Integrable deformations and supergravity backgrounds

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Based on

- **G.I.** 2310.19887
- **G.I.**, K.Sfetsos & K.Siampos 2310.17700

Plan of the talk

Introduction

Supergravity backgrounds for λ-models

Deforming the near-horizon limit of NS1-NS5-NS5

Deforming the near-horizon limit of NS1-NS5

Conclusions

Introduction

- In the past few years there have been various advances in searching for integrable deformations of 2D non-linear σ-models
- **Notable example:** λ-model [Sfetsos '13]
  - ➥ Integrable deformation of a CFT, interpolating between a WZW model and the non-Abelian T-dual of the PCM
- Building blocks: WZW and PCM

$$S = S_{WZW,k}(g) + S_{PCM}(\tilde{g})\,, \qquad g,\, \tilde{g} \in \mathcal{G}$$

- Derivation via gauging procedure that resembles Bucher's approach for T-duality

$$S_{k,\lambda}(g) = S_{WZW,k}(g) - \frac{1}{\pi}\int d^2\sigma J_+\left(\lambda^{-1} - D^T\right)^{-1} J_-$$

$$J_+^a = i\sqrt{k}\,\mathrm{Tr}\left(t^a\partial_+ g\, g^{-1}\right)\,, \quad J_-^a = -i\sqrt{k}\,\mathrm{Tr}\left(t^a g^{-1}\partial_- g\right)$$

where $g \in \mathcal{G}$, $t^a \in \mathrm{Lie}(\mathcal{G})$ and D is the adjoint action of $\mathcal{G}$

Properties:

- Weak/strong coupling symmetry at the level of the action [**G.I.**, Sfetsos & Siampos '14]

$$S_{-k,\lambda^{-1}}(g^{-1}) = S_{k,\lambda}(g)$$

This symmetry should reflect into physical quantities

- Classical integrability [Sfetsos '13; Hollowood, Miramontes & Schmidtt '14; **G.I.**, Sfetsos, Siampos & Torrielli '14]
The equations of motion can be written in a Lax form

$$\partial_+\mathcal{L}_- - \partial_-\mathcal{L}_+ = [\mathcal{L}_+, \mathcal{L}_-]$$

$$\mathcal{L}_\pm = -\frac{2}{\sqrt{k}(1+\lambda)}\frac{z}{z\mp 1}\left(\lambda^{-1} - D^{\pm 1}\right)^{-1} J_\pm\,, \qquad z\in\mathbb{C}$$

and the conserved charges are independent

- For $\lambda = 0$ we find the WZW and for $\lambda \ll 1$ we recover the NATM

$$S = S_{WZW,k}(g) - \underbrace{\frac{\lambda}{\pi}\int d^2\sigma J_+^a J_-^a}_{interaction} \qquad (NATM)$$

where the currents at the CFT point satisfy two commuting algebras

$$J_\pm^a(z)J_\pm^b(0) = \frac{\delta^{ab}}{z^2} + \frac{1}{\sqrt{k}}\frac{f^{abc}J_\pm^c(0)}{z} + \dots$$

- The interaction triggers an RG flow $\Rightarrow G_{\mu\nu}$ and $B_{\mu\nu}$ change. Here the coupling flows with the energy [**G.I.**, Sfetsos & Siampos '14]

$$\beta^\lambda_{1-\text{loop}} = \frac{d\lambda}{d\ln\mu^2} = -\frac{c_\mathcal{G}}{2k}\frac{\lambda^2}{(1+\lambda)^2}$$

Here $c_\mathcal{G}$ is the quadratic Casimir in the adjoint representation

- The UV is at $\lambda = 0$ (CFT point)

- When $\lambda \to 1$ we obtain the NATD of the PCM

$$S = \frac{1}{2\pi} \int d^2\sigma \, \partial_+ u \left(\kappa^2 \mathbb{1} + \mathbb{f}\right)^{-1} \partial_- u \,, \quad \mathbb{f}^{ab} = f^{abc} \, u^c$$

where

$$\lambda = 1 - \frac{\kappa^2}{k} + \dots \,, \qquad g = \mathbb{1} + i \, \frac{u^a t^a}{k} + \dots \,, \qquad k \gg 1$$

- When $\lambda \to -1$ one recovers Nappi's model [Nappi '79]

$$S = \frac{1}{4\pi} \int d^2\sigma \, \partial_+ u \left(b^{-2/3} \mathbb{1} + \frac{\mathbb{f}}{3}\right) \partial_- u \,, \quad \mathbb{f}^{ab} = f^{abc} \, u^c$$

where

$$\lambda = -1 + \frac{1}{b^{2/3} k^{1/3}} + \dots \,, \qquad g = \mathbb{1} + i \, \frac{u^a t^a}{k^{1/3}} + \dots \,, \qquad k \gg 1$$

Example: The $SU(2)$ λ-model

Let us take an $SU(2)$ group element parametrized as

$$g = e^{i\alpha n_i \sigma_i}\,, \qquad n = (-\sin\beta\sin\gamma, \sin\beta\cos\gamma, \cos\beta)$$

The corresponding metric for the λ-model is

$$ds^2 = k\Big(\frac{1+\lambda}{1-\lambda}d\alpha^2 + \frac{1-\lambda^2}{\Delta(\alpha)}\sin^2\alpha\, ds^2(S^2)\Big)$$

where

$$\Delta(\alpha) = (1-\lambda)^2\,\cos^2\alpha + (1+\lambda)^2\,\sin^2\alpha$$

The antisymmetric tensor is

$$B = k\Big(-\alpha + \frac{(1-\lambda)^2}{\Delta(\alpha)}\,\cos\alpha\,\sin\alpha\Big)\mathrm{Vol}(S^2)$$

There is also a scalar field such that

$$e^{-2\Phi} = \Delta(\alpha)$$

Questions:

- Can we take advantage of the deformed σ-models to provide more paradigms of the AdS/CFT correspondence? $\Rightarrow$

  To make contact with holography we will promote some of the deformed σ-models to full solutions of the type-II supergravity

  Embeddings of λ-deformed cosets:

  [Sfetsos, Thompson, '14; Demulder, Sfetsos & Thompson, '15; Hoare & Tseytlin, '15; Borsato, Tseytlin & Wulff, '16; Chervonyi & Lunin, '16, **G.I.** & Sfetsos, '19]

  ➥ Probably non-supersymmetric

- Can we construct embeddings for λ-models on groups?

- If yes, are they supersymmetric?

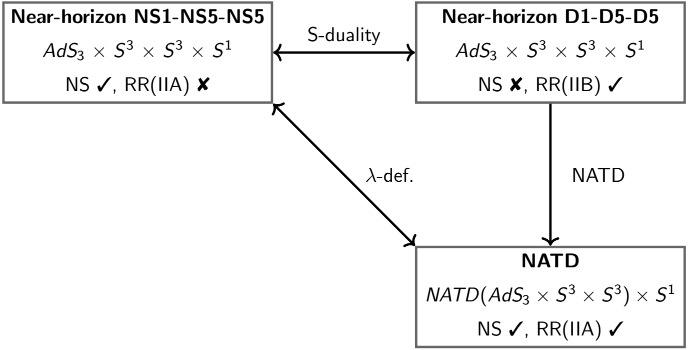
Deforming the near-horizon limit of NS1-NS5-NS5

Based on

- **G.I.** 2310.19887

The goal: Construct the λ-deformation for the near-horizon limit of the NS1-NS5-NS5 intersection $\Rightarrow$ WZW interpretation

- Recall that the λ-model interpolates between a WZW model and the NATD of the PCM
- Our starting point is the S-dual type-IIB solution $\Rightarrow$ Describes the near-horizon limit of the D1-D5-D5 intersection



The solution

Constructing the NS sector

- We will need two copies of the $SU(2)$ λ-model (one for each S^3) and one copy of the $SL(2,\mathbb{R})$ λ-model for the AdS_3
- The λ-model on $SL(2,\mathbb{R})$ can be obtained from the one on $SU(2)$ via

$$\alpha \mapsto \frac{\pi}{2} + i\tilde{\alpha}\,, \qquad \beta \mapsto i\tilde{\beta} - \frac{\pi}{2}\,, \qquad \gamma \mapsto \tilde{\gamma}\,, \qquad k \mapsto -k$$

- For the metric we find

$$ds^2 = L_0^2\, k\Big(\frac{1+\lambda}{1-\lambda}\, d\tilde{\alpha}^2 + \frac{1-\lambda^2}{\tilde{\Delta}(\tilde{\alpha})}\cosh^2\tilde{\alpha}\big(d\tilde{\beta}^2 - \cosh^2\tilde{\beta}\, d\tilde{\gamma}^2\big)\Big)$$

$$+ L_1^2\, k\Big(\frac{1+\lambda}{1-\lambda}\, d\alpha_1^2 + \frac{1-\lambda^2}{\Delta(\alpha_1)}\sin^2\alpha_1\big(d\beta_1^2 + \sin^2\beta_1\, d\gamma_1^2\big)\Big)$$

$$+ L_2^2\, k\Big(\frac{1+\lambda}{1-\lambda}\, d\alpha_2^2 + \frac{1-\lambda^2}{\Delta(\alpha_2)}\sin^2\alpha_2\big(d\beta_2^2 + \sin^2\beta_2\, d\gamma_2^2\big)\Big) + d\omega^2$$

Notice that:

(i) For $\lambda = 0$ the geometry is $AdS_3 \times S^3 \times S^3 \times S^1$

(ii) The radii for the AdS_3 and the two S^3's are restricted as

$$\frac{1}{L_0^2} = \frac{1}{L_1^2} + \frac{1}{L_2^2}$$

(iii) The geometry $AdS_3 \times S^3 \times T^4$ can be obtained by a zoom-in limit

$$\alpha_2 = \frac{\rho}{L_2}\,, \qquad L_2 \to \infty$$

(iv) The deformed geometry is non-singular for $0 \leqslant \lambda < 1$

(v) The deformation breaks isometries

$$AdS_3 \times S^3 \times S^3 \quad \mapsto \quad AdS_2 \times S^2 \times S^2$$

- For the dilaton we simply add the contributions of the scalars for the λ-models on $SL(2,\mathbb{R})$ and $SU(2)$

$$\Phi = -\frac{1}{2}\ln\Big(\tilde{\Delta}(\tilde{\alpha})\Delta(\alpha_1)\Delta(\alpha_2)\Big)$$

  Notice that when $\lambda = 0$ the dilaton vanishes

- Similarly for the NS 2-form

$$B_2 = L_0^2\, k\Big(\tilde{\alpha} + \frac{(1-\lambda)^2}{\tilde{\Delta}(\tilde{\alpha})}\cosh\tilde{\alpha}\,\sinh\tilde{\alpha}\Big)\cosh\tilde{\beta}\, d\tilde{\beta}\wedge d\tilde{\gamma}$$

$$+L_1^2\, k\Big(-\alpha_1 + \frac{(1-\lambda)^2}{\Delta(\alpha_1)}\cos\alpha_1\,\sin\alpha_1\Big)\sin\beta_1\, d\beta_1\wedge d\gamma_1$$

$$+L_2^2\, k\Big(-\alpha_2 + \frac{(1-\lambda)^2}{\Delta(\alpha_2)}\cos\alpha_2\,\sin\alpha_2\Big)\sin\beta_2\, d\beta_2\wedge d\gamma_2$$

  Notice that for $\lambda = 0$ the field strength is given in terms of the volume forms of AdS_3 and S^3's

Constructing the RR sector

The RR poly-form for the deformed solution is obtained from the one for D1-D5-D5 through [**G.I.**'23]

$$e^{\Phi}\,\not{\mathbb{F}}_{\lambda} = \mu\,\not{\mathbb{F}}_{D1-D5-D5}\,\Omega^{-1}$$

- Here Ω can be written in terms of the Γ-matrices in 10D as

$$\Omega = \frac{1}{\sqrt{\tilde{\Delta}(\tilde{\alpha})\Delta(\alpha_1)\Delta(\alpha_2)}}\Big((1-\lambda)\sinh\tilde{\alpha}\Gamma^{012} + (1+\lambda)\cosh\tilde{\alpha}\Gamma^{2}\Big)$$
$$\times\Big((1-\lambda)\cos\alpha_1\Gamma^{345} - (1+\lambda)\sin\alpha_1\Gamma^{3}\Big)$$
$$\times\Big((1-\lambda)\cos\alpha_2\Gamma^{678} - (1+\lambda)\sin\alpha_2\Gamma^{6}\Big)$$

- The constant μ depends only on k and λ and vanishes for $\lambda = 0$
- The poly-form $\mathbb{F}_{\lambda}$ contains RR forms of even rank $\Rightarrow$ type-IIA
  The type-IIB counterpart is obtained by T-duality along S^1
- The transformation mimics the transformation of the RR fields under NATD [Sfetsos, Thompson '10, '14]

Supersymmetry

Dilatino: Treat the $\lambda = 0$ and $\lambda \neq 0$ cases separately

- For $\lambda = 0$ the dilatino is solved by a single projection $\Rightarrow$ 16 SUSYs
- For $\lambda \neq 0$ an extra projection is required $\Rightarrow$ 8 SUSYs

Gravitino: Does not impose further projections. It is solved by

$$\epsilon = \exp\left(-\frac{1}{2}\tanh^{-1}\left(\frac{1-\lambda}{1+\lambda}\tanh\tilde{\alpha}\right)\Gamma^{01}\sigma_3\right)\exp\left(\frac{\tilde{\beta}}{2}\Gamma^{02}\sigma_3\right)\left(-\frac{\tilde{\gamma}}{2}\Gamma^{12}\sigma_3\right)$$

$$\times\exp\left(-\frac{1}{2}\tan^{-1}\left(\frac{1+\lambda}{1-\lambda}\tan\alpha_1\right)\Gamma^{45}\sigma_3\right)\exp\left(\frac{\beta_1}{2}\Gamma^{34}\right)\left(\frac{\gamma_1}{2}\Gamma^{45}\right)$$

$$\times\exp\left(-\frac{1}{2}\tan^{-1}\left(\frac{1+\lambda}{1-\lambda}\tan\alpha_2\right)\Gamma^{78}\sigma_3\right)\exp\left(\frac{\beta_2}{2}\Gamma^{67}\right)\left(\frac{\gamma_2}{2}\Gamma^{78}\right)\eta$$

where η is a constant spinor that satisfies

$$\left(\frac{L_0}{L_1}\Gamma^{012345} + \frac{L_0}{L_2}\Gamma^{012678}\right)\eta = -\eta\,, \quad \Gamma^{019}\sigma_1\eta = -\eta \text{ (only for } \lambda \neq 0)$$

Deforming the near-horizon limit of NS1-NS5

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The λ-deformed $AdS_3 \times S^3 \times T^4$ [G.I., Sfetsos, Siampos '23]

Promoting the λ-model on $SL(2,\mathbb{R}) \times SU(2)$ into a full solution of type-II supergravity results into a background that:

- Interpolates between:
  (a) The near-horizon limit in the NS1-NS5 intersection ($AdS_3 \times S^3 \times T^4$)
  (b) The NATD of the near-horizon limit in the D1-D5 inresection
- The deformation breaks isometries

$$AdS_3 \times S^3 \quad \mapsto \quad AdS_2 \times S^2$$

- Supersymmetry is broken by 1/2: 16 SUSYs for $\lambda = 0$ & 8 for $\lambda \neq 0$
- Fits in the class of type-IIB solutions on $AdS_2 \times CY_2 \times \Sigma_2$ preserving 8 SUSYs [Legramandi, Macpherson, Passias '23], [Lozano, Nunez, Ramirez '21]
- The deformed solution can also be obtained from the λ-deformed $AdS_3 \times S^3 \times S^3 \times S^1$ via zoom-in limit [G.I.'23]

Conclusions

- We promoted the λ-models based on $SL(2,\mathbb{R}) \times SU(2)$ and $SL(2,\mathbb{R}) \times SU(2) \times SU(2)$ to full solutions of type-II supergravity
- When $\lambda = 0$ we recover the near-horizon limits of NS1-NS5 and NS1-NS5-NS5
- When $\lambda \to 1$ we recover the NATD for the near-horizon limits of D1-D5 and D1-D5-D5
- The deformed geometries maintain $AdS_2 \times S^2$ and $AdS_2 \times S^2 \times S^2$ subspaces
- In the presence of deformation SUSY breaks $\Rightarrow$ 8 SUSYs

For the future

- Understanding of the holographic dual theory
- Integrability in the presence of fluxes
- Check whether the solutions admit supersymmetric probe branes

Thank you!