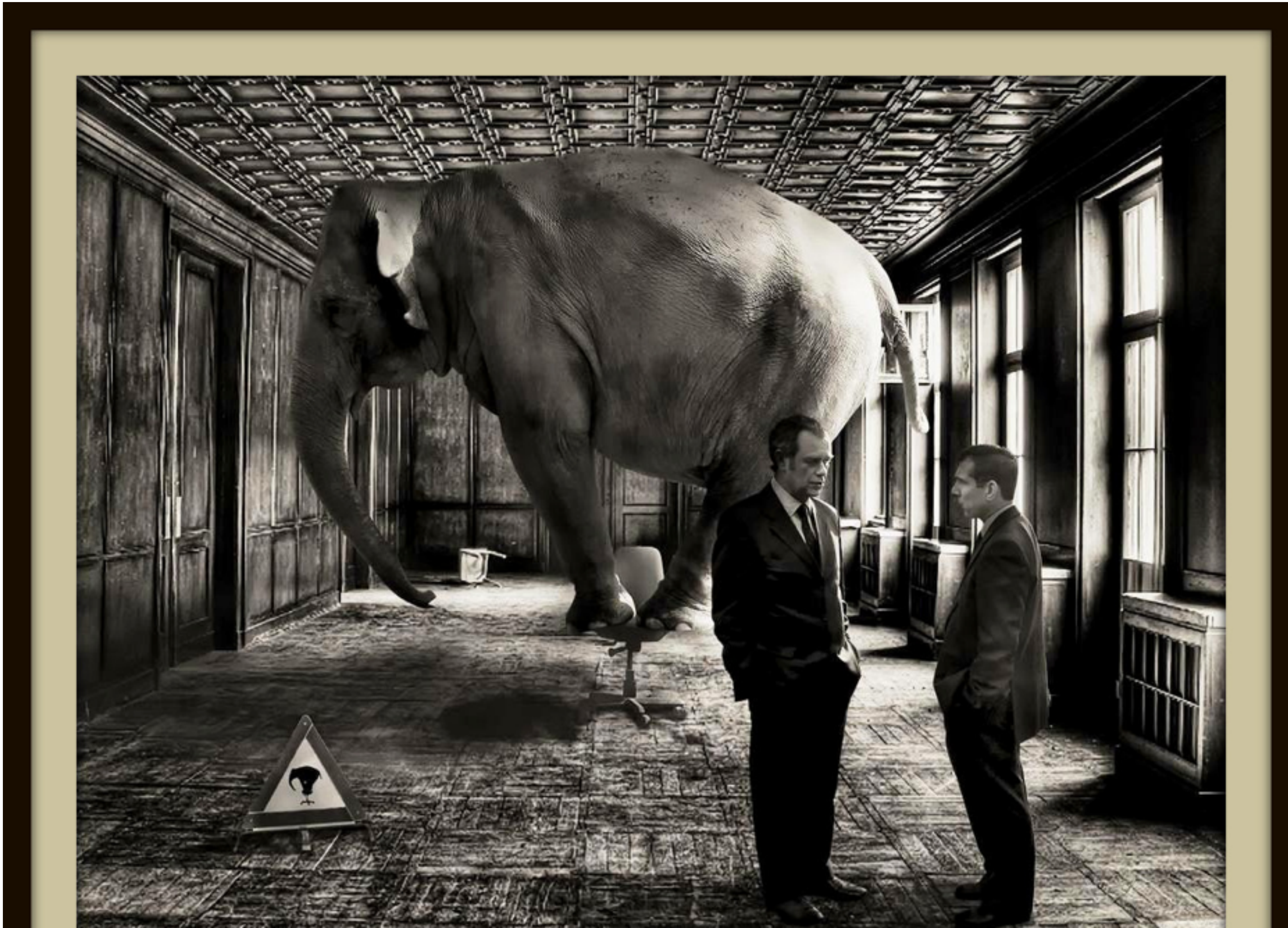


Finite and
finite derivatives!!

$$\phi = -\frac{m_{pp}}{8\pi M_0^2 R}\left[1 + \tilde{\alpha}\, e^{-R/\tilde{\lambda}}\right]$$

$M_\star^{-1} \gtrsim \mu m$?

Matter Lagrangian



Matter Lagrangian

Ingredients: u_μ , $g_{\mu\nu}$ + SM Fields + DM + DE

$$\mathcal{L}_m = \mathcal{L}_{LI}(\mathrm{SM}, \mathrm{DM}, \mathrm{DE}, g_{\mu\nu}) + \kappa_{SM}\, \mathcal{L}_{LV}(\mathrm{SM}, g_{\mu\nu}, u_\mu)$$
$$+\kappa_{DM}\, \mathcal{L}_{LV}(\mathrm{DM}, g_{\mu\nu}, u_\mu) + \kappa_{DE}\, \mathcal{L}_{LV}(\mathrm{DE}, g_{\mu\nu}, u_\mu)$$

SM: e.g. $\bar{\psi}\, u^\mu u^\nu \gamma_\mu \partial_\nu \psi$ ➡ $\omega_\psi^2 = m_\psi^2 + c_\psi^2 \mathrm{k}^2$

$|1 - c_{p,n}/c_\gamma| < 10^{-22}$ dynamical explanation? review by Liberati 13

Kostelecky, Liberati, Mattingly, ... in the following $\kappa_{SM} = 0$

DM, DE: κ_{DM}, κ_{DE}? to be answered by cosmology

1412.4828 [gr-qc] (review article w/ E. Lim)

Matter Lagrangian

Ingredients: u_μ , $g_{\mu\nu}$ + SM Fields + DM + DE

$$\mathcal{L}_m = \mathcal{L}_{LI}(\mathrm{SM}, \mathrm{DM}, \mathrm{DE}, g_{\mu\nu}) + \kappa_{SM}$$ [redacted]

$+\kappa_{DM}$ [redacted] $+ \kappa_{DE}$ [redacted]

SM: [redacted] ➡ [redacted]

[redacted]

review by Liberati 13

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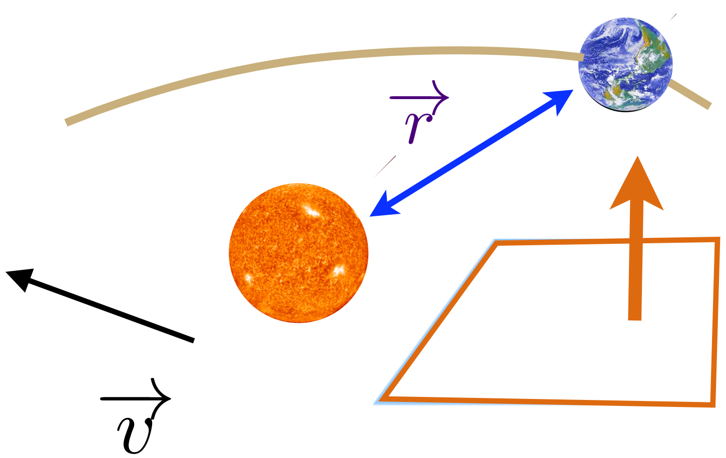
Solar System Constraints (Kh)

Solar system (PN gravity)

DB, Pujolas, Sibiryakov 10



$$h_{00} = -2G_N \frac{M}{r}\left(1 - \frac{(\alpha_1^{PPN} - \alpha_2^{PPN})v^2}{2} - \frac{\alpha_2^{PPN}}{2}\frac{(x^i v^i)^2}{r^2}\right)$$

$$h_{0i} = \frac{\alpha_1^{PPN}}{2} G_N \frac{m}{r} v^i$$

$$\alpha_1^{PPN} = -4(\alpha - 2\beta) \qquad G_N \equiv \frac{1}{8\pi M_P^2(1-\alpha/2)}$$

$$\alpha_2^{PPN} = \frac{(\alpha - 2\beta)(\alpha - \lambda - 3\beta)}{2(\lambda + \beta)}$$

Will 05

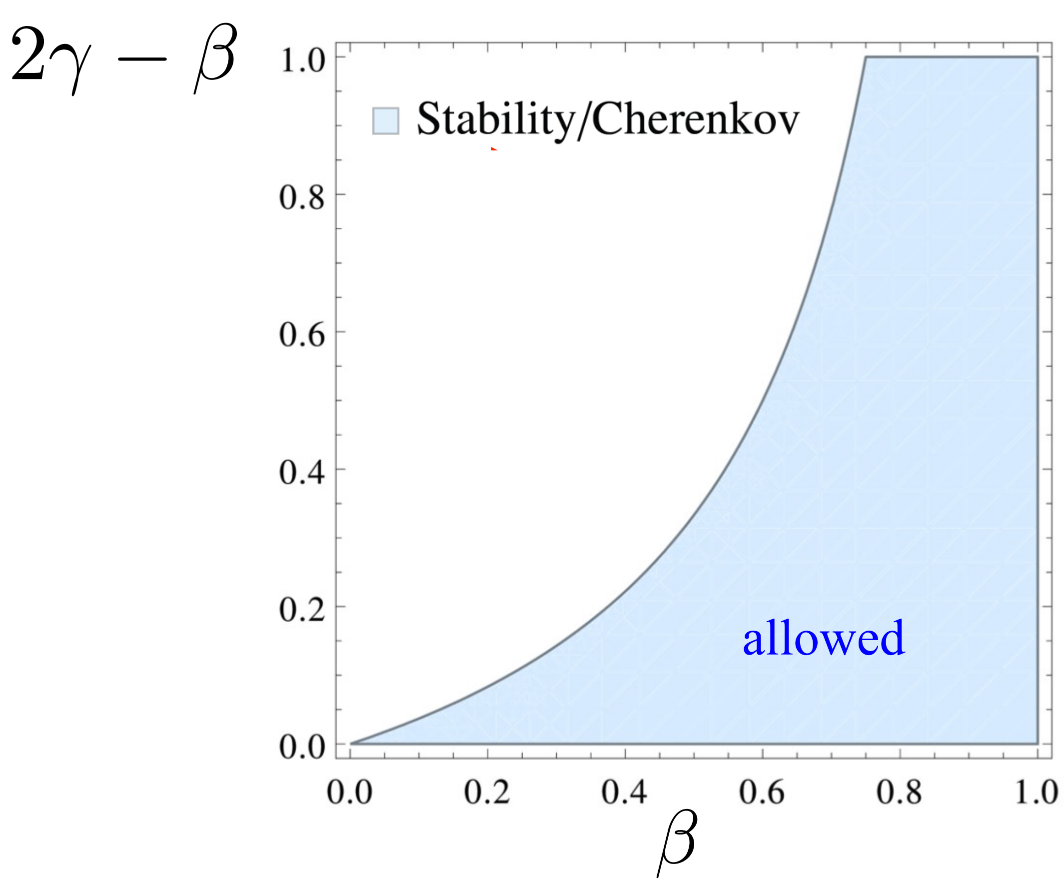
With respect to CMB frame $\vec{v} \sim 10^{-2}$ $\begin{cases} \alpha_1^{PPN} \lesssim 10^{-4} \\ \alpha_2^{PPN} \lesssim 10^{-7} \end{cases}$

$\alpha = 2\beta$ identical to GR in the Solar System!

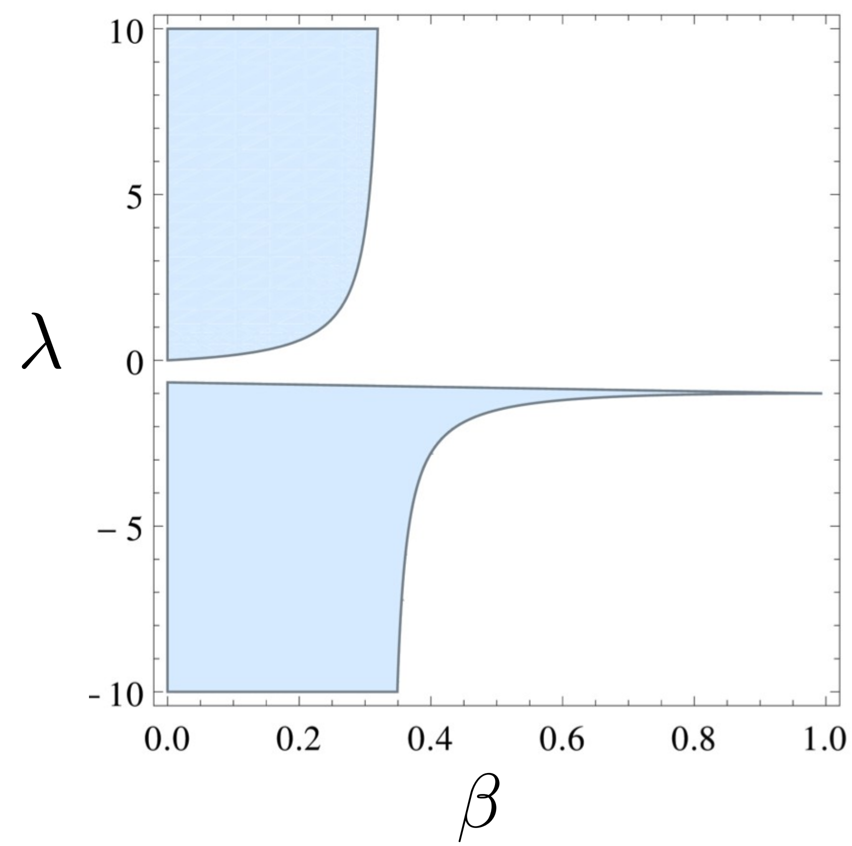
Weak field constraints summary

Solar System constraints leave 2 free parameters

Einstein-æther



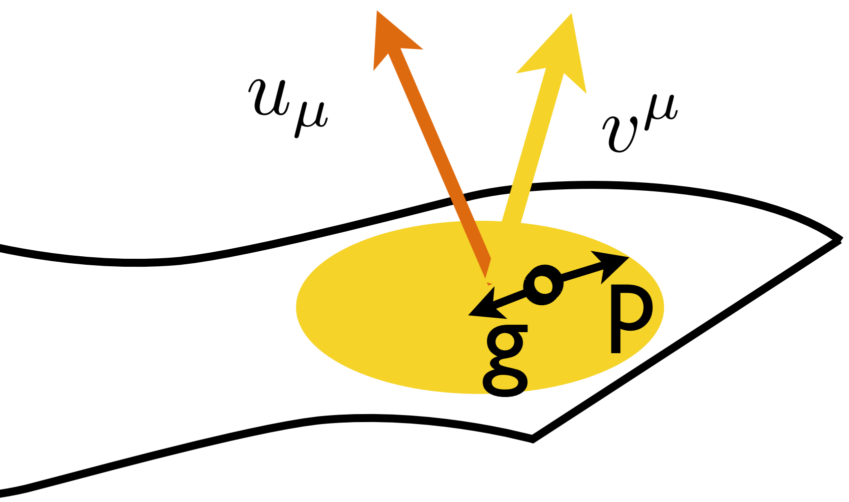
Khronometric



derived from situations with **weak** gravitational fields

Tests with compact objects improve both aspects!

Effects on astrophysical objects



Matter is not modified

Gravitation modified (coupling between gravitons and u_μ)

Violation of **strong equivalence principle (SEP)**
(Nordtvedt effect)

$$T_{\mu\nu} = T^m_{\mu\nu} + T^g_{\mu\nu} + T^u_{\mu\nu}$$

produces $g_{\mu\nu}$, u^μ

Far away: point-particle description with extra coupling

$$S_{pp} = -\tilde{m} \int \mathrm{ds} \quad \Rightarrow \quad S_{pp} = -\tilde{m} \int \mathrm{ds}\, f(u_\mu v^\mu)$$

the **orbital** equations depend on $u_\mu v^\mu$

(analogous to the scalar-tensor case Damour Esposito-Farese 92)

Orbital effects: PN analysis

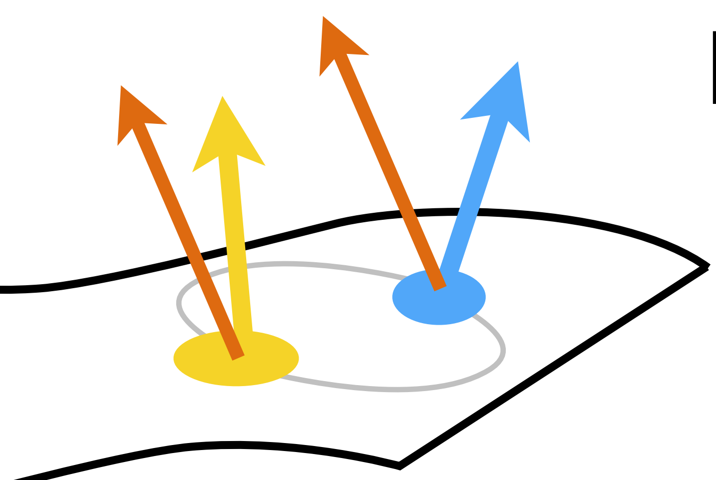
$$S_{pp} = -\tilde{m} \int \mathrm{d}s\, f(u_\mu v^\mu)$$

Slowly moving $u^\mu v_\mu \ll 1$

$$S_{ppA} = -\tilde{m}_A \int ds_A \left[(1 + \sigma_A(1 - u_\mu v^\mu) + O(u_\mu v^\mu - 1)^2\right]$$

sensitivity: encapsulates the strong-field effects

Newtonian limit of N-particles



Foster 07

$$\dot{v}^i_A = \sum_{B \neq A} \frac{-\mathcal{G}_{AB} m_B}{r^3_{AB}} r^i_{AB}$$

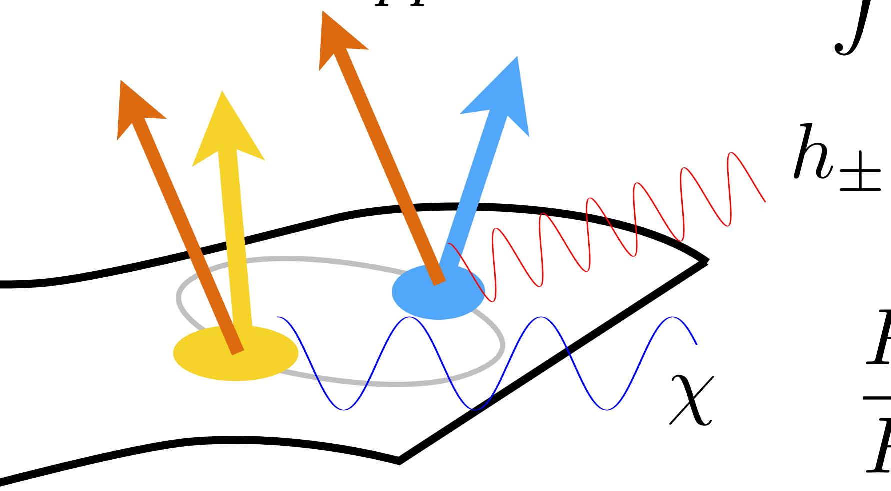
$$m_A \equiv \tilde{m}_A(1 + \sigma_A) \qquad \mathcal{G}_{AB} \equiv \frac{G_N}{(1 + \sigma_A)(1 + \sigma_B)}$$

SEP violated! $P^i = \tilde{m}_1 v^i_1 + \tilde{m}_2 v^i_2$ not conserved!

dipolar radiation expected

Orbital effects: Dissipative sector

$$S_{ppA} = -\tilde{m}_A \int ds_A \left[(1 + \sigma_A (1 - u_\mu v^\mu) + O(u_\mu v^\mu - 1)^2 \right]$$



Yagi, DB, Barausse, Yunes 13

$$\frac{\dot{P}_b}{P_b} = -\frac{3}{2}\frac{\dot{E}}{E} = -\frac{192\pi}{5}\left(\frac{2\pi G_N m}{P_b}\right)^{5/3}\left(\frac{\mu}{m}\right)\frac{1}{P_b}\langle\mathcal{A}\rangle$$

$$s_A \equiv \frac{\sigma_A}{1+\sigma_A}\,, \quad \mathcal{S} \equiv s_1 m_1/m + s_2 m_2/m\,, \quad m \equiv m_1 + m_2\,, \quad \mu \equiv \frac{m_1 m_2}{m}$$

LV: $\langle\mathcal{A}\rangle \equiv (s_1 - s_2)^2 \left(\frac{P_b}{2\pi G_N m}\right)^{2/3} \mathcal{C}$ → dipolar term; $G_N m / P_b \sim \mathcal{O}(10^{-10})$

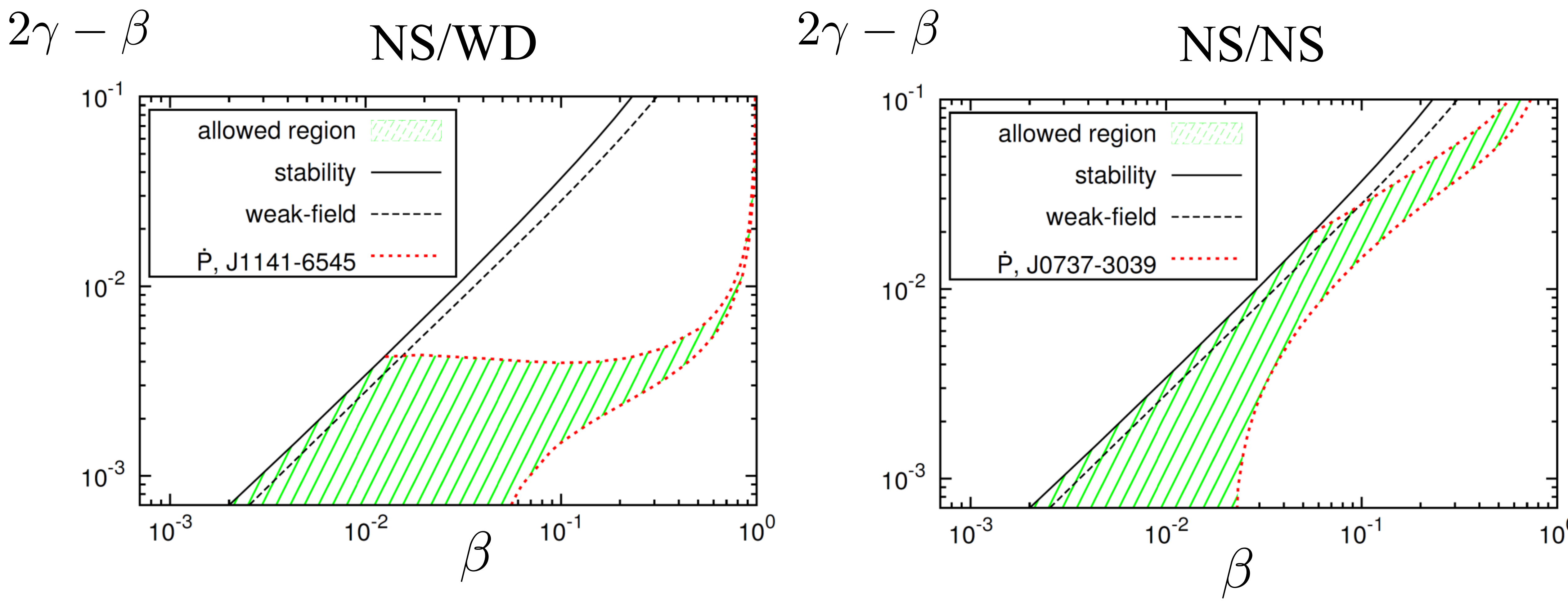
$$+ \left[(1 - s_1)(1 - s_2) \right]^{2/3} \left(\mathcal{A}_1 + \mathcal{S}\mathcal{A}_2 + \mathcal{S}^2 \mathcal{A}_3 \right)$$

$\mathcal{A}_i,\ \mathcal{C}$ functions of the LV parameters

GR: $\mathcal{A} = 1$ no dipole

Constraints from damping of binaries (EA)

Yagi, DB, Yunes, Barausse 13



$$\langle \mathcal{A} \rangle \equiv (s_1 - s_2)^2 \left(\frac{P_b}{2\pi G_N m} \right)^{2/3} \mathcal{C}$$

$$+ \left[(1 - s_1)(1 - s_2) \right]^{2/3} \left(\mathcal{A}_1 + \mathcal{S}\mathcal{A}_2 + \mathcal{S}^2 \mathcal{A}_3 \right)$$

Mostly dipolar

Quadrupolar + dipolar

Conservative dynamics of binaries (EA)

Strong 'effective' constraints on LV through 'strong' PPN

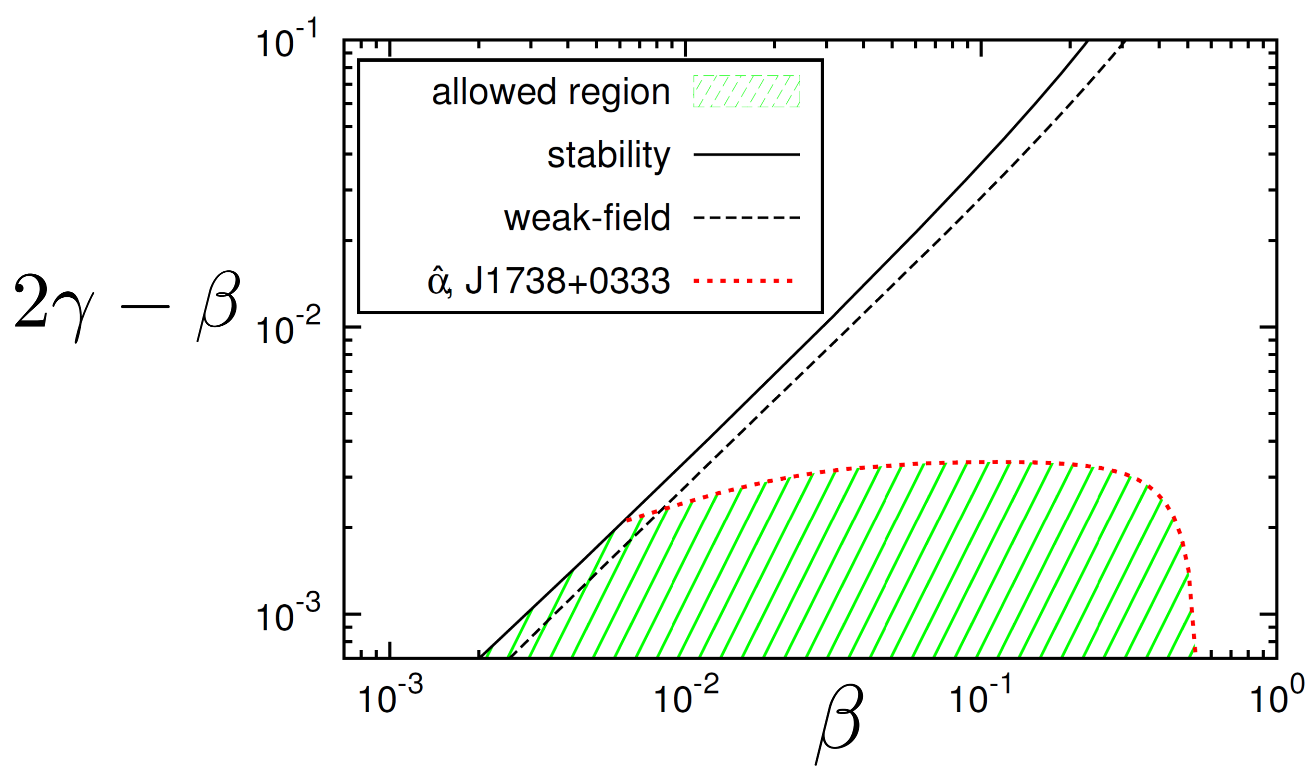
$$g_{0i} = -\frac{1}{c^3}\left[B_1^- \frac{G_N \tilde{m}_1}{r_1} v_1^i + \dots\right] \begin{cases} B_A^- = -\frac{7}{2} - \frac{1}{4}(\hat{\alpha}_1 - 2\hat{\alpha}_2) - \frac{1}{4}\hat{\alpha}_1 \\ B_A^- = -\frac{7}{2} - \frac{1}{4}(\alpha_1 - 2\alpha_2)\left(1 + \frac{2-\alpha}{2\beta - \alpha}\right) - \dots \end{cases}$$

from J1738+0333 $\quad \hat{\alpha}_2 \lesssim 10^{-4} \quad \hat{\alpha}_1 \lesssim 10^{-5}$

Damour, Esposito-Farese 92
Shao Wex, Freire et al 12

source dependent and independent of weak PPN!

but we know the sensitivities if we know the masses!

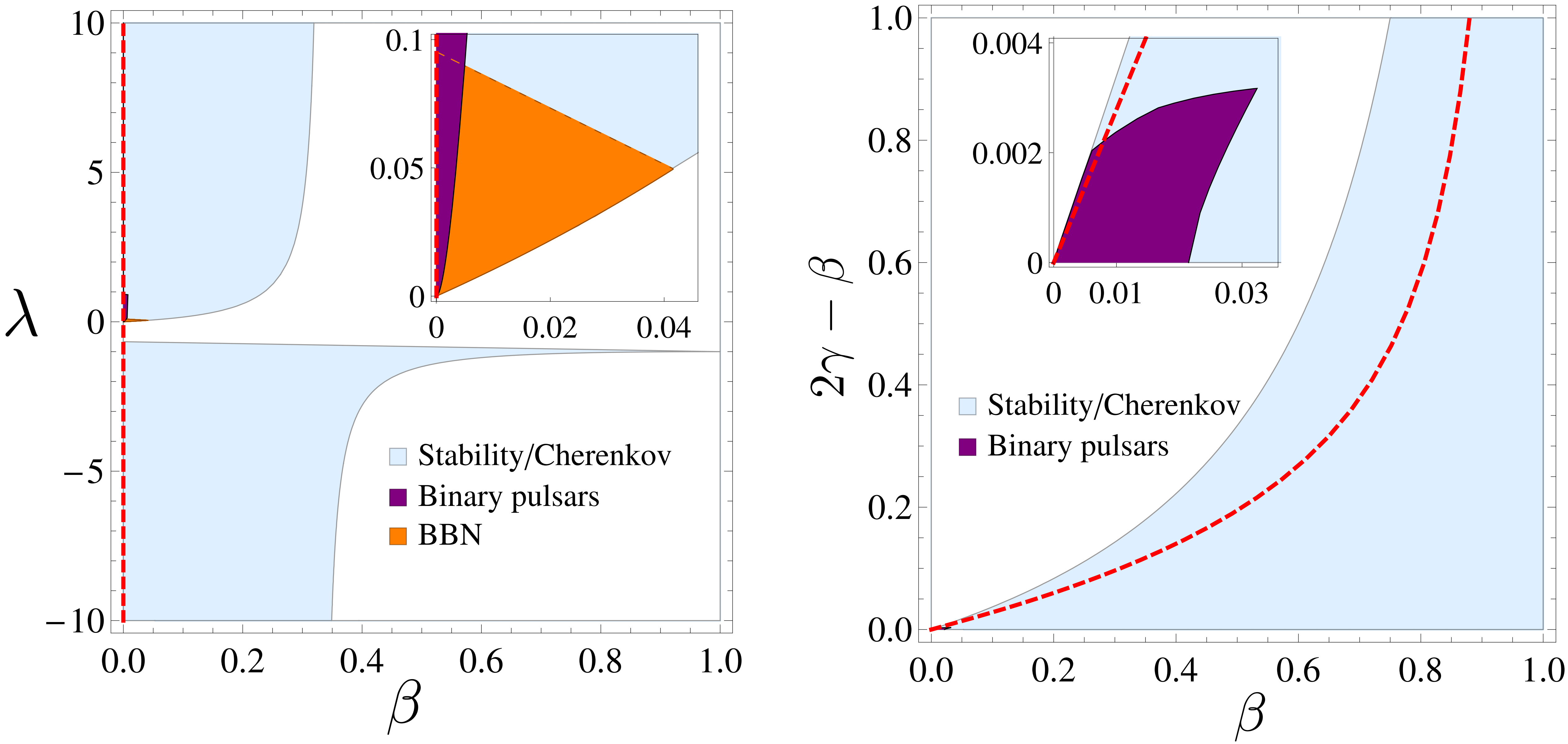


Yagi, DB, Yunes, Barausse 13

Constraints from binaries

(Solar system constraints imposed)

Yagi, DB, Yunes, Barausse 13



Combined constraints from PSR J1141-6545,
PSR J0348+0432, PSR J0737-3039, J1738+0333

A glimpse of cosmological constraints

Modified Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G_c \rho \longrightarrow \text{modified BBN}$$

$G_c(\alpha, \beta, \lambda) \neq G_N(\alpha)$ controls Newtonian dynamics (collapse)

- Modified clustering at large scales

There are extra degrees of freedom: enhanced dissipation

- Modified primordial plasma (anisotropic stress)

CMB and (linear) **scale structure** changes

Linear structure formation

Kobayashi, Urakawa, Yamaguchi 10

$$\mathrm{d}s^2 = a(t)^2\left[(1+2\phi)\mathrm{d}t^2 - \delta_{ij}(1-2\psi)\mathrm{d}x^i\mathrm{d}x^j\right]; \qquad \delta \equiv \frac{\rho}{\bar{\rho}} - 1$$

- Faster Jeans instability: DM dom, subhorizon

$$\frac{k^2\phi}{a^2} = \frac{3H^2(1+\beta/2+3\lambda/2)}{2(1-\alpha/2)}\delta = \frac{3G_N}{2G_c}H^2\delta \quad ; \qquad \delta'' + 2H\delta' = -\frac{k^2\phi}{a^2}$$

$$\delta \sim t^{\frac{1}{6}\left(-1+\sqrt{1+24\frac{G_N}{G_c}}\right)}$$

Audren, Blas, Ivanov, Lesgourgues, Sibiryakov 14

- Anisotropic stress

$$\phi - \psi = O(\beta)$$

Cosmological Constraints (Kh)

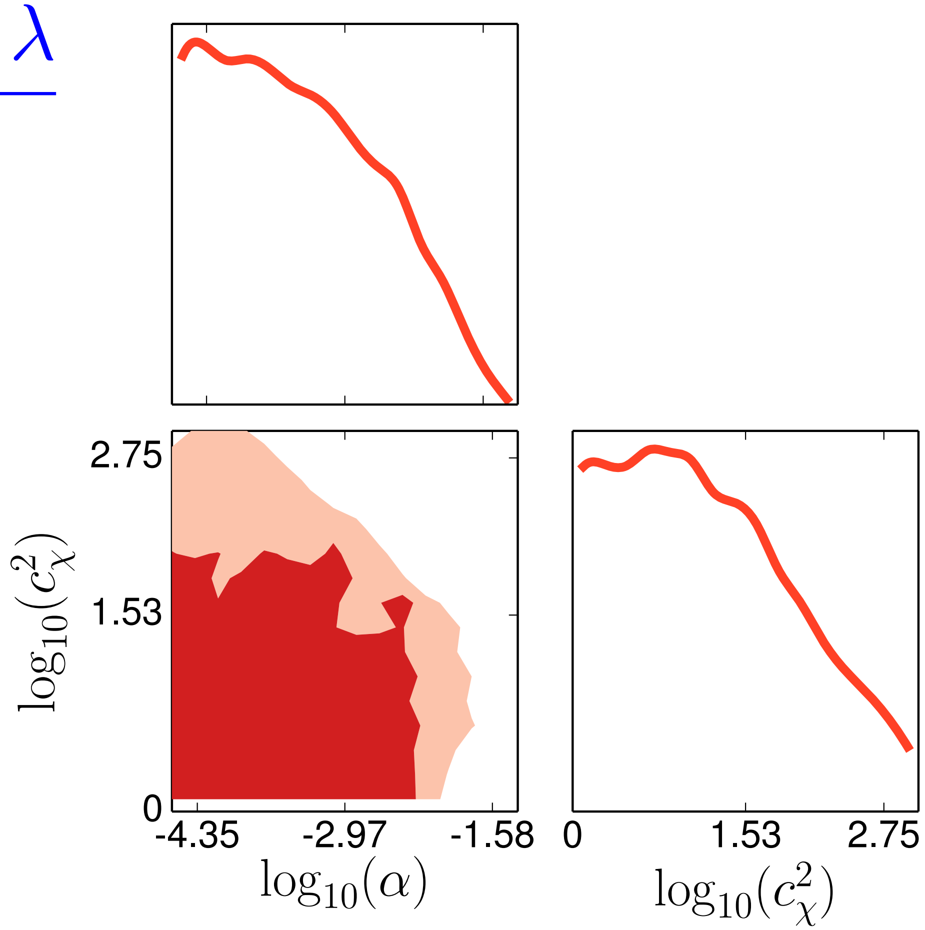
http://montepython.net/

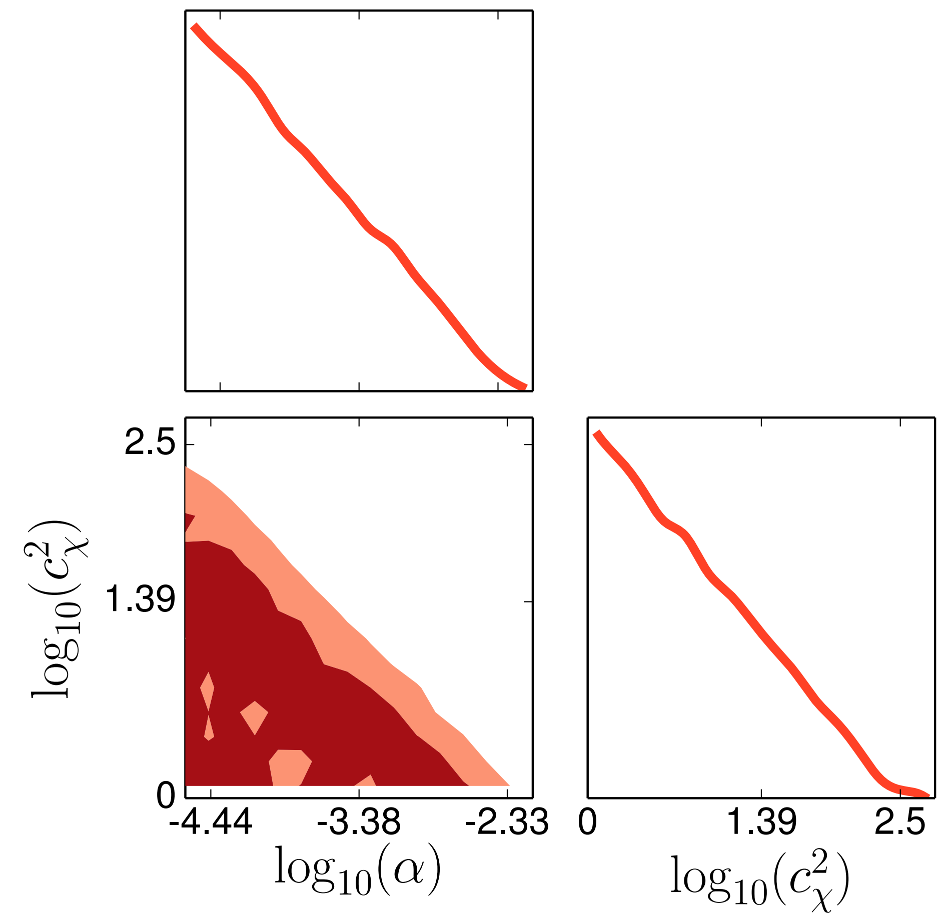
Audren, Blas, Ivanov, Lesgourgues, Sibiryakov 14

(also DM)

Planck, SPT, WiggleZ

$$c_\chi^2 = \frac{\beta + \lambda}{\alpha}$$





$$\alpha = 2\beta$$

	EA	Kh
α	$< 1.0 \cdot 10^{-2}$	$< 1.8 \cdot 10^{-3}$
c_χ^2	427	91

Conclusions

- Exploring Lorentz violation yields a rich phenomenology with strong theoretical motivations (effective or fundamental)
- Lorentz violation modifies gravity at every scale (extra massless d.o.f. $\varphi = t + \chi$ and modified graviton)
- Solar system tests $\alpha_1^{PPN} \lesssim 10^{-4}$ $\alpha_2^{PPN} \lesssim 10^{-7}$

  two unconstrained parameters!
- SEP violated: compact objects develop a u_μ charge (sensitivities)
- Modified orbits and dipolar GWs emission: both constrained by observation of **pulsars in binaries**

$$\beta,\ \lambda \lesssim O(.01)$$

- **All** parameters constrained at percent level! (~ to cosmology)

For the future…

- Sensitivities for other objects! (including BHs)
- More work on the waveform
- Cosmological constraints beyond linear theory
- More fundamental issues: BHs and emergence

Backup slides

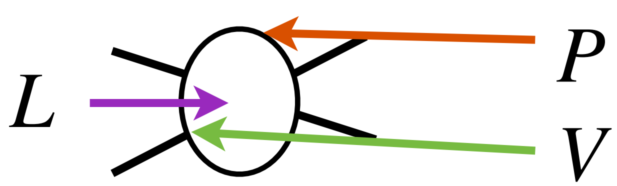
How I: Hořava Gravity in a Nutshell

Hořava 09

Toy model: Lifshitz scalar

$$\mathcal{L} = \phi \left[\partial_0^2 - \left(\frac{-\Delta}{M_\star^2}\right)^z \Delta\right] \left\{\phi + \sum_n a_n \left(\frac{\phi}{M_P}\right)^n\right\}$$

$$\Delta \equiv \partial_i \partial_i$$



To compute amplitudes

$$I \sim \left(\int^{\Lambda_0} \mathrm{d}\omega \int^{\Lambda_i} \mathrm{d}^3 k_i\right)^L \left(\frac{1}{\omega^2 - \bar{\mathrm{k}}^2 \left(\frac{\bar{\mathrm{k}}^2}{M_\star^2}\right)^z + i\epsilon}\right)^{P-V} \sim \Lambda_i^{(2-z)L+2(z+1)}$$

$$\Lambda_0 \sim \Lambda_i^{z+1}$$

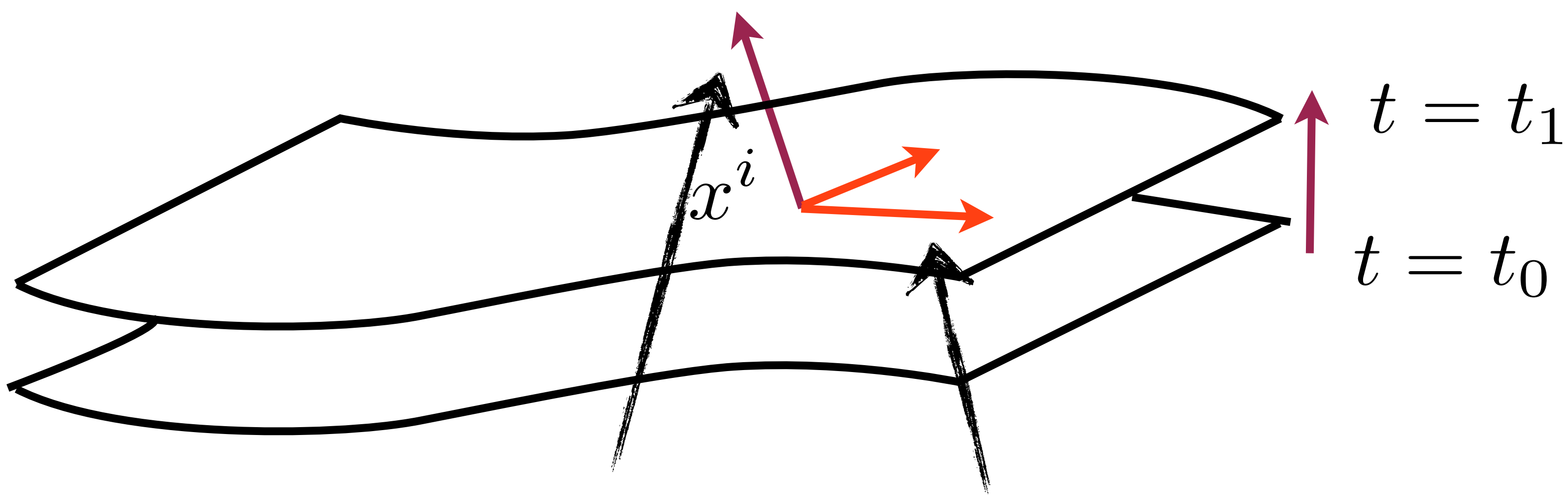
- $z = 0$ **(LI/GR):** $\sim \Lambda_i^{2(L+1)}$ grows with L !
- $z = 2$ **(LV)** $\sim \Lambda_i^6$ fixed !

\# of counterterms may be finite

How I: Hořava Gravity in a Nutshell

Hořava 09

Preferred foliation of space-time



Absolute time and space intervals

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = N^2\mathrm{d}t^2 - \gamma_{ij}(\mathrm{d}x^i + N^i\mathrm{d}t)(\mathrm{d}x^j + N^j\mathrm{d}t)$$

Broken diffeomorphisms: new group of covariance

$$x^i \mapsto \tilde{x}^i(x^j, t) \qquad\qquad t \mapsto \tilde{t}(t)$$

FDiff: Foliation preserving Diff

Extra (gapless?) polarization expected

How I: Hořava Gravity in a Nutshell

Blas, Pujolàs, Sibiryakov 09

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = N^2\mathrm{d}t^2 - \gamma_{ij}(\mathrm{d}x^i + N^i\mathrm{d}t)(\mathrm{d}x^j + N^j\mathrm{d}t)$$

Covariant objects under FDiff

$$K_{ij} \sim \frac{\partial \gamma_{ij}}{\partial t} \sim \omega\,\gamma_{ij} \qquad {}^{(3)}R^i{}_{jkl} \sim \bar{\mathrm{k}}^2 \gamma_{ij} \qquad a_i \equiv \frac{\partial_i N}{N} \sim \mathrm{k}_i\,\phi$$

GR Lagrangian extended to

$$\mathcal{L} = M_P^2 N\sqrt{\gamma}\Big(\underbrace{K_{ij}K^{ij} - (1-\lambda')(\gamma_{ij}K^{ij})^2}_{\partial_0^2} - (1-\beta')\,{}^{(3)}R + \alpha'\,a_i a^i \ldots + \frac{\Delta^{2\,(3)}R}{M_\star^4}\Big)$$

Low energy (IR)	Renormalizability

(Naive) GR limit: $\lambda' = \beta' = \alpha' = 0$

Finite # of counterterms

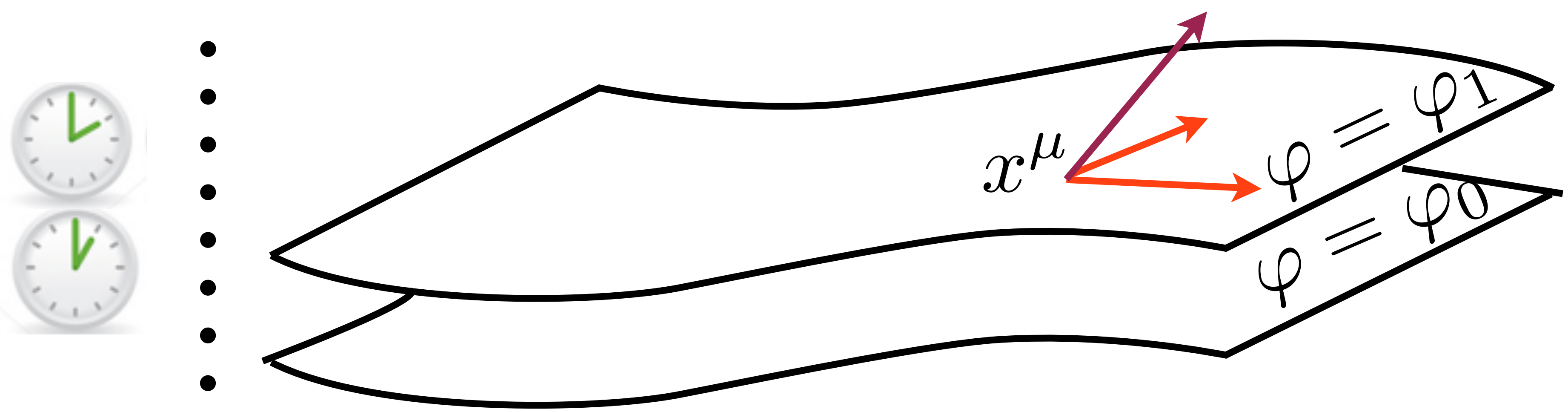
How II: Khronometric Theory

Blas, Pujolàs, Sibiryakov 09

Diff invariance restored by adding a compensator: φ

khronon

$\chi\rho\acute{o}\nu o\varsigma$



$t \to \tilde{t}(t)$

$x^i \mapsto \tilde{x}^i(x^j, t)$

$\varphi \mapsto f(\varphi)$

Diff

$$u_\mu \equiv \frac{\partial_\mu \varphi}{\sqrt{\partial_\alpha \varphi \partial^\alpha \varphi}}$$

.....

K_{ij}

$$\mathcal{K}_{\mu\nu} \equiv \left(\delta^\sigma_\mu - u^\sigma u_\mu\right) \nabla_\sigma u_\nu$$

$\mathcal{L}$

$$\mathcal{L}_{EH} + M_P^2 \sqrt{-g} \left(\lambda \left(\nabla^\mu u_\mu\right)^2 + \alpha \left(u^\nu \nabla_\nu u_\mu\right)^2 + \beta \nabla_\mu u_\nu \nabla^\nu u^\mu \right)$$

$\varphi = t$

$+O(1/M_\star)$

Unitary gauge

Neutron Stars at O(v)

We consider an irrotational fluid for the NS

$$ds^2 = O(v^0) + 2vV(r,\theta)dtdr + 2vrS(r,\theta)dtd\theta + O(v^2)\,, ...$$

In terms of Legendre polynomials

$$V(r,\theta) = \sum v_n(r)P_n(cos\theta)\,,...$$

the different modes decouple! **3 ODE** per mode!

To derive σ_A remind

$$g_{0i} \supset -\frac{1}{c^3}B_1^-(\sigma_A)\frac{G_N\tilde{m}_1}{r_1}v_1^i$$

NS gauge

$$ds^2 = O(v^0) - 2v\left(1 + (B^- + B^+ + 2)\frac{G_N m}{r}\right)\cos\theta dtdr + \ldots$$

$n = 1$ is enough!

$$v_1(r) = v_1^\infty(r) \equiv -1 + A\frac{G_N m}{r} + (k_{A_2}A + k_{c_2})\frac{G_N^2 m^2}{r^2} + ..$$

all known (depend on m)

Computing the sensitivities

Matching of real solution to the effective one

$$S_{ppA} = -\tilde{m}_A \int ds_A \left[(1 + \sigma_A (1 - u_\mu v^\mu) + O(u_\mu v^\mu - 1)^2 \right]$$

Slowly moving star: $v^i \ll 1$ (velocity wrt æther)

Far-away from the star

$$g_{00} = 1 - \frac{1}{c^2} \frac{2G_N \tilde{m}_1}{r_1} + \frac{1}{c^4} \left[\frac{2G_N^2 \tilde{m}_1^2}{r_1^2} - \frac{3G_N \tilde{m}_1}{r_1} v_1^2 (1 + \sigma_1) \right],$$

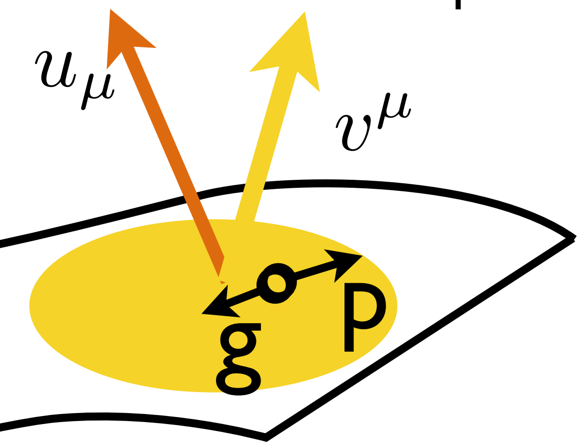
$$g_{0i} = -\frac{1}{c^3} \left[B_1^- \frac{G_N \tilde{m}_1}{r_1} v_1^i + B_1^+ \frac{G_N \tilde{m}_1}{r_1} v_1^j \hat{r}_1^j \hat{r}_1^i \right], \quad g_{ij} = -\left(1 + \frac{1}{c^2} \frac{2G_N \tilde{m}_1}{r_1} \right) \delta_{ij}$$

$$B_A^\pm(\sigma_A^{kh}) \equiv \pm\frac{3}{2} - 2 \pm \frac{1}{4}(\alpha_1^{kh} - 2\alpha_2^{kh}) \left(1 + \frac{2 - \alpha}{2\beta - \alpha} \sigma_A^{kh} \right) - 2\sigma_A^{kh} - \frac{1}{4} \alpha_1^{kh} (1 + \sigma_A^{kh}),$$

LV parameters

Neutron Stars at O(v)

Static, spherically symmetric, asympt. flat, moving with v^i

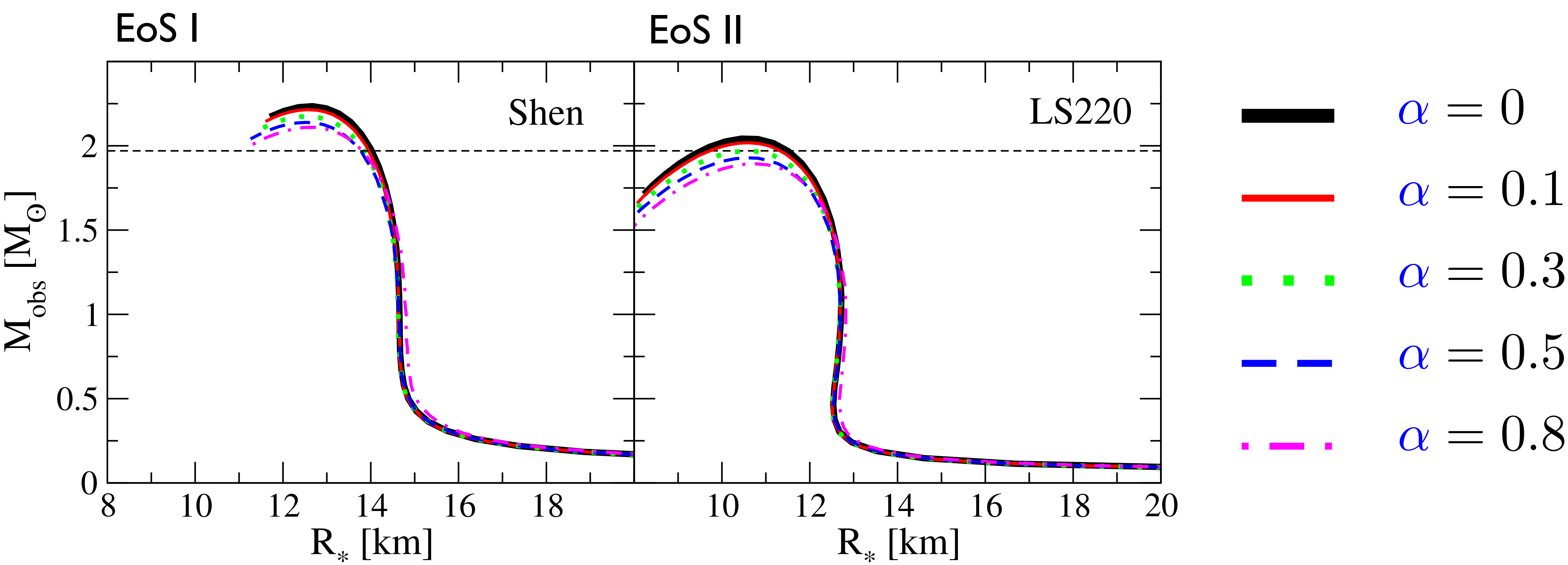


$$ds^2 = e^{\nu(r)}dt^2 - e^{\mu(r)}dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$
$$+ 2vV(r,\theta)dtdr + 2vrS(r,\theta)dtd\theta + \mathcal{O}(v^2)\,,$$

$$u_\mu = e^{\nu(r)/2}\delta^t_\mu + vW(r,\theta)\delta^r_\mu + \mathcal{O}(v^2)$$

At $O(v^0)$: regularity at the center + EoS + continuity at R_*
modified TOV

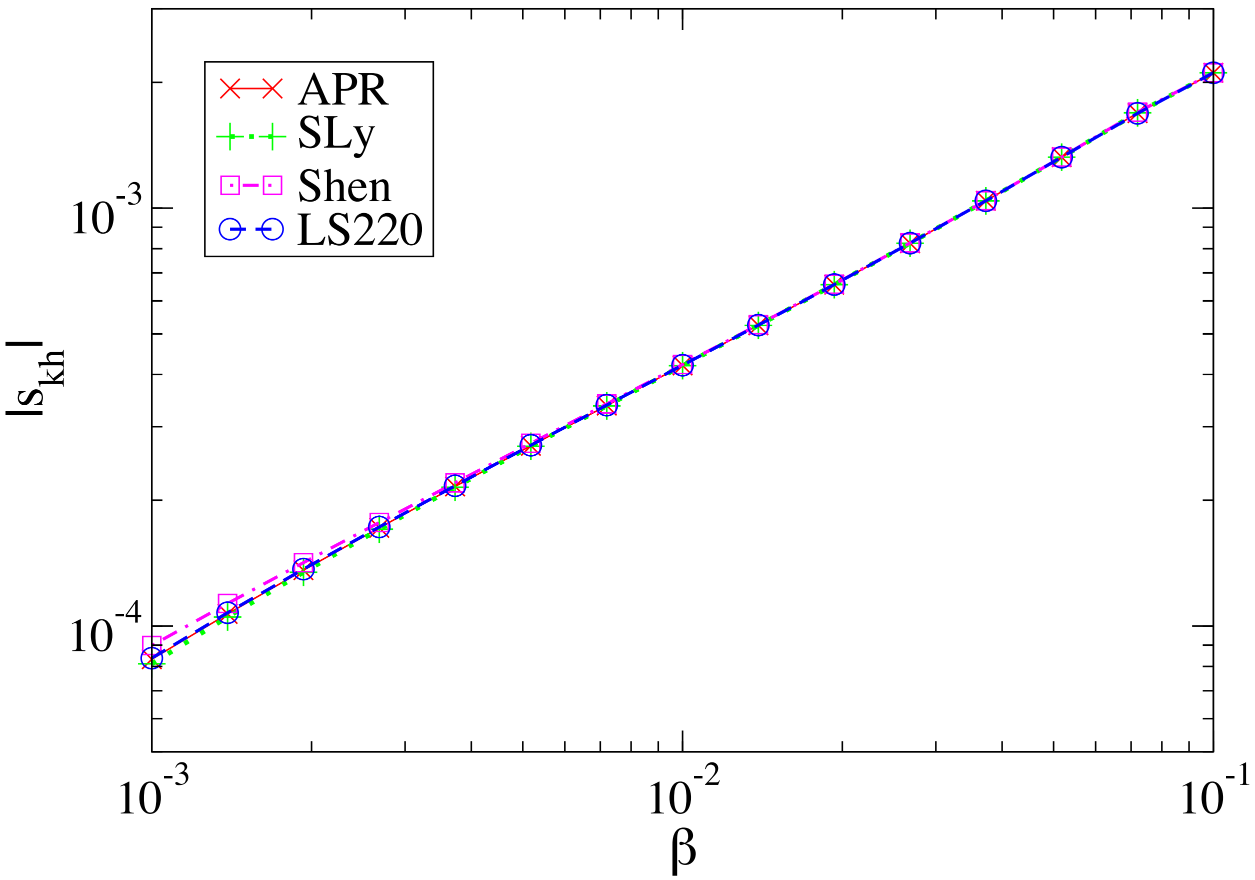
Yagi, DB, Barausse, Yunes 13



Neutron Stars at O(v): Results

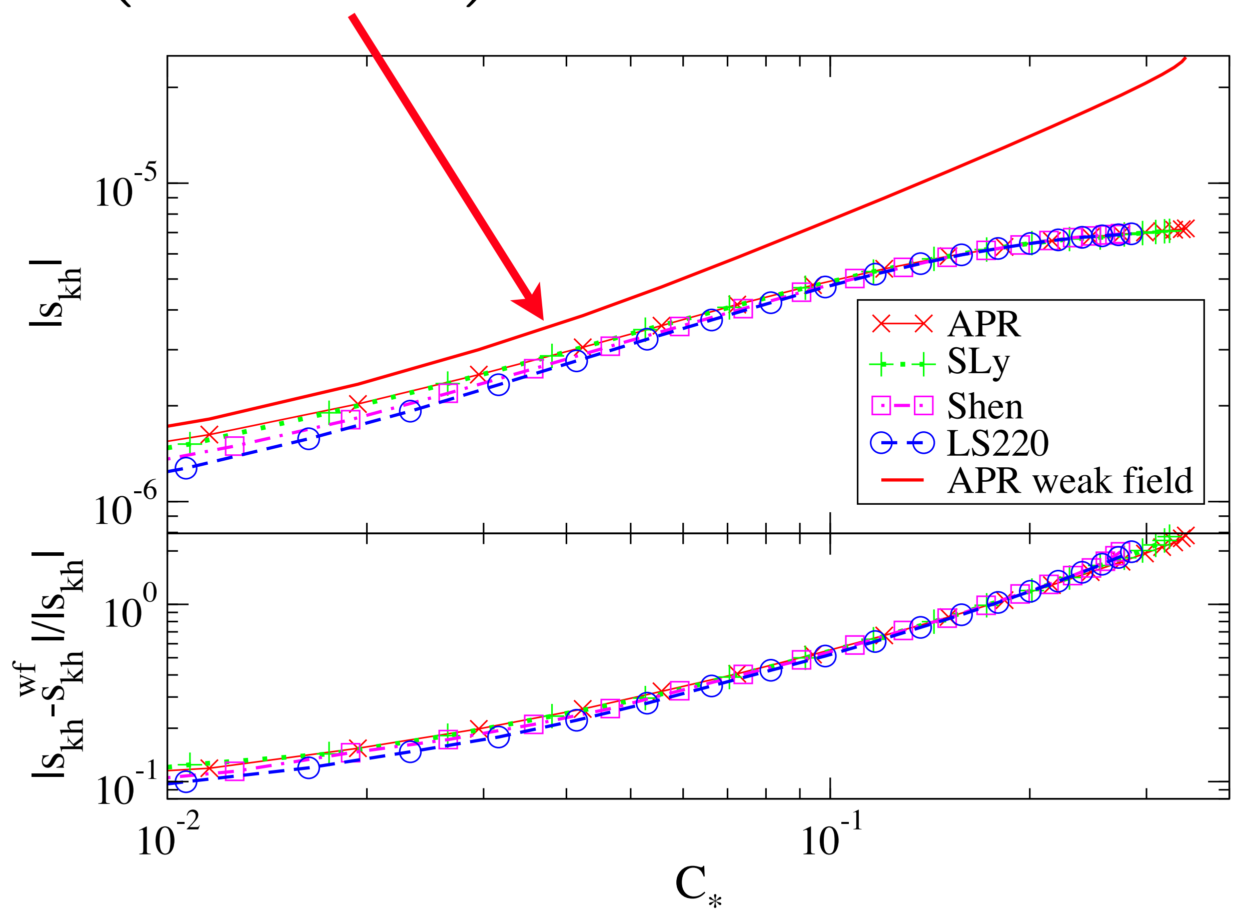
Yagi, DB, Yunes, Barausse 13

Weak field result $s = \left(\alpha_1 - \frac{2}{3}\alpha_2\right) C_* + O(C_*^2)$ Foster 07



$m = 1.4 M_\odot$

$\alpha_1 = \alpha_2 = 0$



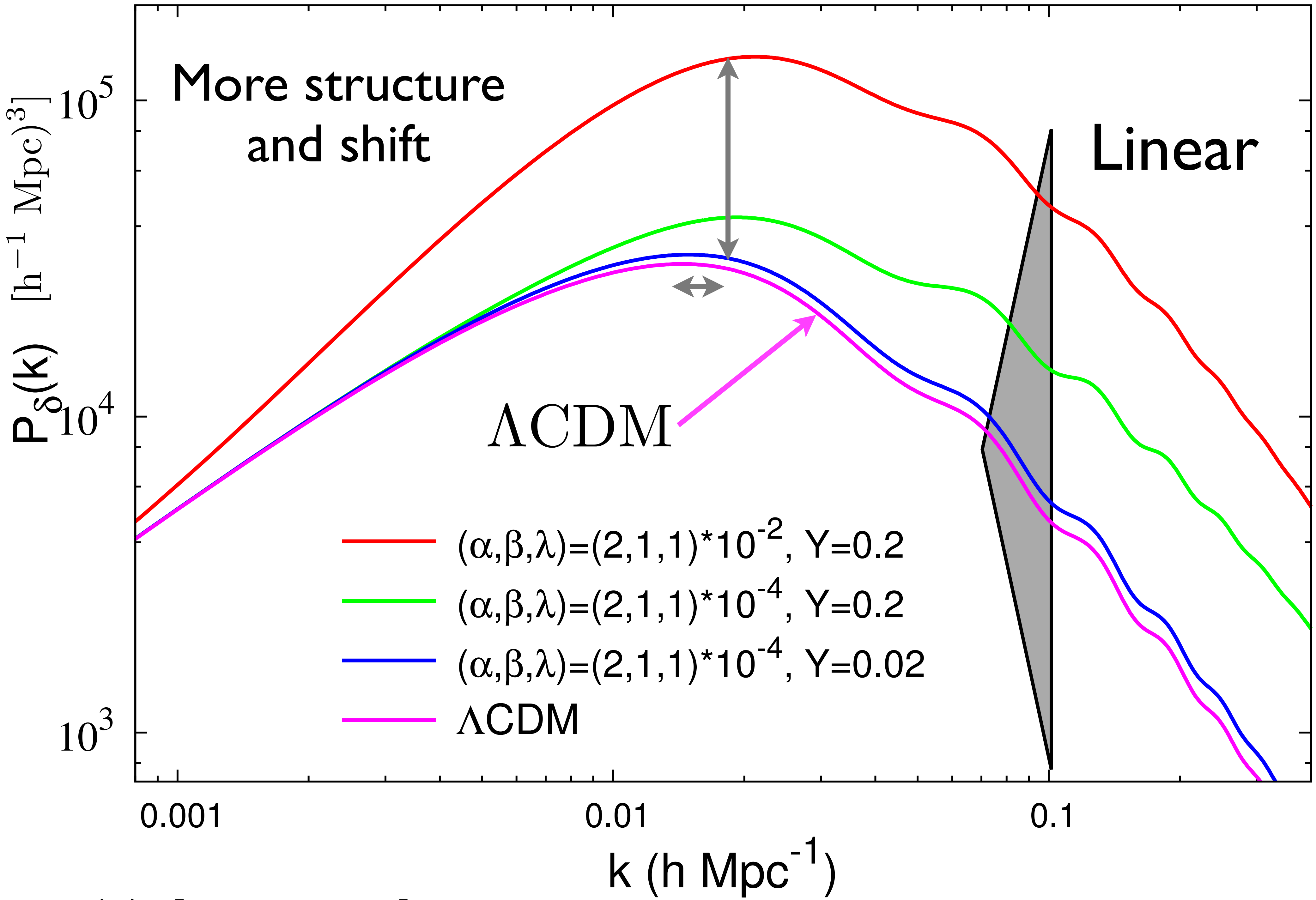
$C_* \equiv G_N m / R$

$\beta = 10^{-4}$

Matter Power Spectrum

$$\langle \delta(k)\delta(k') \rangle \equiv \delta^{(3)}(k+k')P(k)k^3$$

Blas, Ivanov, Sibiryakov 12



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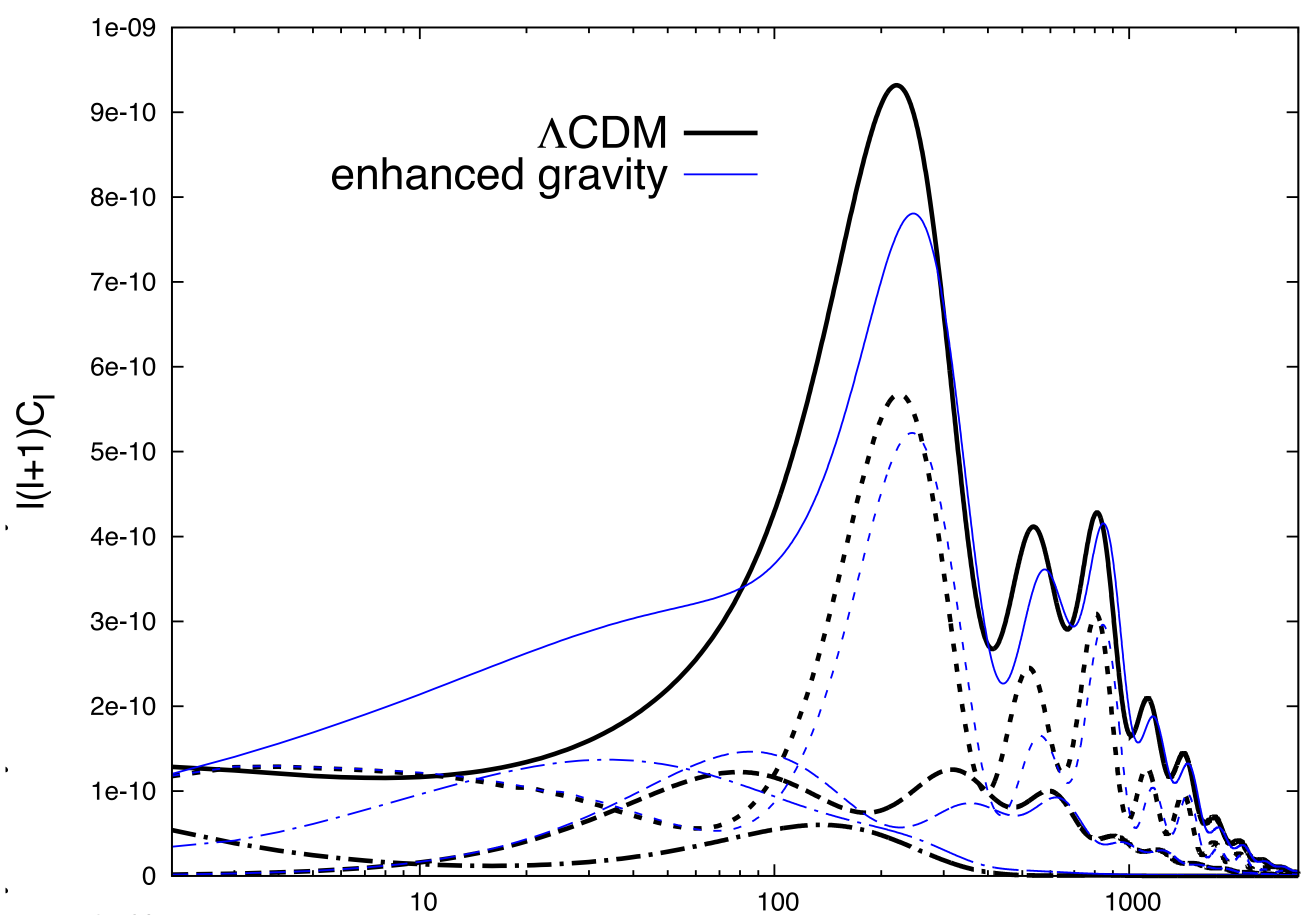
Cosmic Microwave Background

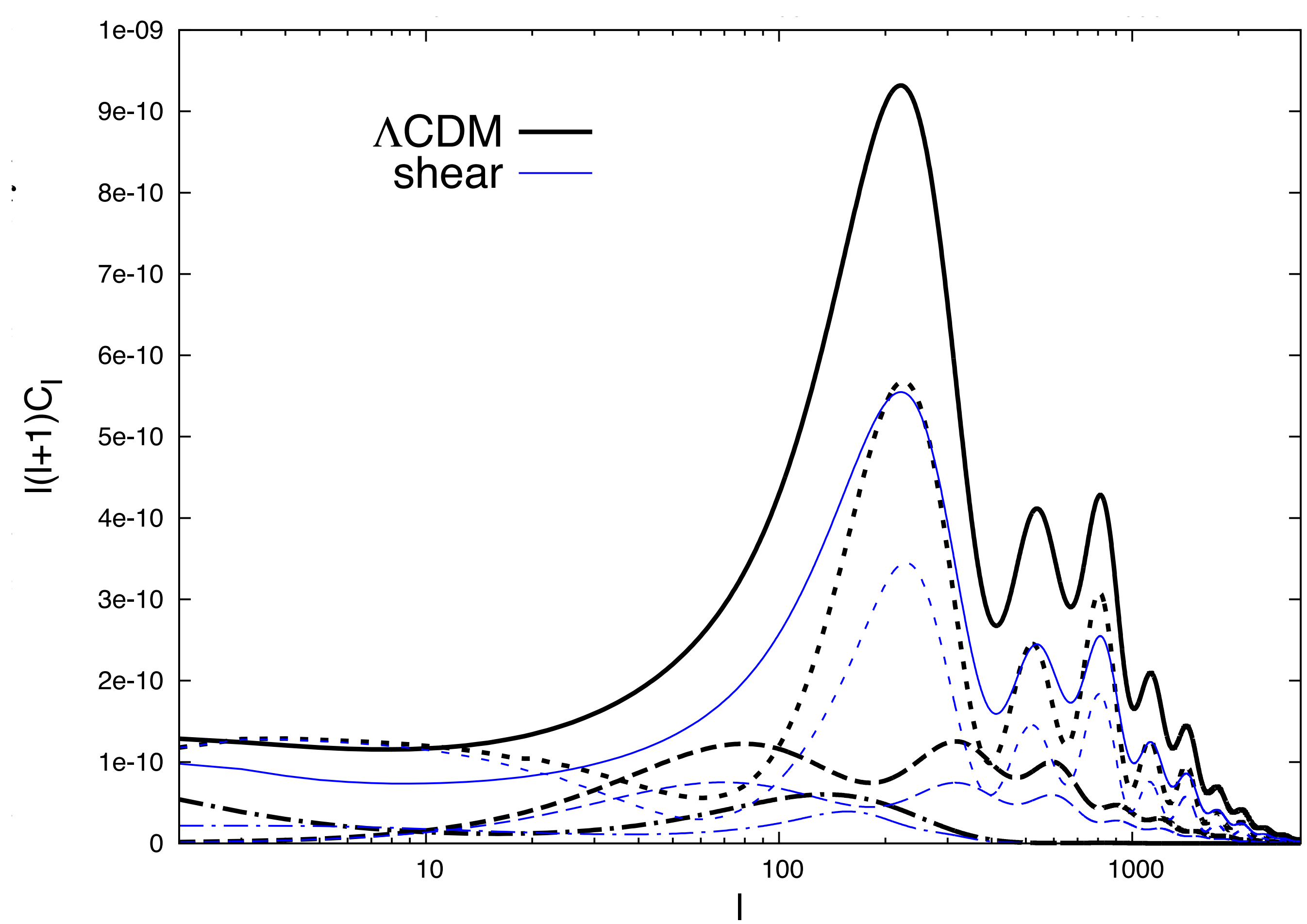
Audren, Blas, Lesgourgues, Sibiryakov 13

$$\ddot{\delta}_\gamma + k^2 c_s^2 \delta_\gamma \supset -\left(\frac{4k^2}{3}\psi\right)$$

$$k^2\psi \sim \frac{G_N}{G_c}\delta_\gamma \quad \rightarrow \quad c_s^{eff}$$

Shift of the peaks, change of zero point of oscillation and amplitude





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