Structure of magnetized transonic accretion disks

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Abstract

Using general relativistic analytical treatment in Kerr metric and numerical simulations with the public HARM2D code, we consider the vertical structure and velocity field in the inner parts of a black hole accretion disk, both outside and inside the last stable orbit.

Chaotic magnetic fields frozen into the accreting matter easily become the dominant pressure source inside the sonic point that allows to predict the equilibrium thickness of the flow. Numerical simulations, however, reveal instabilities in the magnetic fields leading to formation of current sheets in the super-sonic part of the flow and ultimately to accretion through thin layers (or wisps, in 3D) with the vertical spatial scale much smaller.

If disk thickness is large, any consideration of its vertical structure should use some assumptions about the rotation velocity field inside the disk. The latter seems to be strongly dependent upon the presence of a regular magnetic field component near the black hole. If the magnetic flux through the black hole horizon is small, rotation close to the sonic surface conforms very well to Keplerian, and isorotational surfaces have practically cylindrical shapes ($r \sin\theta = const$). We use the assumption of Keplerian cylindrical rotation $\Omega(r,\theta) = \left((r\sin\theta)^{3/2} + a\right)^{-1}$ to estimate the effects of disk thickness upon the Eddington limit and accretion efficiency in black hole accretion disks.

1 Introduction

At the time of its appearence, the standard thin accretion disk model by [1] significantly increased our understanding of accretion upon black holes and other compact objects. But what are the limits of standard disk accretion in the parameter space? For black hole accretion, it is the Eddington limit at the high mass accretion rates and inefficiency of energy release for low accretion rates. In both cases, the disk becomes thick, and its structure should be treated as two-dimensional. Besides, the inner parts of the accretion disk should be considered both taking into account the effects of General Relativity (relativistic thin disk model was first made by [2]) and the transonic nature of the flow near the last stable orbit and inside it. Beskin and Tchekhovskoy [3] considered the motion inside the last stable orbit as ballistic motion of individual particles falling in Kerr space-time. Orbits of the falling particles easily intersected each other in this approximation that can be interpreted as formation of a system of shock waves. Another approach to the transonic solution was the slim disk [4] that operates vertically integrated Euler equations.

In [5], we consider the inner parts of a thin transonic disk containing small-scale magnetic fields and show that, in this case, the supersonic part of the flow becomes magnetically-supported in vertical direction and thus has a distinct vertical scale. Vertical oscillations are damped by dissipative processes. Here, we discuss the analytical results given in this paper and numerical simulations of the transonic accretion flows in Kerr metric presented in [5] and [6].

In section 2, we describe the model of a falling disk-like flow supported by magnetic pressure in vertical direction. In section 3, we discuss the results of numerical simulations aimed to reproduce the structure of the transonic flow.

2 Magnetically-supported falling flow

In the accretion disk, there are several pressure sources, including radiation pressure $p_{\rm r}$, gas pressure $p_{\rm g}$ and magnetic stress $B^2/8\pi$. MHD instabilities tend to amplify the magnetic field stress up to equipartition, $B^2/8\pi \sim p_{\rm r,g}/\beta$. On the other hand, transition to the supersonic flow inside the last stable orbit produces a strong rarefaction of gas that consequently looses large fraction of its pressure (decreasing roughly as $P \propto \rho^\gamma$, where γ is adiabatic index), as well as of the radiation pressure (decreasing due to adiabatic losses as well as due to decreasing optical depth). The only pressure source remaining continuous near the sonic surface is radial magnetic field component. Differential rotation efficiently transforms this field into azimuthal that makes the inner flow supported in vertical direction by magnetic fields. Thickness of such a disk is approximately [5]

$$\frac{H}{R} \simeq (0.2 \div 0.3) \left(\frac{1}{\alpha\beta} \left(\frac{H}{R}\right)_{\rm disk} \right)^{1/3}, \qquad (1)$$

if the magnetic fields are uniform and if vertical balance is close to magnetohydrostatics. Here, H is the geometrical thickness of the disc at radius R, α is dimensionless viscosity parameter (see [1]), and β is magnetization parameter introduced above.

The probable caveat of this approach is the assumption that the magnetic field is assumed uniform within the disk in vertical direction. Real magnetic loops should be closed, thus the magnetic field in the falling disk-like flow should be alternate in direction. The magnetic field, as well as its pressure, should have at least one minimum in this case, that leads to formation of a denser and less magnetized flow inside the magnetic loop. This suggestion is supported by the numerical results.

3 Simulations

The analytical estimates of the vertical spatial scale (1) were compared to relativistic simulations in Kerr metric performed with the public code `HARM2d` [7, 8]. Apart from the models used in [5], here we present one more, D1, with a rapidly rotating black hole $a = 0.99$ (see Fig. 1). All the models, especially those with a single magnetic loop (A1, B1, C1, D1 and the one with higher resolution, B1h; the models differ in Kerr parameters) show a very thin stream inside the last stable orbit, with a thickness much smaller than that predicted by the analytical model.

There are two possible explanations for this discrepancy, both equally plausible:

- the thickness measured with the help of density is lower because most of the accreting matter follows the least magnetized streamlines; the inner flow is in this sense a current sheet that must have a lower-magnetized layer in the middle
- the simulations we use are two-dimensional, and thus could miss some of the important details of MHD turbulence in 3D (A. Tchekhovskoy, private communication).

3.1 Disk thickness and rotation

One interesting by-product of our simulations is rotational law for a magnetized disk that seems to conform well with Keplerian law and also with rotation constant on coordinate cylinders (to accuracy better than $\left(\frac{H}{R}\right)^2 \sim 0.01 \div 0.04$). This is probably connected to the fact that the equation of state of the accreting gas in the simulation is barotropic. In barotropic stationary flows, isostrophes, or "von Zeipel cylinders", are exact cylinders in Newtonian approximation [9]. In Kerr metric, the shapes of

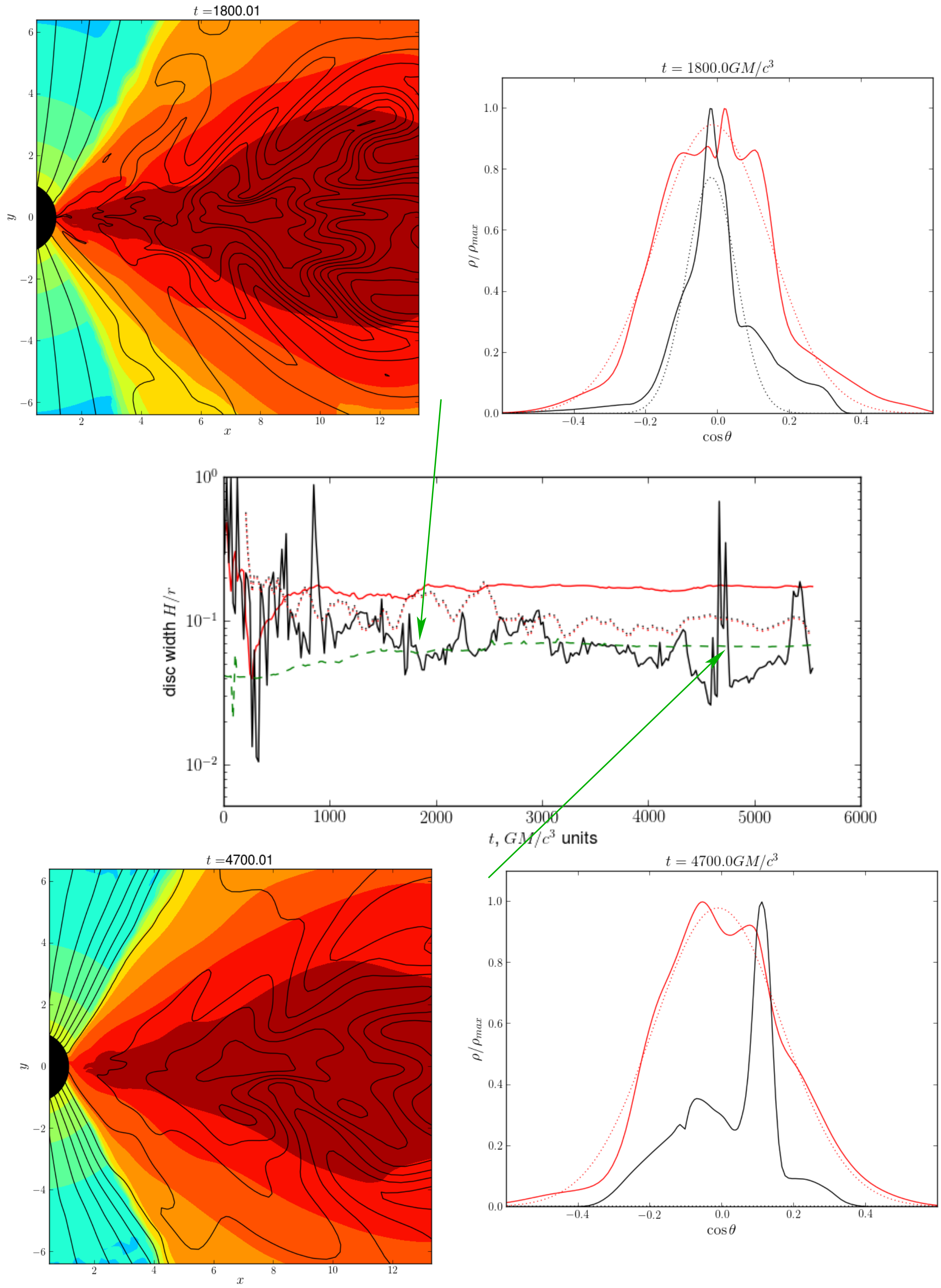

Figure 1: *Central:* disc relative thicknesses for $a = 0.99$ (D1 model) calculated between 6 and 12 GM/c^2 (red/grey solid line) and between 2 and 6 GM/c^2 (black solid line). Equilibrium magnetically-supported disc thicknesses calculated near the event horizon radius (red/grey dotted line) and for the last stable orbit (black dotted) are also shown. Green dashed curve shows the estimated equilibrium thickness of a disc supported by thermal pressure at the marginally bound radius. Upper and lower *left panels* show two representative snapshots at the instances shown by the arrows. Density is shown by colors/shades (logarithmic scale), and magnetic field lines are marked by contours. Corresponding radially-integrated density profiles are shown in the corresponding right panels: inner region (between the horizon and the marginally bound radius) is shown with a black solid curve, disc region (between the last stable orbit and double last stable orbit) with a red/grey solid curve.

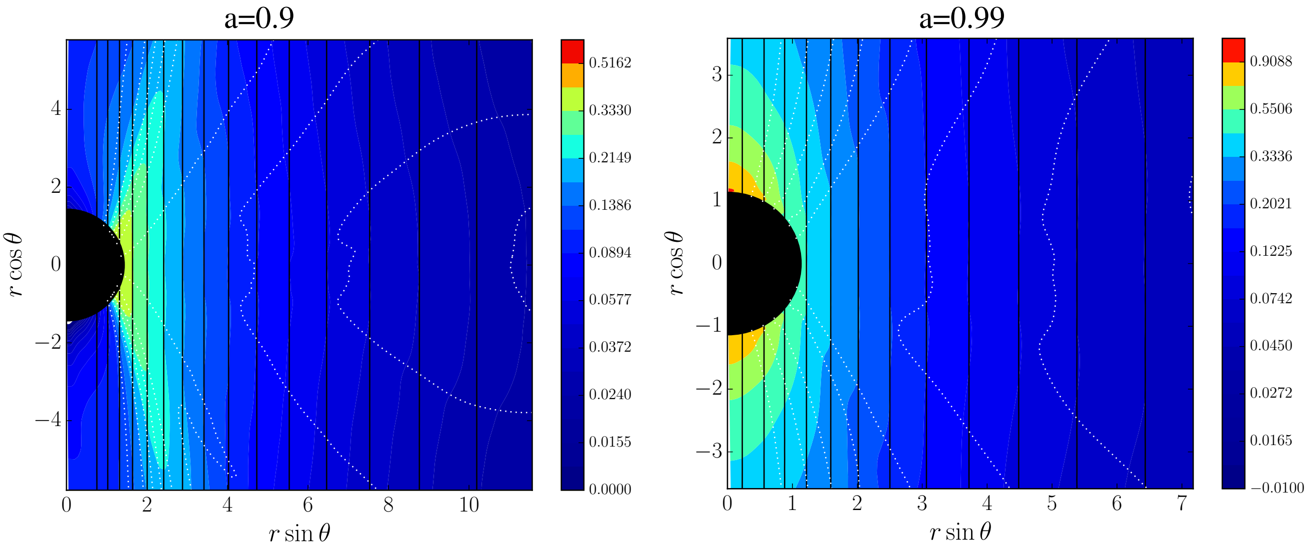

Figure 2: Time-averaged angular frequency $\Omega = u^\varphi/u^t$ map (color-coded) in c^3/GM units, model B1h ($a = 0.9$) in left panel, model D1 ($a = 0.99$) in the right. Black vertical lines correspond to the values $\Omega = \left((r\sin\theta)^{3/2} + a\right)^{-1}$ identical to the color boundary values for the simulated Ω. White dotted lines are the lines of constant net angular momentum $l = -u_\varphi/u_t$. Evidently, above the disk plane, angular momentum is constant along radial rather than vertical lines.

isostrophes (isorotational surfaces) of a barotropic gas are predicted to be concave due to space-time curvature. However, the effect of magnetic field may work in opposite direction. In Fig. 2, it is clearly visible that while the angular velocities are constant on cylindrical surfaces $r\sin\theta = const$, the angular momentum decreases with height. Partially, this may be attributed to the outflows existing further from the disk plane.

Small deviations from Keplerian rotation are expected near the sonic surface. Radial component of Euler equation implies

$$\left(v - \frac{c_s^2}{v}\right) v' = \Omega^2 R - \frac{GM}{R} + c_s^2 \frac{d}{dR}(RH)\,, \tag{2}$$

where v is the radial velocity, and c_s is the speed of sound. When the left-hand side changes its sign, rotation is Keplerian up to $\sim \left(\frac{H}{R}\right)^2$. It is hardly probable that cylindrical surfaces in Boyer-Lindquist coordinates have any physical importance, hence we propose that the real curvatures of the isostrophes cancel out to second order in $\left(\frac{H}{R}\right)^2$.

4 Conclusions

Inner parts of black hole accretion disks in many cases tend to deviate from the standard model. Even for the radiatively efficient flows, most of these deviations are connected to disk thickness and transonic flow near the last stable orbit. The other effect important near the black hole is large-scale magnetic field that we do not consider here. Even a small-scale magnetic field may be dynamically important as it determines the thickness of the supersonic part of the flow which is likely to be magnetically supported. In the sub-sonic part of a moderately thick disk we however expect the flow to be close to Keplerian. More precisely, numerical simulations argue for rotation law of the form $\Omega \simeq \left((r\sin\theta)^{3/2} + a\right)^{-1}$ holding up to $\sim \left(\frac{H}{R}\right)^2$ at one to several last stable orbit radii.

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