First hundred years of GR: successes, status and prospects

T.Padmanabhan

IUCAA, Pune, India

"It ... represents probably most beautiful of all existing physical theories."

"It ... represents probably most beautiful of all existing physical theories."

Landau and Liftshitz
Volume II of the Course of Theoretical Physics

The Genius of Einstein

Principle of Equivalence

+

Special Relativity

↓

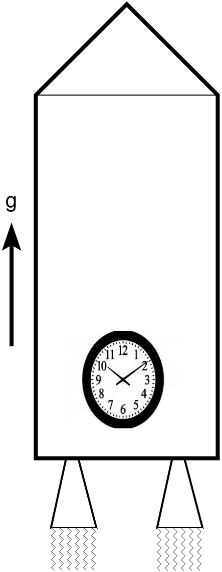
Gravity = Curvature of Spacetime

The Genius of Einstein

Time shown by a reference clock

$$c^2 dt^2 = c^2 dt_{\mathsf{R}}^2 - d\boldsymbol{x}^2$$

Time shown by a clock which moves a distance $d\boldsymbol{x}$ in time dt_{R}



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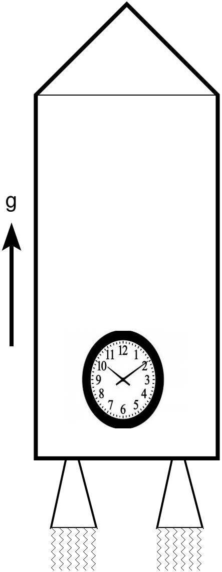
The Genius of Einstein



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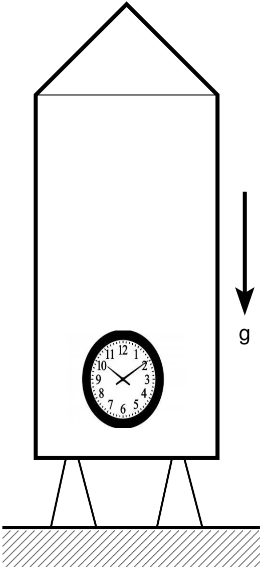

$$V^2 = v^2 + 2gx = v^2 - 2\phi$$

$$c^2dt^2 = c^2\,dt_R^2 + 2\phi\,dt_R^2 - v^2dt_R^2$$

$$\boldsymbol{c^2 dt^2 = c^2 \left(1 + \frac{2\phi}{c^2}\right) dt_R^2 - d\boldsymbol{x}^2}$$

$$c^2 dt^2 = c^2 \left(1 + \frac{2\phi}{c^2}\right) dt_R^2 - d\boldsymbol{x}^2$$

This metric represents a curved spacetime

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$$\phi(t, \boldsymbol{x}) \to g_{ab}(t, \boldsymbol{x})$$

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This metric represents a curved spacetime

$$\phi(t, \boldsymbol{x}) \to g_{ab}(t, \boldsymbol{x})$$

"This fusion of ... metric and gravitation must be considered as the most beautiful achievement of GR." *– W. Pauli*

$$\boldsymbol{\nabla}^2\phi \propto \boldsymbol{\rho} = \boldsymbol{T}_{ab}\, \boldsymbol{u}^a\, \boldsymbol{u}^b = \left(\begin{array}{l}\text{Energy density}\\ \text{measured by}\\ \text{that observer}\end{array}\right)$$

$$\left(\begin{array}{l}\text{Radius of curvature}\\ \text{of space orthogonal}\\ \text{to an observer}\end{array}\right)^{-2} \propto \left(\begin{array}{l}\text{Energy density}\\ \text{measured by}\\ \text{that observer}\end{array}\right)$$

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$$\mathcal{R}_{ijkl} \equiv R_{abcd}\, u^a_i\, u^b_j\, u^c_k\, u^d_l$$

$$R^{-2}_{\text{curv}} \equiv \mathcal{R}^{ij}_{ij} = 2G^a_b u^b u_a$$

$$\rho = T_{ab}\, u^a\, u^b$$

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$$\rho = T_{ab}\, u^a\, u^b$$

$$\boxed{R^{-2}_{\text{curv}} = 16\pi G\rho}$$

Three New Effects

PRECESSION OF MERCURY

BENDING OF LIGHT

GRAVITATIONAL WAVES

Observed precession** $\sim 574''$/**cent

Planetary perturbations** $\sim 532''$/**cent

Difference** $\sim 42''$/**cent

Einstein used this as a bench mark!

First attempt was with wrong field equation and with approximate metric!

Discovered within few months; understood after few decades!

Discovered within few months; understood after few decades!

$$ds^2 = -dt^2 + \frac{dx^2}{\rho^4(x)} + \rho^2 \left(\frac{d\psi^2}{\sin^2\theta} + \sin^2\theta d\phi^2 \right)$$

$$x \equiv \rho^3/3; \quad \psi \equiv -\cos\theta$$

Discovered within few months; understood after few decades!

$$ds^2 = -f_0 dt^2 + f_1 dx^2 + f_2 \left(\frac{d\psi^2}{\sin^2\theta} + \sin^2\theta d\phi^2 \right)$$

$$f_0 f_1 f_2^2 = -1$$

The First Exact Solution

Schwarzschild metric

Discovered within few months; understood after few decades!

$$ds^2 = -f_0 dt^2 + f_1 dx^2 + f_2 \left(\frac{d\psi^2}{\sin^2\theta} + \sin^2\theta d\phi^2 \right)$$

$$f_0 f_1 f_2^2 = -1$$

Looked non-singular at origin, causing considerable confusion!

A Curious Fact

Come back, Newton! All is forgiven!

$$ds^2 = -(1+2\Phi)dt^2 + \frac{dr^2}{(1+2\Phi)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

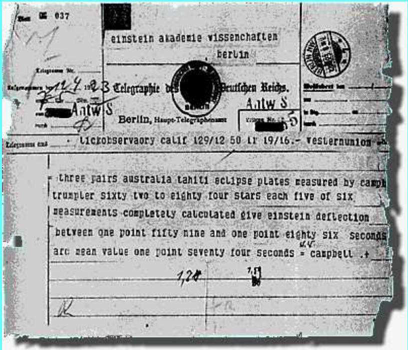
$$T^0_0 = T^r_r = \rho(r); \quad T^\theta_\theta = T^\phi_\phi = \mu(r)$$

Einstein's equations become linear in Φ!

$$\Phi = -\frac{\alpha}{r} + \frac{G}{r}\int 4\pi r^2 \rho(r) dr; \quad \mu = \rho + \frac{1}{2} r \rho'(r)$$

Bending of Light

A small effect in 1919 ...



einstein akademie wissenchaften
berlin

Telegraphie des Deutschen Reichs.

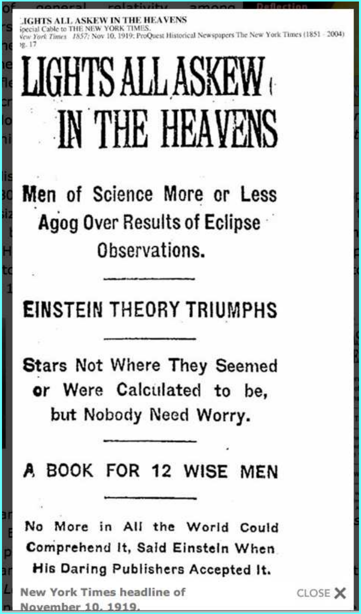
Antw S

Berlin, Haupt-Telegraphenamt

Licrobservaory calif 129/12 50 11 19/16.- westernunion

three pairs australia tahiti eclipse plates measured by campb
trumpler sixty two to eighty four stars each five of six
measurements completely calculated give einstein deflection
between one point fifty nine and one point eighty six seconds
arc mean value one point seventy four seconds = campbell .+

Eddington's telegram to Einstein announcing the observation of the bending of light



LIGHTS ALL ASKEW IN THE HEAVENS
Special Cable to THE NEW YORK TIMES.
New York Times (1857-); Nov 10, 1919; ProQuest Historical Newspapers The New York Times (1851 - 2004)
pg. 17

LIGHTS ALL ASKEW IN THE HEAVENS

Men of Science More or Less Agog Over Results of Eclipse Observations.

EINSTEIN THEORY TRIUMPHS

Stars Not Where They Seemed or Were Calculated to be, but Nobody Need Worry.

A BOOK FOR 12 WISE MEN

No More in All the World Could Comprehend It, Said Einstein When His Daring Publishers Accepted It.

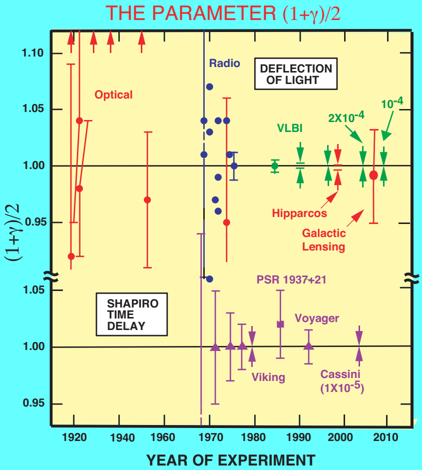
New York Times headline of November 10, 1919.

CLOSE ✕

Bending of Light

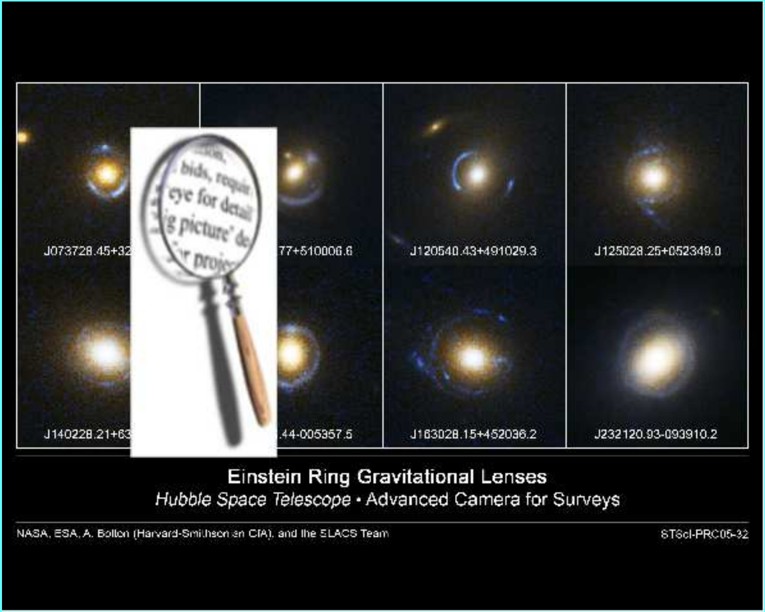
...a large effect today!

$$\theta = 1.7505 \text{ arc-seconds } \frac{1}{2}(1+\gamma)\frac{1+\cos\Phi}{2}$$



C.M. Will, 1409.7871

Gravitational Lensing



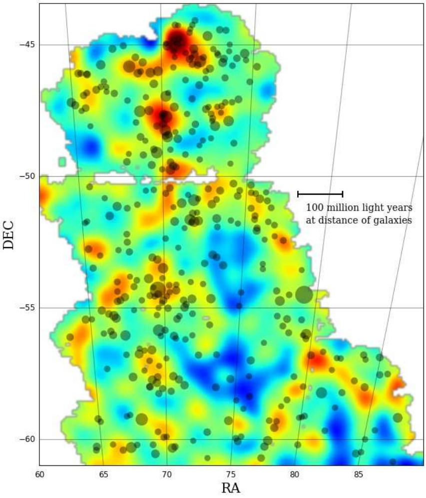
Einstein (1936): Lensing by stars:

"It is of little value."

Zwicky (1937): Lensing by galaxies:

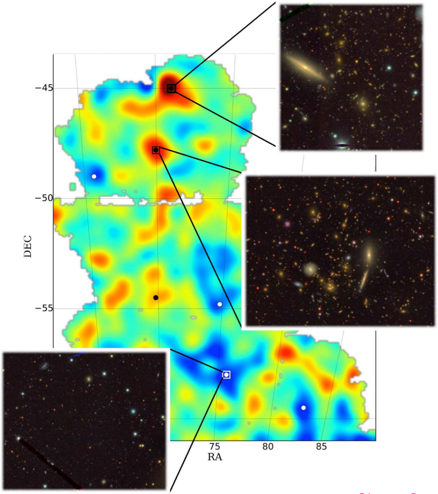
Visionary 1-page paper: testing relativity, magnifying faint objects, measuring masses

Gravitational Lensing



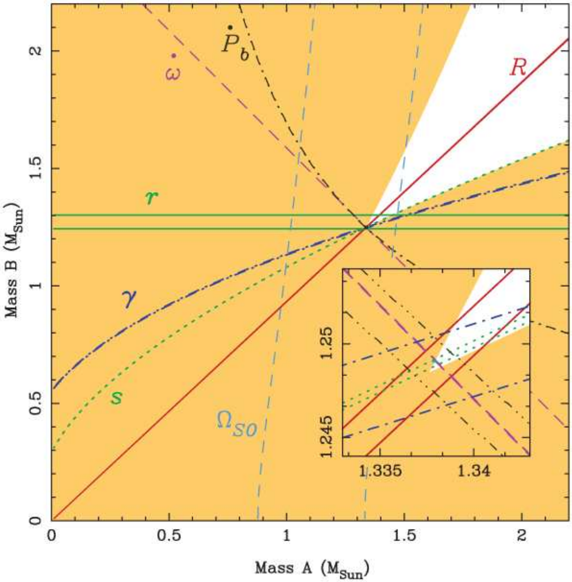
Chang C. et al., 2015; Vikram V. et al., 2015

Gravitational Lensing



Chang C. et al., 2015; Vikram V. et al., 2015

B1913 + 16



Source: M. Kramer

Gravitational Wave Emission

$$m_1 = 1.4398 \pm 0.0002$$

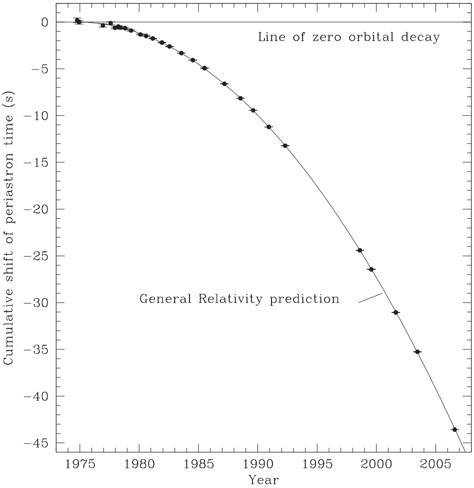
$$m_2 = 1.3886 \pm 0.0002$$

$$\dot{P} = (-2.402531 \pm 0.000014) \times 10^{-12}$$

$$P_{obs}/P_{GR} = 0.997 \pm 0.002$$

Gravitational Wave Emission

B1913 + 16



C.M. Will, 1409.7871

Two Major Areas Of Impact

BLACK HOLES IN ASTROPHYSICS

THE EXPANDING UNIVERSE

"...the relativists...are suddenly experts in a field they hardly knew existed;..."

Thomas Gold
1st Texas Astrophysics

Powers the AGN: $M \approx (10^6 - 10^{10})M_\odot$

X-ray Binaries: $M \approx (5 - 20)M_\odot$

Represents a rotating black hole with an angular momentum parameter $a = J/M$

$$
\begin{aligned}
ds^2 &= -\left(1 - \frac{2\mu r}{\rho^2}\right) dt^2 - \frac{4\mu a r \sin^2\theta}{\rho^2} dt d\phi + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 \\
&\quad + \left(r^2 + a^2 + \frac{2\mu r a^2 \sin^2\theta}{\rho^2}\right) \sin^2\theta d\phi^2
\end{aligned}
$$

$$
\rho^2 \equiv r^2 + a^2 \cos^2\theta \,, \qquad \Delta \equiv r^2 - 2\mu r + a^2
$$

Kerr Metric in the Sky

BH Masses and Spins

Source Name	BH Mass ($M_\odot$)	BH Spin (a_*)
A0620-00	6.3—6.9	0.12 ± 0.19
LMC X-3	5.9—9.2	~0.25
XTE J1550-564	8.5—9.7	0.34±0.24
GRO J1655-40	6.0—6.6	0.70 ± 0.05
4U1543-47	8.4—10.4	0.80 ± 0.05
M33 X-7	14.2—17.1	0.84 ± 0.05
LMC X-1	9.4—12.4	0.92 ± 0.06
Cyg X-1	13.8—15.8	> 0.97
GRS 1915+105	10—18	> 0.98

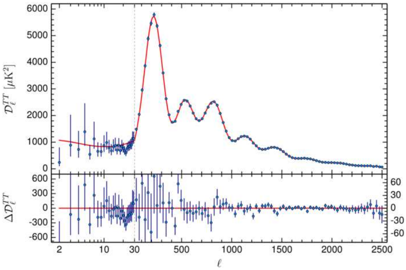
Shafee et al. (2006); McClintock et al. (2006); Davis et al. (2006); Liu et al. (2007,2009); Gou et al. (2009,2010, 2011); Steiner et al. (2010)

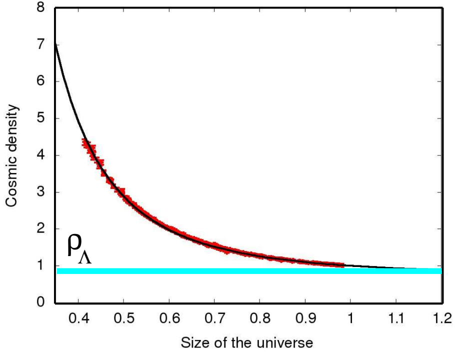
Source: J. E. McClintock, et al., arXiv:1303.1583

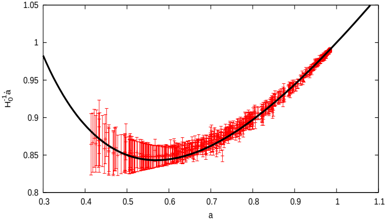
A missed opportunity by Einstein!

Tremendous progress in recent years

The Expanding Universe







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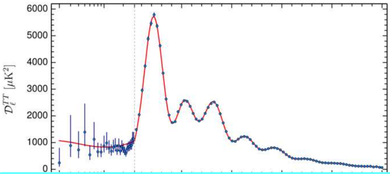
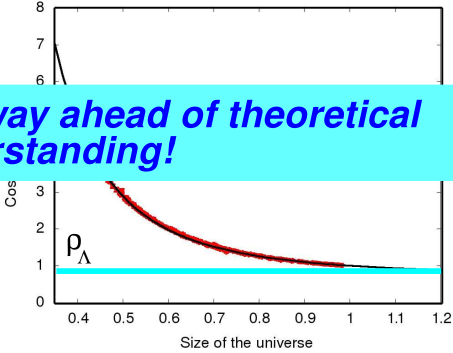
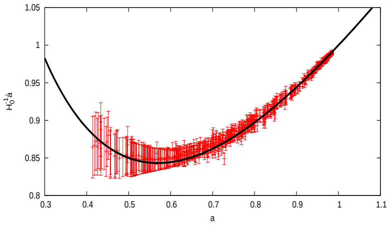




Observations are way ahead of theoretical understanding!



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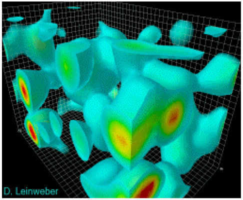
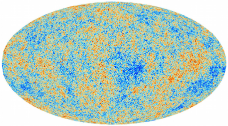
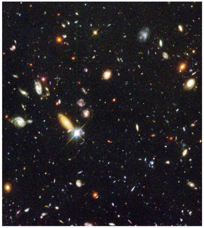
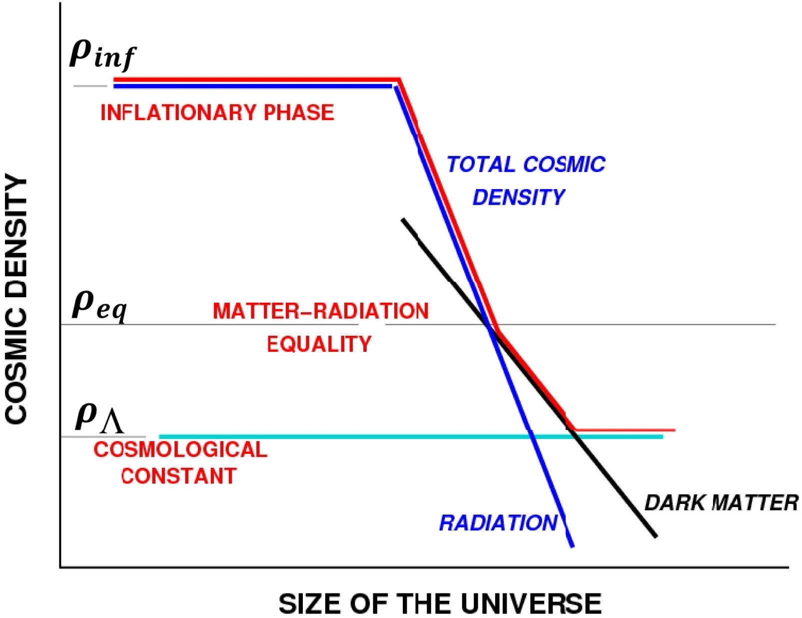


VACUUM FLUCTUATIONS IN THE VERY EARLY UNIVERSE





COSMIC STRUCTURES SEEN TODAY



ρ_{inf}
INFLATIONARY PHASE
TOTAL COSMIC
DENSITY
ρ_{eq}
MATTER–RADIATION
EQUALITY
ρ_{Λ}
COSMOLOGICAL
CONSTANT
DARK MATTER
RADIATION
COSMIC DENSITY
SIZE OF THE UNIVERSE

The Signature Of The Universe

$$\left(\frac{\dot{a}}{a}\right)^2 = \begin{cases} \frac{8\pi G}{3}\rho_{\rm inf} \\ \\ \frac{8\pi G}{3}\left[\rho_\Lambda + \rho_{\rm eq}\left(\left(\frac{a_{\rm eq}}{a}\right)^3 + \left(\frac{a_{\rm eq}}{a}\right)^4\right)\right] \end{cases}$$

The Signature Of The Universe

$$\rho_{\text{inf}} < (1.94 \times 10^{16}\ \text{GeV})^4$$

$$\rho_{\text{eq}} = \frac{\rho_m^4}{\rho_R^3} = [(0.86 \pm 0.09)\ \text{eV}]^4$$

$$\rho_\Lambda = [(2.26 \pm 0.05) \times 10^{-3} \text{eV}]^4$$

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Three Major Challenges

SINGULARITIES: BLACK HOLES, UNIVERSE

COSMOLOGICAL CONSTANT

THE THERMODYNAMIC CONNECTION

"The existence of spacetime singularities represents an end to ... the predictability gained by science. How could physics lead to ... no physics?"

– John Wheeler

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QUANTUM EFFECTS AT $A_{Planck} = (G\hbar/c^3)$ **?**

Gravity breaks a symmetry $H \to H + C$ of the matter sector

Gravity breaks a symmetry $H \to H + C$ of the matter sector

But then:

- *It seems unaffected by changes in the zero-level of the energy*
- *It couples to a small cosmological constant*

$$\Lambda \left(\frac{G\hbar}{c^3} \right) \approx 10^{-123}$$

Spacetimes, Like Matter, can be Hot

***The* most beautiful result in**
the interface of quantum theory and gravity

Spacetimes, Like Matter, can be Hot

***The* most beautiful result in**
the interface of quantum theory and gravity

OBSERVERS WHO PERCEIVE A HORIZON ATTRIBUTE A TEMPERATURE TO SPACETIME

$$k_B T = \frac{\hbar}{c}\left(\frac{g}{2\pi}\right)$$

[Davies (1975), Unruh (1976)]

Spacetimes, Like Matter, can be Hot

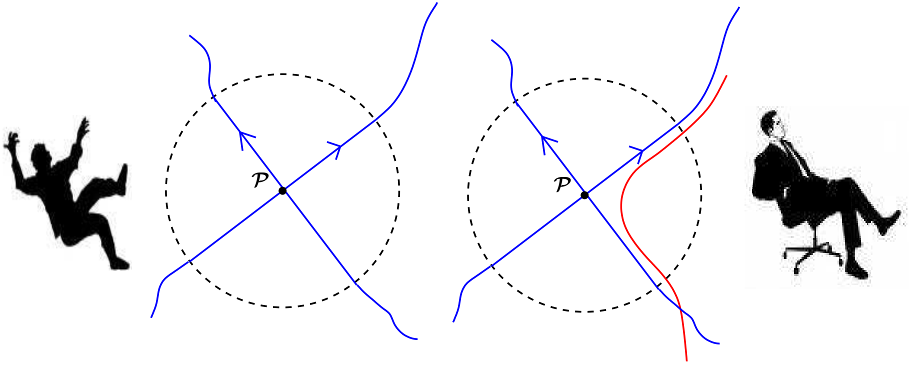
The most beautiful result in the interface of quantum theory and gravity

OBSERVERS WHO PERCEIVE A HORIZON ATTRIBUTE A TEMPERATURE TO SPACETIME

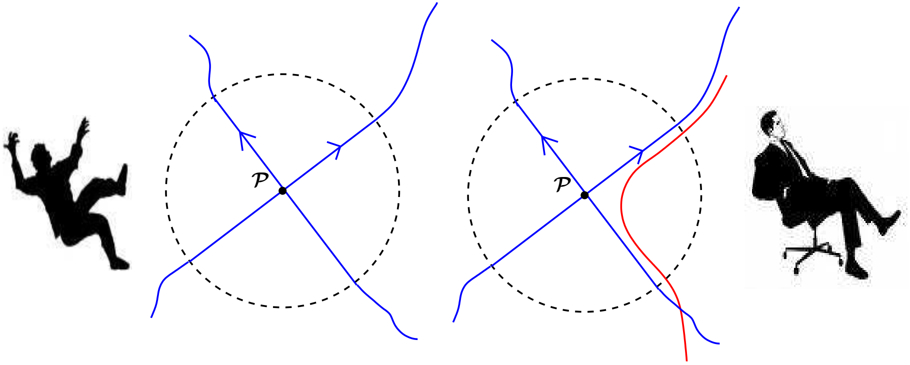
$$k_B T = \frac{\hbar}{c}\left(\frac{g}{2\pi}\right)$$

[Davies (1975), Unruh (1976)]

This allows you to associate a heat density $\mathcal{H} = Ts$ with every event of spacetime!



Vacuum fluctuations $\Longleftrightarrow$ Thermal fluctuations



Vacuum fluctuations $\Longleftrightarrow$ Thermal fluctuations

Why does spacetime exhibit thermal properties?!

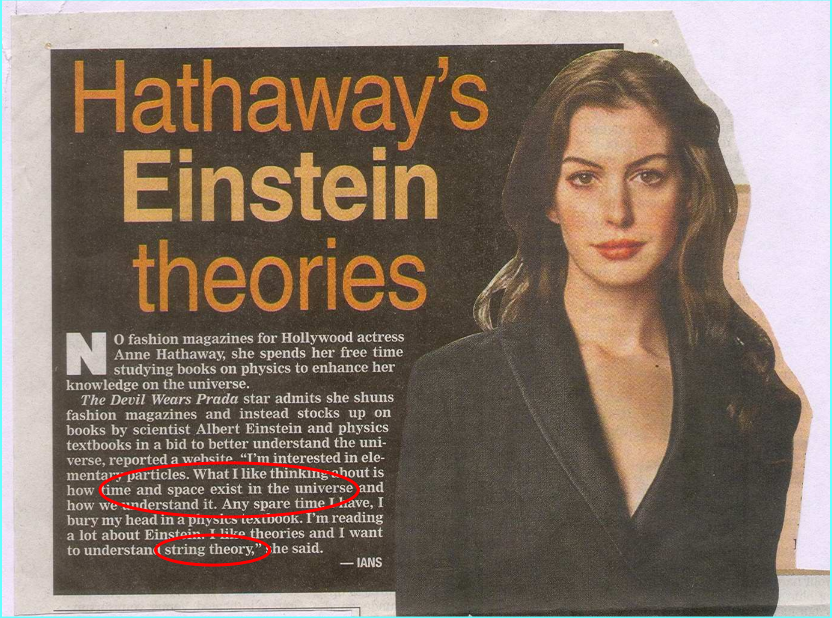
The three challenges involve $\hbar$!

$$A_{Planck} = \frac{G\hbar}{c^3}; \quad \Lambda\left(\frac{G\hbar}{c^3}\right) \approx 10^{-123}; \quad k_B T = \frac{\hbar}{c}\left(\frac{g}{2\pi}\right)$$

HOW DO WE PUT TOGETHER THE PRINCIPLES OF QUANTUM THEORY AND GRAVITY?

Everybody Wants To Quantize Gravity!

Everybody Wants To Quantize Gravity!



Hathaway's Einstein theories

NO fashion magazines for Hollywood actress Anne Hathaway, she spends her free time studying books on physics to enhance her knowledge on the universe.

The Devil Wears Prada star admits she shuns fashion magazines and instead stocks up on books by scientist Albert Einstein and physics textbooks in a bid to better understand the universe, reported a website. "I'm interested in elementary particles. What I like thinking about is how time and space exist in the universe and how we understand it. Any spare time I have, I bury my head in a physics textbook. I'm reading a lot about Einstein. I like theories and I want to understand string theory," she said.

— IANS

... But Nobody Has Succeeded!

... But Nobody Has Succeeded!

- ***The perturbative approach does not work***
- ***Virtually every interesting question about gravity is non-perturbative by nature***
- ***No guiding principle; metric is assumed to be a quantum variable***

GR: The Next 100 Years

Needs another paradigm shift!

If gravity is immune to zero level of energy it must have a thermodynamic interpretation!

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Connects two features usually thought to be completely separate!

[TP, arxiv:1508.06286]

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[TP, arxiv:1508.06286]

Classical Gravity has the same conceptual status as elasticity/hydrodynamics

If gravity is immune to zero level of energy it must have a thermodynamic interpretation!

Connects two features usually thought to be completely separate!

[TP, arxiv:1508.06286]

Study spacetime dynamics the way physicists studied fluids before knowing the atomic structure of matter

Atoms Of Spacetime

TP, arxiv: 1003.5665, 1508.06286

Boltzmann: anything you can heat up has micro-structure!

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- ***To store energy ΔE at temperature T, you need $\Delta n = \Delta E/(1/2)k_B T$ degrees of freedom.***

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- ***To store energy ΔE at temperature T, you need $\Delta n = \Delta E/(1/2)k_BT$ degrees of freedom.***
- ***You can heat up spacetime!***

Boltzmann: anything you can heat up has micro-structure!

- ***To store energy ΔE at temperature T, you need $\Delta n = \Delta E/(1/2)k_B T$ degrees of freedom.***
- ***You can heat up spacetime!***
- ***Count the microscopic spacetime degrees of freedom in terms of distribution function for atoms of spacetime!***

Boltzmann: anything you can heat up has micro-structure!

- ***To store energy ΔE at temperature T, you need $\Delta n = \Delta E/(1/2)k_B T$ degrees of freedom.***
- ***You can heat up spacetime!***
- ***Count the microscopic spacetime degrees of freedom in terms of distribution function for atoms of spacetime!***
- ***The distribution function determines the entropy density of spacetime.***

Nature Of Gravity

Field equations arise from maximizing entropy/heat density of gravity plus matter on all null surfaces.

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$$\mathcal{H} = \frac{Q}{V} = \frac{TS}{V} = \frac{1}{V}(E - F) = Ts$$

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$$\mathcal{H} = \frac{Q}{V} = \frac{TS}{V} = \frac{1}{V}(E - F) = Ts$$

$$Q = \int d\lambda\, d^2x\, \sqrt{\gamma}\, \left(\mathcal{H}_g + \mathcal{H}_m\right)$$

Field equations arise from maximizing entropy/heat density of gravity plus matter on all null surfaces.

$$\mathcal{H} = \frac{Q}{V} = \frac{TS}{V} = \frac{1}{V}(E - F) = Ts$$

$$Q = \int d\lambda \, d^2x \, \sqrt{\gamma} \, (\mathcal{H}_g + \mathcal{H}_m)$$

Works for a wide class gravitational theories; entropy decides the theory.

Gravity responds to heat density
($Ts = p + \rho$) — not energy density!

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Cosmological constant arises as an integration constant.

Gravity responds to heat density ($Ts = p + \rho$) — not energy density!

Cosmological constant arises as an integration constant.

Its value is determined by a new conserved quantity for the universe!

What Makes Spacetime Evolve ?

T.P., arXiv:1312.3253

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$$\underbrace{\int \frac{d\Sigma_a}{8\pi L_P^2}\left[q^{\ell m}\partial\, p^a_{\ell m}\right]}_{} = -\frac{1}{2}k_B T_{\rm av}\ \underbrace{(N_{\rm sur} - N_{\rm bulk})}_{}$$

time evolution of spacetime

= heating of spacetime

deviation from

holographic equipartition

What Makes Spacetime Evolve ?

T.P., arXiv:1312.3253

$$\underbrace{\int \frac{d\Sigma_a}{8\pi L_P^2}\left[q^{\ell m}\partial\, p^a_{\ell m}\right]}_{\text{time evolution of spacetime = heating of spacetime}} = -\frac{1}{2}k_B T_{\rm av}\ \underbrace{(N_{\rm sur} - N_{\rm bulk})}_{\text{deviation from holographic equipartition}}$$

time evolution of spacetime

= heating of spacetime

deviation from

holographic equipartition

This replaces the field equation for gravity

What Makes Spacetime Evolve ?

T.P., arXiv:1312.3253

$$\underbrace{\int \frac{d\Sigma_a}{8\pi L_P^2}\left[q^{\ell m}\partial\, p^a_{\ell m}\right]} = -\frac{1}{2}k_B T_{\rm av}\ \underbrace{\left(N_{\rm sur} - N_{\rm bulk}\right)}$$

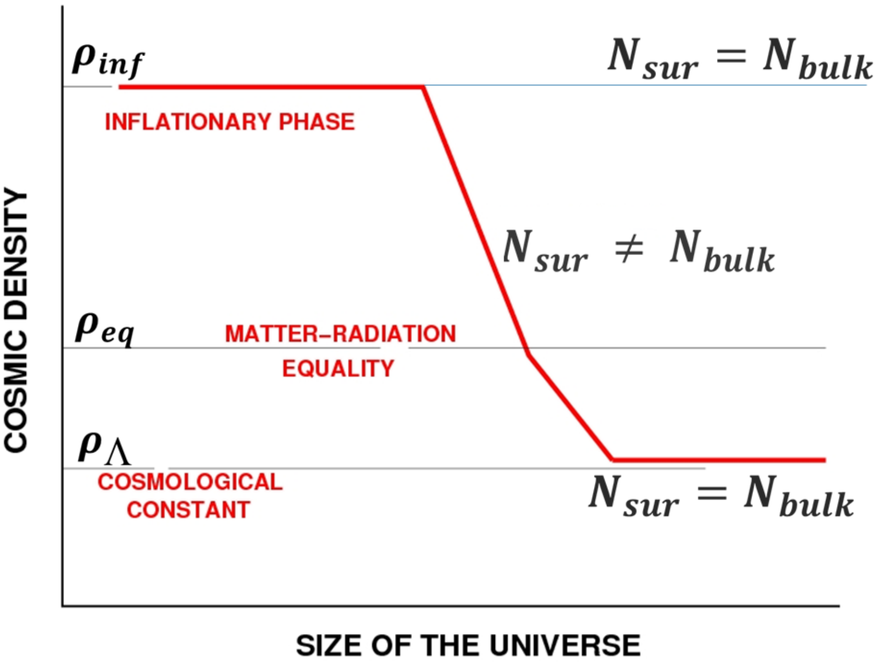
time evolution of spacetime

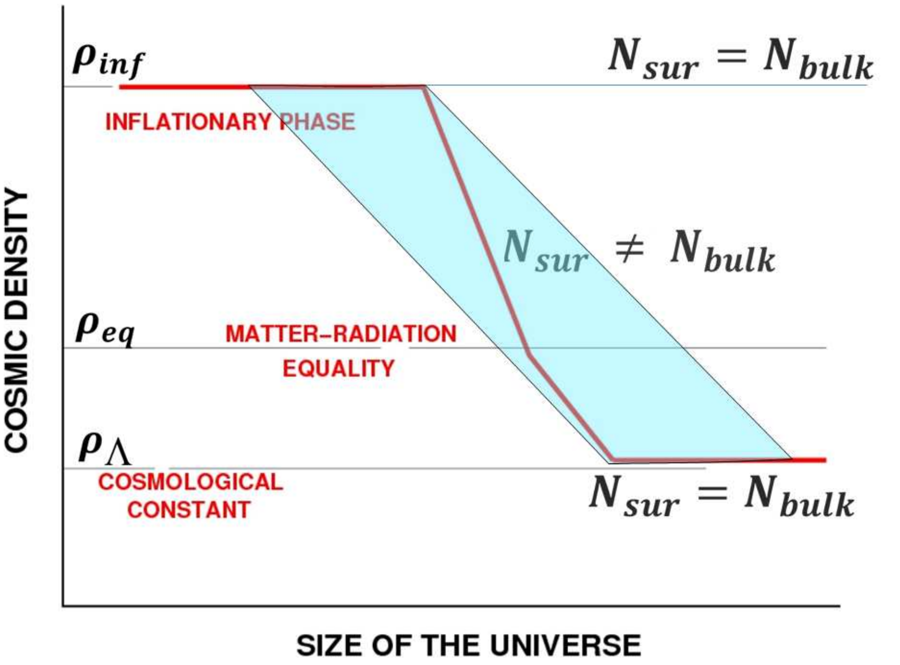
= heating of spacetime

deviation from

holographic equipartition

$$\frac{d\mathcal{R}}{dt} = 1 - \frac{N_{\rm bulk}}{N_{\rm sur}}$$


ρ_{inf}
$N_{sur} = N_{bulk}$
INFLATIONARY PHASE
$N_{sur} \neq N_{bulk}$
ρ_{eq}
MATTER–RADIATION
EQUALITY
ρ_{Λ}
COSMOLOGICAL
CONSTANT
$N_{sur} = N_{bulk}$
COSMIC DENSITY
SIZE OF THE UNIVERSE



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Take-Home Message

Hamsa Padmanabhan, T.P. [arXiv:1302.3226]

$$\rho_\Lambda = \frac{4}{27}\frac{\rho_{inf}^{3/2}}{\rho_{eq}^{1/2}}\ \exp(-36\pi^2)$$

Makes a falsifiable prediction!

$$\rho_{inf} = (1-6)\times 10^{15} GeV$$