“28th Texas Symposium on Relativistic Astrophysics”
Geneva, Dec 16th 2015

“First numerical simulations of the dynamo effect in chiral MHD”

Jennifer Schober
(Nordita fellow)
&
Collaborators:
Axel Brandenburg, Igor Rogachevskii,
Oleg Ruchayskiy, Alexey Boyarsky

NORDITA
(Stockholm)

“Dynamos in Chiral MHD”

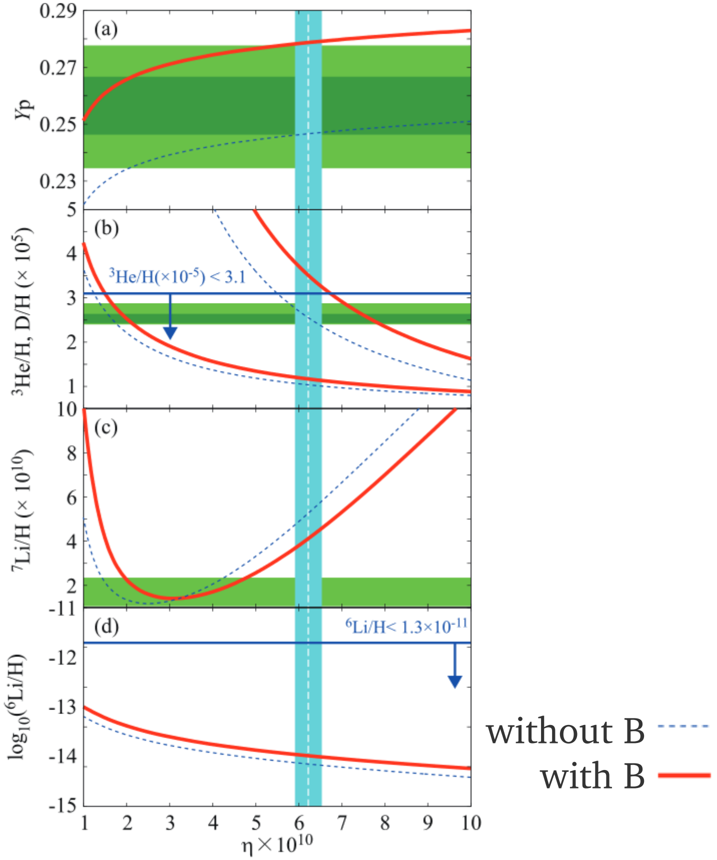
J. Schober (Nordita fellow)

1. Introduction

Importance of primordial B-fields

In the early Universe:

- effects on primordial nucleosynthesis
  [*Yamazaki & Kusakabe 2012*]



- imprint on CMB
  [*Kahniashvili & Ratra 2007*]

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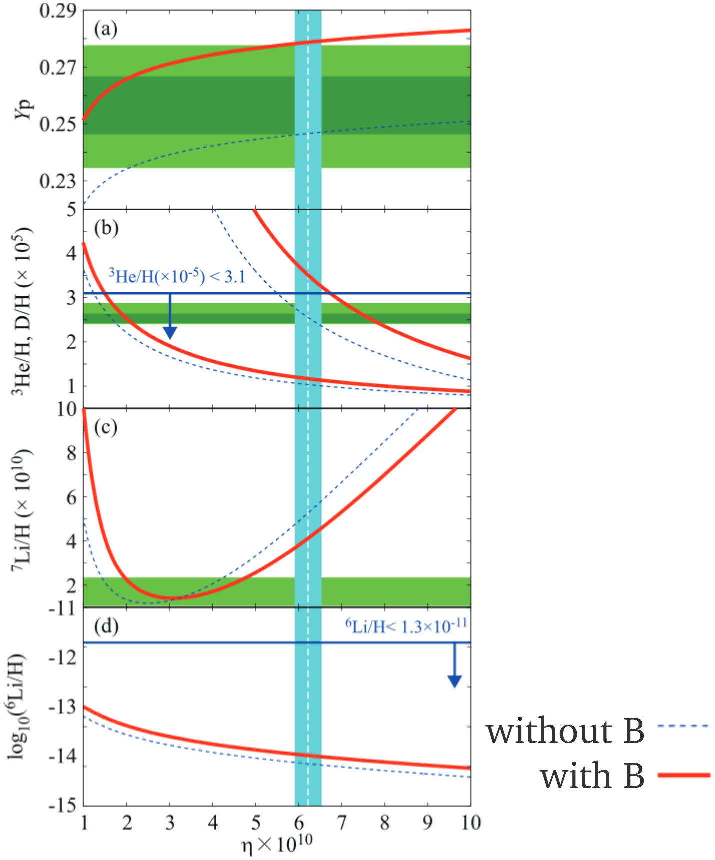

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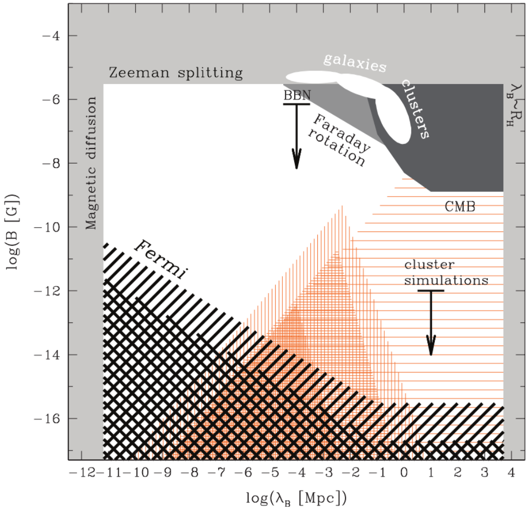
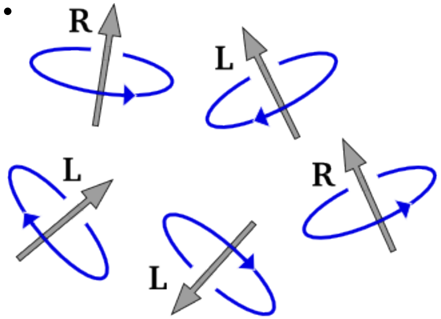
At later times:

- seed fields for dynamo amplification during structure formation
  [*Latif et al. 2012*; *Schober et al. 2012,2013; Machida & Doi 2013, Pakmor & Springel 2013*]
- explanation for intergalactic magnetic fields
  [Fermi observations of TeV blazers, *Neronev & Vovk 2010*]



Chiral magnetic effect

- Spins of fermions have different handedness:
  $\mu_{\rm L}$: "number density" of left-handed fermions
  $\mu_{\rm R}$: "number density" of right-handed fermions



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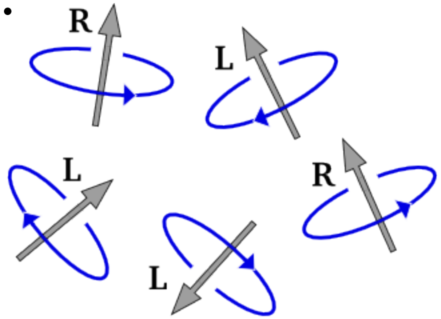

- Energies of interest:

$$10\ \mathrm{MeV} \lesssim E \lesssim 80\ \mathrm{TeV}$$

relativistic limit

flipping reactions in thermal equilibrium

Here: $\mu_{\rm L}$ and $\mu_{\rm R}$ are conserved individually
→ chemical potential μ is constant:

$$\mu \equiv \mu_{\rm L} - \mu_{\rm R} = \mathrm{const}$$

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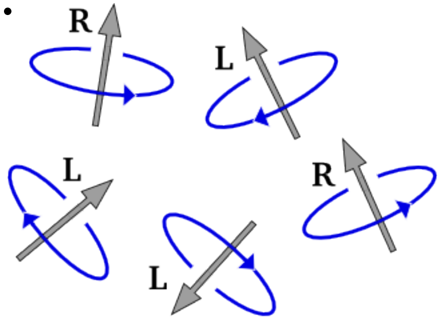

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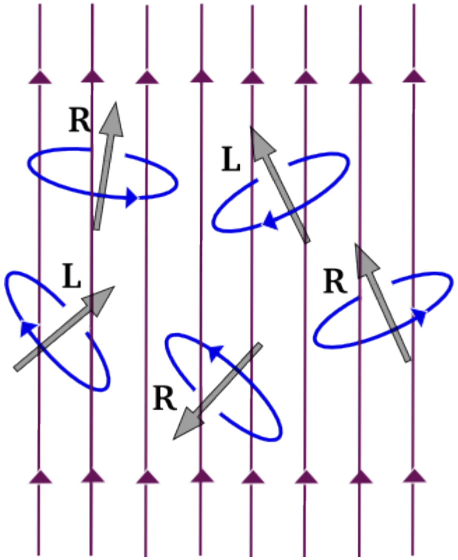
- In presence of external magnetic fields:
  → **chiral anomaly**
  (marcoscopic quantum effect)

→ $$\boxed{\frac{\mathrm{d}\mu}{\mathrm{d}t} \propto \vec{E} \cdot \vec{B}}$$



2. Chiral MHD equations

Chiral MHD equations

- Evolution of the chemical potential:

$$\frac{\partial \mu}{\partial t} = D_5 \, \Delta \mu + \frac{\pi}{\alpha_{\rm em}} \lambda \; \vec{E} \cdot \vec{B}$$

$$\frac{\partial \Theta}{\partial t} + \vec{u} \cdot \nabla \Theta = \frac{\alpha_{em}}{\pi} \mu$$

“Dynamos in Chiral MHD”

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- Maxwell equations (including the coupling to μ):

$$\nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \frac{4\pi}{c} \vec{j} + \frac{\vec{B}}{c} \frac{\partial \Theta}{\partial t} + (\nabla \Theta) \times \vec{E}$$

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- Ohm's law:

$$\vec{j} = \sigma\left(\vec{E} + \frac{1}{c}\vec{U}\times\vec{B}\right)$$

Chiral MHD equations

- Full system of equations:

$$\frac{\partial\mu}{\partial t} = D_5\,\Delta\mu + \lambda\,\eta\left[\vec{B}\cdot(\nabla\times\vec{B}) - \mu\vec{B}^2 + (\vec{U}\cdot\vec{B})(\vec{B}\cdot\nabla\Theta)\right]$$

$$\frac{D\Theta}{Dt} = \mu$$

$$\frac{\partial\vec{B}}{\partial t} = \nabla\times\left\{\vec{u}\times\vec{B} + \eta\left[\mu\vec{B} - \nabla\times\vec{B} - \vec{U}\left(\vec{B}\cdot\nabla\right)\Theta\right]\right\}$$

$$\rho\frac{D\vec{U}}{Dt} = (\nabla\times\vec{B})\times\vec{B} + (\vec{U}\times\vec{B})(\vec{B}\cdot\nabla)\Theta - c_s^2\nabla\rho$$
$$+\nabla\cdot(2\nu\rho S) + \rho\vec{f}$$

$$\frac{D\rho}{Dt} = -\rho\,\nabla\cdot\vec{U}$$

Conservation law

- From chiral MHD equations:

$$\frac{\partial \vec{A}\cdot\vec{B}}{\partial t} + \nabla\cdot \left(\vec{E}\times\vec{A} + \vec{B}\Phi\right) = -2\vec{E}\cdot\vec{B},$$

$$\frac{\partial (2\mu/\lambda)}{\partial t} + \nabla\cdot\big[-\,(2D_5/\lambda)\,\nabla\mu\big] = 2\vec{E}\cdot\vec{B}$$

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$$\frac{\partial(2\mu/\lambda)}{\partial t} + \nabla\cdot\left[-\left(2D_5/\lambda\right)\nabla\mu\right] = 2\vec{E}\cdot\vec{B}$$

- Add upper equations:

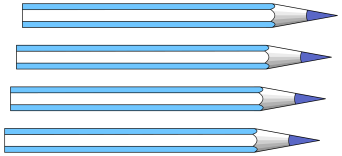
$$\frac{\partial}{\partial t}\Big(\underbrace{\vec{A}\cdot\vec{B} + 2\mu/\lambda}_{\text{conserved}}\Big) + \nabla\cdot\left[\vec{E}\times\vec{A} + \vec{B}\,\Phi - \left(2D_5/\lambda\right)\nabla\mu\right] = 0$$

“Dynamos in Chiral MHD”

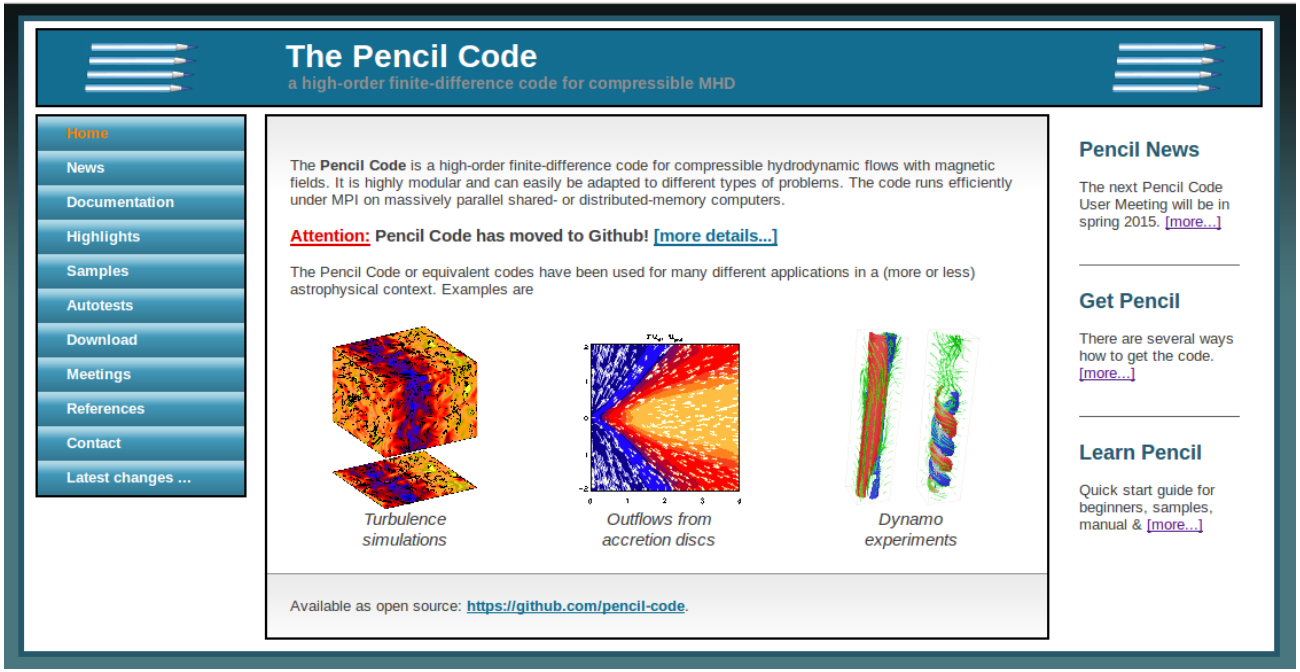
1. Introduction
2. **Chiral MHD equations**
3. α^2 dynamo
4. α - shear dynamo
5. Conclusion & outlook

J. Schober
(Nordita fellow)

Pencil Code



- Open source high-order finite-difference code
- Purpose: solution of (compressive) MHD equations
- Various applications: from fluid dynamics and turbulence to astrophysics
- Webpage: *http://pencil-code.nordita.org/*



The Pencil Code

a high-order finite-difference code for compressible MHD

The **Pencil Code** is a high-order finite-difference code for compressible hydrodynamic flows with magnetic fields. It is highly modular and can easily be adapted to different types of problems. The code runs efficiently under MPI on massively parallel shared- or distributed-memory computers.

Attention: Pencil Code has moved to Github! [more details...]

The Pencil Code or equivalent codes have been used for many different applications in a (more or less) astrophysical context. Examples are

Turbulence simulations

Outflows from accretion discs

Dynamo experiments

Available as open source: https://github.com/pencil-code.

Pencil News

The next Pencil Code User Meeting will be in spring 2015. [more...]

Get Pencil

There are several ways how to get the code. [more...]

Learn Pencil

Quick start guide for beginners, samples, manual & [more...]

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2. α^2 dynamo

Theory

- Linearised induction equation (with $\vec{U}_{\rm eq} = (0,0,0)$):

$$\frac{\partial \vec{B}}{\partial t} = \nabla\times\left\{\vec{U}\times\vec{B} + \eta\left[\mu\vec{B} - \nabla\times\vec{B}\right]\right\}$$

α^2 - dynamo

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α^2 - dynamo

- Ansatz:

$$\vec{B}(t,z) = B_y(t,z)\vec{e}_y + \nabla\times[A(t,z)\vec{e}_y]$$

$$\mathrm{A},\ \mathrm{B}_y \propto \exp[\gamma_2 t + i k_z z]$$

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- Growth rate:

$$\boxed{\gamma_2^{\rm max} = \frac{\eta\mu_{\rm eq}^2}{4}}$$

Simulations

- 2d runs with a resolution of resolution 256^2
- Run parameters:

  $\nu = 10^{-3}$

  $\eta = 10^{-3}$
- Initial conditions:

  $\vec{U} = (0, 0, 0)$

  $\vec{B} = 10^{-4}(0, \sin(x), \cos(x))$

  $\mu = 2$

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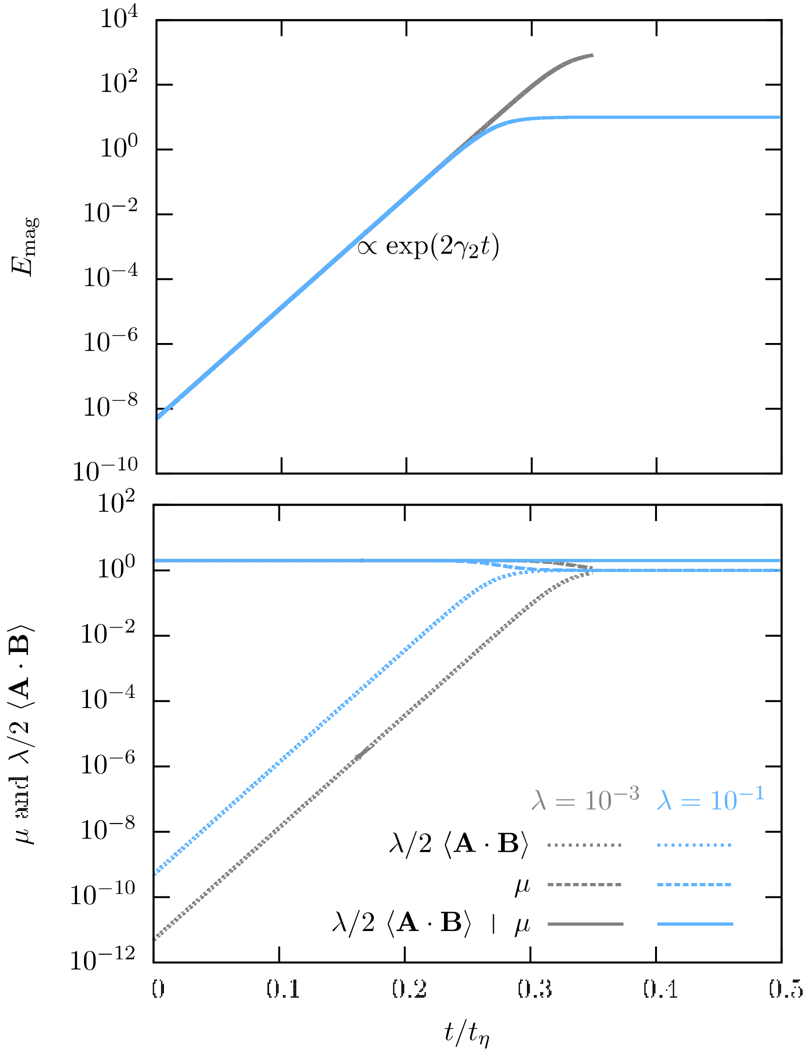

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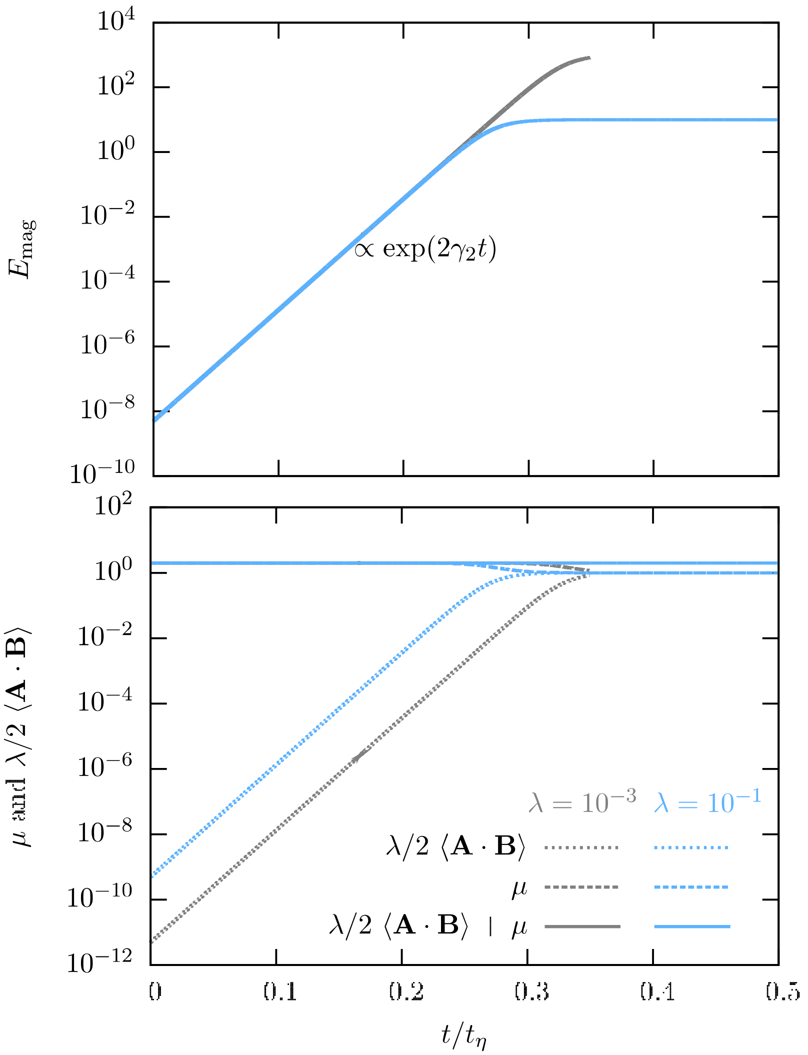
- Resulting growth rate:

  - fit: $\gamma_2 \approx 10^{-3}$

  - theory: $\gamma_2^{\max} = \dfrac{\eta\mu_{\rm eq}^2}{4} = 10^{-3}$



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2. α - shear dynamo

Theory

- Linearised induction equation (with $\vec{U}_{\rm eq} = (0, U_S \cdot x, 0)$):

$$\frac{\partial \vec{B}}{\partial t} = \nabla\times\left\{\vec{U}\times\vec{B} + \eta\left[\mu\vec{B} - \nabla\times\vec{B}\right]\right\}$$

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- Growth rate and dynamo frequency:

$$\gamma_S^{\rm max} = \frac{3}{8}\left(\frac{U_S^2\,\mu_{\rm eq}^2\,\eta}{2}\right)^{1/3}$$

$$\omega_S^{\rm max} = \frac{1}{2}\left(\frac{U_S^2\,\mu_{\rm eq}^2\,\eta}{2}\right)^{1/3}$$

Simulations

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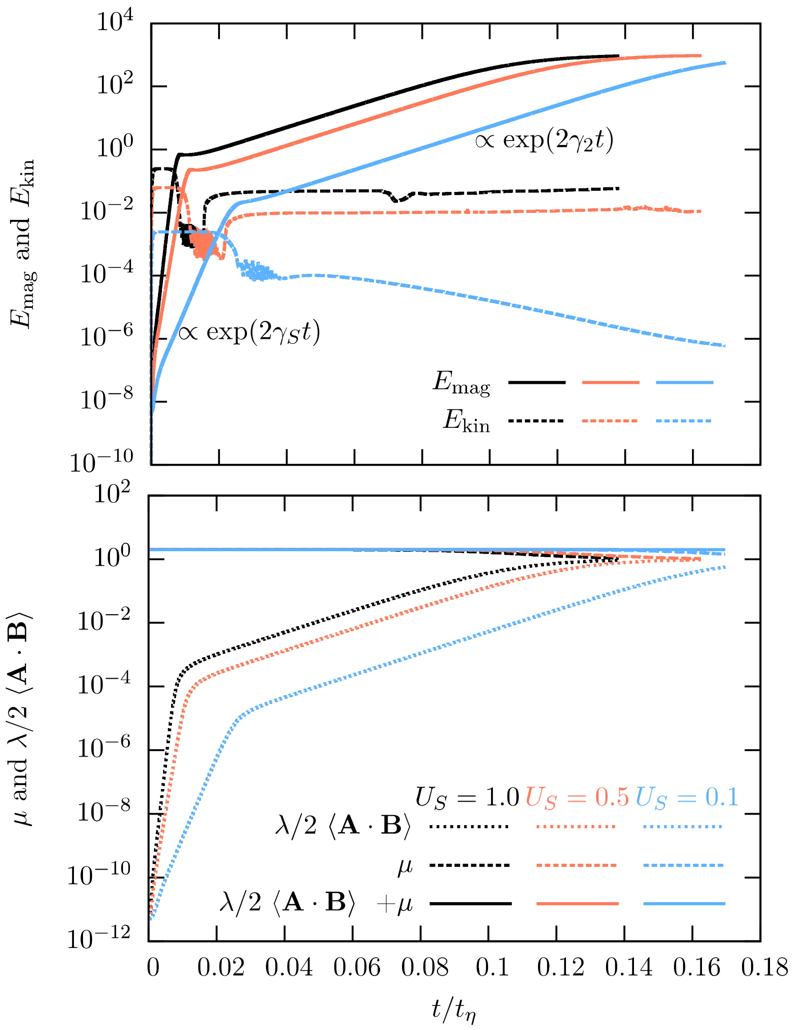

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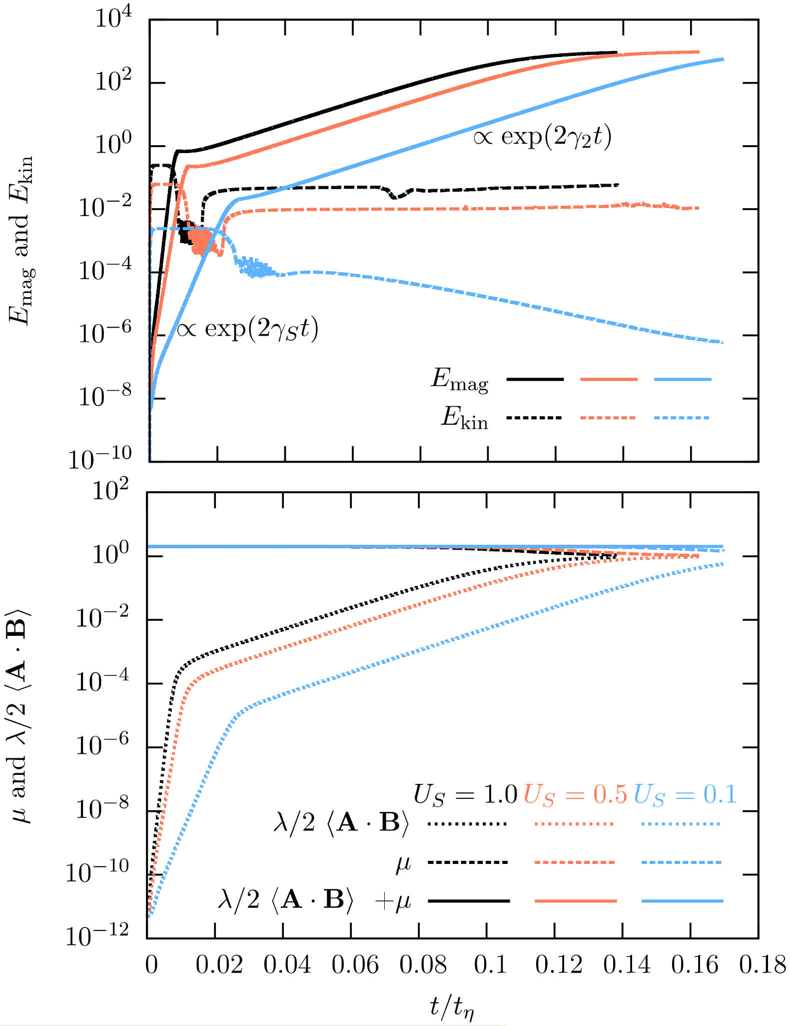


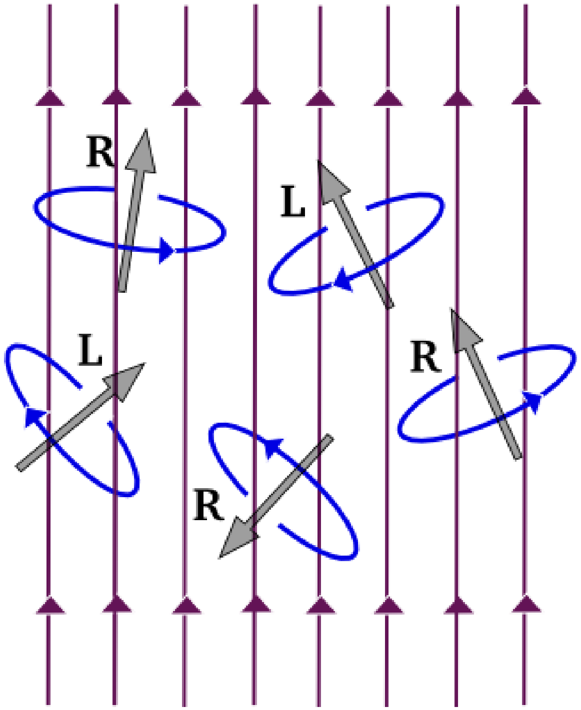
- Results:

U_S	γ_S (fit)	$\gamma_S^{\max}$ (theory)	ω_S(fit)	$\omega_S^{\max}$ (theory)
1.0	2.73×10^{-2}	4.72×10^{-2}	7.06×10^{-2}	6.30×10^{-2}
0.5	1.84×10^{-2}	2.98×10^{-2}	2.67×10^{-2}	3.97×10^{-2}
0.1	7.13×10^{-3}	1.02×10^{-2}	7.84×10^{-3}	1.36×10^{-2}

3. Conclusion & Outlook

Conclusion & Outlook

- Implementation of chiral MHD equations in the *Pencil Code*.

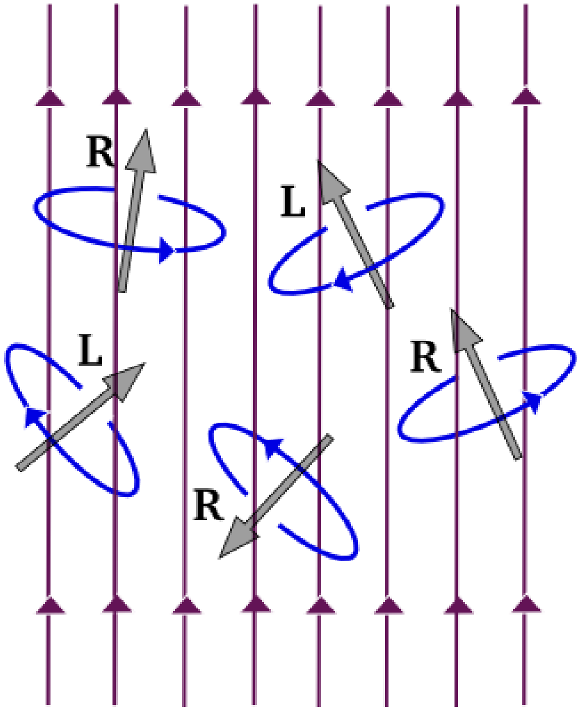


Conclusion & Outlook

- Implementation of chiral MHD equations in the *Pencil Code.*

- Numerical simulations of laminar dynamos:
  - α^2 dynamo
  - α - shear dynamo

  → Confirmation of analytical predictions for growth rates and dynamo waves.



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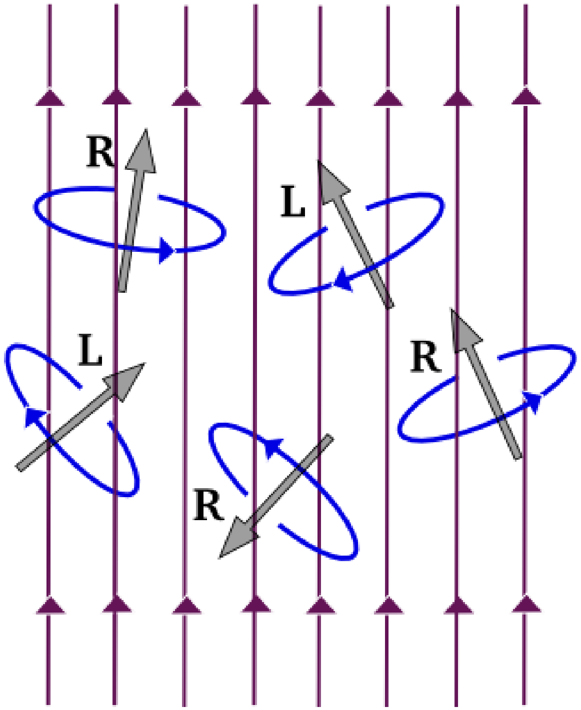

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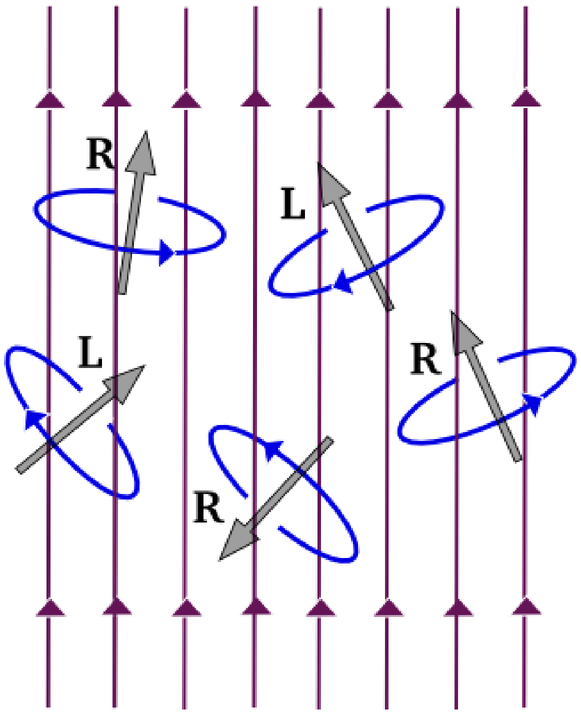

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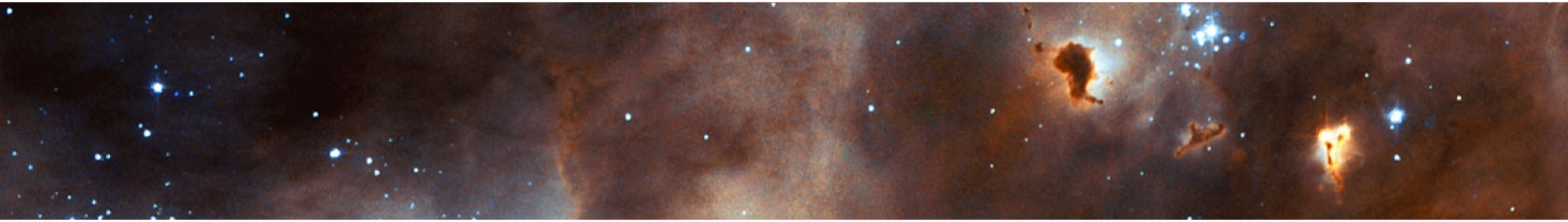
- Open question: How is the magnetic field amplification affected by turbulence?
  → run simulations with forcing



Thanks for your attention!

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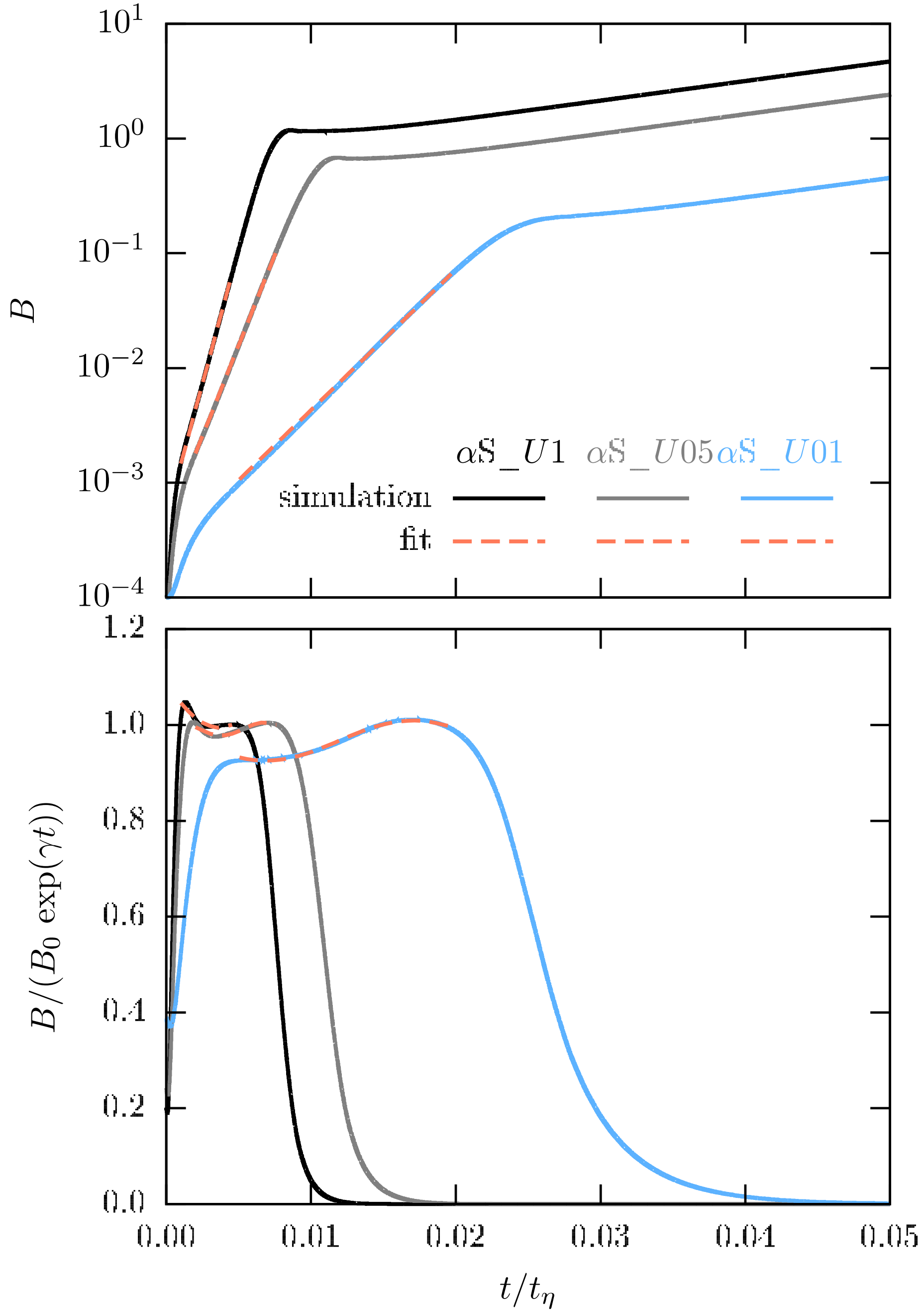
Backup slides

“Dynamos in Chiral MHD”

Dynamo waves



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Initial chemical potential