Rotational tidal Love numbers and their impact on compact object inspirals

Gonçalo Castro

based on
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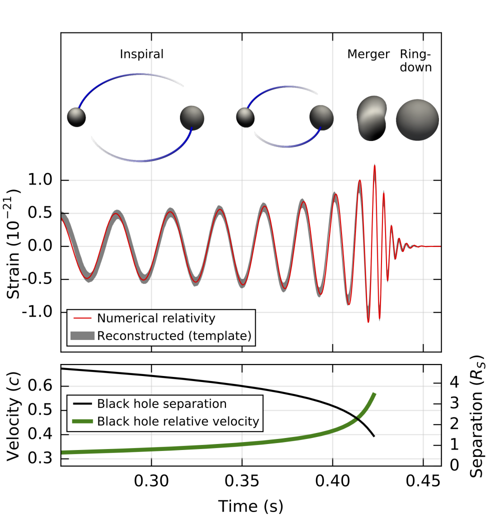
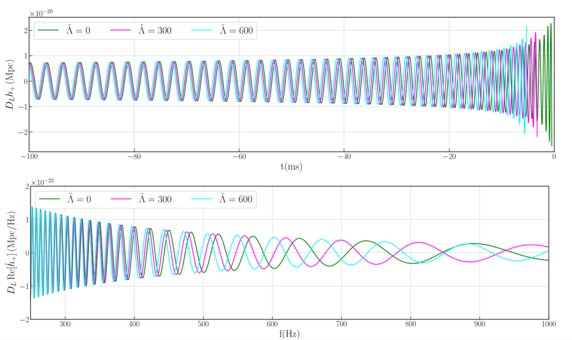
Compact object merger



(B.P. Abbott *et al*, Phys. Rev. Lett. 116, 061102)

Tidal deformability



(Katerina Chatziioannou, Gen Relativ Gravit 52, 109)

- Phase of gravitational waveform depends on tidal deformability

Relativistic tidal Love numbers - Static background

$$g_{00} = g_{00}^{(0)} + \sum_{l} g_{00}^{(l,\text{tidal})} \mathcal{E}^{(l)} + g_{00}^{(l,\text{response})} k_l^{\text{el}} \mathcal{E}^{(l)}$$

$$g_{00} = -1 + \frac{2M}{r} + \frac{3}{r^2} Q_{ij} n^{<i} n^{j>} + O\left(\frac{1}{r^2}\right) - \mathcal{E}_{ij} r^2 n^{<i} n^{j>} + O(r^2)$$

Induced multipole moments (adiabatic relations):

$$M_2 = \lambda_E^2 \mathcal{E}_0^{(2)}$$

$$S_3 = \lambda_M^3 \mathcal{B}_0^{(3)}$$

Love number definition

$$\lambda_E^{(l)} = \frac{\partial M_l}{\partial \mathcal{E}_0^{(l)}}, \quad \lambda_M^{(l)} = \frac{\partial S_l}{\partial \mathcal{B}_0^{(l)}}$$

Effects of rotation

Slowly-rotating: 1st order in spin (Hartle-Thorne metric)

Assumptions:

- Time-independent perturbations
- Axisymmetric perturbations
- Static fluid ($\delta u^t \neq 0$, $\delta u^i = 0$)
- Quadrupolar and octupolar, electric and magnetic external tidal fields

$$g_{00} = g_{00}^{(0)} + (...)\mathcal{E}^{(2)} + g_{00}^{(2,\chi,\text{tidal})}\chi\mathcal{B}^{(3)} + g_{00}^{(2,\chi,\text{response})}\lambda_E^{(23)}\chi\mathcal{B}^{(3)}$$

$$g_{0\phi} = g_{0\phi}^{(0)} + (...)\mathcal{B}^{(3)} + g_{0\phi}^{(3,\chi,\text{tidal})}\chi\mathcal{E}^{(2)} + g_{0\phi}^{(3,\chi,\text{response})}\lambda_M^{(32)}\chi\mathcal{E}^{(2)}$$

Effects of rotation

New induced multipoles sourced by the coupling between spin χ and the tidal field of opposite parity and $l \pm 1$

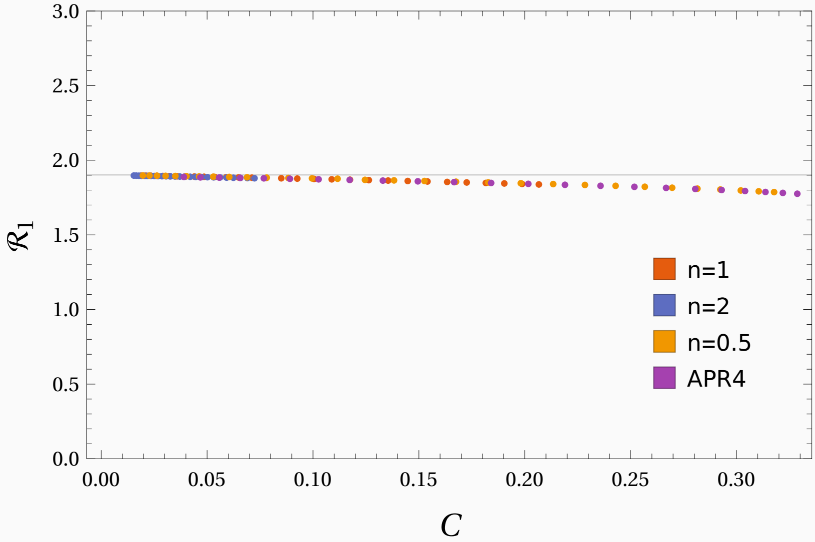
$$M_2 = \lambda_E^2 \mathcal{E}_0^{(2)} + \chi \lambda_E^{(23)} \mathcal{B}_0^{(3)}, \quad M_3 = \lambda_E^3 \mathcal{E}_0^{(3)} + \chi \lambda_E^{(32)} \mathcal{B}_0^{(2)}$$

$$S_2 = \lambda_M^2 \mathcal{B}_0^{(2)} + \chi \lambda_M^{(23)} \mathcal{E}_0^{(3)}, \quad S_3 = \lambda_M^3 \mathcal{B}_0^{(3)} + \chi \lambda_M^{(32)} \mathcal{E}_0^{(2)}$$

Rotational Love numbers' definition

$$\delta\lambda_E^{(l,l')} = \frac{\partial M_l}{\partial \mathcal{B}_0^{(l')}}, \quad \delta\lambda_M^{(l,l')} = \frac{\partial S_l}{\partial \mathcal{E}_0^{(l')}}$$

Ratio between RTLNs



Ratio between RTLNs - what is happening?

Hidden symmetry between RTLNs!

- Ratios of rotational tidal Love numbers of opposite parities are independent of equation of state
- Result valid for non-polytropic EOSs

However:

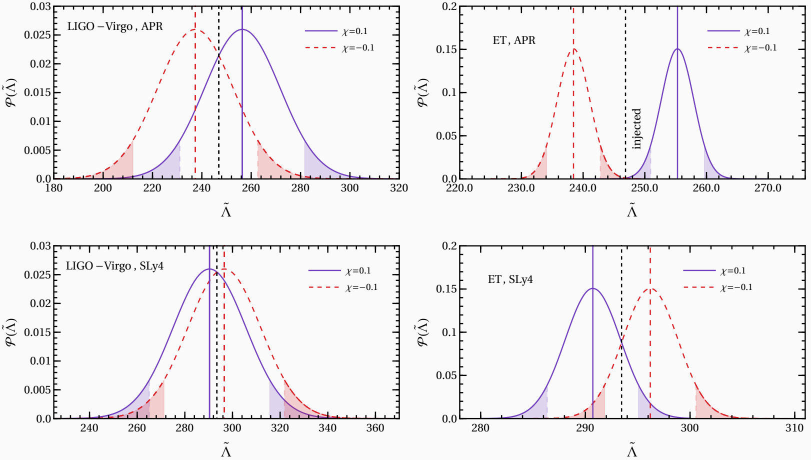
- $\lesssim 5\%$ deviation from predicted value at higher compactness (but still EOS independent)

Gravitational waveform

$$\psi_{\mathsf{T}} = -\frac{39}{2}\tilde{\Lambda}x^5 + \left(-\frac{3115}{64}\tilde{\Lambda} + \delta\frac{6595}{364}\delta\tilde{\Lambda} + \tilde{\Sigma}\right)x^6 + \left(\hat{\Lambda} + \hat{\Sigma} + \hat{K} + \hat{\Gamma}\right)x^{\frac{13}{2}}$$

- $\hat{\Lambda}$, $\hat{\Sigma}$: spin-TLNs
- $\hat{K}$: tidal tail
- $\hat{\Gamma}$: RTLNs

Effect of spin-tidal terms



Measurability of $\hat{\Gamma}$

	EOS	χ_1	χ_2	$\tilde{\Lambda}$	$\hat{\Gamma}$
$\chi = 0.1$	APR4	$0.10^{+0.06}_{-0.12}$	$0.10^{+0.19}_{-0.10}$	550^{+80}_{-90}	$7.7^{+1.8}_{-2.0} \times 10^5$
	SLy4	$0.10^{+0.06}_{-0.10}$	$0.10^{+0.17}_{-0.10}$	690^{+90}_{-90}	$8.4^{+1.9}_{-1.9} \times 10^5$
$\chi = 0.05$	APR4	$0.05^{+0.09}_{-0.11}$	$0.05^{+0.18}_{-0.14}$	550^{+90}_{-90}	$3.9^{+2.0}_{-1.6} \times 10^5$
	SLy4	$0.05^{+0.10}_{-0.10}$	$0.05^{+0.16}_{-0.15}$	690^{+100}_{-90}	$4.2^{+2.2}_{-1.6} \times 10^5$

But spins and RTLNs entangled:

$$\hat{\Gamma} = \frac{\chi_1}{M^4}\left[\left(856\eta_1 - 816\eta_1^2\right)\lambda_{23}^{(1)} - \left(\frac{833\eta_1}{3} - 278\eta_1^2\right)\sigma_{23}^{(1)} - \nu\left(272\lambda_{32}^{(1)} - 204\sigma_{32}^{(1)}\right)\right] + (1 \leftrightarrow 2)\,.$$

Measurability of spins is a work in progress

Conclusions

- We developed tools for RTLN computation
- Hidden symmetry confirmed for opposite parity RTLNs
- 6.5PN tidal terms (RTLNs included) necessary for accurate measurement of tidal deformability $\tilde{\Lambda}$
- $\hat{\Gamma}$ constrainable but innacuracy on spin measurement could prevent its use for EOS studies
- Next step: dynamical tidal perturbations

Newtonian Love numbers

Gravitational potential of deformed body:

$$U_{\text{deformed}} = \frac{M}{r} - \sum_{lm} \frac{4\pi}{2l+1} \frac{I_{lm}}{r^{l+1}} Y_{lm}$$

External potential:

$$U_{\text{tid}} = \sum_{lm} \frac{4\pi}{2l+1} r^l d_{lm} Y_{lm}$$

Love number definition

$$I_{lm} = 2k_l R^{2l+1} d_{lm}, \quad \delta R = \sum_{lm} \frac{4\pi}{2l+1} \frac{R^{l+2} h_l}{M} d_{lm}$$

$$k_l = \frac{l+1-\eta_l(R)}{2(l+\eta_l(R))}, \quad h_l = 1 + 2k_l$$

RTLNs in a Lagrangian formulation

Phys. Rev. D 98, 104046 (2018), Abdelsalhin et al

- Impact of RTLNs in the GW phase at 6.5 PN order
- Studied via a Lagrangian formalism:

$$\mathcal{L}_2^{\rm int} = -\frac{1}{4\lambda_2} Q^{ab} Q^{ab} - \frac{1}{12\lambda_3} Q^{abc} Q^{abc} - \frac{1}{6\sigma_2} S^{ab} S^{ab} - \frac{1}{16\sigma_3} S^{abc} S^{abc} + \alpha J_2^a Q^{bc} S^{abc} + \beta J_2^a S^{bc} Q^{abc} \, ,$$

- Simplification of adiabatic relations
  - $\sigma_{32} = 8\lambda_2\sigma_3\alpha$, $\lambda_{23} = 2\lambda_2\sigma_3\alpha$
  - $\lambda_M^{(32)}/\lambda_E^{(23)} = c^{st}$

Dynamical detour

- Previous computations done under "static" fluid assumption: $\delta u^i = 0$
- Physically motivated conditions are of irrotational fluid: $\delta u^r = \delta u^\theta = 0$, $\delta u^\phi \neq 0$, in the static limit

For a slowly-rotating object, δu^ϕ depends on dynamical quantities; time-dependent perturbations necessary

Impact of RTLNs in measurement of $\tilde{\Lambda}$

$$(h|h_T) = 4\mathcal{R}\int_{f_{\rm min}}^{f_{\rm max}} df\,\frac{\tilde{h}(f)\tilde{h}_T^*(f)}{S_n(f)}$$

- $S_n(f)$: noise spectral density; design aLIGO and ET
- h: "true" signal
- h_T: template signal

$$\mathcal{M}(h(\gamma_0), h_T(\gamma_T)) = \max_{\theta_0,\theta_T}\frac{(h|h_T)}{\sqrt{(h|h)\,(h_T|h_T)}}$$

$$p(\gamma_T) \propto \exp\left[-\rho^2\left(1-\mathcal{M}(h|h_T)\right)\right]$$