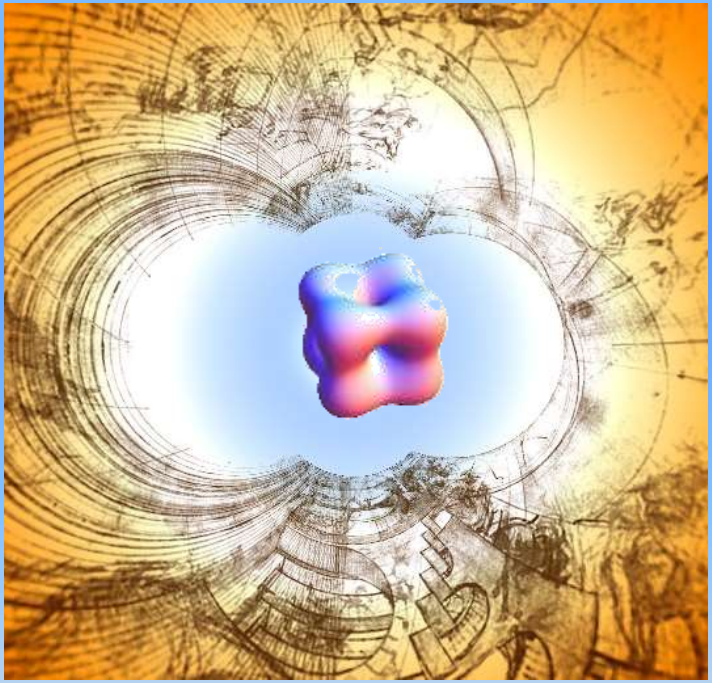
XV Black Hole Workshop - *20.12.2022*

Curing conical singularities with scalar hair



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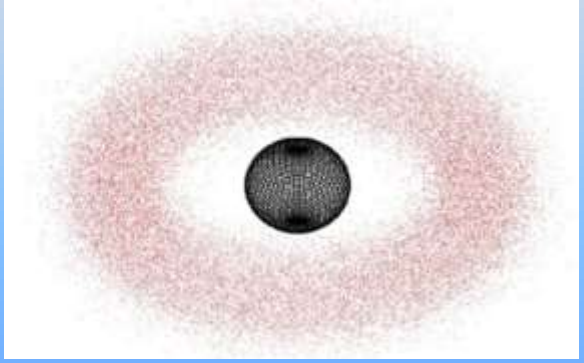
(work done with C. Herdeiro)

the problem we address:

- the ***no hair conjecture*** can be violated
- Schwarschild/Kerr Black Hole + scalar hair:

many interesting solutions



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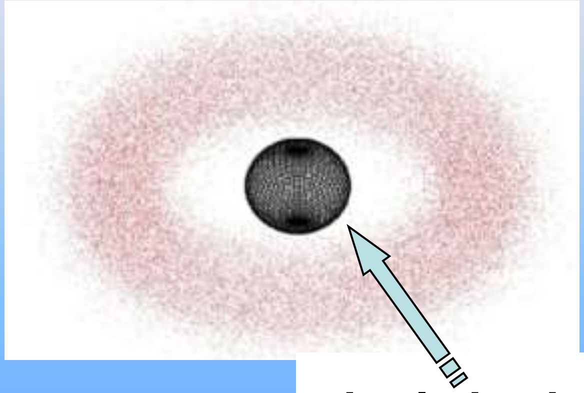

Q1: what about multi-Black Holes?

- **electro-vacuum:** N Black Holes *(exact solutions)*
- simplest case N=2: the Bach-Weyl vacuum solution (2Schwarzschild)
- generic presence of *conical singularities*

the scalar hair can balance the 2BH system

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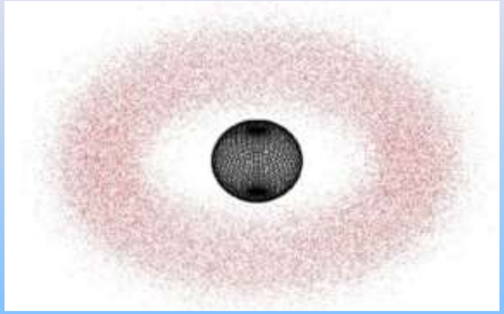

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the scalar hair can balance the system

Q2: what about higher dimensions?

D=5: the static Emparan-Reall vacuum Black Ring

- it possesses conical singularities

static, regular Black Rings with scalar hair

a warning:

- *this is rather a mathematical problem*
  (we do not claim any contact with observations)
- *toy model to test the validity of some results in vacuum GR*

also

- *all configurations are static*
- *no results on stability*

however...

- ***first work in this direction***
- ***we propose a general framework***
  (can be used for other models)

results in C. Herdeiro, E.R: e-Print: 2004.00336
Y. Brihaye, C. Herdeiro, E.R., e-Print: 2207.13114
C. Herdeiro, E.R., *to appear*.

vacuum: the **Israel-Khan** solution (1964):
N-coliniar Schwarzschild BHs

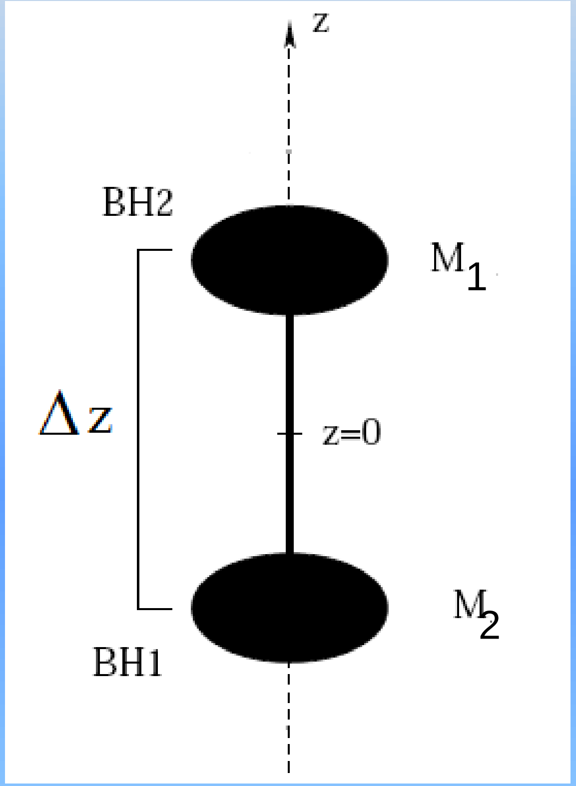
N=2: the **Bach-Weyl** system (1922)

between the BHs (**strut**) $\delta<0$

it provides the extra-force to prevent the collapse

the conical singularity

from the horizons to infinity (**string**) $\delta>0$



'effective stress energy tensor' associated with the singularity.

extra-source

$$T_i^j = -\delta_i^j \frac{1}{8\pi}(2\pi - \alpha)\delta_\Sigma$$

$$(i,j) \neq (\rho, \varphi)$$

the presence of conical singularities: ***undesirable feature***

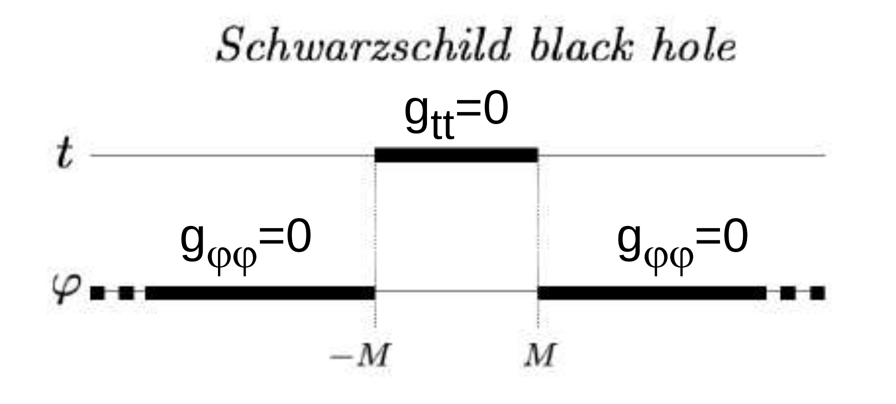
crucial (technical) point:

the use of Weyl-type coordinates+rod structure:

(the z-axis is divided into 3-intervals (called rods of the solution)

$$ds^2 = f_1(\rho,z)(d\rho^2 + dz^2) + f_2(\rho,z)d\varphi^2 - f_0(\rho,z)dt^2$$

$$R_{\mu\nu} = 0$$



e.g.

$$f_0(\rho,z) = \frac{r_+ + r_- - 2M}{r_+ + r_- + 2M}$$

...

$$r_\pm = \sqrt{\rho^2 + (z \pm M)^2}$$

$$\rho = \sqrt{r^2 - 2Mr}\sin\theta$$
$$z = (r - M)\cos\theta$$

$$ds^2 = \frac{dr^2}{1 - \frac{2M}{r}} + r^2(d\theta^2 + \sin^2\theta d\phi^2) - (1 - \frac{2M}{r})dt^2$$

crucial (technical) point:

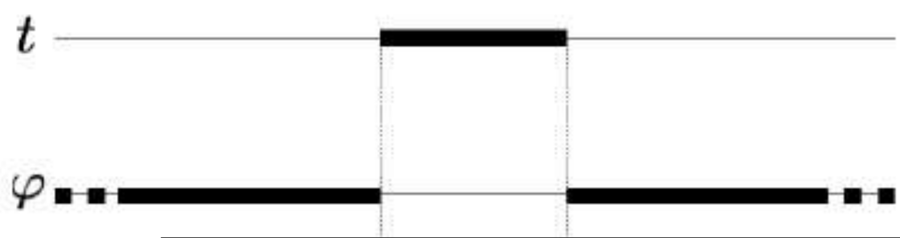
the use of Weyl-type coordinates + rod structure:

(the z-axis is divided into 3-intervals (called rods of the solution))

$$ds^2 = f_1(\rho,z)(d\rho^2 + dz^2) + f_2(\rho,z)d\varphi^2 - f_0(\rho,z)dt^2$$

just a test of the method

Schwarzschild black hole with scalar hair



$$\mathcal{S} = \frac{1}{4\pi}\int d^4x\sqrt{-g}\left[\frac{R}{4G} - \partial_\alpha\Phi\partial^\alpha\Phi - U(\Phi)\right]$$

- single, non-vacuum Black Holes can be expressed in these coordinates
- we recover Schwarzschild Black Hole with scalar hair
- technically much more complicated

  (solve a boundary value problem (set of PDEs))

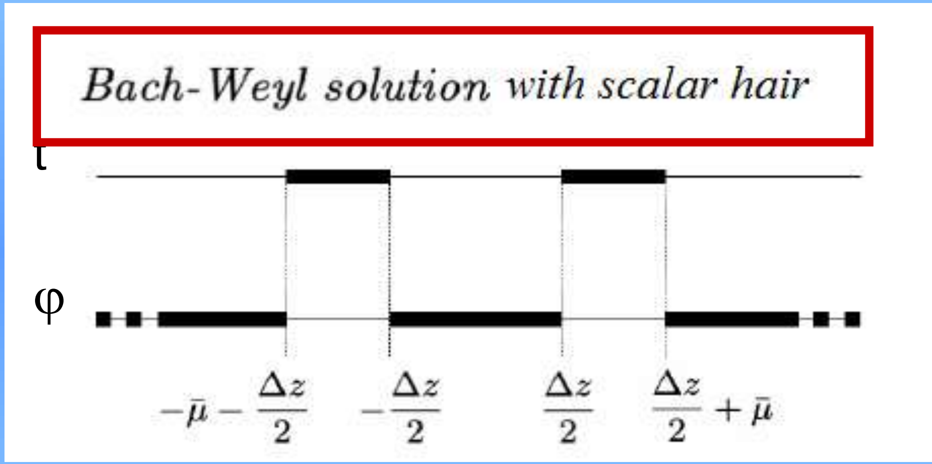
$$\Phi dU/d\Phi < 0$$

crucial (technical) point:

the use of Weyl-type coordinates+rod structure:

(the z-axis is divided into 5-intervals (called rods of the solution)

$$ds^2 = f_1(\rho,z)(d\rho^2 + dz^2) + f_2(\rho,z)d\varphi^2 - f_0(\rho,z)dt^2$$



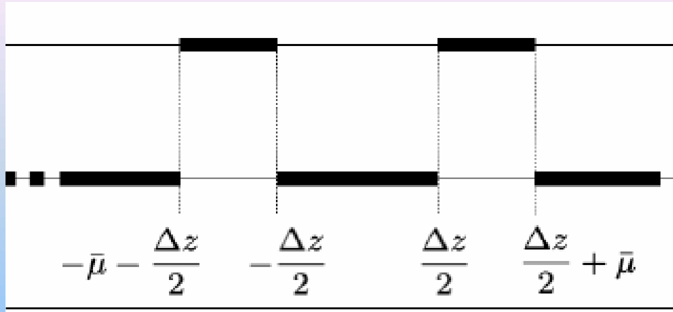
two identical BHs

we propose a general framework

- preserve the rood structure of the vacuum Back-Weyl solution
- work with *'deformed'* functions F_i

$$f_i = f_i^{(0)} e^{2F_i}.$$

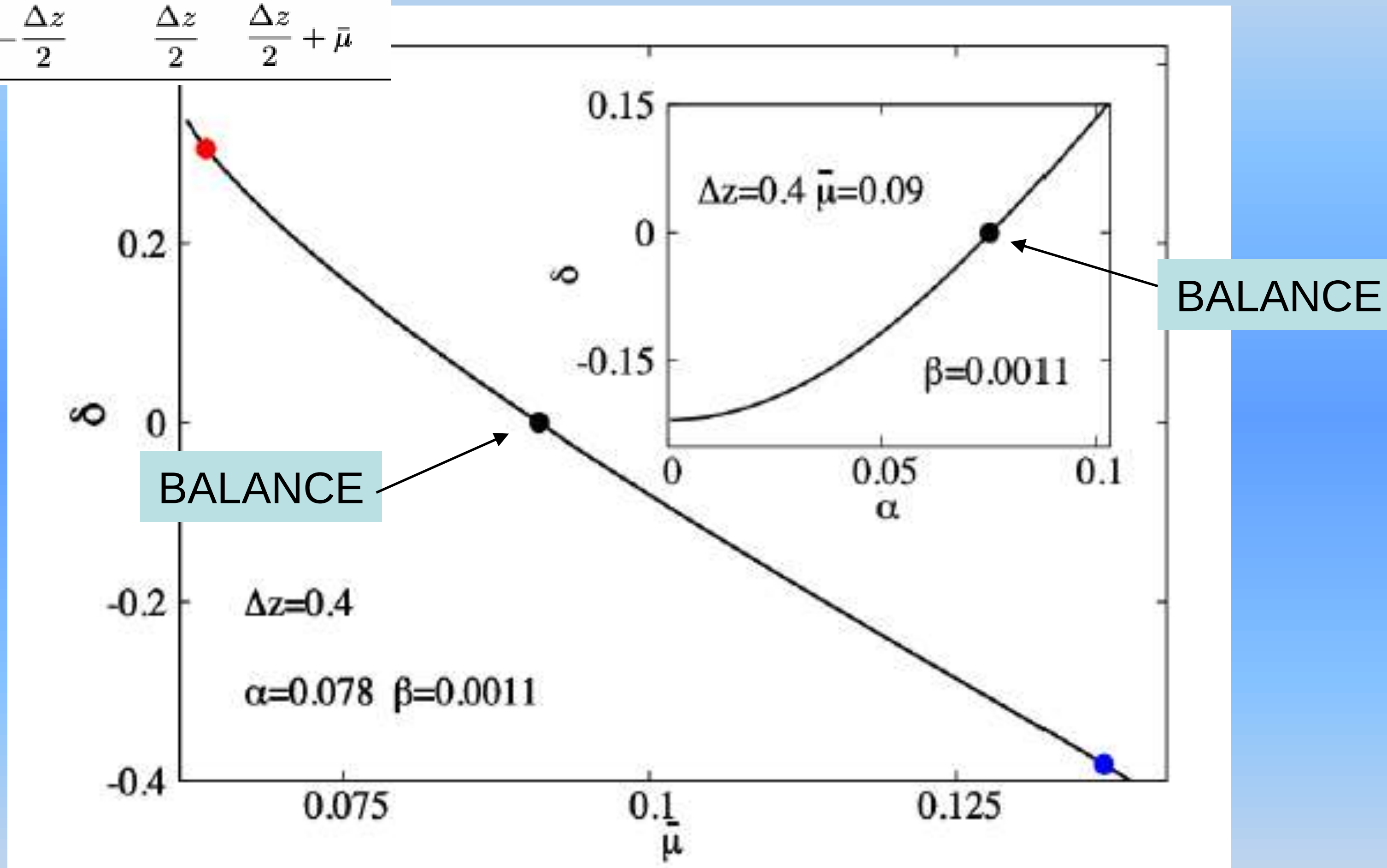
- solve a boundary value problem (set of four PDEs)



$$U = \mu^2\Phi^2 - \lambda\Phi^4 + \nu\Phi^6$$

$$\alpha^2 = \frac{G\mu^2}{\lambda}$$

$$\beta^2 = \frac{\nu\mu^2}{\lambda}$$



the conical excess/deficit δ s shown as a function of the horizon size (input parameter) and .the strength of gravity α (inset)..

the next problem we address:

Q2: what about higher dimensions?

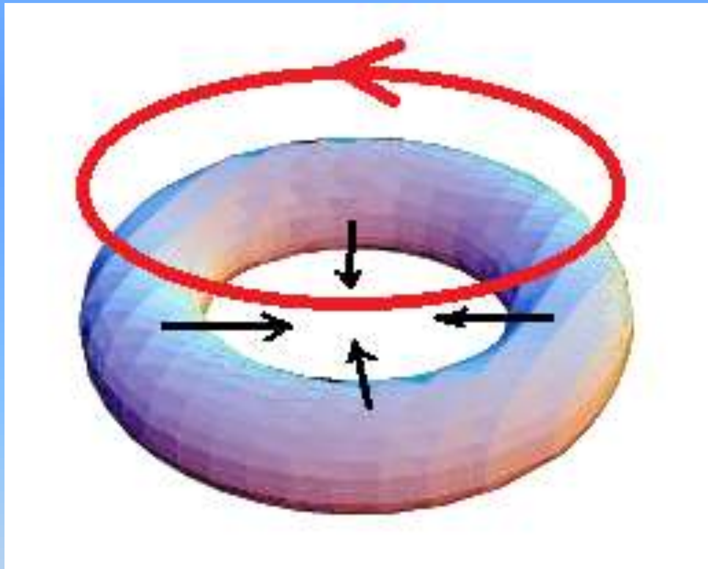
D=5: the static Emparan-Reall vacuum Black Ring: **conical singularities**

how to balance a Black Ring?

simplest solution: add rotation!

$$R_{\mu\nu} = 0$$

Emparan-Reall
exact solution (2002)



static, balanced Black Rings in Einstein-gauged scalar field model

the Einstein-Maxwell-scalar model

a different mechanism

$$\Psi dU/d|\Psi| > 0$$

$$S = \int d^D x \sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{4}F_{\alpha\beta}F^{\alpha\beta} - D_\alpha \Psi^* D^\alpha \Psi - U(|\Psi|)\right]$$

spherically symmetric sector:

gauged boson stars

no black holes

(Mayo-Bekenstein theorem)

however, loophole!

static, spherically symmetric black holes

D=4 result 2020

$$\Psi \sim e^{-iwt}$$

'resonance' condition

$$w = q\Phi_H$$

$$U(\psi) = \mu^2\psi^2 - \lambda\psi^4 + \nu\psi^6$$

- **nonlinearities are crucial**
- **one solves the full EMs problem** *(electric charge)*
- **the solutions are not connected with EM black holes**

D=4

J.-P. Hong, M. Suzuki and M. Yamada, *Phys. Lett. B* **803** (2020) 135324
C.Herdeiro and E. Radu, *Eur. Phys. J. C* **80** (2020) 390

Schwarzschild Black Holes with charged scalar hair

electric charge **Q**

D=5

:

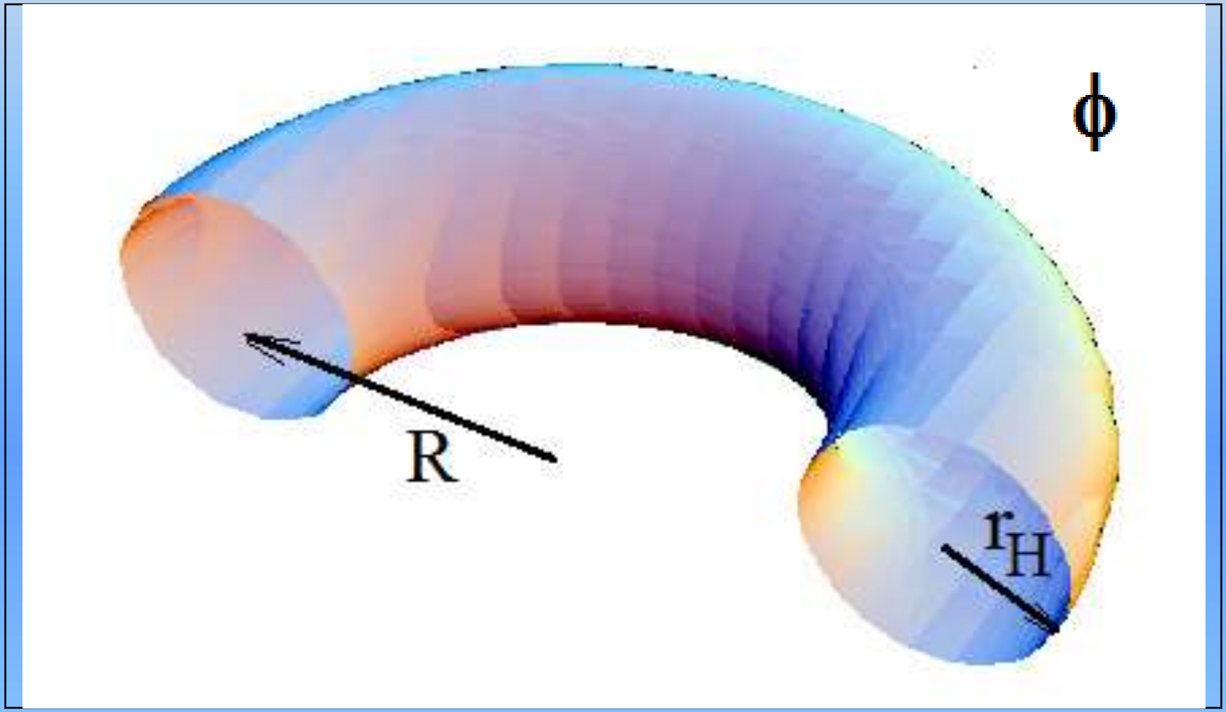
Y. Brihaye, C. Herdeiro and E. Radu, *JHEP* 10 (2022) 153

Schwarzschild-Tangherlini Black Holes, Black Strings and *Black Rings* with charged scalar hair

static *Black Rings* with charged scalar hair

in numerics: **three input parameters:** $R,\ r_H,\ \phi$

+q, scalar potential



generic solutions: conical sing.

very important…

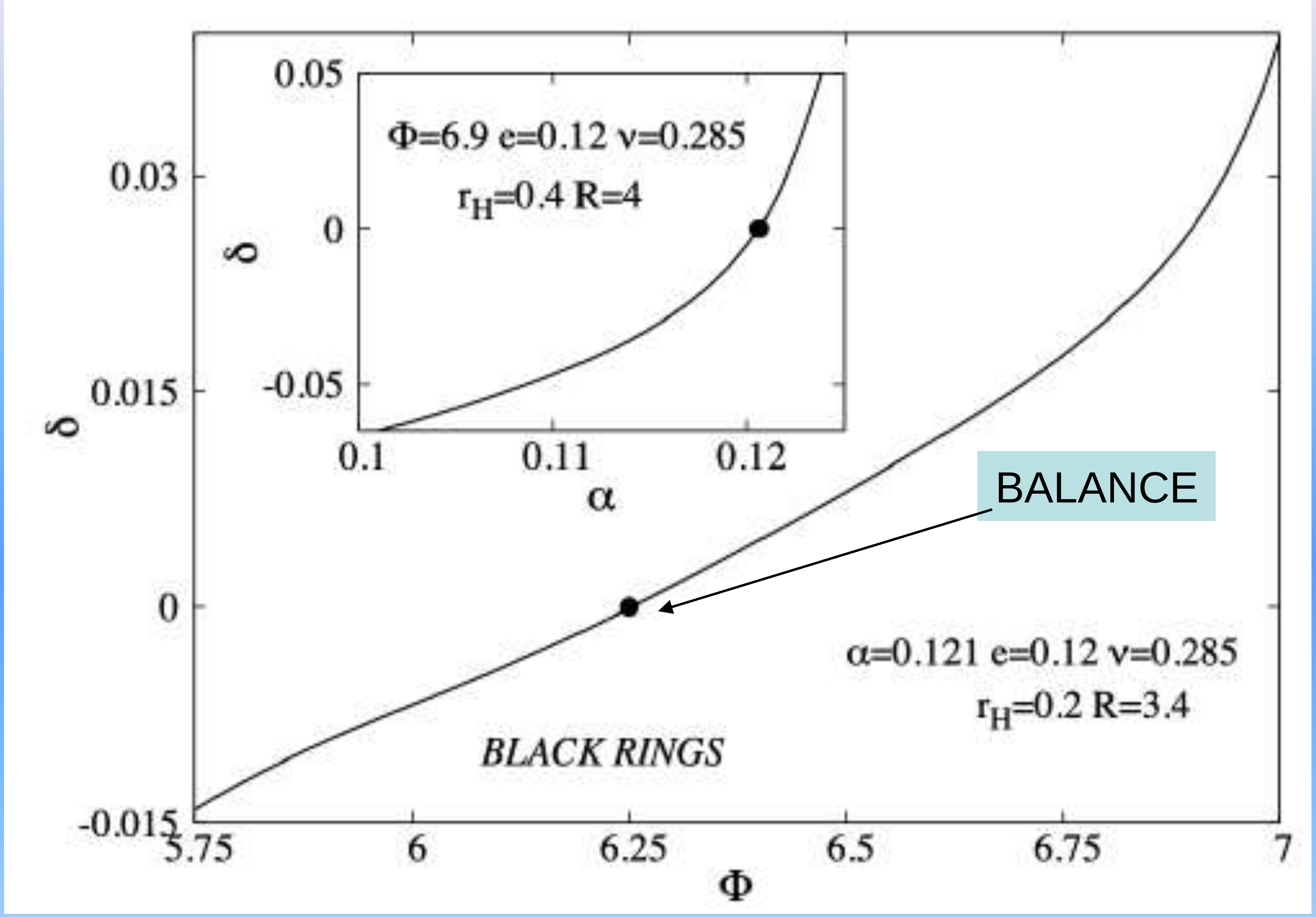
$T_H,\ A_H,\ M,\ Q$ **-computed from numerical output**

in practice:

- **fix the model**
- **vary $R>r_H$ or Φ**
- **regular solutions exist for critical values of Φ only**

$$\alpha^2 = \frac{4\pi G\mu^2}{\lambda}$$

$$\beta^2 = \frac{\nu\mu^2}{\lambda}$$



the conical excess/deficit of a static BR is shown as a function of the electrostatic potential Φ and of the input parameter α which measures the strength of gravity (the inset).

to summarize::

- the (vacuum) two-Black Hole system/the static Black Ring posses conical singularities

- however, the scalar hair can provide the extra-force to balance the system

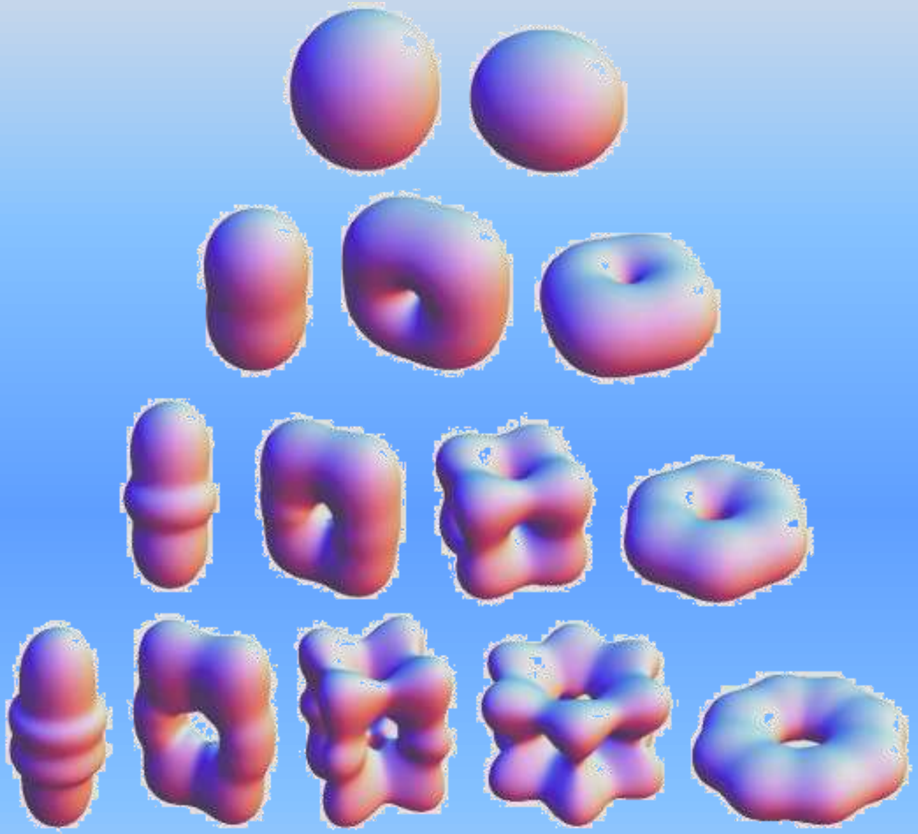
more than an exercise?

many open problems:

- including rotation
- stability?
- a more physical framework (**same methods**)
  (scalar field – *just the simplest case)*

main message:

one cannot safely extrapolate the properties of the (electro-)vacuum Black Holes to any model



Muito obrigado pela vossa atenção!