Observers and flows in black hole spacetimes: The inside story

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Frames and observers: geometry probed by particles

Regular coordinates for Schwarzschild

Eddington-Finkelstein, Kruskal-Szekeres,Lemaitre, etc.

Unification: Fomin 1968 (local Lorentz transformation)
Martel and Poisson 2001
Bronnikov et al 2012
Lemos and Silva 2021
Toporensky and OZ 2022

Dynamics and geometry: physical realization of frame

Two faces: parameter e of transformation and energy/momentum of particle

Frames and limiting transition under horizon

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + r^2 d\Omega^2$$

$$dt = \frac{e_0}{f}\sqrt{F}\,dT - \frac{\sqrt{G}P_0}{f}d\rho$$

$$P_0 = \sqrt{e^2{}_0 - f}$$

$$dr = e_0\sqrt{G}\,d\rho - \sqrt{F}\,P_0\,dT$$

$$e_0 = \frac{1}{\sqrt{1-V^2}}$$

$$ds^2 = -F dT^2 + G d\rho^2 + r^2 d\Omega^2$$

If we put F=1 and will use previous radial coordinate, we have

$$d\tilde{t} = e_0 dt + \frac{P_0 dr}{f},$$

instead of T we use in this particular case $\tilde{t}$

$$ds^2 = -d\tilde{t}^2 + 2\frac{d\tilde{t}drV}{e_0} + \frac{dr^2}{e^2{}_0}$$

Generalization of GP metric

$e_0 \to 0$ Singular transformation. Both coordinates fail to be independent

Impossible to take limit in this metric directly

The proper distance grows indefinitely, metric becomes singular

Another synchronous frame under horizon

T=-r, t=y, g=- f>0 Novikov's approach

$$ds^2 = -\frac{dT^2}{g} + g dy^2 + T^2 d\Omega^2.$$

$$ds^2 = -d\hat{t}^2 + g dy^2 + T^2(\hat{t}) d\Omega^2.$$

What happens in the limit $e_0 \to 0$

Singular transformation. Term with $d\rho$ drops out

Works under horizon only, f<0 $P_0 = \sqrt{e^2{}_0 - f}$

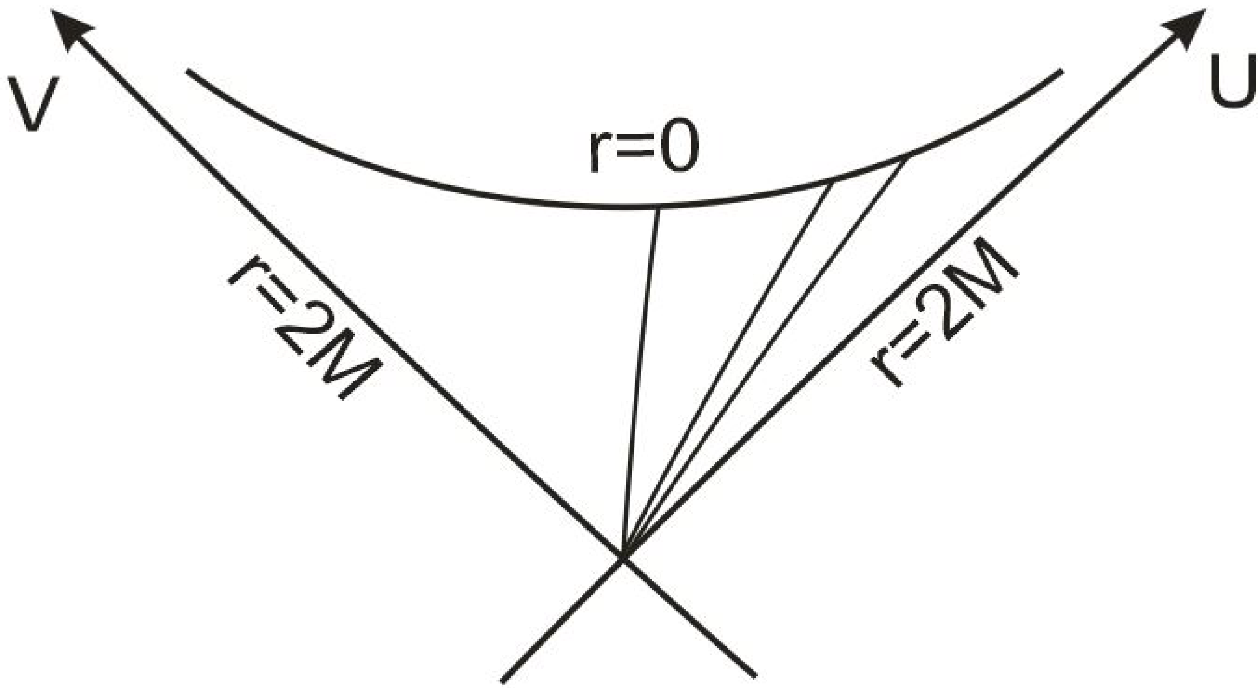
For a synchronous metric the limit is allowed, provided we make rescaling

$$\rho = e_0 \tilde{\rho}$$

$$ds^2 = -dT^2 + g(r(T))d\tilde{\rho}^2 + r^2 d\Omega^2 \qquad g = -f > 0$$

$$T = -\int^r \frac{d\tilde{r}}{g(\tilde{r})}$$

Novikov presentation,
Particular case of Kantowski-Sachs cosmology



Time-like geodesics for e_0=0

Another version: modification of GP system under horizon

From $\tilde{t}, r$ to $\tilde{t}, t$

$$ds^2 = -\frac{g}{P^2{}_0} d\tilde{t}^2 + \frac{g^2 dt^2}{P^2{}_0} + 2\frac{e_0 g dt d\tilde{t}}{P^2{}_0} + r^2(t,\tilde{t}) d\Omega^2$$

$$ds^2 = -d\tilde{t}^2 + \frac{g^2}{P^2{}_0}(dt + \frac{e_0 d\tilde{t}}{g})^2 + r^2(t,\tilde{t}) d\Omega^2$$

Under horizon t is spacelike, so we have 1 spacelike and 1 timelike coordinates
In GP system two timelike under horizon.
Nondiagonal term defines flow velocity $-\frac{e_0}{g}$

can be interpreted as a velocity with respect to frame where fiducial observer has $e_0 = 0$
Metric dual to GP, arranged for region under horizon, has smooth limit to $e_0 = 0$

General radially moving references frames in the black hole background

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + r^2 (d\theta^2 + d\varphi^2 \sin^2\theta).$$

$$d\tilde{t} = e_0 dt + \frac{dr}{f} P_0$$

$$P_0 \equiv \sqrt{e_0^2 - f}.$$

Velocities and their behavior

Frame attached to free falling observer

$$e_0$$

Energy of observer

$$e$$

energy of particle

$$V^{(1)} = \frac{P_0 e - P e_0}{e_0 e - P P_0}$$

$$P_0 \equiv \sqrt{e_0^2 - f}$$

$$V^{(3)} = \frac{\mathbf{L}\sqrt{-f}}{rP}$$

Horizon limit

If e and e_0 have the same sign

$$V^{(1)} \rightarrow V_H^{(1)} = \frac{e_0^2(1+\frac{\mathbf{L}^2}{r_g^2}) - e^2}{e_0^2(1+\frac{\mathbf{L}^2}{r_g^2}) + e^2}$$

$$V^{(3)} \rightarrow V_H^{(3)} = \frac{2ee_0\mathbf{L}}{r_g[e^2 + e_0^2(1+\frac{\mathbf{L}^2}{r_g^2})]}.$$

For different signs

$$|V^{(1)}| \to 1$$

$$V^{(3)} \to 0$$

Classification of frames

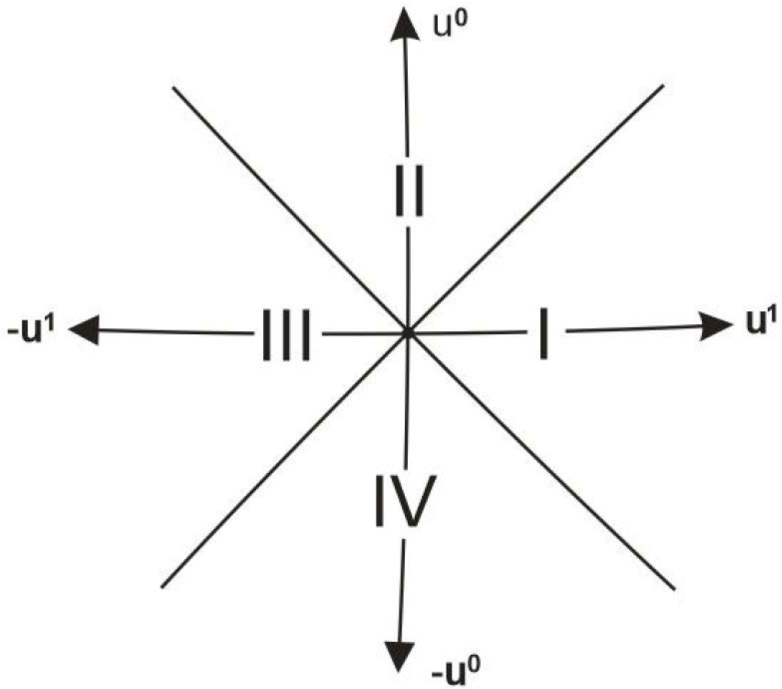


FIG. 1: The Kruskal diagram for the Schwarzschild metric

Frame	Regions covered
$(C, +)$	I, II
$(C, -)$	III, II
$(E, +)$	IV, I
$(E, -)$	IV, III

E – expanding, C – contracting, sign of e_0 indicated

MOTION WITH ANGULAR MOMENTUM AND HORIZON ASYMPTOTICS

Frame / Particle	$e > 0,\ P > 0$	$e < 0,\ P > 0$	$e > 0,\ P < 0$	$e < 0,\ P < 0$
$e_0 > 0,\ P_0 > 0$	$V_H^{(1)};\ V_H^{(3)}$	$+1;\ 0$	$+1;\ 0$	–
$e_0 < 0,\ P_0 > 0$	$-1;\ 0$	$-V_H^{(1)};\ V_H^{(3)}$	–	$-1;\ 0$
$e_0 > 0,\ P_0 < 0$	$-1;\ 0$	–	$-V_H^{(1)};\ V_H^{(3)}$	$-1;\ 0$
$e_0 < 0,\ P_0 < 0$	–	$+1;\ 0$	$+1;\ 0$	$V_H^{(1)};\ V_H^{(3)}$

Near singularity

$$L \neq 0$$

$$V^{(3)} \to sign\mathbf{L} = \pm\mathbf{1}$$

$$V^{(1)} \approx \frac{e_0}{P_0} \approx \frac{e_0}{\sqrt{-f}} \to 0$$

pure radial motion appears to be unstable—an arbitrary small deviation grows infinitely and results in an ultrarelativistic motion in an angular direction. If initially the directions of the vectors *L* are distributed randomly, the corresponding particles have mutual ultrarelativistic relative velocities near a singularity.

If

$$e_0 = 0$$

$$L = 0$$

$$V^{(3)} = 0$$

$$V^{(1)} \approx \frac{e_0 - e}{\sqrt{-f}} \to 0$$

Massive versus massless,
Redshift – blueshift under horizon

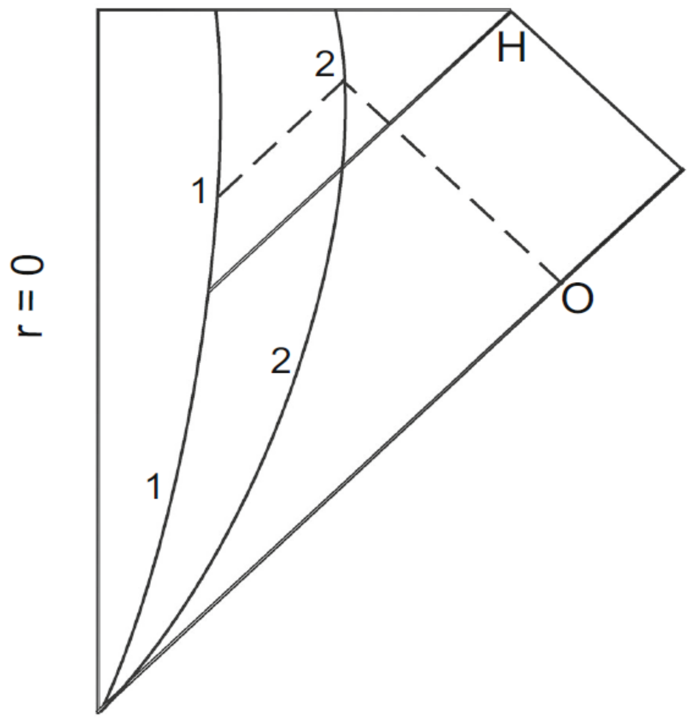


Table 2 Behavior of the frequency near the singularity

	ω
$\mathcal{L}l > 0$	Finite nonzero
$\mathcal{L}l < 0$	Infinite blueshift
$\mathcal{L} = 0, l \neq 0$	Infinite blueshift
$\mathcal{L} \neq 0, l = 0$	Infinite blueshift
$\mathcal{L} = 0 = l$	Infinite redshift

Relation with BSW effect (indefinite growth of energy
In CM frame)

Schwarzschild – no BSW effect, provided both

$$e > 0 \qquad e_0 > 0$$

R - region

White holes (Grib and Pavlov): possible

If particle comes from mirror universe,

$$e_0 < 0$$

BSW possible but in weak version

Lemaitre time to reach horizon is infinite for one particle and finite for another. Bifurcation point, its vicinity.

Frame and particle dynamics

$$ds^2 = -\frac{f}{e_0^2}d\tilde{t}^2 + \frac{2d\tilde{t}dr}{e_0^2}P_0 + \frac{dr^2}{e_0^2} + r^2 d\omega^2.$$

$$d\rho = \frac{dr}{P_0} + d\tilde{t}$$

Generalization of Lemaitre

$$ds^2 = -d\tilde{t}^2 + \frac{P_0^2}{e_0^2}d\rho^2 + r^2 d\omega^2.$$

$$\frac{d\rho}{d\tau} = \frac{e_0(eP_0 - e_0 P)}{f}$$

If $e = e_0$ $\rho = const$

Potential applications

RN metric. Frame with e_0=1 is insufficient beyond the turning point since f grows and f>1

S-dS

$$f = 1 - \frac{2m}{r} - \frac{\Lambda r^2}{3}$$

$$f_{max} < 1 \qquad v = \sqrt{1 - f}$$

Frame either contracting or expanding

$$e^2{}_0 = f_{max}$$

Can describe both

Summary

- General frame with arbitrary e_0
- e-Lemaitre, e-GP
- including e_0<0, classification of frames
- Limiting transition e_0--→0
- General formulas for 3-velocity
- Behavior ner singularity. Blueshift is typical

Thank you!