Numerical models for superfluid neutron stars & application to pulsar glitches

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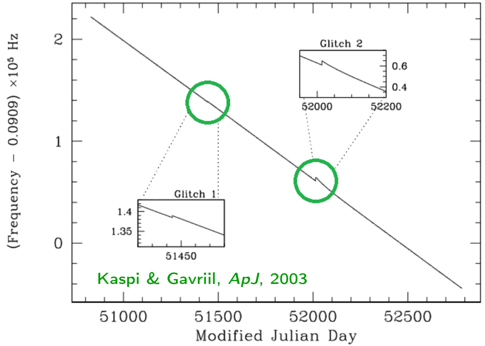
Geneva, December 15, 2015

Pulsar glitches



Observational features

Wong+, *ApJ*, 2001; Espinoza+, *MNRAS*, 2011

- **amplitude:**

$$\Delta\Omega/\Omega \sim 10^{-9} - 10^{-5}$$

- **rise time:**

$$\begin{cases} \tau_r \lesssim 30 \text{ s} & \textit{Vela} \\ \tau_r \sim 0.5 \text{ d} & \textit{Crab} \end{cases}$$

- **exponential relaxation** on several days or months.

Models for pulsar glitches

Haskell & Melatos, *IJMPD*, 2015

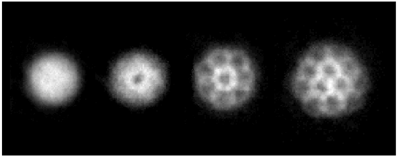
- Rearrangement of the moment of inertia --→ crust **quakes**,
- Angular momentum transfer between *two* fluids --→ **superfluidity**.

Superfluidity in neutron stars

Baym, Pethick & Pines, *Nature*, 1969

Superfluid properties:

- **zero** viscosity,
- **infinite** thermal conductivity,
- angular momentum carried by **vortex lines**.



Madison et al., *PRL*, 2000

Theoretical considerations

$$T_c \simeq 10^9 - 10^{10} \text{ K}$$

⇢ **superfluid neutrons** in the core & in the inner crust of the NS.

Observational supports

- **Long relaxation time scales** in pulsar glitches,
- Fast cooling in Cassiopeia A,
- QPOs from SGRs, ...

Vortex-mediated glitch theory

Anderson & Itoh, *Nature*, 1975

Consequence of superfluidity:

several **dynamically distinct** components inside neutron stars.

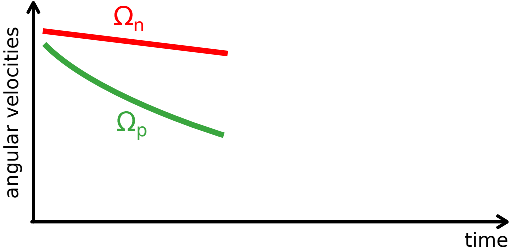
Two-fluid model

- Charged particles:

  $\Omega_p = \Omega \leftrightarrow$ pulsar

- Superfluid neutrons:

  $\Omega_n \gtrsim \Omega_p$



Before the glitch: the **vortices** are pinned to the crust.

radial force $<$ pinning force

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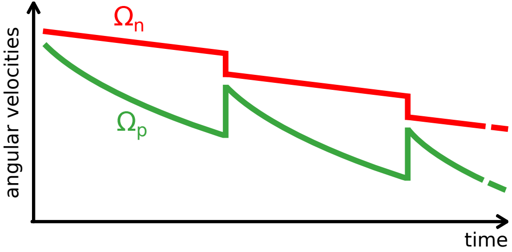


Once a threshold in $\Omega_n - \Omega_p$ is reached:

radial force $=$ pinning force

$\dashrightarrow$ angular momentum transfer between the fluids

Purposes of the present work

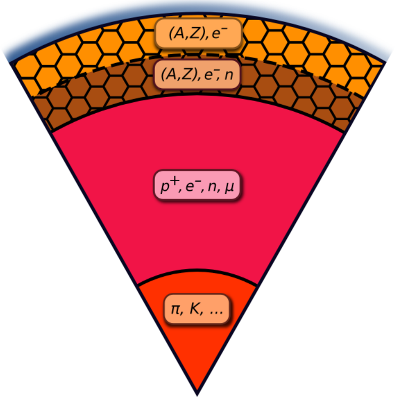
Previous works:

- Theoretical model with two fluids in GR developed by **Carter, Langlois,** ***et al.*** (1990s).
- Prix, Novak & Comer, *PRD*, 2005: Numerical model for stationary superfluid neutron stars implemented in LORENE. ⇢ polytropic EoS (non realistic).

Purposes

- Compute realistic **equilibrium configurations** of rotating superfluid neutron stars,
- Build a simple numerical model concerning **pulsar glitches**.

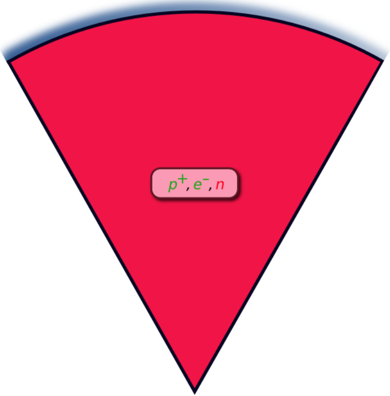
Basic assumptions



Equilibrium configurations

- isolated star,
- $T = 0$,
- no magnetic field,
- dissipative effects are **neglected**,
- **uniform** composition $\rightarrow p, e^-, n$,
- asymptotically flat, **stationary**, **axisymmetric** & **circular** metric,
- **rigid-body** rotation: Ω_n, Ω_p.

Basic assumptions



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Relativistic two-fluid hydrodynamics

Carter, "Covariant theory of conductivity in ideal fluid or solid media", 1989 & Carter & Langlois, *Nuc. Phys. B*, 1998

System = two **perfect** fluids:

- superfluid neutrons $\rightarrow \vec{n}_{\rm n} = n_{\rm n} \vec{u}_{\rm n}$,
- protons & electrons $\rightarrow \vec{n}_{\rm p} = n_{\rm p} \vec{u}_{\rm p}$.

Energy-momentum tensor

$$T_{\alpha\beta} = n_{{\rm n}\alpha} p^{\rm n}_{\beta} + n_{{\rm p}\alpha} p^{\rm p}_{\beta} + \Psi g_{\alpha\beta}$$

$\hookrightarrow$ **conjugate momenta**

Entrainment matrix:

$$\begin{cases} p^{\rm n}_{\alpha} = \mathcal{K}^{\rm nn} n^{\rm n}_{\alpha} + \mathcal{K}^{\rm np} n^{\rm p}_{\alpha} \\ p^{\rm p}_{\alpha} = \mathcal{K}^{\rm pn} n^{\rm n}_{\alpha} + \mathcal{K}^{\rm pp} n^{\rm p}_{\alpha} \end{cases}$$

$\dashrightarrow$ entrainment effect

Equation of state

$$\mathcal{E}(n_{\rm n}, n_{\rm p}, \Delta^2)$$

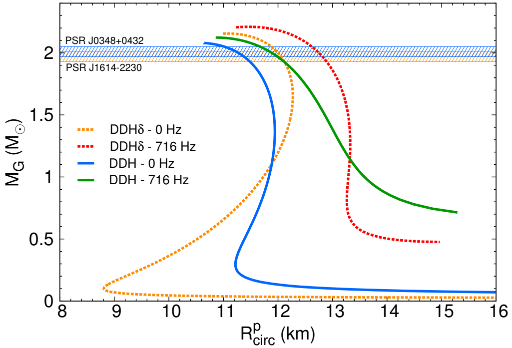
Equations of state

Relativistic Mean-Field Theory:

nucleon-nucleon interactions ⇔ exchange of effective mesons

$$\mathcal{L} = \boxed{\mathcal{L}_b} + \boxed{\mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\rho + \mathcal{L}_\delta} + \boxed{\mathcal{L}_{\text{int}}}$$

free baryons — free mesons — interaction



- Gravitational mass:

$$M_G = M^B + E_{\text{bind}},$$

- Circumferential radius:

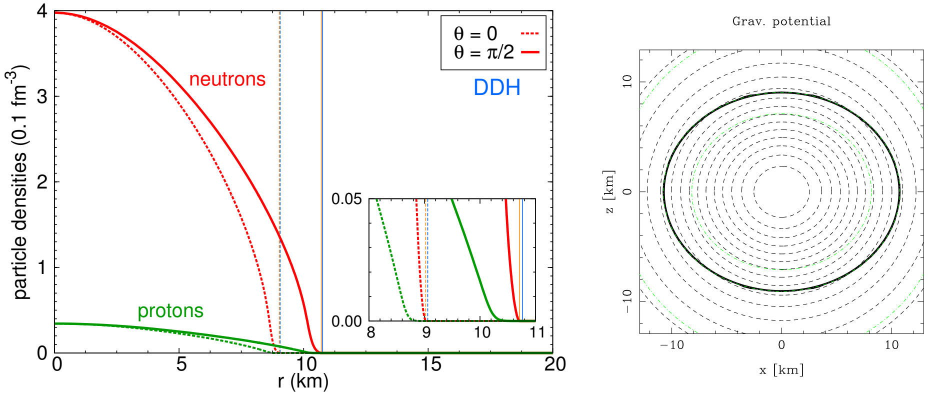
$$R^X_{\text{circ, eq}} = \mathcal{C}^X/2\pi.$$

Virial identities:

$$GRV \sim 10^{-8} - 10^{-5}$$

Density profiles

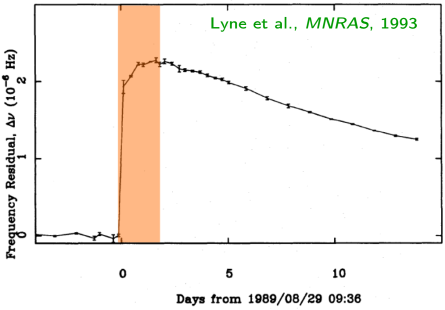
$$M_G = 1.4\ \mathrm{M}_\odot,\ \Omega_\mathrm{n}/2\pi = \Omega_\mathrm{p}/2\pi = 716\ \mathrm{Hz}$$



Modelling glitch rise

Bulk model for pulsar glitches:

$\Omega_{\rm n} - \Omega_{\rm p} = \Delta\Omega_c \Rightarrow$ angular momentum transfer through **mutual friction**



Typical time scales

- **rise time:**

$$\begin{cases} \tau_r < 30 \text{ s} & \textbf{Vela} \\ \tau_r \sim 0.5 \text{ d} & \textbf{Crab} \end{cases}$$

Dodson+, *ApSS*, 2007 & Wong+, *ApJ*, 2001

- **hydrodynamical time:**

$$\tau_{\rm h} \simeq 0.1 \text{ ms}$$

Shapiro, *ApJ*, 2000

⇢ Series of equilibrium configurations with **constant** $M^{\rm B}$ and J.

Angular momentum transfer

Langlois, Sedrakian & Carter, *MNRAS*, 1998 & Sidery, Passamonti & Andersson, *MNRAS*, 2010

Mutual friction moment:

superfluid vorticity → *lag*

$$\Gamma_{int} = -\overline{\mathcal{B}} \int \Gamma_{\rm n} n_{\rm n} \varpi_{\rm n} h_\perp^2 \, \mathrm{d}\Sigma \times (\Omega_{\rm n} - \Omega_{\rm p})$$

mutual friction parameter

Evolution equations:

$$\begin{cases} \dot{J}_{\rm n} = + \Gamma_{\rm int}, \\ \dot{J}_{\rm p} = - \Gamma_{\rm int}. \end{cases}$$

⇢ Computation of $\Omega_{\rm n}(t)$ & $\Omega_{\rm p}(t)$ profiles from $\Omega_{\rm n,0} > \Omega_{\rm p,0}$

Trigger threshold:

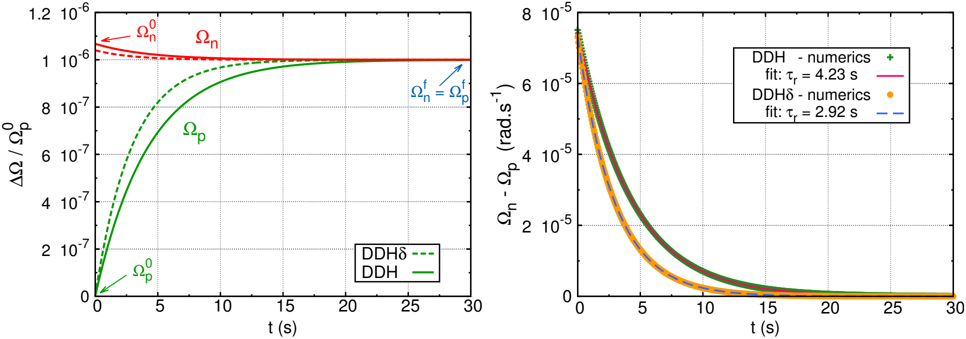
$$\Delta\Omega_c = \Omega_{\rm n}^0 - \Omega_{\rm p}^0 \simeq \frac{I}{I_{\rm n}} \frac{\Delta\Omega}{\Omega} \Omega$$

Input parameters

$M^{\rm B}$, Ω, *EoS*, β-eq., $\Delta\Omega/\Omega$, $\overline{\mathcal{B}}$

Spin-up time scale

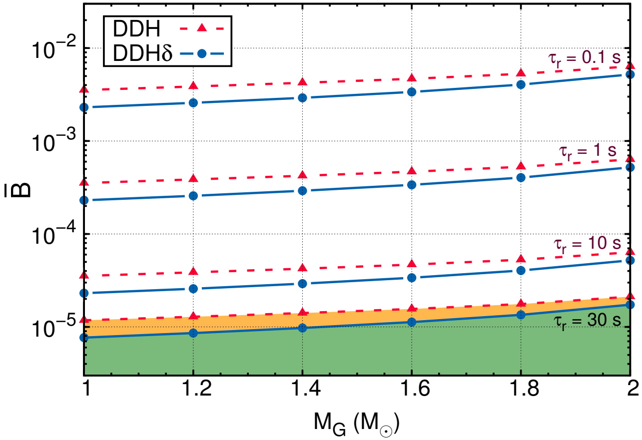
$$\Delta\Omega/\Omega = 10^{-6},\ \Omega_{\rm n}^{f}/2\pi = \Omega_{\rm p}^{f}/2\pi = 11.19\ \text{Hz},\ M_G = 1.4\ \text{M}_\odot\ \&\ \overline{\mathcal{B}} = 10^{-4}$$



$$\Omega_{\rm n}(t) - \Omega_{\rm p}(t) \propto e^{-t/\tau_r}$$

The Vela pulsar

$$\Delta\Omega/\Omega = 10^{-6},\ \Omega_n^f/2\pi = \Omega_p^f/2\pi = 11.19\ \text{Hz}$$



drag-to-lift ratio

$$\overline{\mathcal{B}} \simeq \frac{I_n^g}{I_n} \times \mathcal{R}$$

involved part of the superfluid

Conclusion

- Equilibrium configurations for superfluid neutron stars with realistic EoSs, using LORENE,
- Bulk model for pulsar glitches seen as angular momentum transfers through mutual friction force, in GR.

Future work Ho, Espinoza, Antonopoulou & Andersson, *Science Advances*, 2015

Confrontation with accurate observations of glitches

⇢ to get some **constraints** on the interior of neutron stars

Thank you!