

Constraints on Induced Gravity Dark Energy Models

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OUTLINE

- Modern cosmology and the cosmological constant problem
- How to solve the cosmological constant problem
- Why is difficult to modify gravity
- The scalar-tensor theories:
 - ◆ Brans-Dicke
 - ◆ Induced Gravity
- Constraints on Induced Gravity
- Generalization of the model
- Conclusions



MODERN COSMOLOGY

The standard model of cosmology is based on two assumptions:

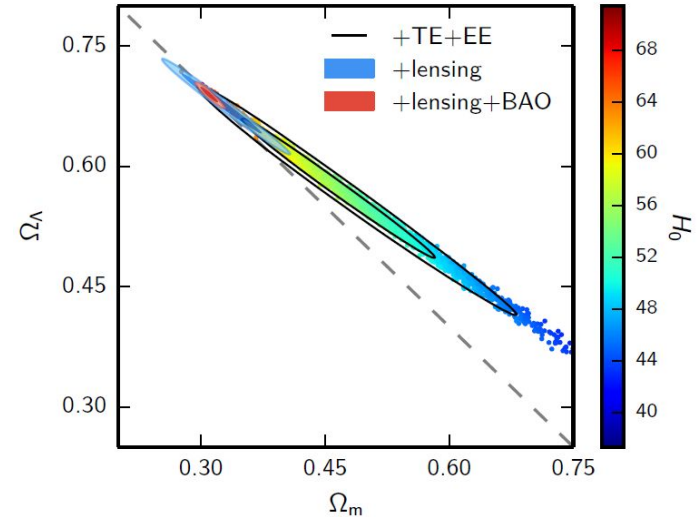
→ our Universe is **homogeneous and isotropic**

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 + Kr^2} + r^2 d\Omega^2 \right)$$

→ gravity is described by **General Relativity**

$$H^2 \equiv \left(\frac{\dot{a}}{a} \right)^2 + \frac{K}{a^2} = \frac{8\pi G}{3} \rho_m + \frac{\Lambda}{3}$$

$$1 = \Omega_m + \Omega_K + \Omega_\Lambda$$



$$\Omega_\Lambda = 0.685 \pm 0.013$$

$$\Omega_m = 0.315 \pm 0.013$$

How to solve the cosmological constant problem

- We have to change one of our fundamental assumptions:
 1. Change the Cosmological Principle;

How to solve the cosmological constant problem

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1. Change the Cosmological Principle;

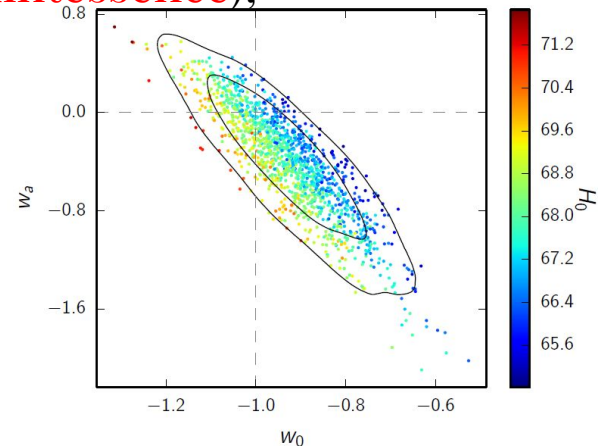
2. Introduce a fifth fundamental force (**quintessence**);

→ “Why now” problem;

→ The hierarchy problem;

→ Often treated as a fluid without specifying its physical origin.

Ratra & Peebles (1988)

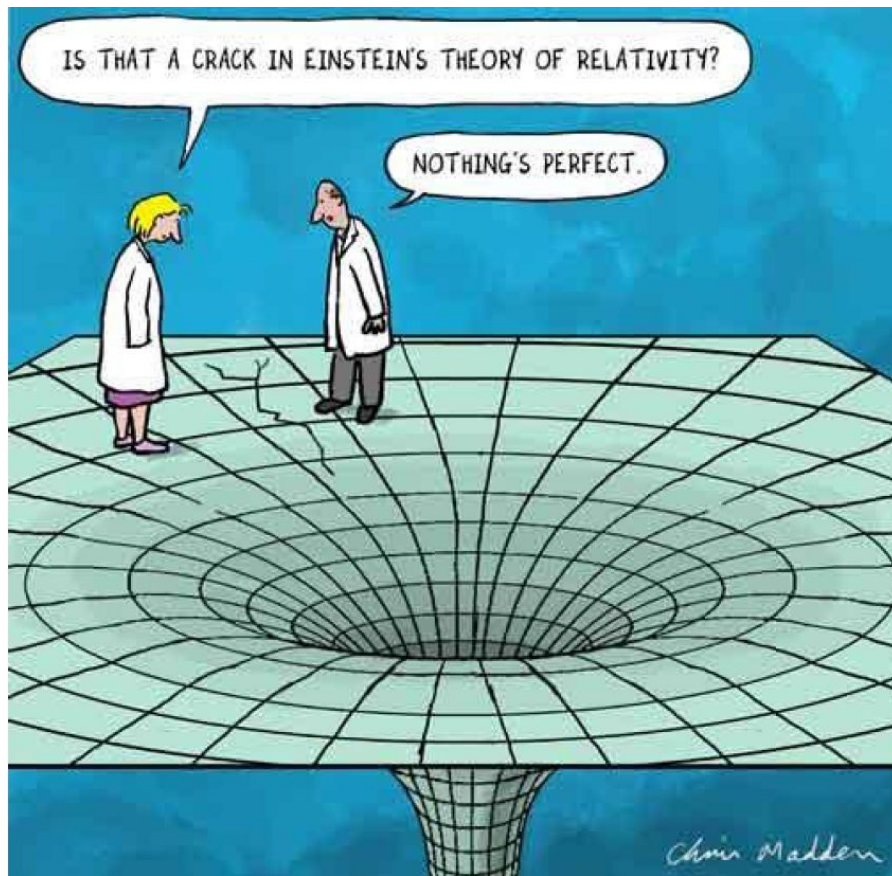


Planck 2015 results. XIII. Cosmological parameters (2015)

How to solve the cosmological constant problem

→ We have to change one of our fundamental assumptions:

1. Change the Cosmological Principle;
2. Introduce a fifth fundamental force (**quintessence**);
3. Modified gravity !



HOW TO MODIFY GR ?

WHY IS DIFFICULT TO MODIFY GR?

- The **Lovelock's theorem** (1971):
“Einstein's equations are the only second-order, local equations of motion for a metric derivable from the action in 4D”
- If we modify GR, we need to have one of these:
 1. extra degree of freedom (extra fields);
 2. higher derivatives;
 3. spacetime with more than four dimensions;
 4. non-locality.
- Cosmological observations compatible with Λ CDM model.



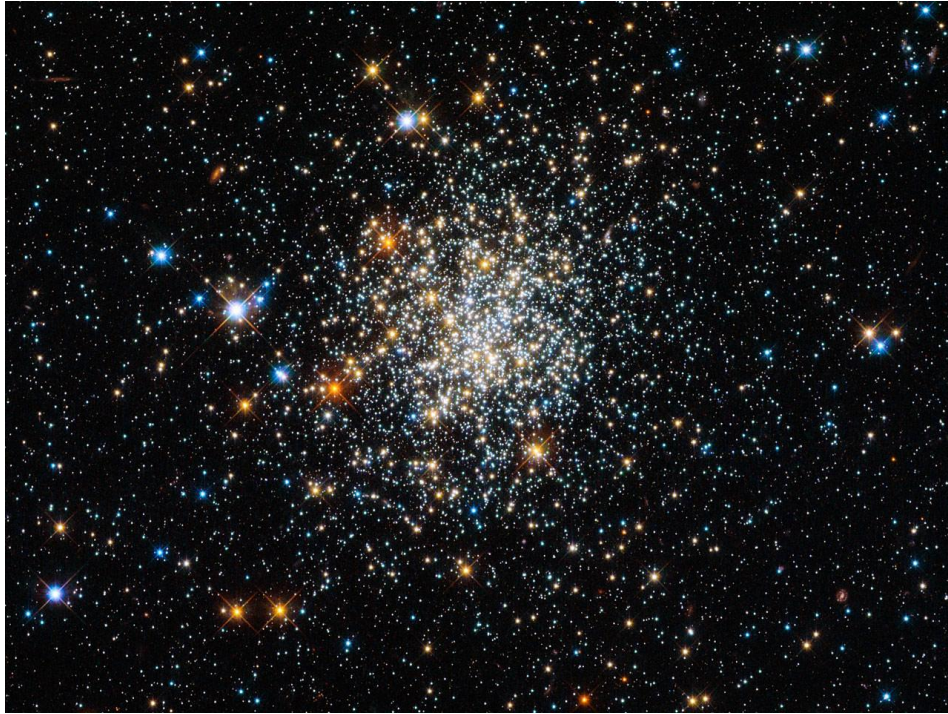
Planck 2013 results. XVI. Cosmological parameters (2013)

Planck 2015 results. XIV. Dark energy and modified gravity (2015)

Planck 2015 results. XIII. Cosmological parameters (2015)

HOW

Let's ϕ it!



+ a scalar field ϕ

SCALAR-TENSOR THEORY

→ GR:

$$\mathcal{S} = \int dx^4 \sqrt{-g} \left[\frac{R}{16\pi G} + \mathcal{L}_m \right]$$

→ Scalar-tensor theory:

$$\mathcal{S} = \int dx^4 \sqrt{-g} \left[\frac{1}{16\pi} (f(\phi)R + g(\phi)\nabla_\mu\phi\nabla^\mu\phi) - 2\Lambda(\phi) + \mathcal{L}_m \right]$$

...after conformal transformation...

$$\mathcal{S} = \int dx^4 \sqrt{-g} \left[\frac{1}{16\pi} \left(\phi R - \frac{\omega(\phi)}{\phi} \nabla_\mu\phi\nabla^\mu\phi \right) - 2\Lambda(\phi) + \mathcal{L}_m \right]$$

Brans-Dicke gravity

Brans & Dicke (1961)

→ Simplest modified gravity model:

$$\omega(\phi) = \omega_{BD} = \text{const}$$

$$\Lambda(\phi) = 0$$

$$\mathcal{S} = \int dx^4 \sqrt{-g} \left[\frac{1}{16\pi} \left(\phi R - \frac{\omega_{BD}}{\phi} \nabla_\mu \phi \nabla^\mu \phi \right) + \mathcal{L}_m \right]$$

→ The gravitational constant G is not presumed to be constant but instead $1/G$ is replaced by a scalar field ϕ which can vary from place to place and with time;

→ GR is recovered in the limit of large values for ω_{BD} .

Induced Gravity

Zee (1980), Cooper & Venturi (1981), Wetterich (1988)

→ Action for IG with a quartic potential:

$$\mathcal{S} = \int dx^4 \sqrt{-g} \left[\frac{\gamma \sigma^2 R}{2} - \frac{1}{2} \nabla_\mu \sigma \nabla^\mu \sigma - \frac{\lambda}{4} \sigma^4 + \mathcal{L}_m \right]$$

→ After redefinition of the field and of the coupling we find the Brans-Dicke model with a quadratic potential:

$$\gamma \sigma^2 = \phi / 8\pi$$

$$\omega_{BD} = 1/4\gamma$$

$$m = \sqrt{2\lambda} / 16\pi\gamma$$

IG as basic and testable example of more...

→ Many (healthy) extensions of GR are based on the inclusion of scalar degree of freedom:

1. Galileon models

Nicolis, Rattazzi & Trincherini (2009)

2. Hordenski theory

Hordenski (1974), Deffayet, Esposito-Farese & Vikman (2009)

3. Beyond Hordenski

Zumalacárregui & García-Bellido (2014), Gleyzes, Langlois, Piazza & Vernizzi (2015)

$$L_2^\phi \equiv G_2(\phi, X)$$

$$L_3^\phi \equiv G_3(\phi, X) \square \phi$$

$$L_4^\phi \equiv G_4(\phi, X)^{(4)} R - 2G_{4,X}(\phi, X) (\square \phi^2 - \phi^{\mu\nu} \phi_{\mu\nu}) \\ + F_4(\phi, X) \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\mu'\nu'\rho'\sigma'} \phi_\mu \phi_{\mu'} \phi_{\nu\nu'} \phi_{\rho\rho'}$$

$$L_5^\phi \equiv G_5(\phi, X)^{(4)} G_{\mu\nu} \phi^{\mu\nu} + \frac{1}{3} G_{5,X}(\phi, X) (\square \phi^3 - 3\square \phi \phi_{\mu\nu} \phi^{\mu\nu} + 2\phi_{\mu\nu} \phi^{\mu\rho} \phi_\sigma^\nu) \\ + F_5(\phi, X) \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\mu'\nu'\rho'\sigma'} \phi_\mu \phi_{\mu'} \phi_{\nu\nu'} \phi_{\rho\rho'} \phi_{\sigma\sigma'}$$

Background dynamics (1)

- We initialize the scalar field deep in the radiation era (early DE/MG model) with adiabatic initial conditions;
- We fine tuned the initial value of the scalar field, σ_1 , in order to get the exact value of the gravitational constant today, which implies that the model satisfies the Solar System constraints on GR;
- We studied predictions for cosmological observables by solving self-consistently the background dynamics and the system of linear fluctuations without any use of parameterization nor the quasi-static approximation;
- The evolution of the background cosmology can be easily compared with DE in Einstein gravity with a Newtonian constant given by: $\tilde{G} = 1/8\pi\gamma\sigma_0^2$

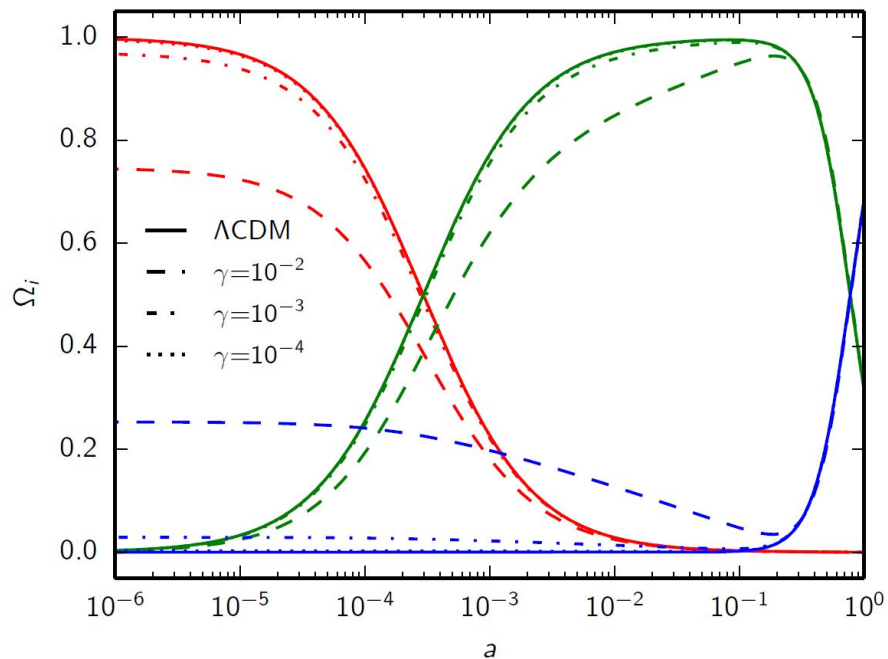
$$\rho_{\text{DE}} = \frac{\sigma_0^2}{\sigma^2} \left(\frac{\dot{\sigma}}{2} - 6\gamma H \dot{\sigma} \sigma + \lambda \frac{\sigma^4}{4} \right) + \sum_i \rho_i \left(\frac{\sigma_0^2}{\sigma^2} - 1 \right)$$

$$p_{\text{DE}} = \frac{\sigma_0^2}{\sigma^2} \left[\frac{\dot{\sigma}}{2} - 2\gamma H \dot{\sigma} \sigma - \lambda \frac{\sigma^4}{4} + \sum_i \frac{2\gamma\rho_i + p_i}{1 + 6\gamma} \right] - \sum_i p_i$$

Cerioni, Finelli, Tronconi & Venturi (2009)

Umiltà, Ballardini, Finelli & Paoletti (2015) 15

Background dynamics (2)

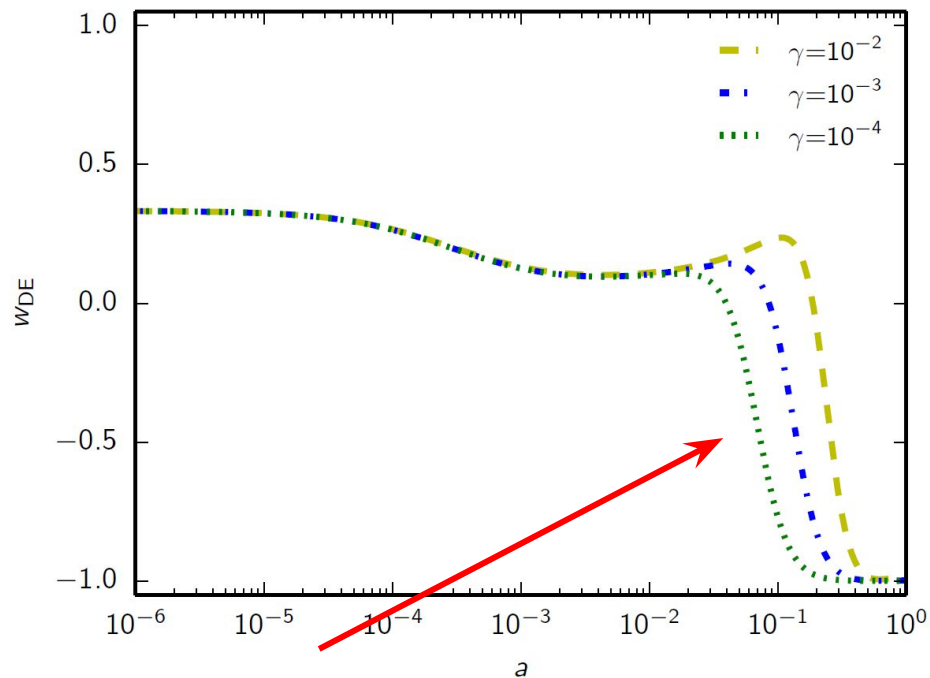
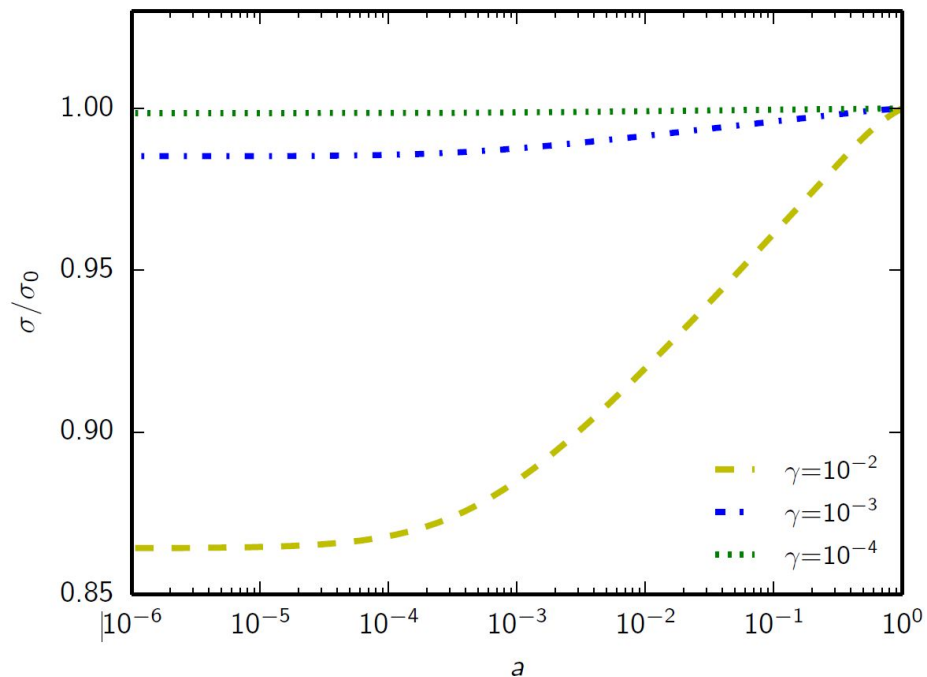


$$\Omega_R \equiv \frac{\rho_\nu + \rho_\gamma}{3\gamma\sigma_0^2 H^2}$$

$$\Omega_M \equiv \frac{\rho_b + \rho_c}{3\gamma\sigma_0^2 H^2}$$

$$\Omega_{DE} \equiv \frac{\rho_{DE}}{3\gamma\sigma_0^2 H^2}$$

Background dynamics (3)



acceleration start at different times

Cosmological fluctuations

- CMB anisotropies as best probe to test perturbations;
- How to evolve background and linear fluctuations in a high precision framework?

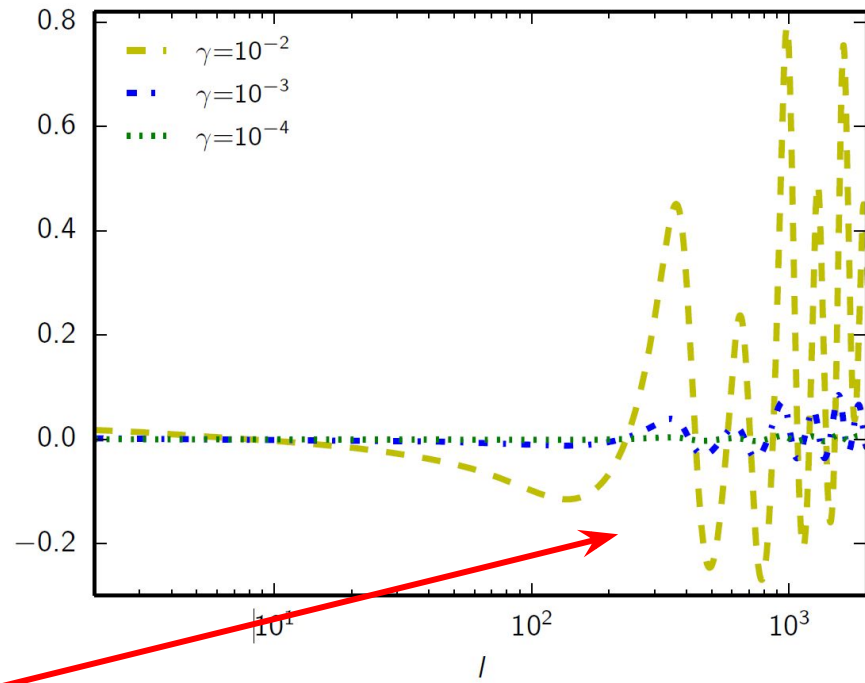
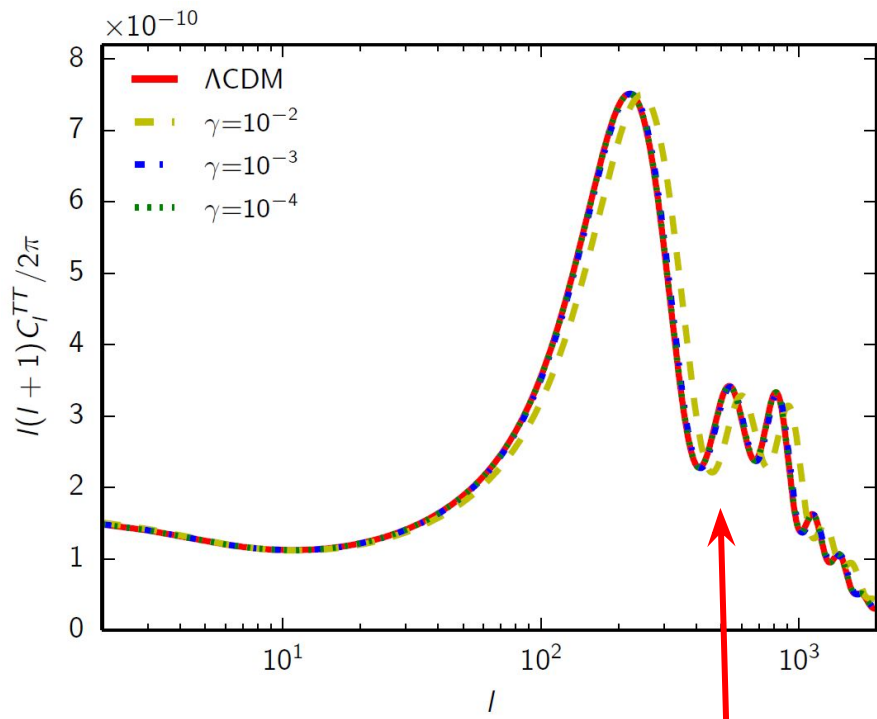


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Monte Python

Audren, Lesgourgues, Benabed & Prunet (2103)
https://github.com/baudren/montepython_public

CMB angular power spectrum (TT case)



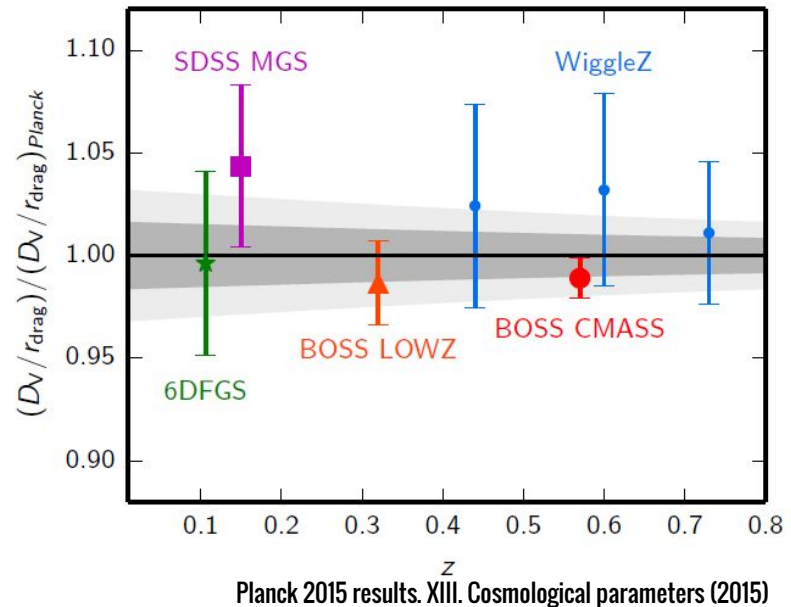
changes the position of the acoustic peaks

ONLY CMB?

Baryon Acoustic Oscillation

→ We considered a compilation of BAO data from:

1. 6dFGS at $z=0.106$;
Beutler et al. (2011)
2. SDSS-MGS at $z=0.15$;
Ross et al. (2015)
3. SDSS-DR11 LOWZ at $z=0.32$;
BOSS Collaboration (2014)
4. SDSS-DR11 CMASS at $z=0.56$.
BOSS Collaboration (2014)

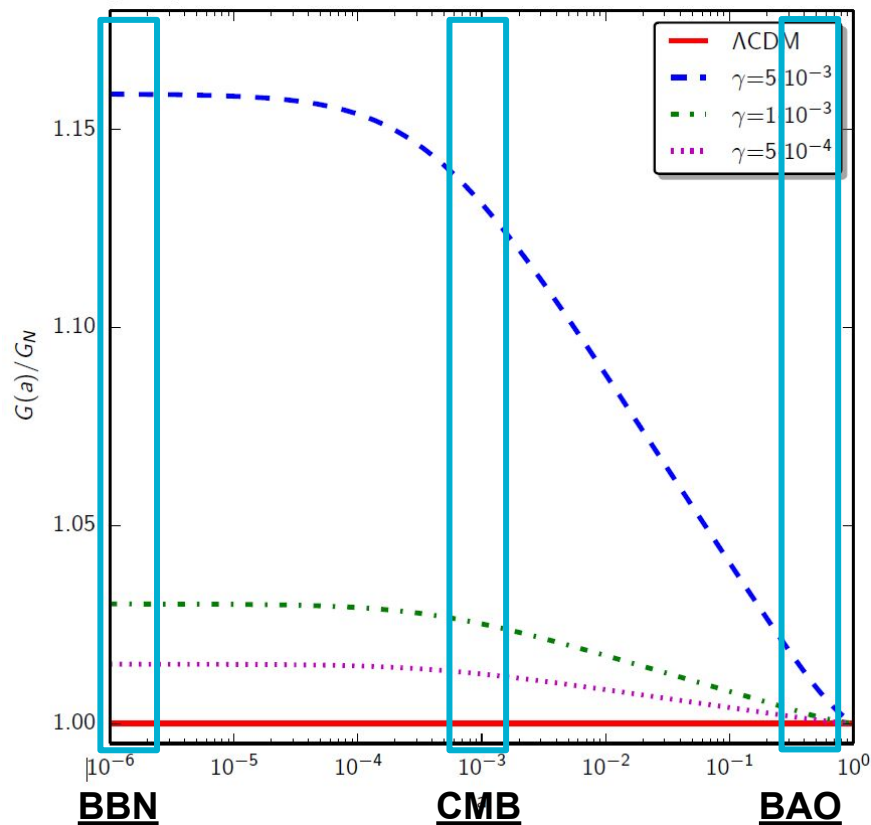


Big Bang Nucleosynthesis

- The energy density in the early Universe can also be proved by the BBN predictions;
- The observed primordial light-element abundances constrain the value of G during the BBN epoch from the time of weak reaction freezeout (~ 1 sec, 1 MeV) to the freezeout of the nuclear reaction ($\sim 10^4$ sec, 10 keV);
- The increasing of Newton's constant causes the increase of the expansion rate and this yields a larger ${}^4\text{He}$ abundance;
- We use the PARthENoPE code (Pisani et al. 2007) to provide a careful determination of light nuclei abundances, with a very small uncertainties.

Combined constraints from three different scales

$$G = \frac{1}{8\pi\gamma\sigma^2}$$



RESULTS

What we found

	PLANCK 2013 + BAO Λ CDM	PLANCK 2013 + BAO IG
$10^5 \Omega_b h^2$	2215^{+24}_{-25}	2203 ± 25
$10^4 \Omega_c h^2$	1187^{+13}_{-14}	1207^{+18}_{-22}
H_0 [km s ⁻¹ Mpc ⁻¹]	$68.4^{+0.6}_{-0.7}$	$69.5^{+0.9}_{-1.2}$
τ	$0.091^{+0.012}_{-0.014}$	$0.088^{+0.012}_{-0.013}$
$\ln(10^{10} A_s)$	$3.089^{+0.024}_{-0.027}$	$3.090^{+0.024}_{-0.026}$
n_s	0.9626 ± 0.0053	0.9611 ± 0.0053
ζ	...	< 0.0047 (95% CL)
$10^3 \gamma$...	< 1.2 (95% CL)
γ_{PN}	...	> 0.9953 (95% CL)
Ω_m	0.301 ± 0.008	0.295 ± 0.009
$\delta G_N / G_N$...	$-0.015^{+0.013}_{-0.006}$
$10^{13} \dot{G}_N(z=0) / G_N$ [yr ⁻¹]	...	$-0.61^{+0.55}_{-0.25}$
$10^{23} \ddot{G}_N(z=0) / G_N$ [yr ⁻²]	...	$0.86^{+0.33}_{-0.78}$

→ Cosmological parameters compatible with the values obtained for the Λ CDM model;

→ A slightly larger value for the Hubble parameter today;

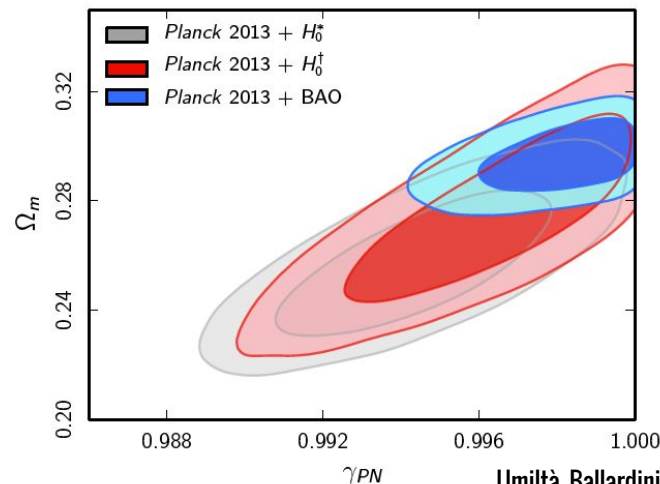
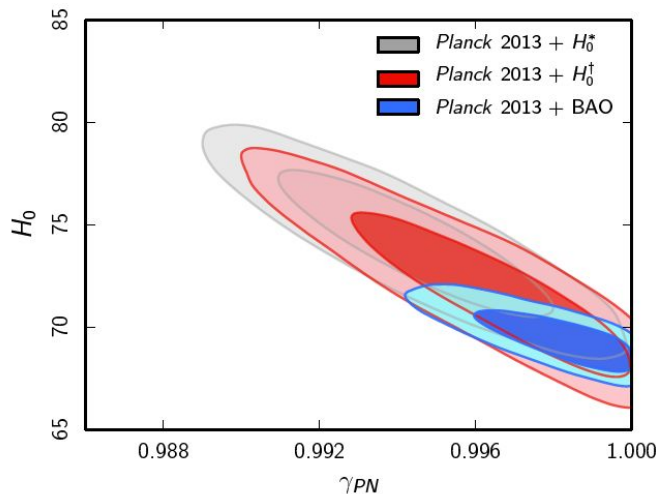
→ IG model with a quartic potential is not preferred over Einstein gravity, $\Delta\chi^2 \sim 0.7$.

Combination with local measurements

- IG prefers a higher value of the Hubble parameter H_0 with respect to Λ CDM;
- We considered separately the impact of two different local estimates of H_0 :

1. $H_0^* = 73.8 \pm 2.4$ km/s Mpc Riess et al. (2011)

2. $H_0^+ = 70.6 \pm 3.0$ km/s Mpc Efstathiou (2014)



Combination with local measurements

	PLANCK 2013 + H_0^*	PLANCK 2013 + H_0^\dagger	PLANCK 2013 + BAO + H_0^\dagger
$10^5 \Omega_b h^2$	2219 ± 28	2213_{-29}^{+28}	2203 ± 26
$10^4 \Omega_c h^2$	1188_{-26}^{+25}	1194_{-25}^{+25}	1207_{-22}^{+18}
H_0 (km s $^{-1}$ Mpc $^{-1}$)	$74.1_{-2.4}^{+2.3}$	$72.1_{-3.1}^{+2.2}$	$69.64_{-1.11}^{+0.88}$
τ	$0.092_{-0.014}^{+0.013}$	$0.091_{-0.015}^{+0.013}$	$0.088_{-0.014}^{+0.012}$
$\ln(10^{10} A_s)$	$3.098_{-0.027}^{+0.025}$	$3.095_{-0.028}^{+0.025}$	$3.091_{-0.027}^{+0.024}$
n_s	$0.9704_{-0.0072}^{+0.0070}$	$0.9667_{-0.0078}^{+0.0075}$	$0.9613_{-0.0054}^{+0.0055}$
ζ	0.0056 ± 0.0023	< 0.0083 (95% CL)	0.0047 (95% CL)
$10^3 \gamma$	1.4 ± 0.6	< 2.1 (95% CL)	< 1.2 (95% CL)
γ_{PN}	$0.9944_{-0.0022}^{+0.0023}$	> 0.9918 (95% CL)	> 0.9954 (95% CL)
Ω_m	$0.257_{-0.019}^{+0.016}$	$0.274_{-0.021}^{+0.022}$	$0.294_{-0.008}^{+0.009}$
$\delta G_N / G_N$	$-0.041_{-0.016}^{+0.017}$	-0.028 ± 0.012	$-0.016_{-0.006}^{+0.010}$
$10^{13} \dot{G}_N(z=0) / G_N$ [yr $^{-1}$]	$-1.56_{-0.58}^{+0.61}$	$-1.10_{-0.49}^{+0.83}$	$-0.64_{-0.25}^{+0.52}$
$10^{23} \ddot{G}_N(z=0) / G_N$ [yr $^{-2}$]	$2.4_{-1.0}^{+0.9}$	$1.7_{-1.5}^{+0.7}$	$0.89_{-0.75}^{+0.24}$

deviation from GR at more than 2σ

WHAT'S NEW

Planck 2015 data

- We analyze the model with the new Planck data release with more combinations of dataset (high-L CMB polarization, CMB lensing);
- By using the combination of the polarization and of the lensing obtained constraints based only on Planck/CMB data.
- Better constraint on γ , the coupling with the Ricci scalar.



Planck & BBN

Hamann, Lesgourgues & Mangano (2008)

- In BBN theory the primordial abundance of helium depends from the baryon density, extra relativistic degree of freedom and non-zero chemical potential;
- We implement the variation of G during radiation domination as a component of extra radiation:

$$H^2 \simeq \frac{8\pi G}{3} \rho_\gamma \left[1 + \frac{7}{8} \left(\frac{4}{11} \right)^{\frac{4}{3}} (3.046 + \Delta N) \right]$$

- We found that the BBN consistency condition doesn't change the constraints on γ obtained from the CMB and BAO, but it gives a different value of primordial helium abundance.

*GENERALIZE THE
POTENTIAL*

Monomial potential

$$V(\sigma) \propto \lambda \sigma^n$$

- The self-interacting potential, in the IG Lagrangian density, drives the scalar field in presence of non-relativistic matter to a time-independent value at recent times;
- For values different from $n=4$ we got an extra contribution to the Klein-Gordon equation:

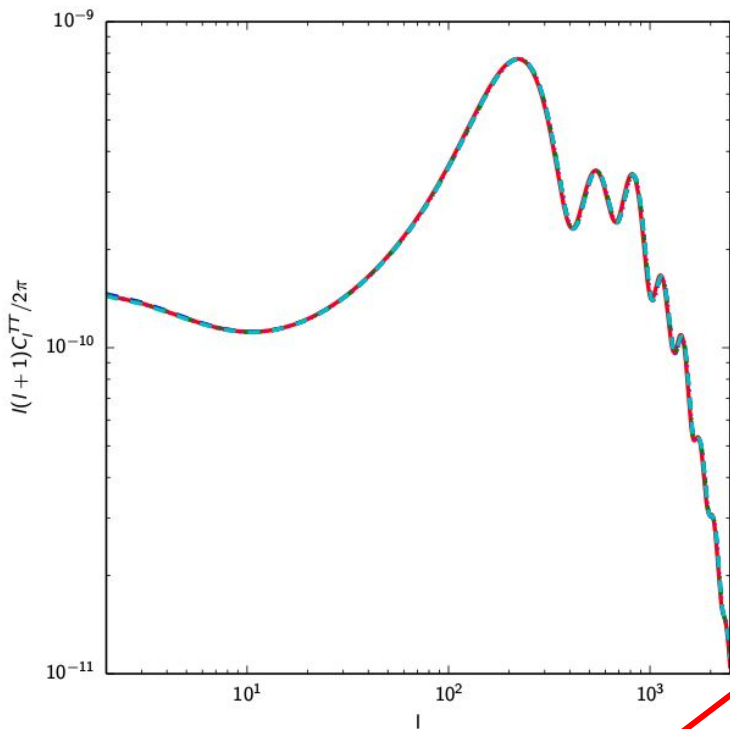
$$\ddot{\sigma} + 3H\dot{\sigma} + \frac{\dot{\sigma}^2}{\sigma} + \frac{1}{(1+6\gamma)} \left(V_{,\sigma} - \frac{4V}{\sigma} \right) = \frac{1}{(1+6\gamma)} \frac{\sum_i (\rho_i - 3p_i)}{\sigma}$$

- Also the equation for the field fluctuation does not depend on the potential for the self-interacting case:

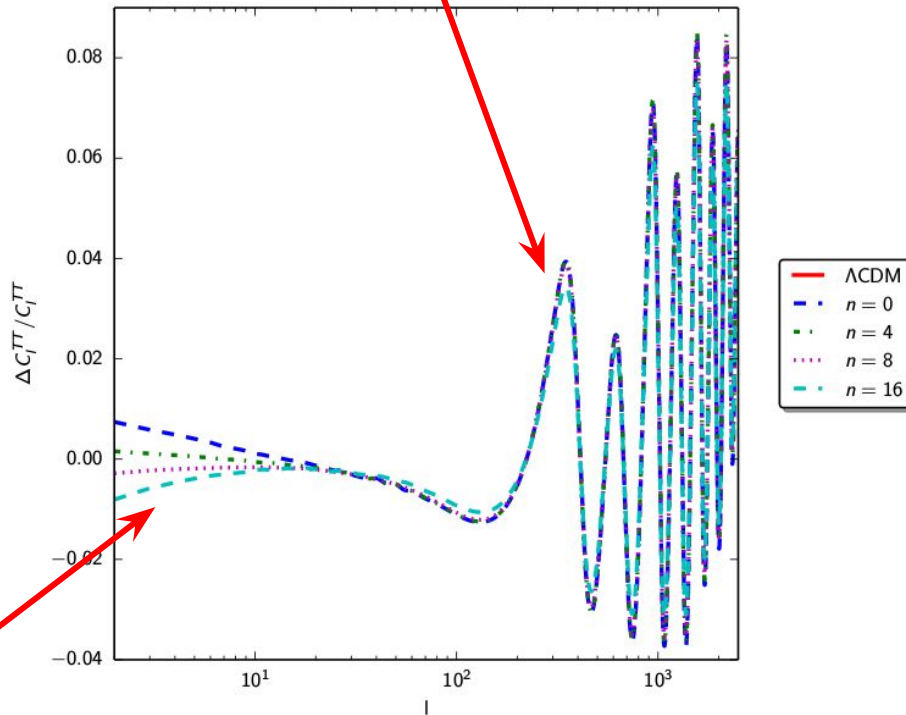
$$\begin{aligned} \delta\ddot{\sigma} + \delta\dot{\sigma} \left(3H + 2\frac{\dot{\sigma}}{\sigma} \right) + \left[\frac{k^2}{a^2} - \frac{\dot{\sigma}^2}{\sigma^2} + \frac{\sum_i (\rho_i - 3p_i)}{(1+6\gamma)\sigma^2} + \frac{1}{(1+6\gamma)} \left(V_{,\sigma\sigma} + \frac{4V}{\sigma^2} - \frac{4V_{,\sigma}}{\sigma} \right) \right] \delta\sigma \\ = \frac{2\Psi \sum_i (\rho_i - 3p_i)}{(1+6\gamma)\sigma} + \frac{\sum_i (\delta\rho_i - 3\delta p_i)}{(1+6\gamma)\sigma} + \dot{\sigma} (3\dot{\Phi} + \dot{\Psi}) \end{aligned}$$

Observational effects

different amplitude of the peaks for large values of 'n'



$\gamma = 10^{-3}$



modify the plateau at large scales

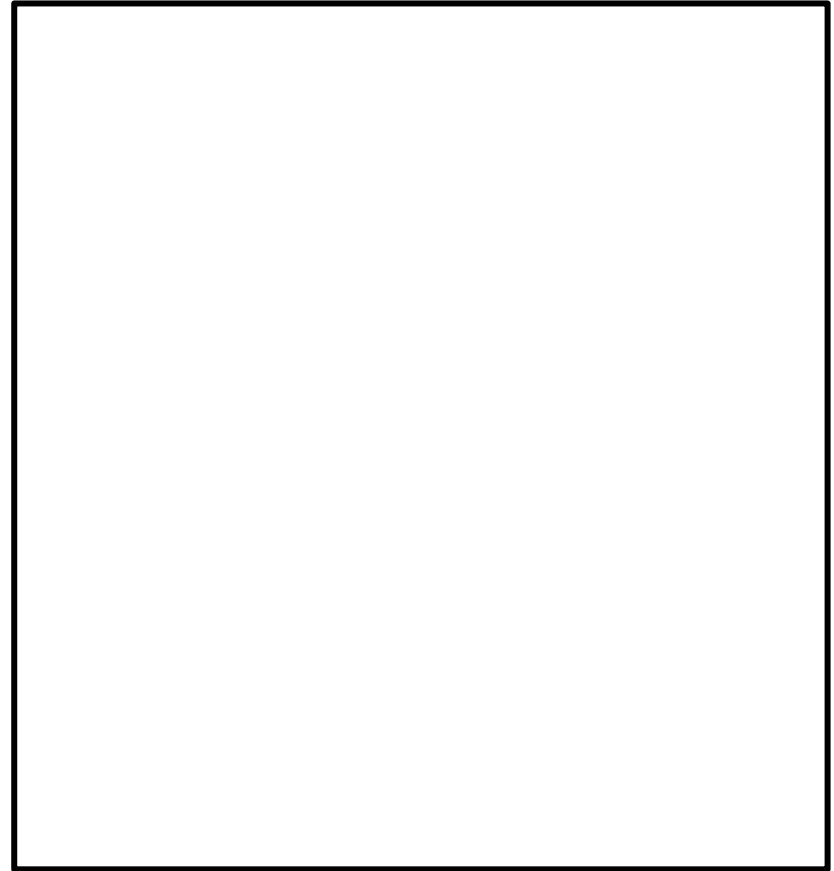
Effects on the cosmological parameters

→ Constraints on the cosmological parameters quite stable to different potentials;

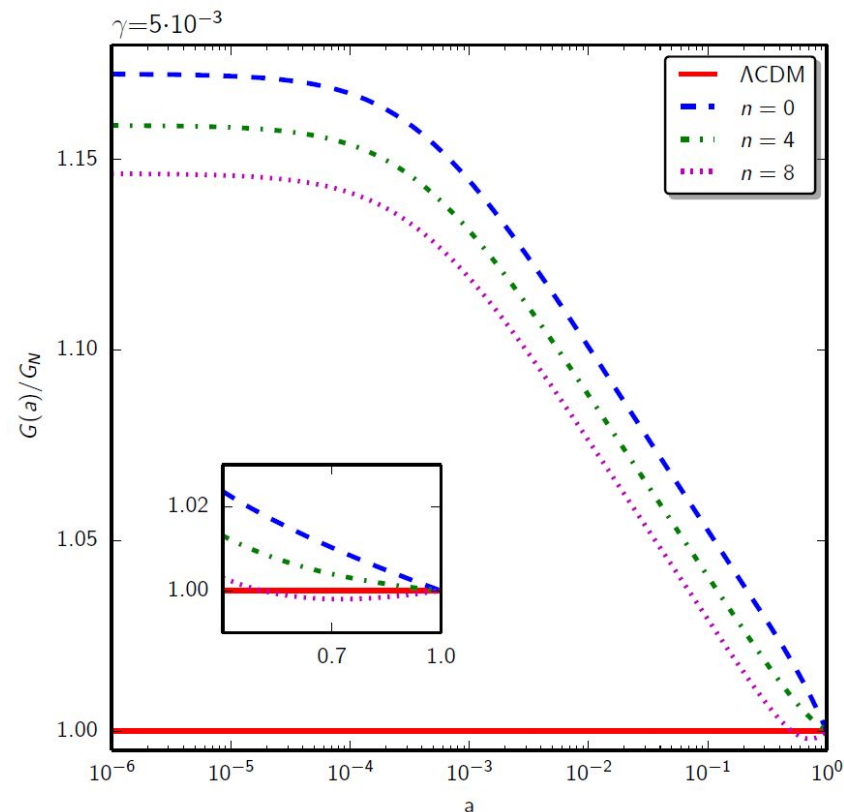
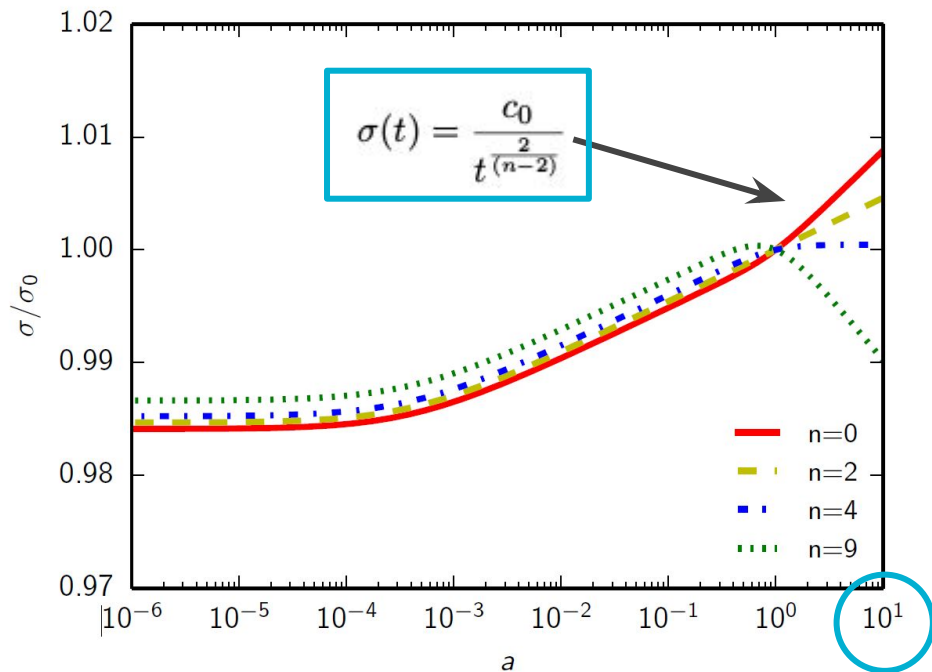
→ Strong effects on the derivatives of G , i.e. on the evolution of the scalar field:

$$\frac{\delta G_N}{G_N} = \frac{\sigma_i^2 - \sigma_0^2}{\sigma_0^2}$$

$$\frac{\dot{G}_N}{G_N} = -2 \frac{\dot{\sigma}_0}{\sigma_0}$$



Evolution of the scalar field



→ The scalar field is effectively massless during the radiation and most of the matter era.

Cerioni, Finelli, Tronconi & Venturi (2009)

Ballardini, Finelli, Paoletti & Umiltà (2015/16) 35

CONCLUSIONS (1)

- We have studied an induced gravity, or Brans-Dicke-like, dark energy models with a monomial non-negative potential, with particular emphasis on the quartic potential ($n=4$);
- In this class of models the effective gravitational constant decrease during the matter era from the constant value it had in the radiation era;
- These are the simplest models within scalar-tensor theories with seven cosmological parameters, just one more than Λ CDM model, with a parameter of state similar to early dark energy models in the framework of GR with the current gravitational constant;
- We studied predictions for cosmological observables by solving self-consistently the background dynamics and the system of linear fluctuations without any use of parameterization nor the quasi-static approximation;

CONCLUSIONS (2)

- These models require a value for H_0 larger than Λ CDM, which is closer to the one obtained from local measurement;
- We have obtained the following 95% CL constraint:
 $\gamma < 0.0012$ Planck 2013 +BAO
and anticipated how with Planck 2015 data a CMB only bound can be obtained;
- The constraints from CMB and BAO are much tighter than those from BBN;
- Little dependence on 'n' for the cosmological constraints, except for the derived constraint on the time variation of the effective Newton's constant, which is directly linked to the time evolution of the scalar field at recent times.

THANKS

EXTRA SLIDES

Quasi-static approximation

$$k^2[\Phi - \delta(k, a)\Psi] = 12\pi G a^2 \mu(k, a)(\rho + p)\bar{\sigma}$$

$$k^2\Psi = -4\pi G a^2 \mu(k, a) [\Delta + 3(\rho + p)\bar{\sigma}]$$

$\gamma = 10^{-2}, k = 0.005 \text{ Mpc}^{-1}$

