GBU close to spatial infinity: a log story

Edgar Gasperín

arXiv:2209.12247

(work in collaboration with M. Duarte, J. Feng & D. Hilditch)

Instituto Superior Técnico, U. Lisboa

XV Black Holes Workshop, Lisbon

edgar.gasperin@tecnico.ulisboa.pt

Lisbon, ISCTE, December 2022.

Prelude: A log affair

Peeling

- Classical Penrose's Peeling theorem, $n = 0, 1, 2, 3, 4$, "NP-gauge"

$$\psi_n = \mathcal{O}(\tilde{r}^{-5+n})$$

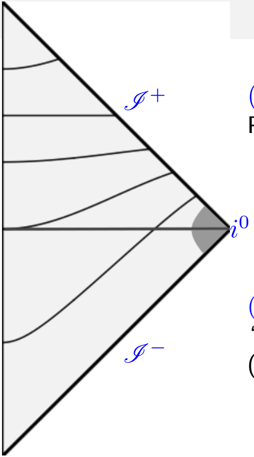
- H. Friedrich's Conformal Approach : [G. & Valiente Kroon 17]
  "F$\rightarrow$NP-gauge"

$$\psi_2 = \mathcal{O}(\tilde{r}^{-3}\log(\tilde{r}))$$

- ASH-GHG Approach : [Duarte, Feng, G. & Hilditch 22 (a)] "GHG-gauge"

$$\psi_2 = \mathcal{O}(\tilde{\rho}^{-3}\log(\tilde{\rho}))$$

- GHG-logs can be gauged away.



$(\tilde{\mathcal{M}}, \tilde{g})$, $\{\tilde{x}^\mu\}$
Physical

$(\mathcal{M}, g)$, $\{x^\mu\}$
"Unphysical"
(Conformal)

GHG-logs vs i^0-Logs

- Same logs?
- Are they related?

Scene 1: GBU model & ASH-logs

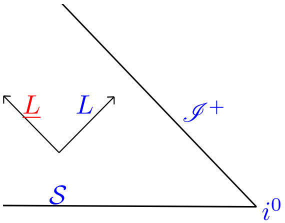
GBU

Physical Minkowski spt $(\mathbb{R}^4, \tilde{\boldsymbol{\eta}})$

$$\tilde{\Box}\tilde{g} = 0$$
$$\tilde{\Box}\tilde{b} = (\nabla_{\tilde{t}}\tilde{g})^2$$
$$\tilde{\Box}\tilde{u} = \tfrac{2}{\tilde{\rho}}\nabla_{\tilde{t}}\tilde{u}$$

Asympt. Syst. Heuristics (ASH)

[Hörmander, Lindblad]



GBU higher order

[Duarte, Feng, G., Hilditch 21]

- $$\boxed{\tilde{g} = \sum_{n=1}^{\infty} \frac{G_n}{\tilde{\rho}^n}}$$
  $$\tilde{b} = \sum_{n=1}^{\infty} \frac{B_n}{\tilde{\rho}^n},$$
  $$\tilde{u} = \frac{m}{\tilde{\rho}} + \sum_{n=2}^{\infty} \frac{U_n}{\tilde{\rho}^n},$$

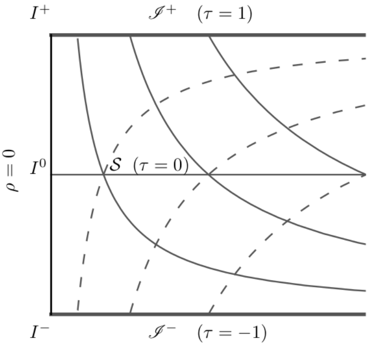
  where B_n, U_n split as $\alpha_n + \beta_n \log\tilde{\rho}$
- Good has no log!
- $\rightsquigarrow$ Generalised to the EFE in GHG [Duarte, Feng, G, Hilditch 22 (a)]

Scene 2: The cylinder at i^0

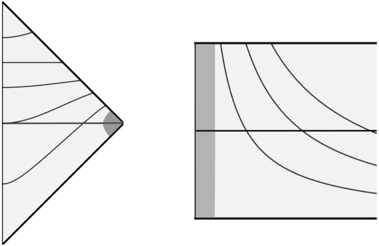
Cylinder at spatial infinity

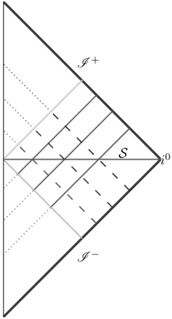
[Friedrich, Valiente Kroon]

- $\boldsymbol{g} = \Theta^2 \tilde{\boldsymbol{\eta}}, \quad \Theta = \rho(1-\tau^2)$
- $i^0 \to$ blown up to $I \approx \mathbb{R} \times \mathbb{S}^2$
- $\mathscr{I}^{\pm} = \{\tau = \pm 1\}$, $\mathcal{S} = \{\tau = 0\}$
- $I^{\pm}$: Critical sets; i^0 "touches" $\mathscr{I}^{\pm}$



Hyperboloidal slices in i^0-Cylinder diagram





Scene 3: GBU in the cylinder at i^0

Physical to Unphysical (Conformal)

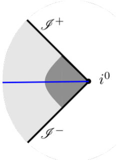
- Mink $(\mathbb{R}^4, \tilde{\boldsymbol{\eta}}) \to i^0$-Cyl $(\mathbb{R}^4, \boldsymbol{g})$
- $\boldsymbol{g} = \Theta^2 \tilde{\boldsymbol{\eta}}$
- $\tilde{\phi} = \Theta \phi$

Unphysical GBU

- Solve ϕ in $(\mathbb{R}^4, \boldsymbol{g})$
- Translate back to $\tilde{\phi}$ in $(\tilde{t}, \tilde{\rho})$
- Eval. @ outgoing null $\tilde{u} = -|\tilde{u}_*|$

GBU-i^0 [Duarte, Feng, G., Hilditch 22 (b)]

- $$\tilde{\phi} = \sum_{n=1}^{\infty} \frac{A_n}{\tilde{\rho}^n}$$



  where $A_n = \alpha_n + \beta_n \log \tilde{\rho}$

- $i^0 \rightsquigarrow$ Good has logs!

- Can ASH recover i^0-logs?
- Yes, (improved) ASH can predict the *leading* log.
- $\partial_{\tilde{v}}(\tilde{\rho}\tilde{\phi}) \sim \mathcal{O}(\tilde{v}^{-1})$

Epi-log

- Higher order i^0-logs with ASH? —Future work
- Relation with (Spin-0) NP constants? —**Rafael Pinto**'s master thesis work