CAFE a new relativistic MHD code

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General Relativistic MHD equations

The relativistic Magnetohydrodynamics equations are derived from the mass conservation, stress-energy tensor local conservation and the Maxwell equation.

$$\nabla_\mu(\rho u^\mu) = 0$$
$$\nabla_\mu(T^{\mu\nu}) = 0$$
$$\nabla_\mu(u^\mu b^\nu - u^\nu b^\mu) = 0$$

$$T^{\mu\nu} = (\rho h + b^2)u^\mu u^\nu + (p + b^2/2)g^{\mu\nu} - b^\mu b^\mu$$

Perfect fluid and electromagnetic Stress-Energy tensor

$$b^0 = \frac{W B^i v_i}{\alpha}$$
$$b^i = \frac{B^i + \alpha b^0(v^i - \frac{\beta^i}{\alpha})}{W}$$

Magnetic field component comoving to the fluid

W is the Lorentz factor

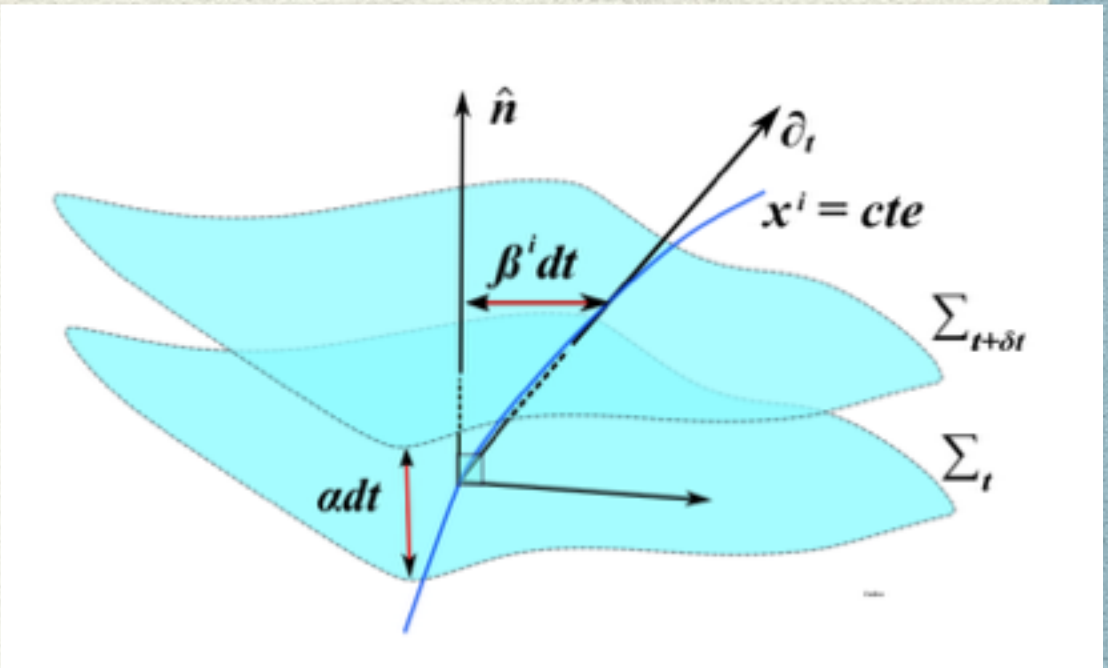
$$h = 1 + e + p/\rho$$ Specific enthalpy

$$p = \rho e(\Gamma - 1)$$ Pressure

$\alpha,\ \beta,\ g^{\mu\nu}$ Are the space-time parameters

Space-time and the physical quantities

$$ds^2 = -(\alpha^2 - \beta_i\beta^i)dt^2 + 2\beta_i dx^i dt + \gamma_{ij}dx^i dx^j$$



α -> Lapse function
β_i -> Shift vector
γ_{ij} -> Three metric

We can construct a base of the space-time

$$e_{(\mu)} = \{\mathbf{n}, \partial_\mathbf{i}\}, \qquad \mathbf{n}_\mu = (-\alpha, \mathbf{0}, \mathbf{0}, \mathbf{0})$$

We project the density current and energy-momentum tensor on this base, obtaining the physical quantities on the spacial hypersurfaces and along the proper time direction.

$$\begin{aligned} D &\equiv -\mathbf{J}\cdot\mathbf{n} = \rho W, \\ S_j &\equiv \mathbf{T}(\mathbf{n}, e_j) = \rho h^* W^2 v_j - \alpha b^0 b_j, \\ E &\equiv \mathbf{T}(\mathbf{n}, \mathbf{n}) = \rho h^* W^2 - p^* - (\alpha b^0)^2, \\ \tau &\equiv E - D. \end{aligned}$$

$$J = \rho u^\mu, \quad h^* = h + b^2/\rho, \quad p^* = p + b^2/2.$$

We write the ideal MHD in Valencia formulation, where we have a conservative form of the equations and additionally the constraint on the divergence of the magnetic field (no monopoles).

$$\frac{1}{\sqrt{-g}}\left[\partial_t(\sqrt{\gamma}\mathbf{U}) + \partial_i(\sqrt{\gamma}\mathbf{F}^i)\right] = \mathbf{S},$$
$$\nabla \cdot \mathbf{B} = 0,$$

Where the components of the conservative variables **U,** numerical fluxes **F** and sources **S** are:

$$\mathbf{U} = \begin{bmatrix} D \\ S_j \\ \tau \\ B^k \end{bmatrix} = \begin{bmatrix} \rho W \\ \rho h^* W^2 v_j - \alpha b^0 b_j \\ \rho h^* W^2 - p^* - (\alpha b^0)^2 \\ B^k \end{bmatrix},$$

$$\mathbf{F}^i = \begin{bmatrix} D(v^i - \frac{\beta^i}{\alpha}) \\ S_j\left(v^i - \frac{\beta^i}{\alpha}\right) + p^*\delta^i_j - b^i b_j \\ \tau\left(v^i - \frac{\beta^i}{\alpha}\right) + p^* v^i - b^0 b_j \\ \left(v^i - \frac{\beta^i}{\alpha}\right)B^k - \left(v^k - \frac{\beta^k}{\alpha}\right)B^i \end{bmatrix},$$

$$\mathbf{S} = \begin{bmatrix} 0 \\ T^{\mu\nu} g_{\nu\sigma}\Gamma^\sigma_{\mu j} \\ T^{\mu 0}\partial_\mu \alpha - T^{\mu\nu}\Gamma^0_{\mu\nu}\alpha \\ 0^k \end{bmatrix}.$$

The fluxes depends on the conservative variables **U** and primitive variables $\mathbf{w} = (\rho, p, v^i, B^j)$.

The characteristic decomposition of the GRHMD, can be obtain from the Anile (1989) variables $\tilde{U} = (u^\alpha, b^\alpha, p, s)^T$, we the GRMHD take the form

$$(A^\alpha)^\lambda_\sigma \nabla_\alpha \tilde{U}^\sigma = 0$$

$$\lambda = \alpha(v^x - \beta^x/\alpha),$$
$$\lambda_\pm = \frac{b^x \pm \sqrt{E}u^x}{b^0 \pm \sqrt{E}u^0},$$
$$\lambda_{ms\pm} = \frac{u^0 u^x(1-\Omega^2) \pm \sqrt{\Omega^2(\Omega^2 + (1-\Omega^2)((u^0)^2 - (u^x)^2))}}{\Omega^2 + (1-\Omega^2)(u^0)^2}.$$

where $\Omega^2 = C_s^2 + C_A^2 - C_s^2 C_A^2$ and $C_A^2 = b^2/E$

Numerical methods

The numerical methods used in CAFE are:

- The Methods of Lines.
- Runge-Kutta of the third order.
- HRSC: HLLE flux formula; minmod, mc, weno5 cell reconstructors.
- Flux Constrained Transport and Divergence Cleaning methods.
- We use a root finder to recovery the primitive variables.

CAFE characteristic:

- Unigrid in 3D cartesian coordinates
- Fix uniform grid.
- Parallelised with OpenMPI
- Constructed in fortran 90
- Special relativity

We also have a 2D version in hydro ; one in the equatorial symmetry and other in the axial symmetry

Hydro tests

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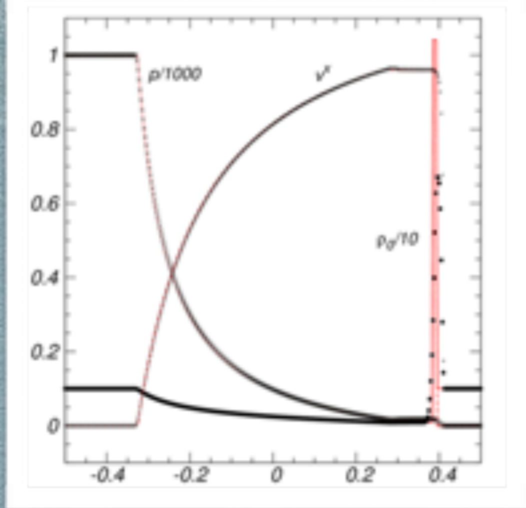
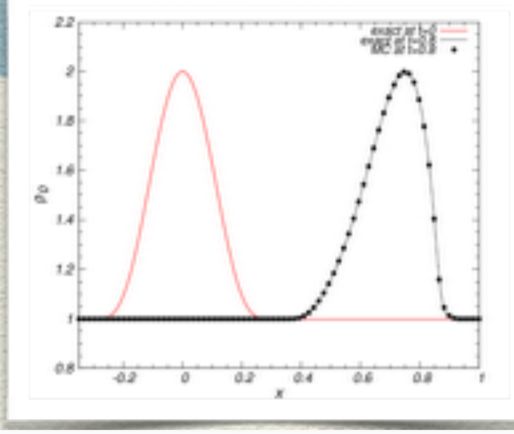


Strong relativistic Blast Wave compared with
Del Zanna & Bucciantini 2002;
Martí & Müller 2003

Smooth Initial Profile

Zhang & MacFadyen (2006) and

Radice & Rezzolla (2012)

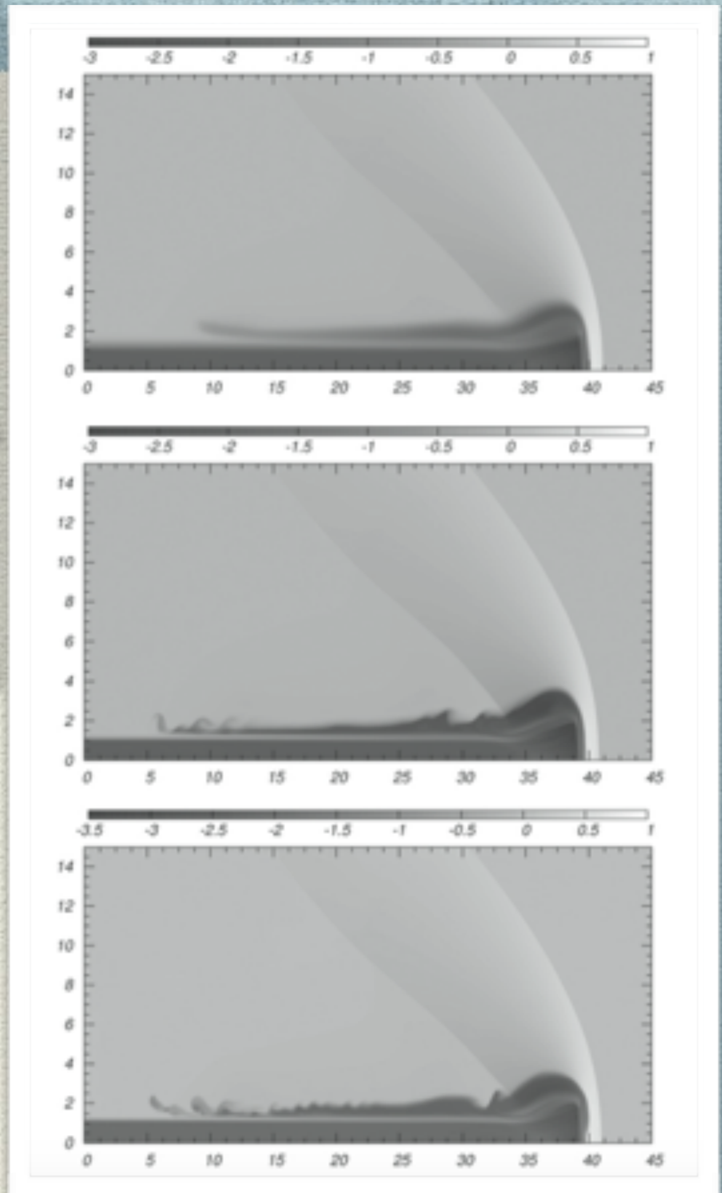


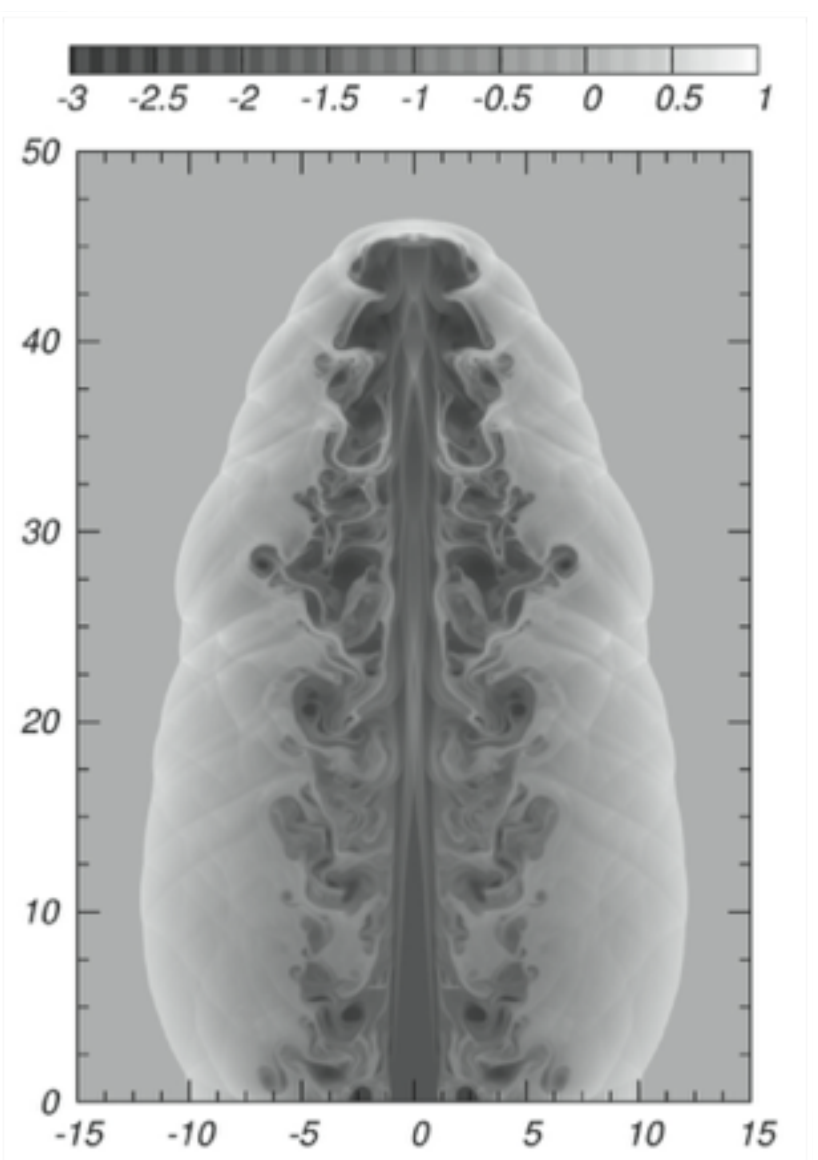
Cells	MM	MC	WENO5	PPM
Smooth Profile Test				
80	...	...	...	...
160	2.11	2.57	2.27	1.90
320	2.09	2.37	2.30	1.86
640	2.04	2.18	2.48	1.91
1280	2.00	2.10	2.60	1.95

Logarithm of the rest mass density for the hot jet model A1 in Martí et al. (1997) at time t = 48.82.

Using MINMOD, MC, and WENO5.

$v_b = 0.99$, Mach 1.72.

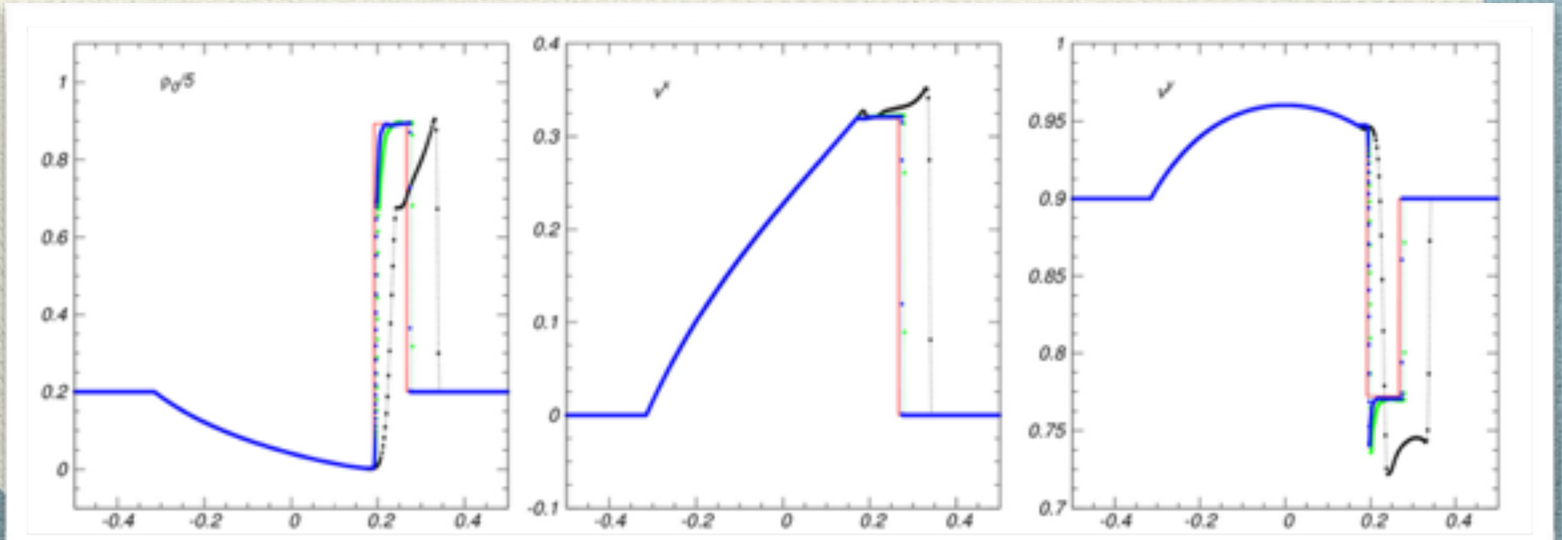




Model C2 in Martí et al. (1997)

$v_b = 0.99$, Mach 6.

Nonzero Transverse Velocity: Hard Test, using 400, 3200 and 6400 cells in agreement with Zhang & MacFadyen (2006)



RMHD tests

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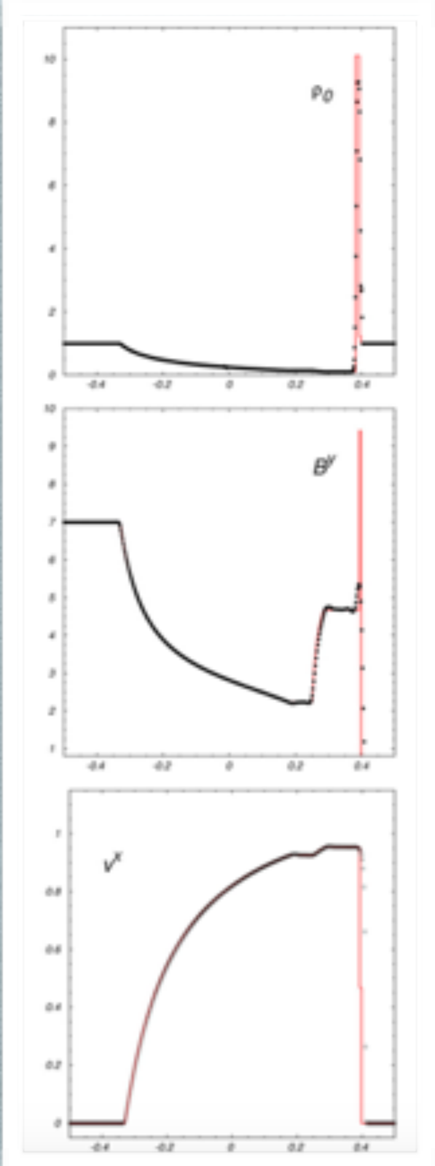
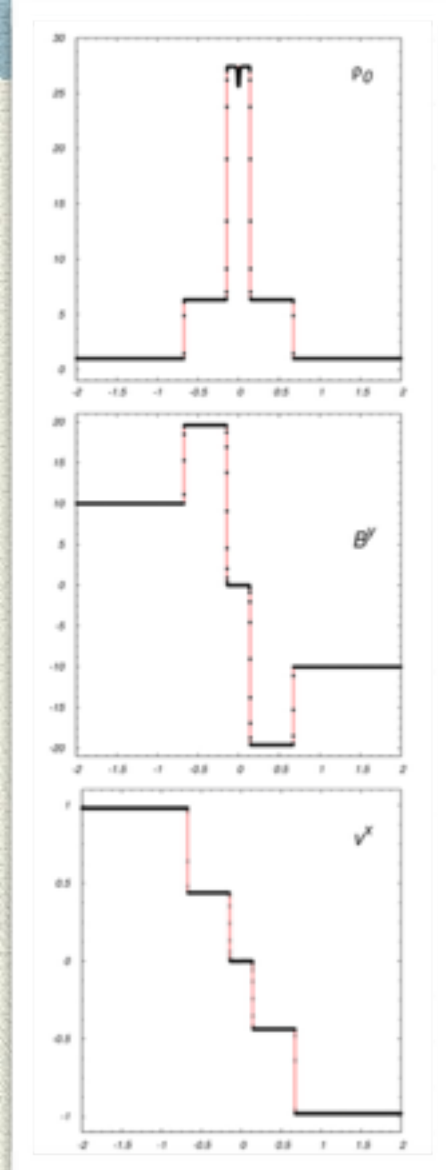
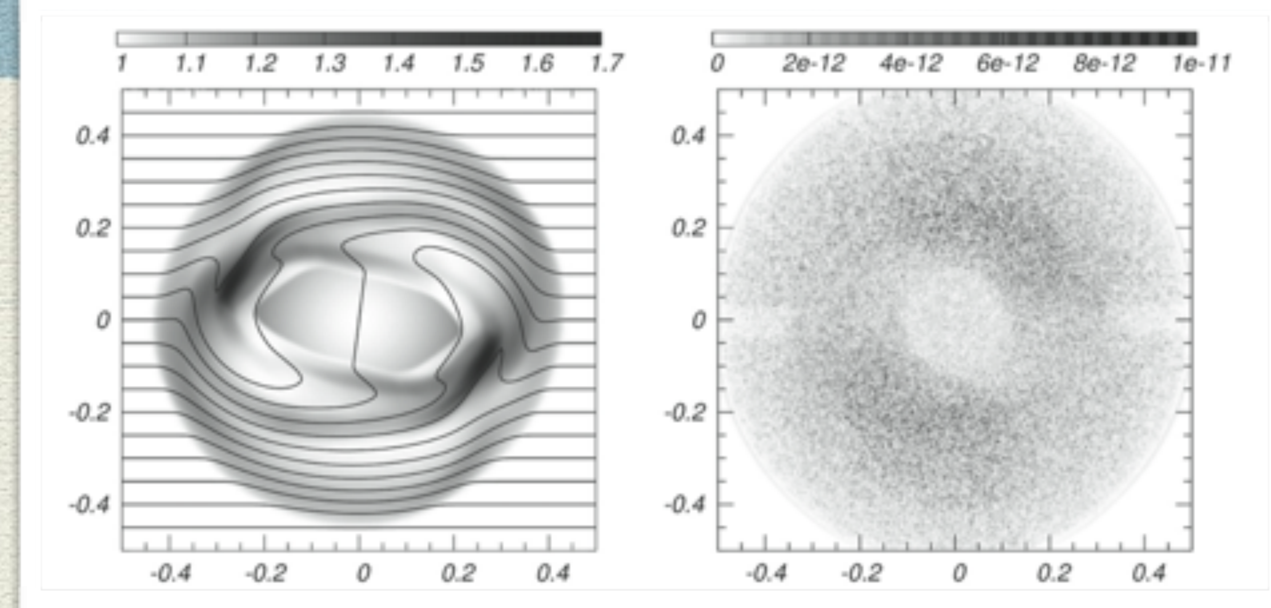


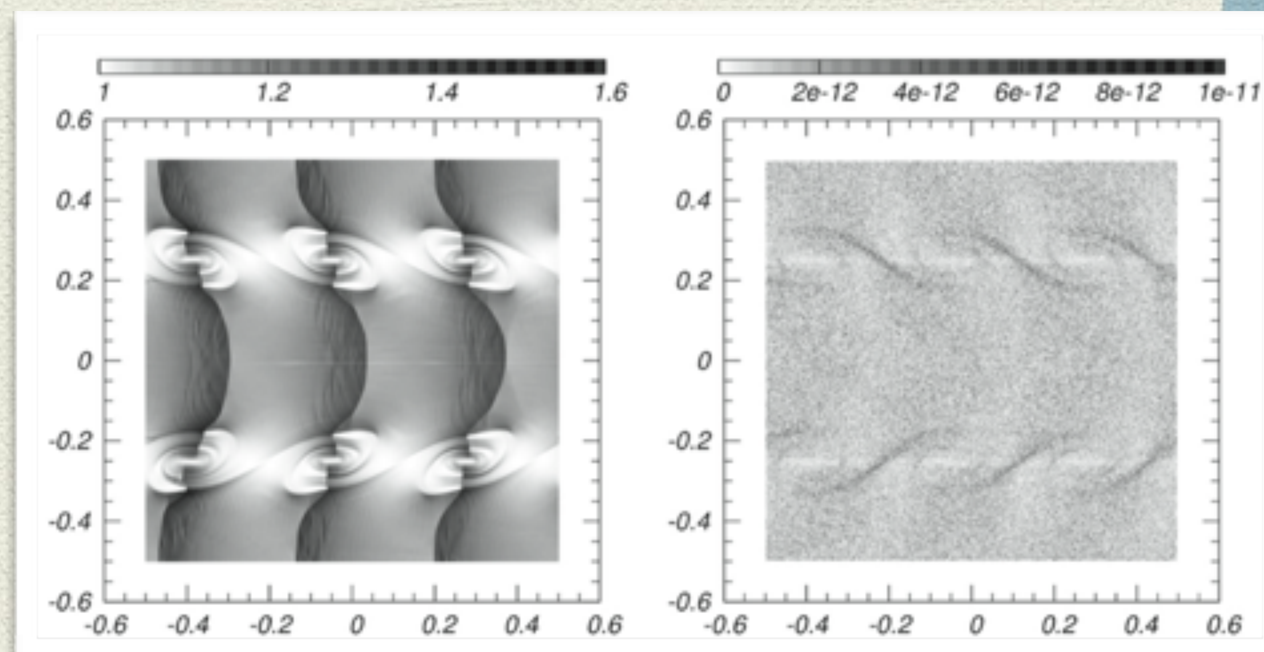
Balsara 3 blast wave



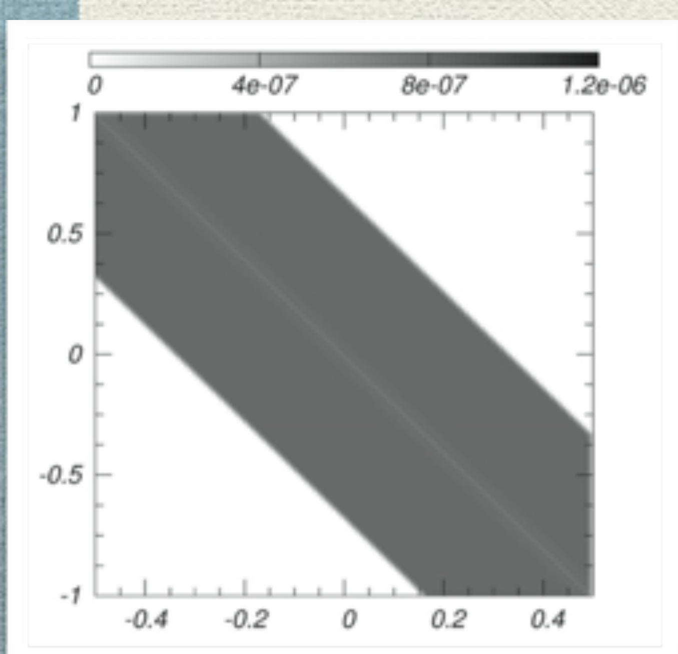
Komissarov collision test

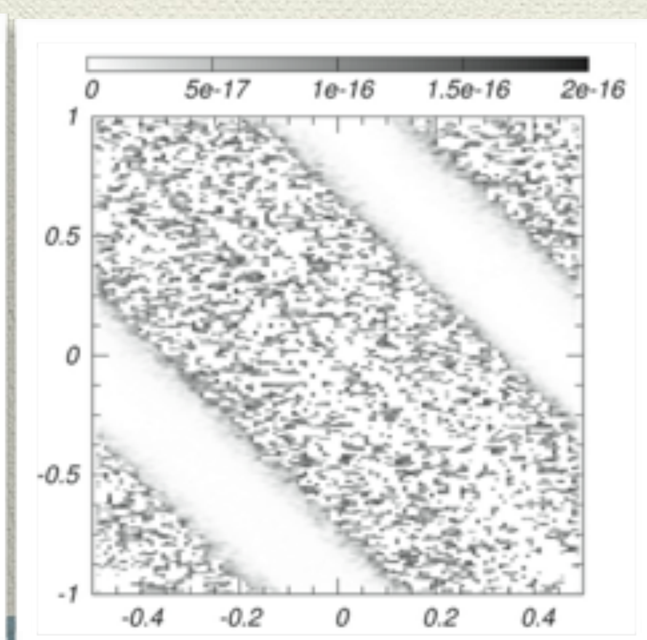


2D magnetic rotor



Magnetized Kelvin–Helmholtz





Oblique magnetic loop field advection test (3D)

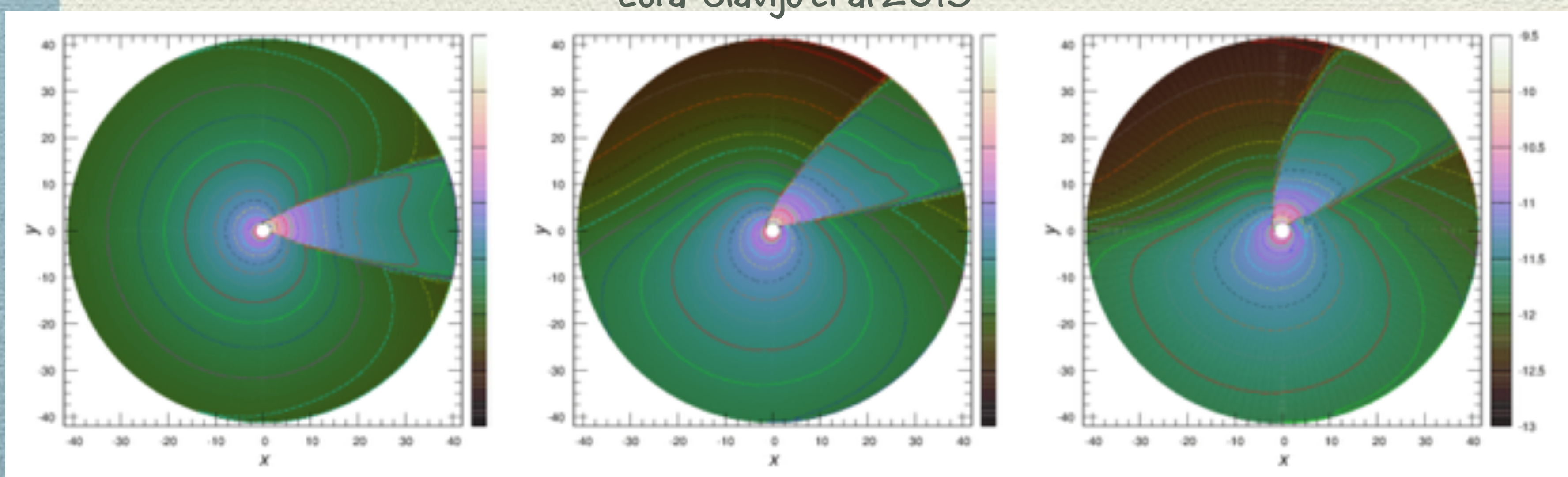
L_1 Norm of the Error in Density for Each Reconstructor and the Flux-CT Method to Control the Magnetic Field Divergence-free Constraint

Resolution	MM	MC	WENO5	PPM	MM	MC	WENO5	PPM
	Flux-CT Error				Order of Convergence			
				Test 1				
Δx_1	1.18e-1	1.10e-1	1.14e-1	1.18e-1	···	···	···	···
Δx_1	8.09e-2	6.97e-2	7.14e-2	1.02e-1	0.54	0.65	0.67	0.21
Δx_2	5.20e-2	3.96e-2	3.68e-2	8.21e-2	0.63	0.81	0.95	0.31
Δx_3	3.05e-2	2.15e-2	1.95e-2	6.39e-2	0.77	0.88	0.91	0.36
Δx_4	1.65e-2	1.12e-3	9.99e-3	4.95e-2	0.88	0.94	0.96	0.37
				Test 2				
Δx_1	3.39e-1	3.16e-1	3.15e-1	3.45e-1	···	···	···	···
Δx_1	2.25e-1	2.50e-1	2.30e-1	2.35e-1	0.59	0.62	0.65	0.55
Δx_2	1.47e-1	1.57e-1	1.42e-2	1.53e-1	0.63	0.67	0.69	0.62
Δx_3	9.41e-2	9.80e-2	8.60e-2	9.87e-2	0.64	0.68	0.72	0.63
Δx_4	5.91e-2	6.04e-2	4.97e-2	6.24e-2	0.67	0.69	0.79	0.66
				Test 3				
Δx_1	2.10e-2	1.56e-2	1.59e-2	2.09e-2	···	···	···	···
Δx_2	1.33e-2	8.86e-3	9.36e-3	1.33e-2	0.65	0.81	0.76	0.65
Δx_3	8.04e-3	4.76e-3	5.05e-3	8.00e-3	0.72	0.90	0.89	0.73
Δx_4	4.83e-3	2.48e-3	2.68e-3	4.83e-3	0.73	0.89	0.91	0.73
Δx_5	2.89e-3	1.39e-3	1.40e-3	2.89e-3	0.74	0.89	0.93	0.74
				Test 4				
Δx_1	1.65e-1	1.51e-1	1.49e-1	1.60e-1	···	···	···	···
Δx_2	1.25e-1	1.14e-1	1.12e-1	1.22e-1	0.40	0.40	0.41	0.39
Δx_3	7.98e-2	7.11e-2	6.76e-1	7.86e-1	0.64	0.68	0.72	0.63
Δx_4	5.05e-2	4.37e-2	3.98e-2	5.00e-2	0.66	0.70	0.76	0.65
Δx_5	3.15e-2	2.57e-2	2.23e-2	3.13e-2	0.68	0.76	0.83	0.67
				Test 5				
Δx_1	2.26e-1	2.24e-1	2.24e-1	2.94e-1	···	···	···	···
Δx_2	1.84e-1	1.58e-1	1.48e-1	2.28e-1	0.47	0.50	0.59	0.37
Δx_3	1.28e-1	1.04e-1	9.39e-2	1.63e-1	0.52	0.60	0.65	0.44
Δx_4	8.70e-2	6.62e-2	5.74e-2	1.12e-1	0.55	0.65	0.71	0.54
Δx_5	5.41e-2	3.96e-2	3.36e-2	0.68e-2	0.68	0.74	0.77	0.71
				Test 6				
Δx_1	2.31e0	2.15e0	2.24e0	2.31e0	···	···	···	···
Δx_2	1.56e0	1.41e0	1.46e0	1.58e0	0.56	0.60	0.61	0.54
Δx_3	1.05e0	8.89e-1	9.08e-1	1.07e0	0.57	0.66	0.68	0.56
Δx_4	6.68e-1	5.33e-1	5.30e-1	6.89e-1	0.65	0.73	0.77	0.63
Δx_5	4.13e-1	3.11e-1	2.99e-1	5.50e-1	0.71	0.77	0.82	0.69
				Test 7				
Δx_1	1.66e-1	1.64e-1	1.67e-1	1.75e-1	···	···	···	···
Δx_1	1.14e-1	1.12e-1	1.13e-1	1.22e-1	0.54	0.55	0.56	0.52
Δx_2	7.32e-2	6.96e-2	7.01e-2	7.94e-2	0.63	0.68	0.69	0.62
Δx_3	4.47e-2	4.16e-2	4.01e-2	4.88e-2	0.71	0.74	0.80	0.70
Δx_4	2.58e-2	2.36e-2	2.24e-2	2.89e-3	0.79	0.81	0.84	0.75
				Test 8				
Δx_1	1.85e-1	1.94e-2	1.96e-1	1.80e-1	···	···	···	···
Δx_1	1.20e-1	1.24e-1	1.23e-1	1.16e-1	0.62	0.64	0.67	0.63
Δx_2	7.21e-2	7.41e-2	7.22e-2	7.11e-2	0.73	0.74	0.76	0.70

Current and Future

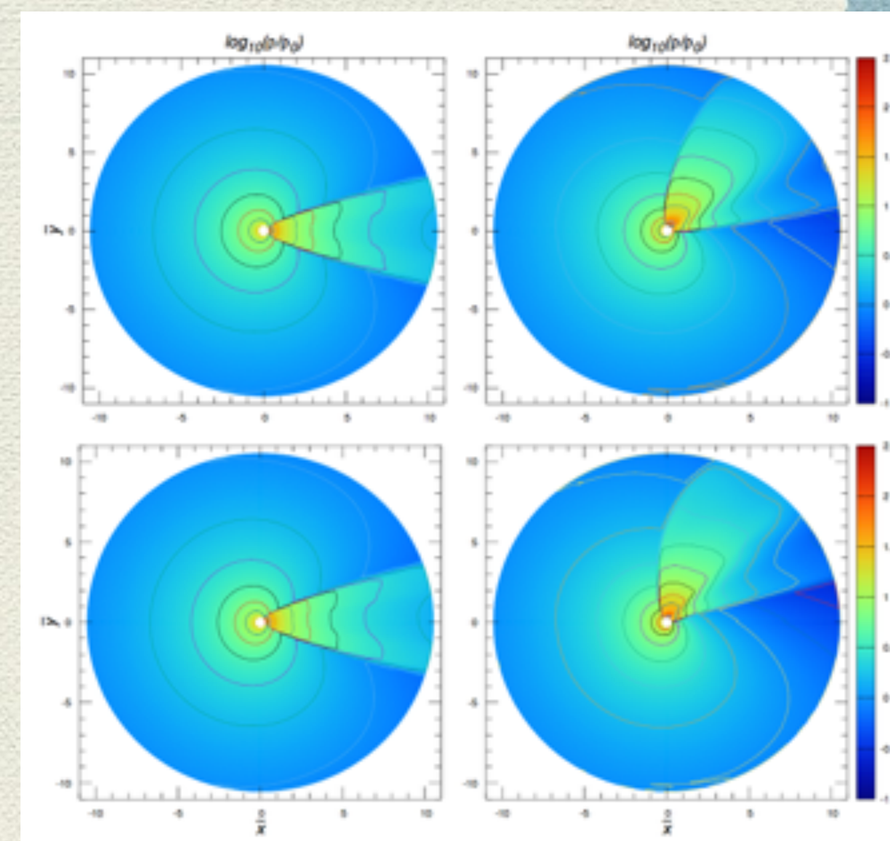
* We adding the curved space-time; Kerr metric in cartesian coordinates, and general metric.
* We studying the Bondi-Hoyle accretion with a 2D version of CAFE

Lora-Clavijo et al 2015



Bondi-hoyle accretion with density gradients

Cruz-Osorio 2016 (in preparation)



Bondi-hoyle accretion with velocity gradients

* We will have the second version of CAFE that includes a generalised space-time
* We will study the magnetised Bondi-Hoyle accretion in 3D

Thanks!

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