Acceleration and radiation of cosmic rays near by astrophysically black hole

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Motivation

No hair theorem: M(mass) , a(spin) and Q(charge).

- Mass:

- can be measured from the orbits near by objects.
- many other methods mass measurement.

- Spin is loosely constrained:

- has no Newtonian effect
- regime of strong gravity is needed
- spin can be determined based on the modelling of e.g. the light curves of a hot spot or a jet base

- Charge Q:

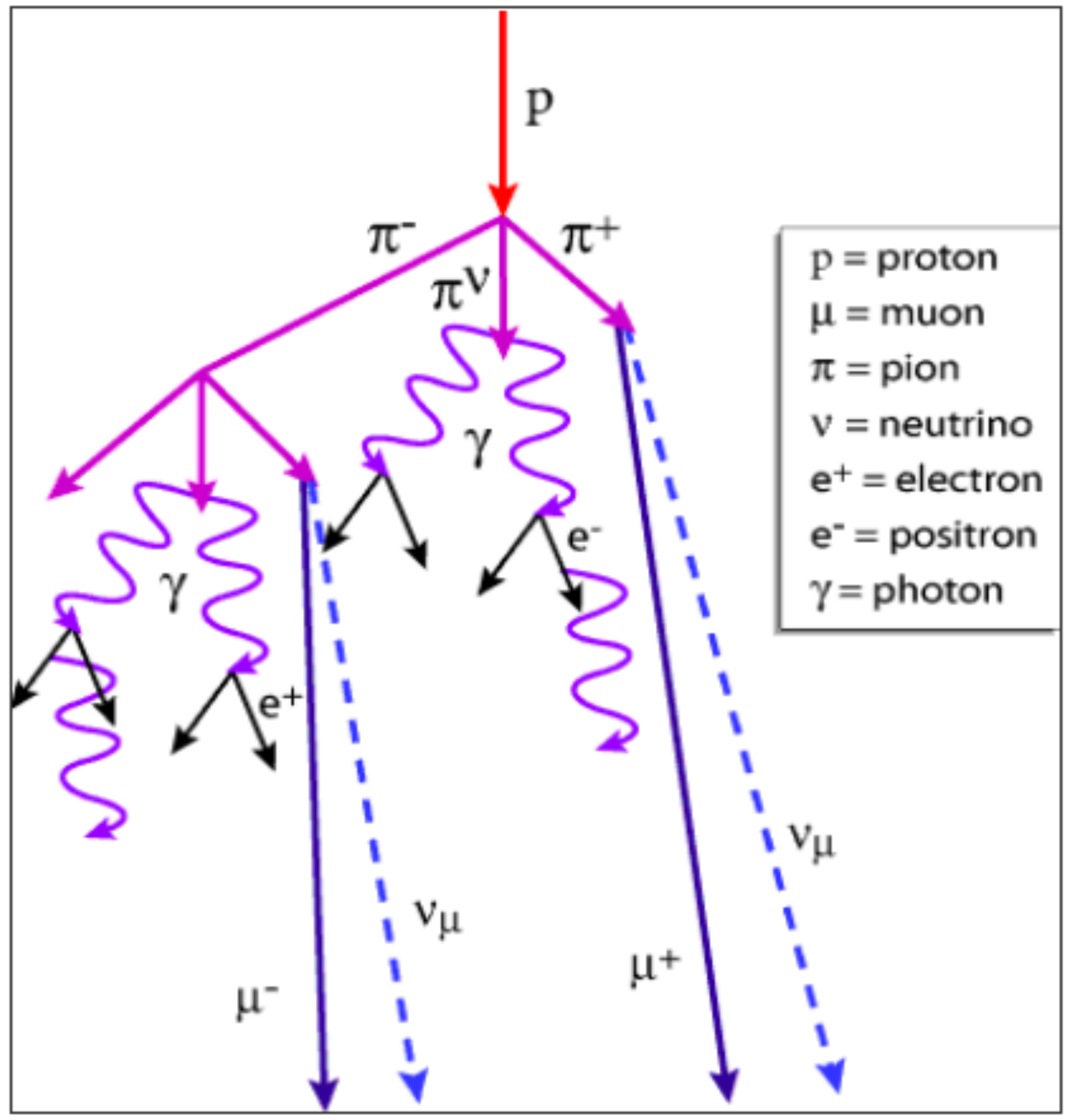
- Maximum value of the charge of the black hole $\sqrt{\frac{Q_G^2 G}{c^4}} = \frac{2GM}{c^2} \Rightarrow Q_G \approx 10^{30} \frac{M}{M_\odot}$Fr.
- Realistic value of the black hole's charge $10^{11} \frac{M}{M_\odot}$Fr $\leq Q_{\mathrm{BH}} \leq 10^{18} \frac{M}{M_\odot}$Fr. *Zajaček M. et al.(2018)*

CRs and UHECRs observations

UHECRs:

- Unreachable energy by Earth based experiments
- Charged particles
- Spectrum has knees and ankle
- Extremely rare at ultra-high energies
- Extra-Galactic origin
- Detected mostly on Earth - Composition at high energy

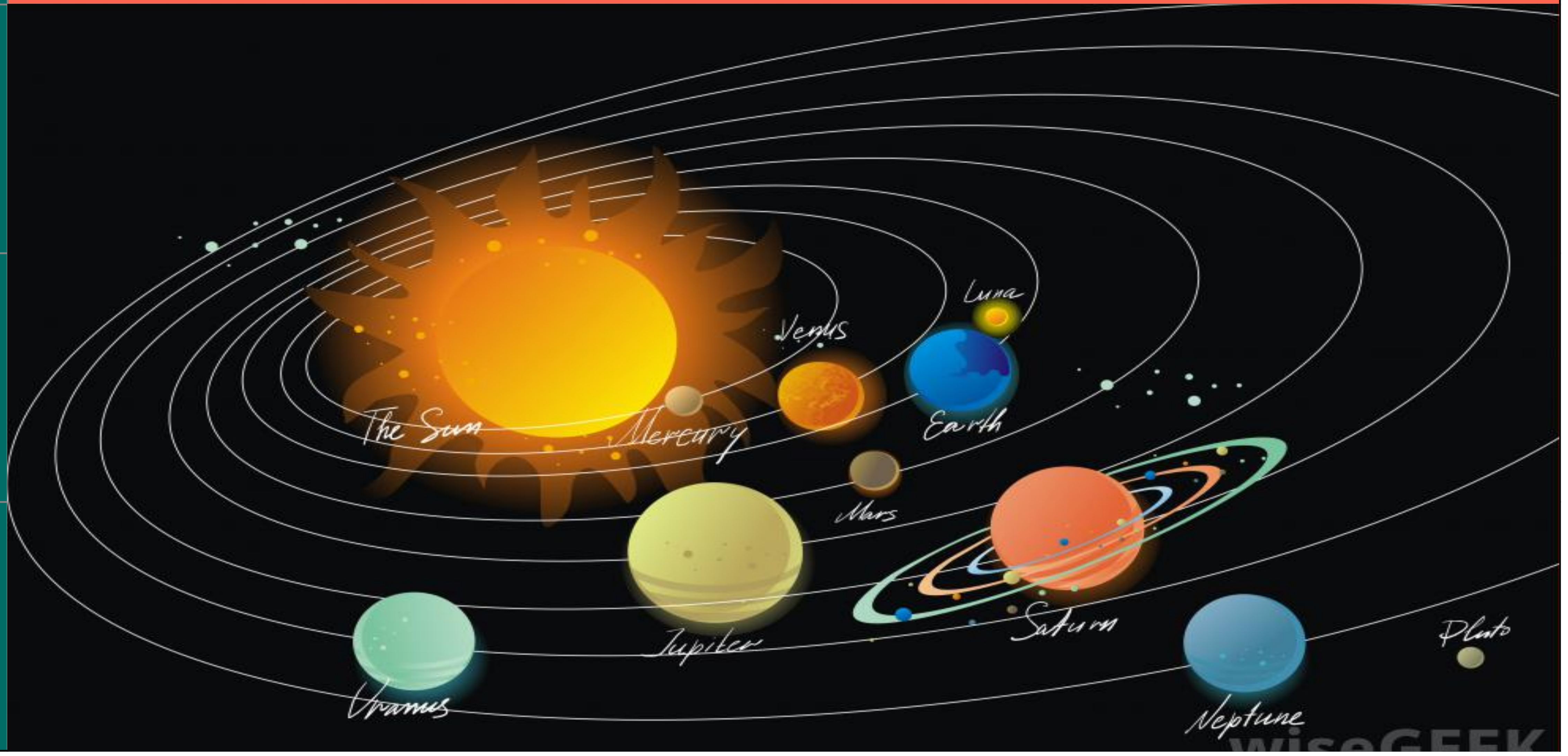
* Mechanism is unknown - most energetic accelerator in the universe!



Production of a cosmic-ray extensive air shower.

How to create UHECRs?

Exotic scenarios	Acceleration scenarios
Extra dimensions scenarios	Powerful source with enough available energy
Lorentz invariance violation	Build accelerator of ~400mln km size with LHC technology:
Existence of new particles	
Along magnetic field lines	
Topological defects, strings, SUSY…	



Motion around weakly charged black hole

Schwarzschild metric:

$$\mathrm{d}s^2 = -\left(1 - \frac{2M}{r}\right)\mathrm{d}t^2 + \left(1 - \frac{2M}{r}\right)^{-1}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2)$$

Four potential:

$$A_\mu = \left(-\frac{Q}{r}, 0, 0, 0\right)$$

Specific energy and angular momentum:

$$\mathscr{E} = \frac{E}{m}, \quad \mathscr{L} = \frac{L}{m}$$

Super Hamiltonian and equations of motion:

$$H = \frac{1}{2}g^{\alpha\beta}(\pi_\alpha - qA_\alpha)(\pi_\beta - qA_\beta) + \frac{1}{2}m^2 = 0, \quad \pi_\mu = p_\mu + qA_\mu$$

$$\frac{\mathrm{d}X^\alpha}{\mathrm{d}\zeta} = \frac{\partial H}{\partial P_\mu}, \quad \frac{\mathrm{d}P_\mu}{\mathrm{d}\zeta} = -\frac{\partial H}{\partial X^\mu}$$

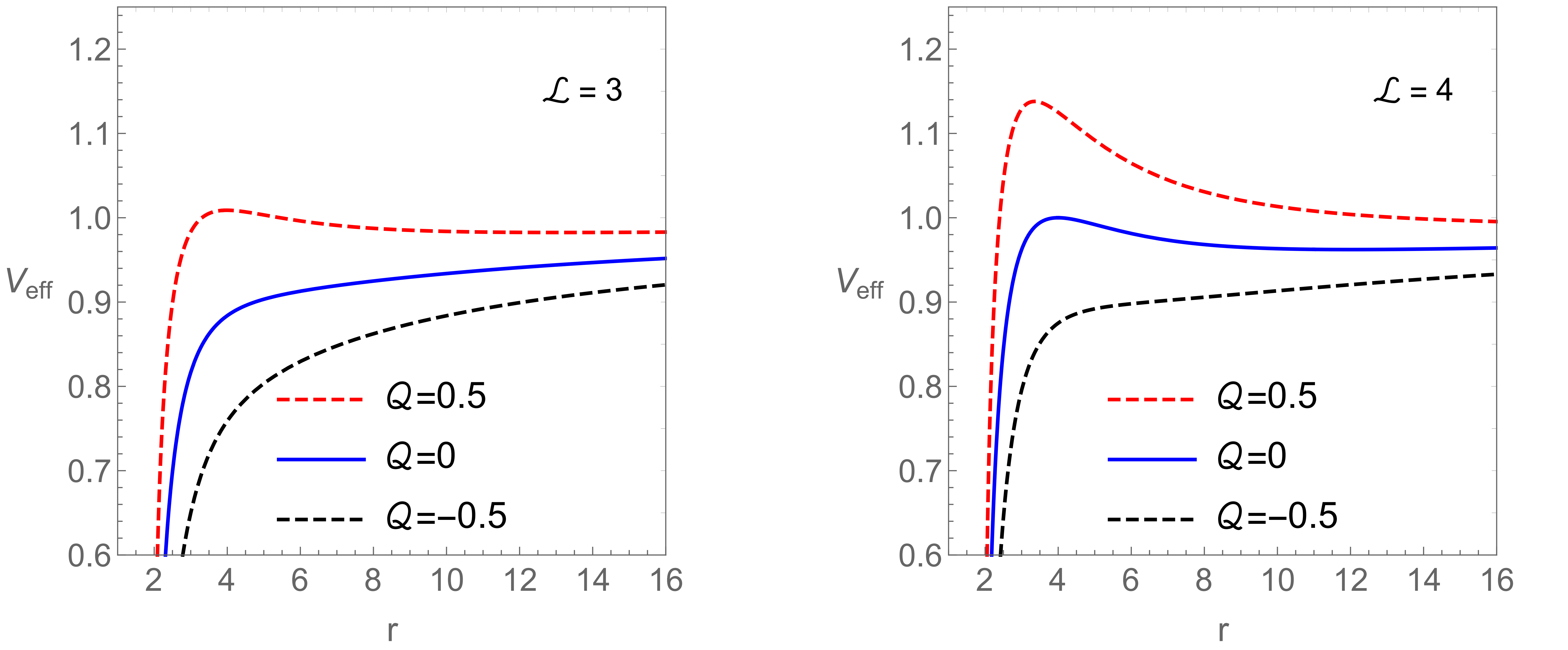
Motion around weakly charged black hole

Effective potential:

$$V_{eff}(r) = \frac{\mathcal{Q}}{r} + \sqrt{\left(1 - \frac{2M}{r}\right)\left(1 + \frac{\mathcal{L}^2}{r^2 \sin^2\theta}\right)},$$

Electric parameter:

$$\mathcal{Q} = \frac{Qq}{m}$$



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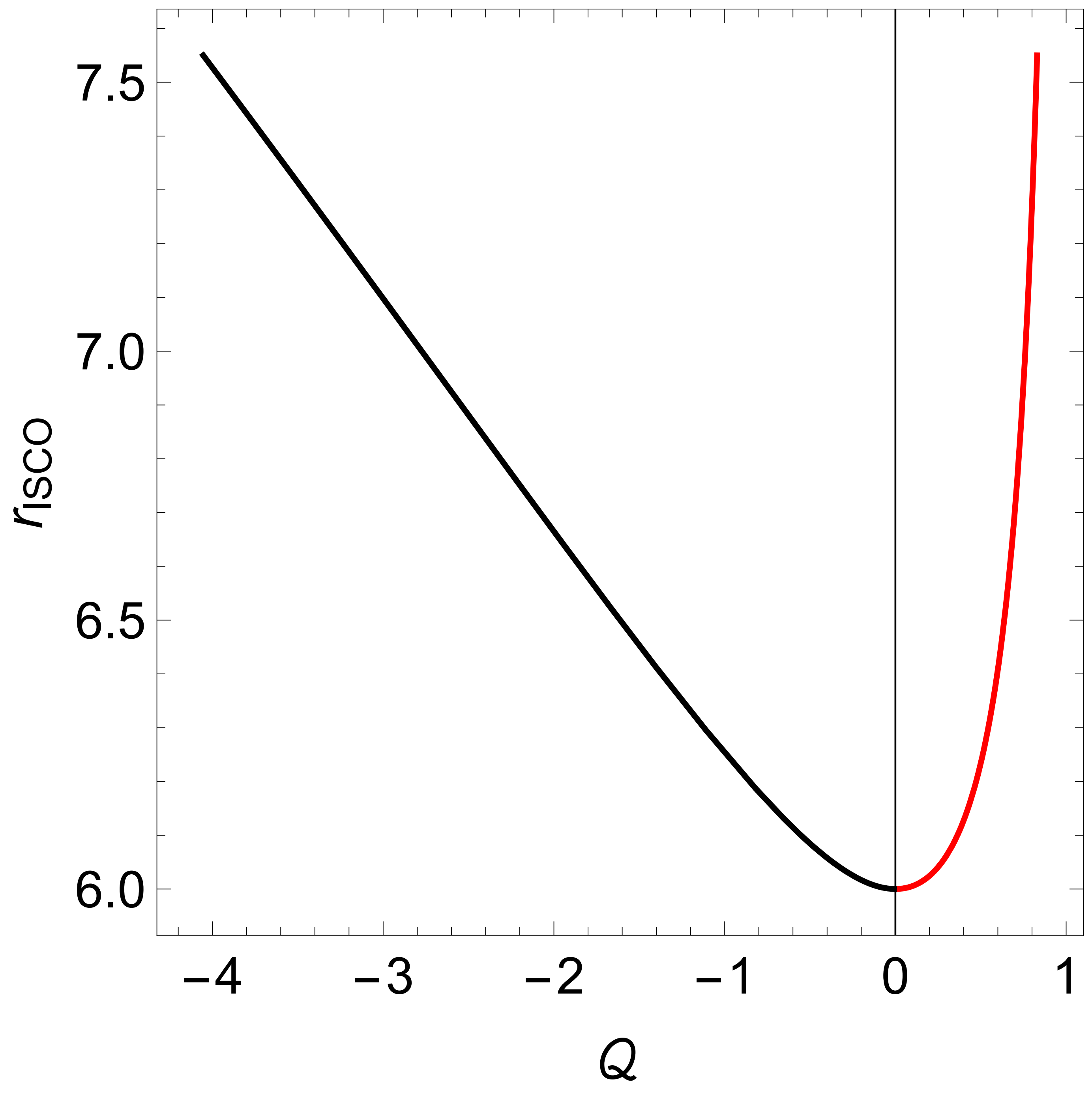
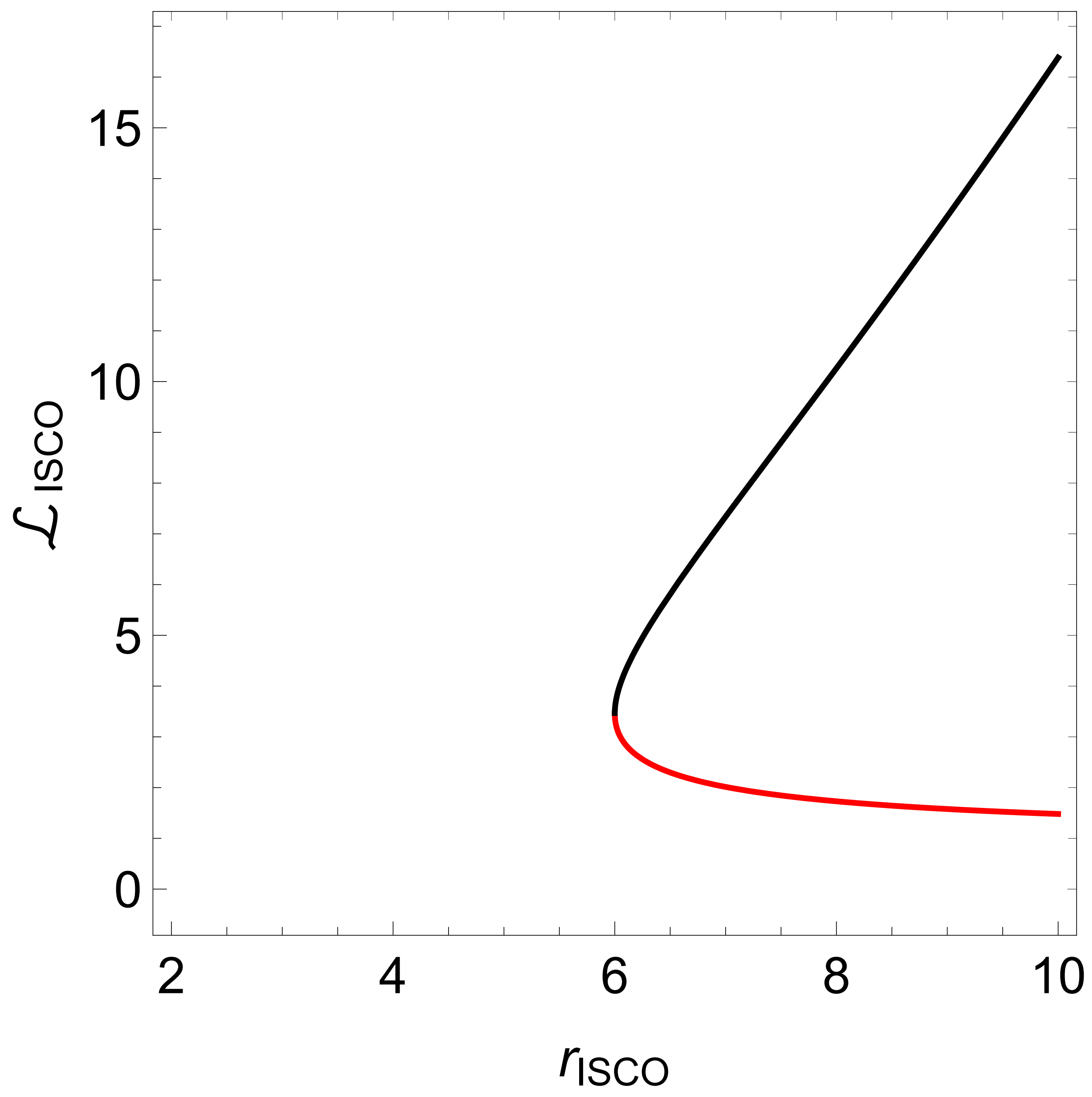
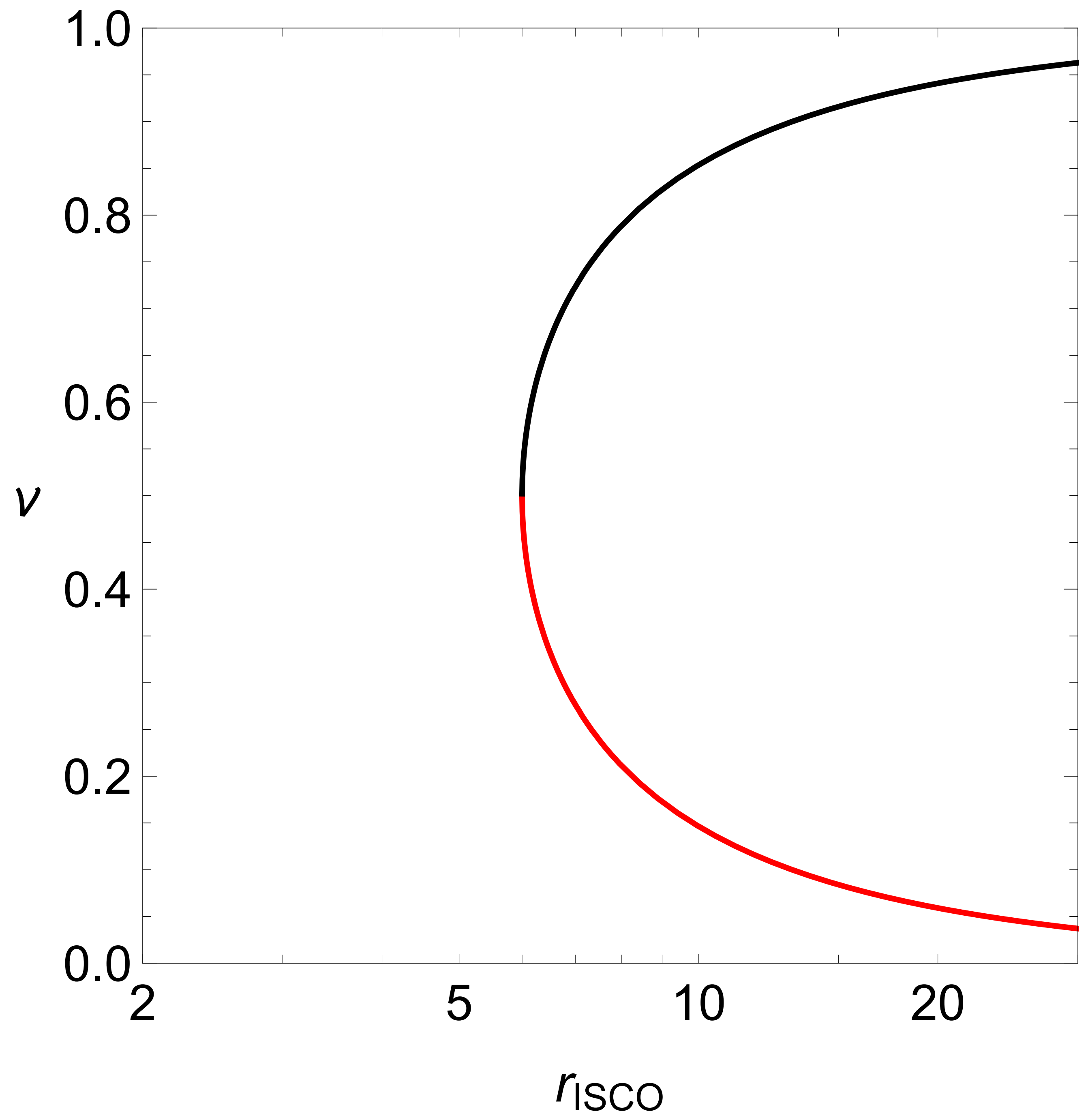
ISCO and velocity at ISCO of the charged particle

$$\partial_r^2 V_{eff}(r, \mathscr{L}, \mathscr{Q}) = \mathscr{L}^2 r^2 (J(r-2)+2) + r^4 (J(r-2) - r + 3) + \mathscr{L}^4((r-3)r+3) = 0,$$

$$v = \sqrt{\frac{1}{1 + r_{isco}^2 / \mathscr{L}_{isco}^2}}$$







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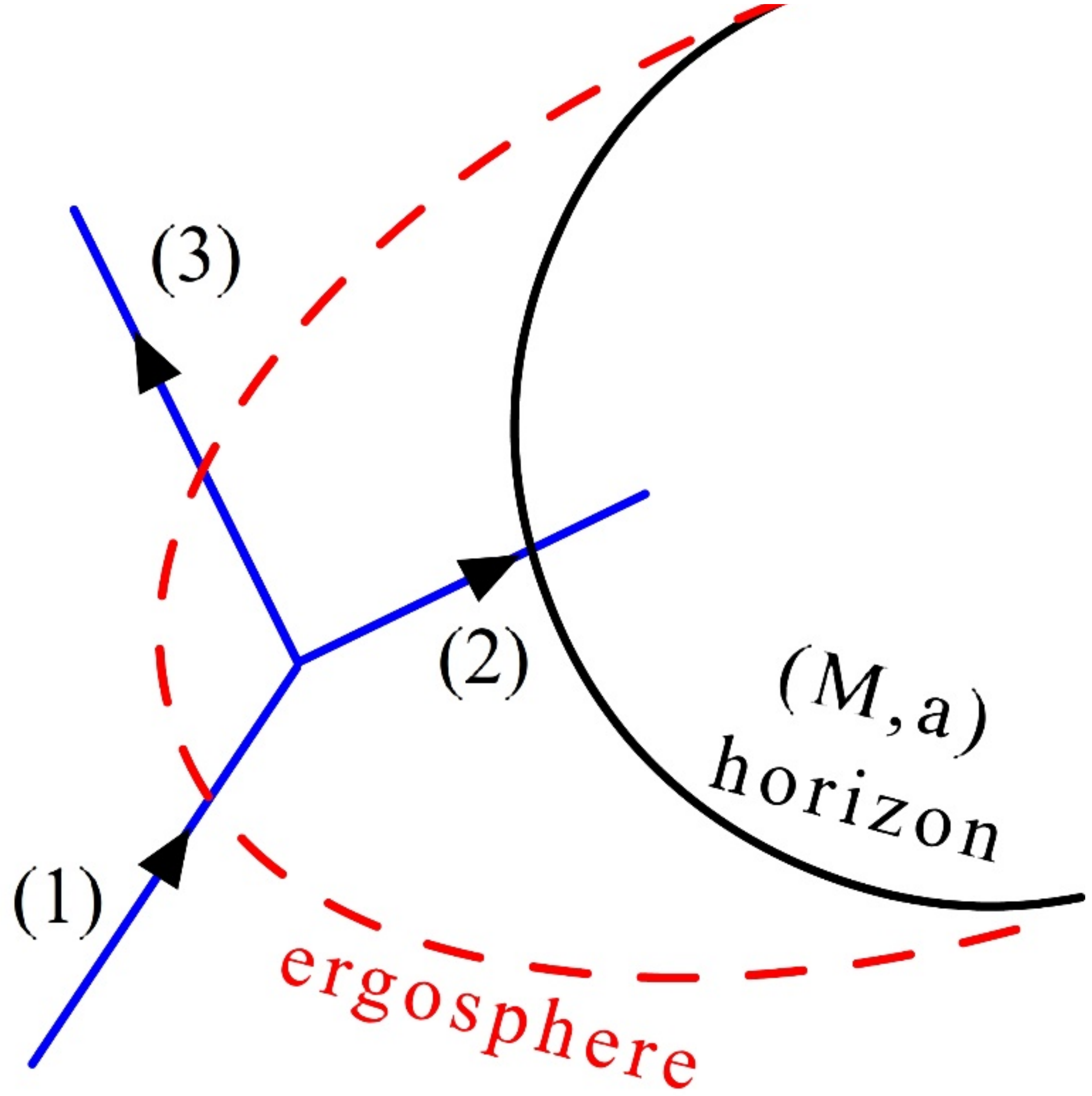
Original Penrose Process



Conservation laws:

$$E_1 = E_2 + E_3, \quad L_1 = L_2 + L_3,$$

$$m_1 \dot{r}_1 = m_2 \dot{r}_2 + m_3 \dot{r}_3,$$

$$0 = m_2 \dot{\theta}_2 + m_3 \dot{\theta}_3,$$

$$m_1 \geq m_2 + m_3,$$

$$m_1 u_1^{\phi} = m_2 u_2^{\phi} + m_3 u_3^{\phi}$$

Efficiency of Penrose process:

$$\eta = \frac{E_3 - E_1}{E_1} = \frac{-E_2}{E_1}, \qquad \eta_{max} = 21\,\%$$

Electric Penrose Process

Particle 1 splits into 2 fragments 2 and 3 close to the horizon:

$$E_1 = E_2 + E_3, \quad L_1 = L_2 + L_3, \quad q_1 = q_2 + q_3,$$
$$m_1 \dot{r}_1 = m_2 \dot{r}_2 + m_3 \dot{r}_3, \quad m_1 \geqslant m_2 + m_3$$

$$m_1 u_1^\phi = m_2 u_2^\phi + m_3 u_3^\phi$$

Noticing that $u^\phi = \Omega u^t = \Omega e / f(r)$, where $e_i = (E_i + q_i A_t)/m_i$

$$E_3 = \left(\frac{1}{\sqrt{2 r_{ion}}} + \frac{1}{2} \right) E_1 + \frac{q_3 Q}{r_{ion}}$$

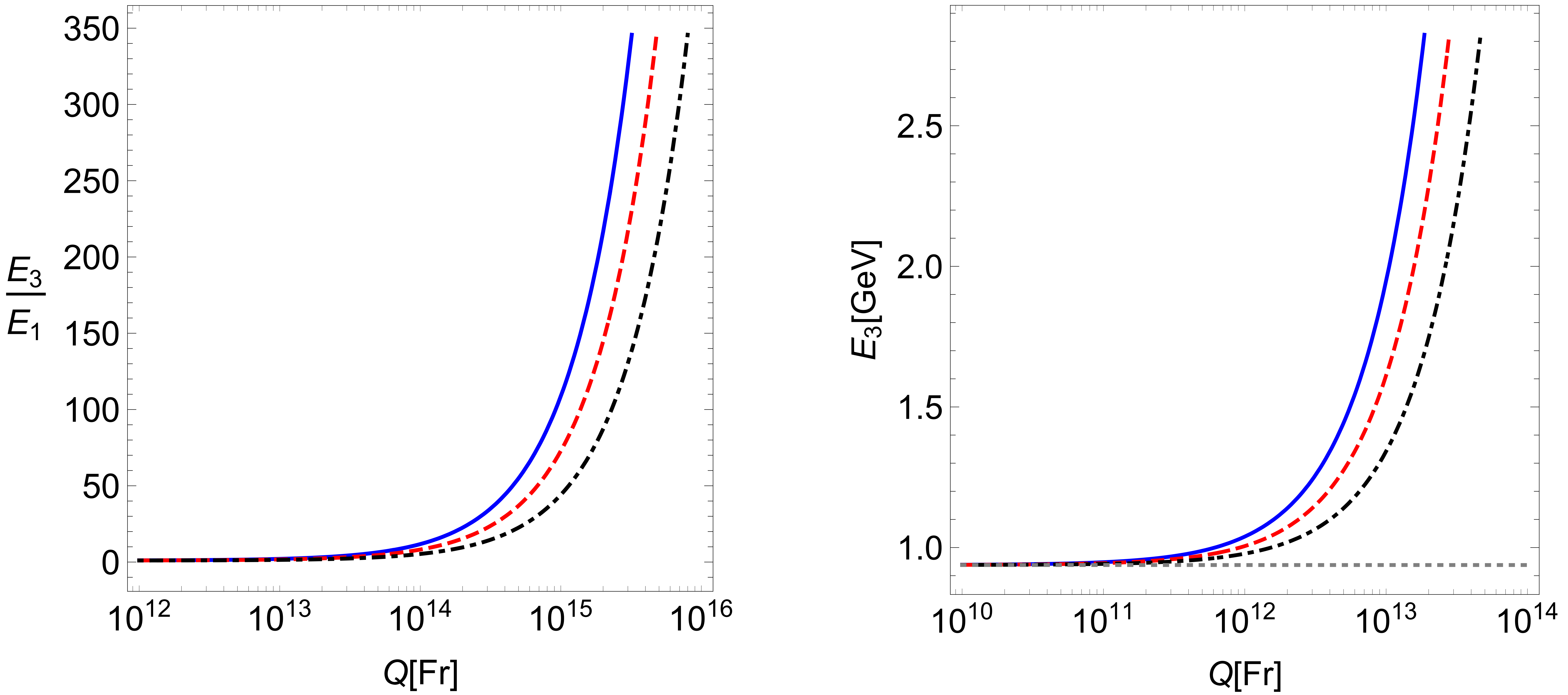
Acceleration of particles by weakly charged black hole

$$\eta = \frac{E_3}{E_1} = \frac{1}{2} + \sqrt{\frac{GM}{2r_{ion}c^2}} + \frac{ZeQ}{Am_n c^2 r_{ion}}$$

Blue: $r_{ion} = 2GM/c^2$

Red dashed: $r_{ion} = 3GM/c^2$

Black dot-dashed: $r_{ion} = 4GM/c^2$



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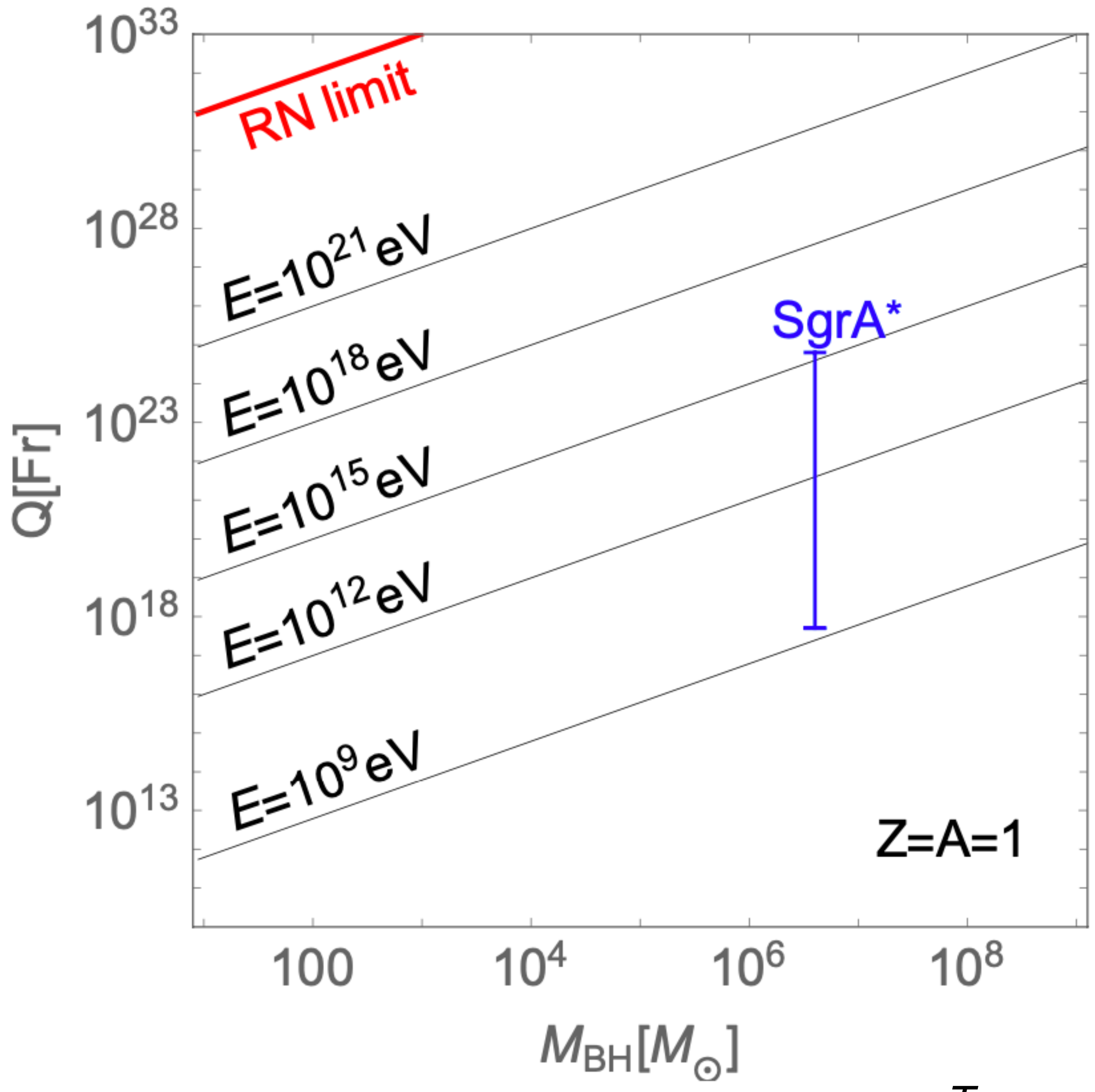

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Blandford-Znajek mechanism and Electric Penrose Process

Penrose Process:	**Blandford-Znajek mechanism**	**Electric Penrose Process**
Type of black hole:	Rotating black hole	Non-rotating black hole
Extracted:	Rotational energy	Electrostatic energy
Field:	Magnetic field	Electric field
Escape:	Along magnetic field lines	Isotropic escape
Trajectory:	Curled	Straight

Particle radiation near a weakly charged black hole

Electric field — Gravity — Radiation reaction

$$\frac{du^\mu}{d\tau} = \frac{q}{m} g^{\mu\rho} F_{\rho\nu} u^\nu + \frac{q}{m} g^{\mu\rho} \mathscr{F}_{\rho\nu} u^\nu - \Gamma^\mu_{\alpha\beta} u^\alpha u^\beta$$

$u^\mu = dx^\mu / d\tau$ – Particle four velocity

$\Gamma^\mu_{\alpha\beta} = \frac{1}{2} g^{\mu\gamma}(g_{\gamma\alpha,\beta} + g_{\gamma\beta,\alpha} - g_{\alpha\beta,\gamma})$ – Christoffel symbols

$F_{\rho\nu} = \partial_\rho A_\nu - \partial_\nu A_\rho$ – Tensor of electromagnetic field

$\mathscr{F}_{\rho\nu} = \partial_\rho \mathscr{A}_\nu - \partial_\nu \mathscr{A}_\rho$ – Self-force of charged particle

Four potential:

$$A_\mu = \left(-\frac{Q}{r}, 0, 0, 0\right)$$

Particle radiation near a weakly charged black hole

DeWitt & Brehme equation (1960):

$$\frac{Du^\mu}{d\tau} = \frac{q}{m}F^\mu{}_\nu u^\nu + \frac{2q^2}{m^2}\left(\frac{D^2u^\mu}{d\tau^2} + u^\mu u_\nu \frac{D^2u^\nu}{d\tau^2}\right) + \frac{q^2}{3m}(R^\mu{}_\lambda u^\nu + R^\nu{}_\lambda u_\nu u^\lambda u^\mu) + \frac{q^2}{m}u_\nu \int_{-\infty}^{\tau} D^{[\mu}G^{\nu]}_{+\lambda'}(\tau,\tau')u^{\lambda'}(\tau')d\tau'$$

- Ricci terms are irrelevant in vacuum metrics
- Tail term can estimated, e.g. around Schwarzschild black hole:

$$\frac{F_{tail}}{F_N} \sim \frac{q^2}{mMG} \sim 10^{-19}\left(\frac{q}{e}\right)\left(\frac{m_e}{m}\right)\left(\frac{10M_\odot}{M}\right)$$

e.g. Dewitt & Dewitt (1964), Smith & Will (1980), Gal'tsov (1982), …

For elementary particles in astrophysical scenarios the equation of motion can be simplified to the covariant form of LD equation:

$$\frac{Du^\mu}{d\tau} = \frac{q}{m}F^\mu{}_\nu u^\nu + \frac{2q^2}{m^2}\left(\frac{D^2u^\mu}{d\tau^2} + u^\mu u_\nu \frac{D^2u^\nu}{d\tau^2}\right)$$

Particle radiation near a weakly charged black hole

$$\frac{Du^\mu}{d\tau} = \frac{q}{m} F^\mu{}_\nu u^\nu + \frac{2q^2}{m^2} \left(\frac{D^2 u^\mu}{d\tau^2} + u^\mu u_\nu \frac{D^2 u^\nu}{d\tau^2} \right)$$

Solutions

1. Direct integration - runaway solutions!:

   Requires properly chosen initial conditions

   Time dispersion error - backward integration helps

2. Reduction of order of equation - covariant Landau - Lifshitz equation:

$$\frac{Du^\mu}{d\tau} = \frac{q}{m} F^\mu{}_\nu u^\nu + k\tilde{q} \left(\frac{DF^\alpha{}_\beta}{dx^\mu} u^\beta u^\mu + \tilde{q} (F^\alpha{}_\beta F^\beta{}_\beta + F_{\mu\nu} F^\nu{}_\sigma u^\sigma u^\alpha) u^\mu \right)$$

Equations of motion:

$$\frac{du^t}{d\tau} = \frac{Qu^r}{r^2 f} + \frac{k\mathcal{Q}}{r^4 f}\left[fr^3(u^\phi)^2 + (u^r)^2\{\mathcal{Q}u^t - 2r\} + fu^t\{\mathcal{Q} - f(u^t)^2\}\right] - \frac{2}{r^2 f}u^r u^t$$

$$\frac{du^r}{d\tau} = \frac{\mathcal{Q}fu^t}{r^2} + \frac{k\mathcal{Q}u^r}{r^4 f}\left[\mathcal{Q}f + \mathcal{Q}(u^r)^2 - f^2 u^t\{2r + \mathcal{Q}u^t\}\right] + \frac{(u^r)^2}{r^2 f} + f\left[r(u^\phi)^2 - \frac{(u^t)^2}{r^2}\right]$$

$$\frac{du^\phi}{d\tau} = \frac{k\mathcal{Q}u^\phi}{r^4 f}\left[\mathcal{Q}(u^r)^2 + f^2 u^t\{r - \mathcal{Q}u^t\}\right]$$

Electric parameter:

$$\mathcal{Q} = \frac{Qq}{m}$$

Radiation parameter:

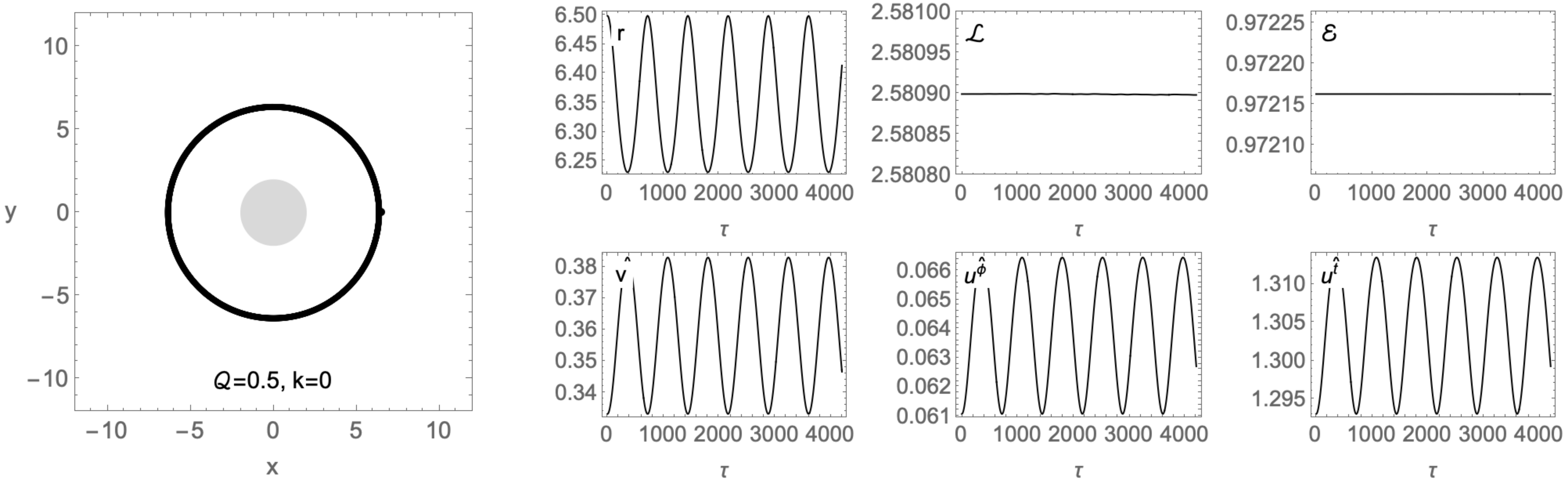
$$k = \frac{2q^2}{3m}$$

Orbital widening

Without radiation:

$$\mathcal{Q} = \frac{Qq}{m} = 0.5$$

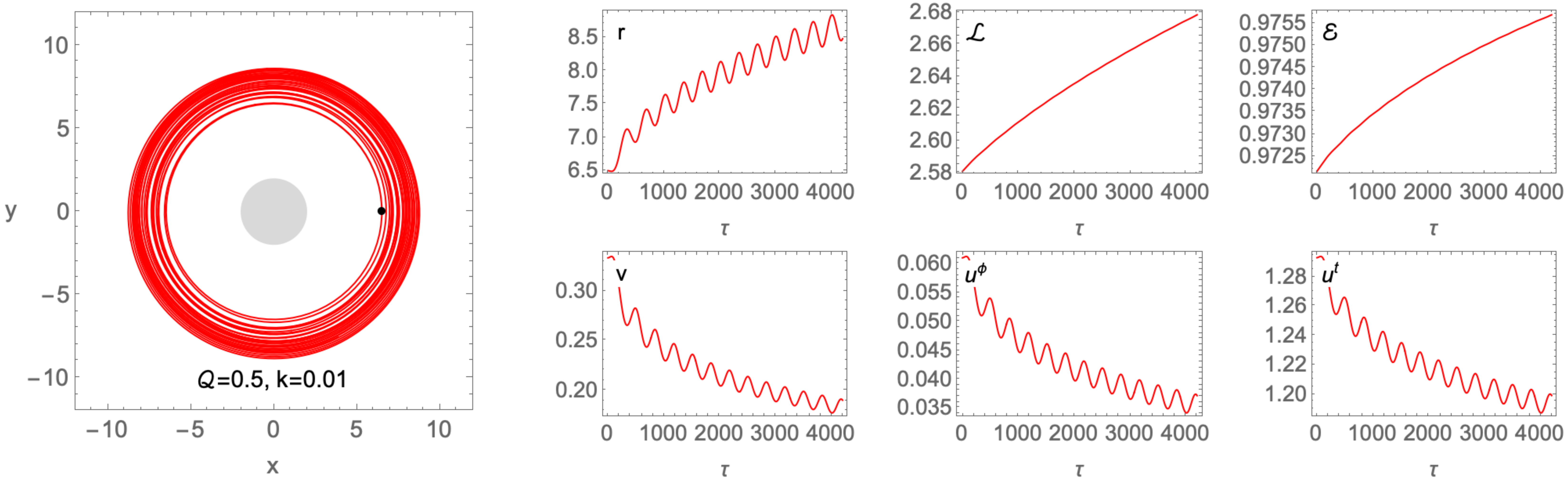
$$k = \frac{2q^2}{3m} = 0$$



With radiation:

$$\mathcal{Q} = \frac{Qq}{m} = 0.5$$

$$k = \frac{2q^2}{3m} = 0.01$$



In Preparation: Juraev B.,Tursunov A., Stuchlík Z. and Kološ M.

Conclusion

- The energy of ionized particle can be much greater than the initial energy of the neutral particle if both charges of the ionized particle and black hole have the same sign.
- No rotation of BH is needed! Energy comes in expense of the electrostatic energy of the BH
- Energy of a charged particle is $E_{ion} \sim Q/M_{BH}$ i.e. *E* is restricted by the charge-to-mass ratio of BH
- Combined gravitational and electric field is spherically symmetric, therefore one would expect isotropic statistics of escaping charged particles with no preferred direction of the motion.
- Particle escapes shifting of circular orbit outwards from the black hole due to radiation reaction.

Thank you for your attention!