Novel Computational Methods for the Determination of Properties of Cosmic Magnetic Fields

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Abstract

In this work we present three different approaches to analyze different aspects of Extragalactic Magnetic Fields. First, GRPropa, a numerical Monte Carlo code to simulate the propagation of electromagnetic cascades in the intergalactic space. Second, a semianalytic code for the time evolution of Primordial Magnetic Fields based on their spectral quantities. And, finally, an MHD code based on Kinetic Consistent Schemes. We describe the methods in detail and show their first applications in astrophyscis.

1 Introduction

Despite all the efforts which have been expended to understand Extragalactic Magnetic Fields (EGMF) their origin, evolution and current state are still unknown. Regarding the first of these three, possible scenarios of magnetogenesis can be subdivided into two categories – the cosmological one, where EGMF are created in the very early Universe, and the astrophysical one, where they emerge during later cosmological times, for example during Galaxy formation. In the cosmological case this is claimed to happen due to a global event like inflation [1] or a cosmological phase transition (for an overview of possible scenarios, especially during the Electroweak or the QCD phase transition, see [2]). The produced field then has a large field strength and a small correlation length, such that one has to analyze how they evolve with time to the EGMF today which, in contrast, are thought to have low field strengths and large correlation lengths. This has been studied by various authors, mostly using numerical simulations [3, 4, 5, 6], which, however, due to limitations of computational power, are not able to resolve the scales in time and space necessary to give a full picture.

In the astrophysical scenario magnetogenesis happens at a time when the Universe has obtained certain structures like (proto)galaxies which are then also responsible for the creation of EGMF, such that, in contrast to the cosmological scenario, magnetic fields are created rather locally instead of globally and subsequently spread into the intergalactic space. Possible causes are for example the vorticity of protogalaxies [7], rotating black holes [8] or the ejection of magnetic fields, along with matter, by galaxies [9, 10].

Finally, another problem studying EGMF is the difficulty to observe them, such that up to the present day only limits of their two main parameters, the average field strength B and the correlation length L_{c}. While solid upper limits for both and lower limits for the latter exist [2], obtaining lower limits for B is a rather difficult problem. Recently, the authors of [11, 12] found, that such limit indeed *can* be obtained by analyzing the flux of high-energy gamma rays form TeV blazars which is lower than expected which they claim to be due to the deflection of the produced electromagnetic cascade by EGMF. Their analysis hence resulted in a lower limit on B being as low as 10^{-16} G. However, this claim is still under debate as others [13, 14, 15] claim that such a suppressed might arise due to the interaction of the cascade electrons with the Intergalactic Medium (IGM).

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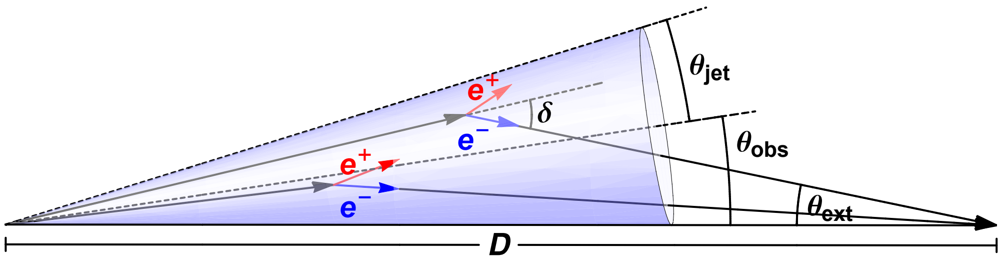

Figure 1: Schematics of an electromagnetic cascade caused, in the way described in the text, by a high-energy photon (marked by black arrows) emitted by a source on the left within a jet with opening angle θ_{jet} tilted by θ_{obs} with respect to the line of sight in a distance D from the observer on the right. Even if it is a point source, it will appear as extensive with an angle θ_{ext} to the observer.

In this work we present three different computational approaches to address the issues ldescribed above. First, in Sec. 2, we discuss GRPropa, a new software to simulate the propagation of electromagnetic cascades in the intergalactic space which, as stated above, might be used to determine the properties of EGMF. Then, in Secs. 3 and 4, we show two methods using which different aspects of the cosmological and astrophysical scenario of magnetogenesis may be analyzed, respectively – on the one hand a semianalytical treatment of the time evolution of Primordial Magnetic Fields and on the other hand a new MHD simulation code based on Kinetic Consistent Schemes (KCS). Finally, in Sec. 5, we summarize our findings and give an outlook on current and future work.

2 GRPropa

The aim of the new GRPropa software we are developing is the simulation of the propagation of electromagnetic cascades in the intergalactic space. The basic principles for that are rather simple: An emitted gamma ray interacts with the low-energy photons of the Extragalactic Background Light (EBL) and produces an electron/positron pair. These electrons and positrons then, in turn, interact with the EBL via Inverse Compton (IC) scattering and by this produce a new gamma ray photon which is repeated until the photon energy drops below the pair production threshold. This process is schematically shown in Fig. 1.

Up to the present days codes like, for example, ELMAG [16], which are simulating the development of electromagnetic cascades, are one-dimensional and work with the small-angle approximation to mimic 3D effects. However, in order to simulate the full impact of magnetic fields onto the propagation a full three-dimensional treatment is necessary since turbulent magnetic fields present in the intergalactic voids naturally introduce complex three-dimensional trajectories for the electrons/positrons which, if the fields are strong enough, cannot be treated with the small-angle approximation anymore.

These are the limitations which GRPropa is able to overcome in a thorough way for the first time. It is based on the CRPropa 3 [17, 18] code which is commonly used for ultra-high energy cosmic ray propagation. Using the modular structure of the code and the flexibility to handle custom magnetic field configurations and the propagation of particles therein, we have implemented relevant interactions for gamma rays and electrons, namely Pair Production and IC as described above, in the energy range from 1 GeV to $\sim$ PeV. Adiabatic losses due to the expansion of the universe are also taken into account. Synchrotron losses, albeit small in this energy range, are considered as well, for the sake of completeness.

In GRPropa particles are propagated step-by-step. Within each step the probability of a given interaction to occur is computed using tabulated values for the interaction rate. If the particle is charged, deflections due to magnetic fields are calculated by integrating the equations of motion. By doing so, we are adopting a full three-dimensional Monte Carlo approach for the propagation. All tests regarding the performance, stability and physical plausibility so far give excellent results. In order to give an illustrative example of the correct implementation of magnetic fields, we compare the

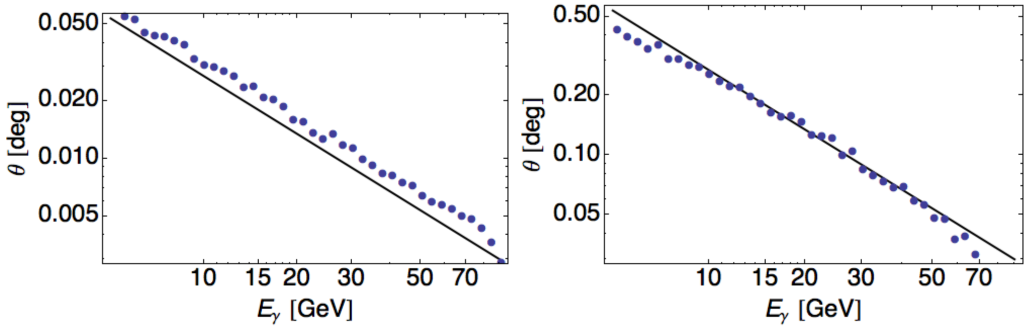

Figure 2: Comparison of $\theta_{\rm ext}$ from analytical estimates of (1) (solid line) with GRPropa output (blue dots) for $E_{\rm TeV} = 10\,{\rm TeV}$, $D = 1\,{\rm Gpc}$ as well as $B = 10^{-16}\,{\rm G}$ (left) and $B = 10^{-15}\,{\rm G}$ (right), respectively.

output for $\theta_{\rm ext}$ from GRPropa with the analytic formula (cf. Fig. 1) [19]

$$\theta_{\rm ext}(E_\gamma) \simeq 0.05^\circ (1+z_{\rm s})^{-4} \times \left(\frac{B}{\rm fG}\right)\left(\frac{E_\gamma}{0.1\,{\rm TeV}}\right)^{-1}\left(\frac{D}{\rm Gpc}\right)^{-1}\left(\frac{E_{\rm TeV}}{10\,{\rm TeV}}\right)^{-1}, \tag{1}$$

where E_γ is the energy of the gamma ray arriving at Earth, z the redshift, B is the EGF field strength, D is the distance from the source to the observer and $E_{\rm TeV}$ is the energy of the gamma ray emitted by the source. This comparison for a single monochromatic source with $E_{\rm TeV} = 10\,{\rm TeV}$, $D = 1\,{\rm Gpc}$ and both $B = 10^{-16}\,{\rm G}$ and $B = 10^{-15}\,{\rm G}$ is shown in Fig. 2. As one can see, they are in very good agreement with each other, which is an additional confirmation of our approach.

To conclude, another distinct feature of GRPropa should be mentioned, namely the consideration of magnetic helicity $\mathcal{H}$ of EGMF, defined as

$$\mathcal{H} = \int \mathbf{A}\cdot\mathbf{B}\,{\rm d}^3x\,, \tag{2}$$

where $\mathbf{A}$ is the magnetic vector potential. It has a great impact on the propagation of electromagnetic cascades such that, as pointed out in [20, 21, 22, 23], performing a statistical analysis of the arrival directions of gamma rays at Earth, one could draw conclusions on the topological structure of EGMF, which we were able to confirm with our simulations [24].

3 Semianalytical Study of the Time Evolution of Primordial Magnetic Fields

As mentioned above, Primordial Magnetic Fields are magnetic fields which originated globally in the Early Universe, for example during the QCD or the Electroweak Phase Transition. The problem when treating their time evolution from their generation until today is the large range of scales involved, both in time and space, such that full MHD simulations often lack the resolution necessary to obtain a full picture.

This is why a semianalytic picture is a more promising approach to obtain meaningful results as it reduces the number of degrees of freedom and thus the computational time by considering directly the spectral quantities M_k, U_k and $\mathcal{H}_k$, defined as

$$\rho\int M_q{\rm d}q \equiv \frac{1}{8\pi V}\int \mathbf{B}^2(\mathbf{x})\,{\rm d}^3x\,,\ \rho\int U_q{\rm d}k \equiv \frac{\rho}{2V}\int \mathbf{v}^2(\mathbf{x})\,\rho\int \mathcal{H}_q{\rm d}q \equiv \frac{1}{V}\int \mathbf{A}(\mathbf{x})\cdot\mathbf{B}(\mathbf{x})\,{\rm d}^3x\,, \tag{3}$$

and thus corresponding to the spectral magnetic energy, kinetic energy and (according to (2)) magnetic helicity densities, respectively. Here, ρ denotes the mass density, V the volume and $\mathbf{v}$ the velocity field of the fluid, while q is the magnitude of the wavevector from the Fourier Transformation in space.

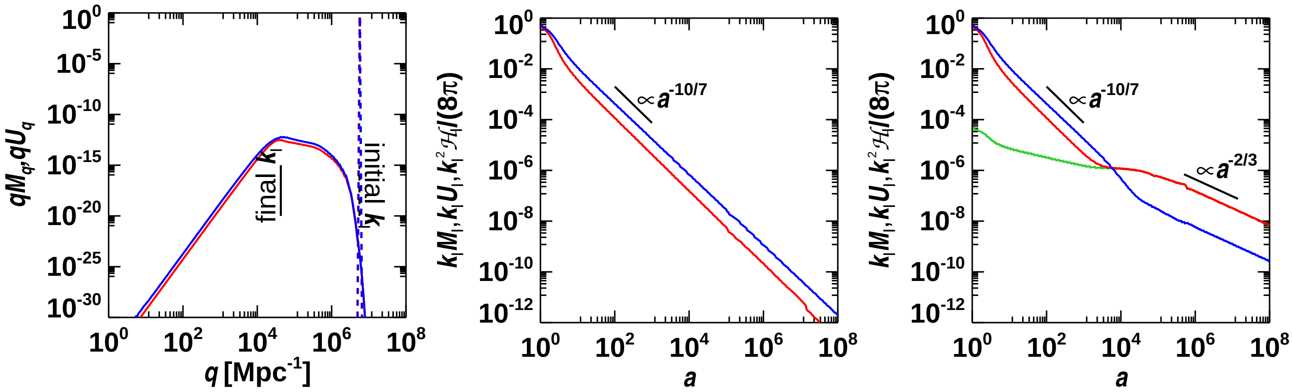

Figure 3: Time evolution of M_q (red), U_q (blue) and $\mathcal{H}_q$ (green), cf. (3). The first panel shows the initial configuration (dashed) at the QCD phase transition as well as the final one at Recombination (solid) in q-space for zero helicity. The middle and right panel show the time evolution of of $M_{\rm I}$, $U_{\rm I}$ and $\mathcal{H}_{\rm I}$ for the case without and with helicity, respectively.

Based on [25] we were able to derive differential time evolution equations for the ensemble average of these three quantities which may be found in [26, 27]. As shown in the left panel of Fig. 3 for initial conditions of the QCD phase transition without helicity, where the magnetic fields are created peaked around an integral scale $q = k_{\rm I}$, we find that, rather independent of the particluar initial shape (dotted) of M_q (red) and U_q (blue), for large scales (i.e. small q) a tail $M_q \propto q^4$ at Recombination (solid) builds up, corresponding to a Batchelor spectrum and thus confirming the findings of [4, 28]. In addition, one can also see that a state close to equipartition is created and then preserved up to the present day.

The picture changes, however, once magnetic helicity is introduced. This is shown in Fig. 3 by the comparison between the non-helical (central panel) and helical (right panel) case of the time evolution of $M_{\rm I} \equiv M_{k_{\rm I}}$, $U_{\rm I} \equiv U_{k_{\rm I}}$ and $\mathcal{H}_{\rm I} \equiv \mathcal{H}_{k_{\rm I}}$ (green), where a is the scale factor defined to be equal to 1 at the moment of the phase transition. As one can see, even values of helicity as small as a fraction of 10^{-4} of the maximal value, given enough time, dramatically change the time evolution by creating a so-called Inverse Cascade due to which more energy is transferred to larger scales. This, therefore, might be important in order to be able to chose an appropriate magnetogenesis scenario.

4 Using Kinetic Consistent Schemes in Astrophysics

The third project to understand EGMF is the use of Kinetic Consistent Schemes as a powerful method to derive the MHD equations in the most natural way in a form which is particularly useful for high performance computing. This method is a well-known tool in hydrodynamics [29], however only recently a way to include electromagnetism has been found [30].

The starting point of this approach is the distribution function $f(\mathbf{x}, \boldsymbol{\xi}, t)$ which gives the probability to find a particle of the fluid with velocity $\boldsymbol{\xi}$ at position $\mathbf{x}$ at time t and for which the time evolution is governed by the Boltzmann Equation

$$\partial_t f(\mathbf{x}, \boldsymbol{\xi}, t) + \boldsymbol{\xi} \cdot \nabla f(\mathbf{x}, \boldsymbol{\xi}, t) = C(f)\,, \tag{4}$$

where $C(f)$ is the collision integral and no external forces are present. Due to the complexity of the Collision Integral, usually an analytical solution cannot be found. That is, however, possible for $C(f) = 0$ and gives the Maxwellian distribution

$$f_{\rm M}(\mathbf{x}, \boldsymbol{\xi}, t) = \frac{\rho(\boldsymbol{\xi}, t)}{(2\pi R T(\boldsymbol{\xi}, t))^{\frac{3}{2}}} \exp\left\{\frac{(\boldsymbol{\xi} - v(\boldsymbol{\xi}, t))^2}{2RT(\boldsymbol{\xi}, t)}\right\}\,, \tag{5}$$

where ρ is the mass density, T the (local) temperature, $\mathbf{v}$ the velocity field and R the Universal Gas Constant.

The main idea now is to start with $f_{\rm M}(\mathbf{x}, \boldsymbol{\xi}, t = t_0)$ at the initial time t_0 and then consider the distribution function at a time $t_0 + \tau$, i.e. $f(\mathbf{x}, \boldsymbol{\xi}, t_0 + \tau)$ for which then a Taylor expansion in time up to the second order is carried out which, together with the Boltzmann Equation, gives

$$\frac{\partial f}{\partial t} + \tau \frac{\partial^2 f}{\partial t^2} + (\boldsymbol{\xi} \cdot \nabla) f_{\rm M}(\mathbf{x}, \boldsymbol{\xi}, t_0) = \frac{\tau}{2} (\boldsymbol{\xi} \cdot \nabla)^2 f_{\rm M}(\mathbf{x}, \boldsymbol{\xi}, t_0) \,. \tag{6}$$

Integrating this expression according to the moments of the distribution function and identifying τ with the correlation time then gives the time evolution equation for the hydrodynamics quantities like the mass, energy and momentum densities including all possible viscosity and energy dissipation terms which here come out naturally instead of introducing them *ad hoc* as done usually.

The final step of this derivation is the transition from hydro- to *magneto*hydrodynamics, i.e. the inclusion of electromagnetism. As has been found out in [30], this may be done in a rather simple and elegant way by considering the velocity as a complex quantity, i.e.

$$\boldsymbol{\xi} \in \mathbb{C}^3 \,, \; \mathbf{v} \to \mathbf{v} + i\mathbf{v}_{\rm A} \,, \tag{7}$$

where $\mathbf{v}_{\rm A}$ is the Alfvén Velocity $\mathbf{v}_{\rm A} = \mathbf{B}\rho^{-\frac{1}{2}}$. Therefore, this indeed brings in magnetic fields, such that a new moment of the distribution function is needed in order to extract it. It turns out that it is given by the summation invariant $m\boldsymbol{\xi}^*$, where the star $*$ indicates the complex conjugation and m is the particle mass, such that we obtain

$$\mathbf{B}(\mathbf{x}, t) = -\rho^{-\frac{1}{2}} \int m\xi^* f(\mathbf{x}, \boldsymbol{\xi}, t) \, \mathrm{d}^3\xi \,. \tag{8}$$

In a similar way, by using this summation invariant on (6) with the new complex quantities (7), we obtain the complete set of MHD equations including all viscosity terms which then can be used to perform full-scale simulations. We have done that by introducing a new code which efficiently exploits the advantages of the method by making it highly parallelizable and fast and thus suitable for high-performance computing. The stability and performance tests have been very successful, such that we were able to apply it to a first astrophysical problem, namely the outflow of matter from Galaxies due to Supernova-driven magnetic fields [31] for which magnetic fields will be included in the next step.

5 Conclusions and Outlook

In this work we presented three different approaches to study Extragalactic Magnetic Fields – GRPropa, a three-dimensional Monte Carlo code for the simulation of electromagnetic cascades; an efficient semianalytical approach to study the time evolution of Primordial Magnetic Fields via their energy and helicity spectra; and finally a novel MHD simulation software which makes use of Kinetic Consistent Schemes for the Boltzmann Equation and its recent extension to electromagnetic fields and is applied to different astrophysical scenarios like galactic winds due to Supernova explosions.

In the future we will apply these three approaches to different problems in astrophysics. GRPropa might be used to analyze fluxes and arrival directions of gamma rays on Earth and by that deduce the structure and magnitude of EGMF. The semianalytic method is currently being improved by updating the formalism and extending it to include kinetic helicity, such that it can be used for more problems. Finally, the new MHD code is fully operational and thus for example can be used to include magnetic fields into the Galactic scenario described above.

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