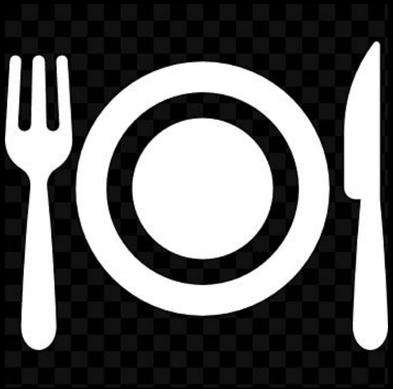
XV Black Holes Workshop

19–20 Dec 2022
ISCTE - University Institute of Lisbon

Virial identities in relativistic gravity

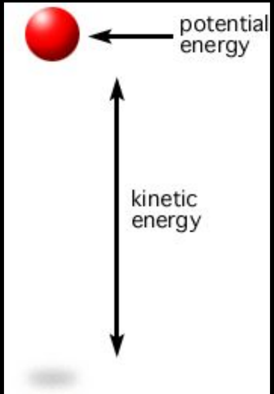
Alexandre M. Pombo

Introduction



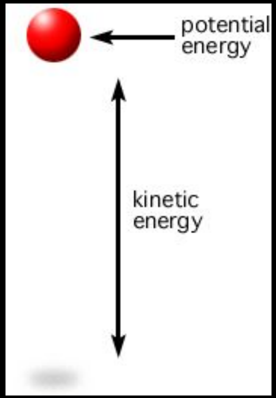
Introduction: Virial Theorem

- The virial theorem relates the **average** kinetic and potential energy
- It allows the average kinetic energy to be calculated even for very complicated systems
- The theorem has found applications in several areas



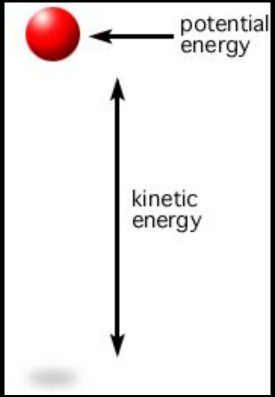
Introduction: Virial Theorem

- The virial theorem relates the average kinetic and potential energy
- It allows the average kinetic energy to be calculated even for very complicated systems
- The theorem has found applications in several areas



Introduction: Virial Theorem

- The virial theorem relates the average kinetic and potential energy
- It allows the average kinetic energy to be calculated even for very complicated systems
- The theorem has found applications in several areas



Virial identity

- Integral identities that are virial-like
- In field theory rather than particle mechanics
- It is obtained from scaling arguments
- Computed independently from the equations of motion
- Applicable to stationary spacetimes
- We present an approach for curved spacetimes

Virial identity

- Integral identities that are virial-like
- In field theory rather than particle mechanics
- It is obtained from scaling arguments
- Computed independently from the equations of motion
- Applicable to stationary spacetimes
- We present an approach for curved spacetimes

Virial identity

- Integral identities that are virial-like
- In field theory rather than particle mechanics
- It is obtained from scaling arguments
- Computed independently from the equations of motion
- Applicable to stationary spacetimes
- We present an approach for curved spacetimes

Virial identity

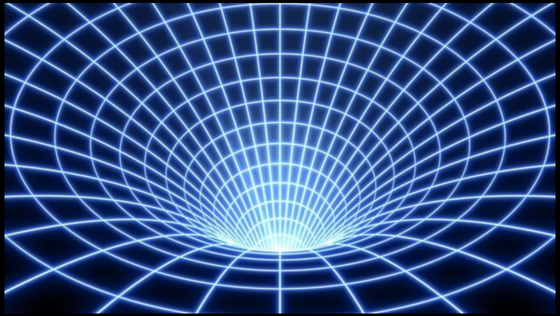
- Integral identities that are virial-like
- In field theory rather than particle mechanics
- It is obtained from scaling arguments
- Computed independently from the equations of motion
- Applicable to stationary spacetimes
- We present an approach for curved spacetimes

Virial identity

- Integral identities that are virial-like
- In field theory rather than particle mechanics
- It is obtained from scaling arguments
- Computed independently from the equations of motion
- Applicable to stationary spacetimes
- We present an approach for curved spacetimes

Virial identity

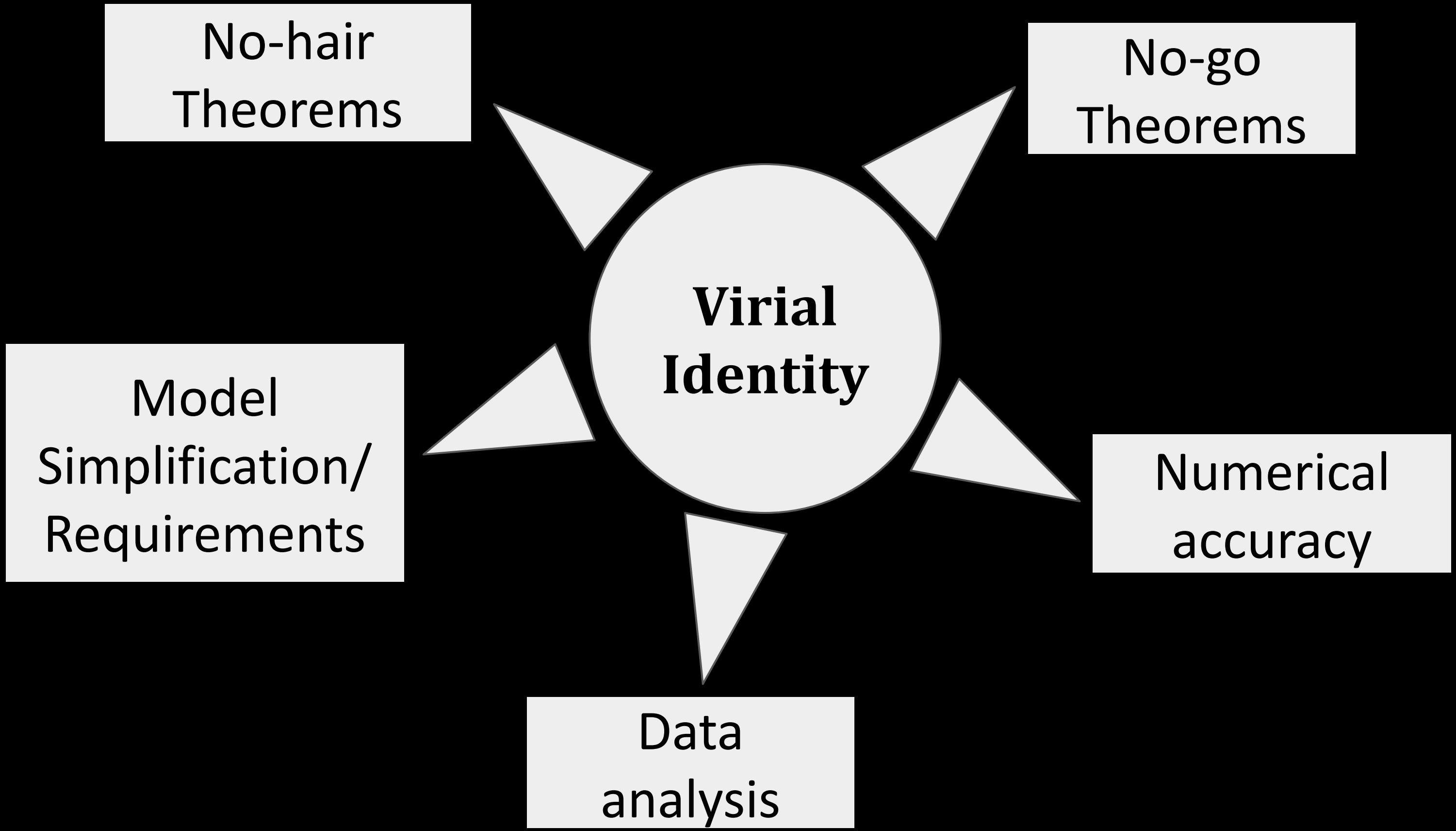
- Integral identities that are virial-like
- In field theory rather than particle mechanics
- It is obtained from scaling arguments
- Computed independently from the equations of motion
- Applicable to stationary spacetimes
- We present an approach for curved spacetimes



Importance

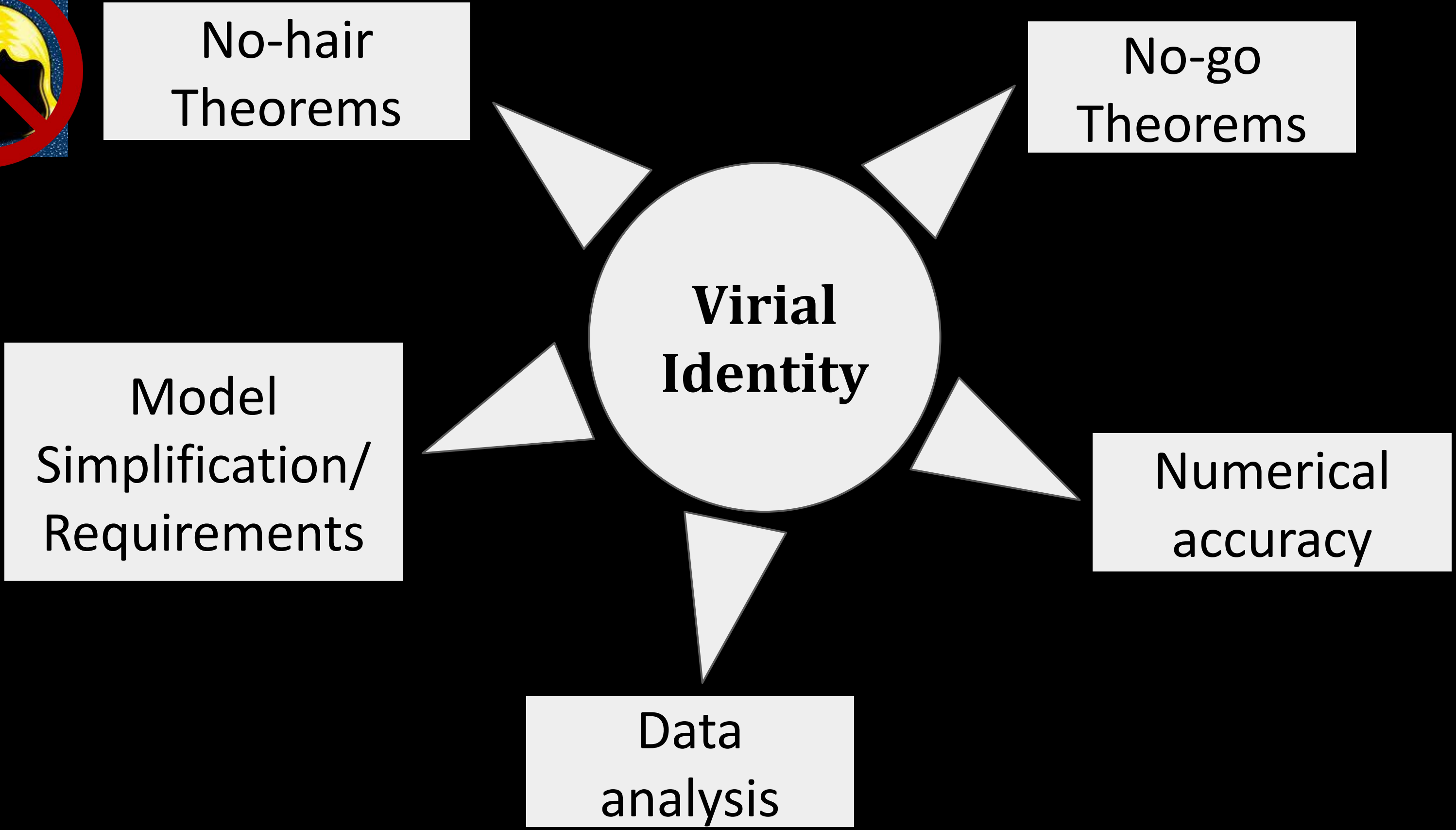
Virial Identity

Importance

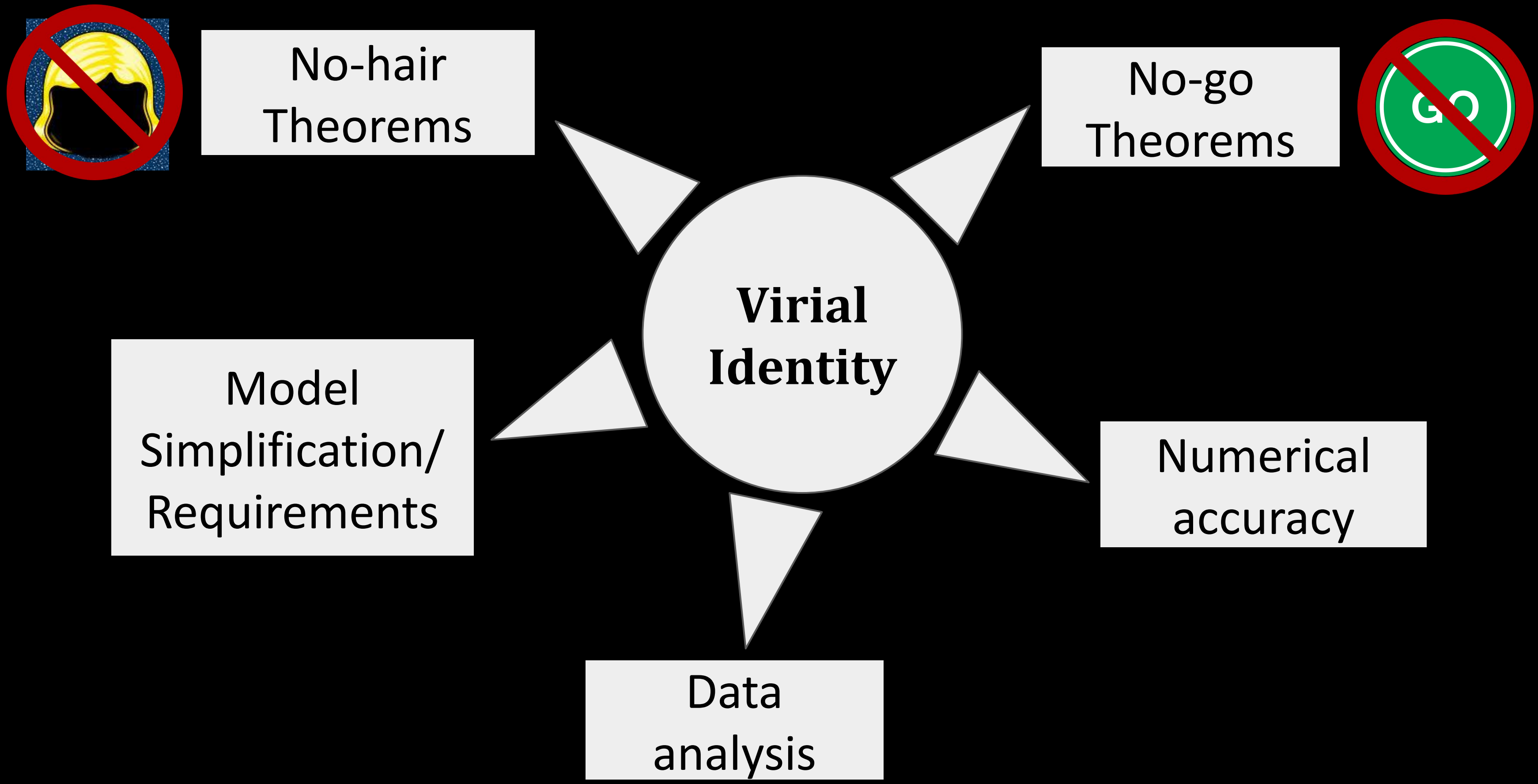


No-hair Theorems
No-go Theorems
Virial Identity
Model Simplification/ Requirements
Numerical accuracy
Data analysis

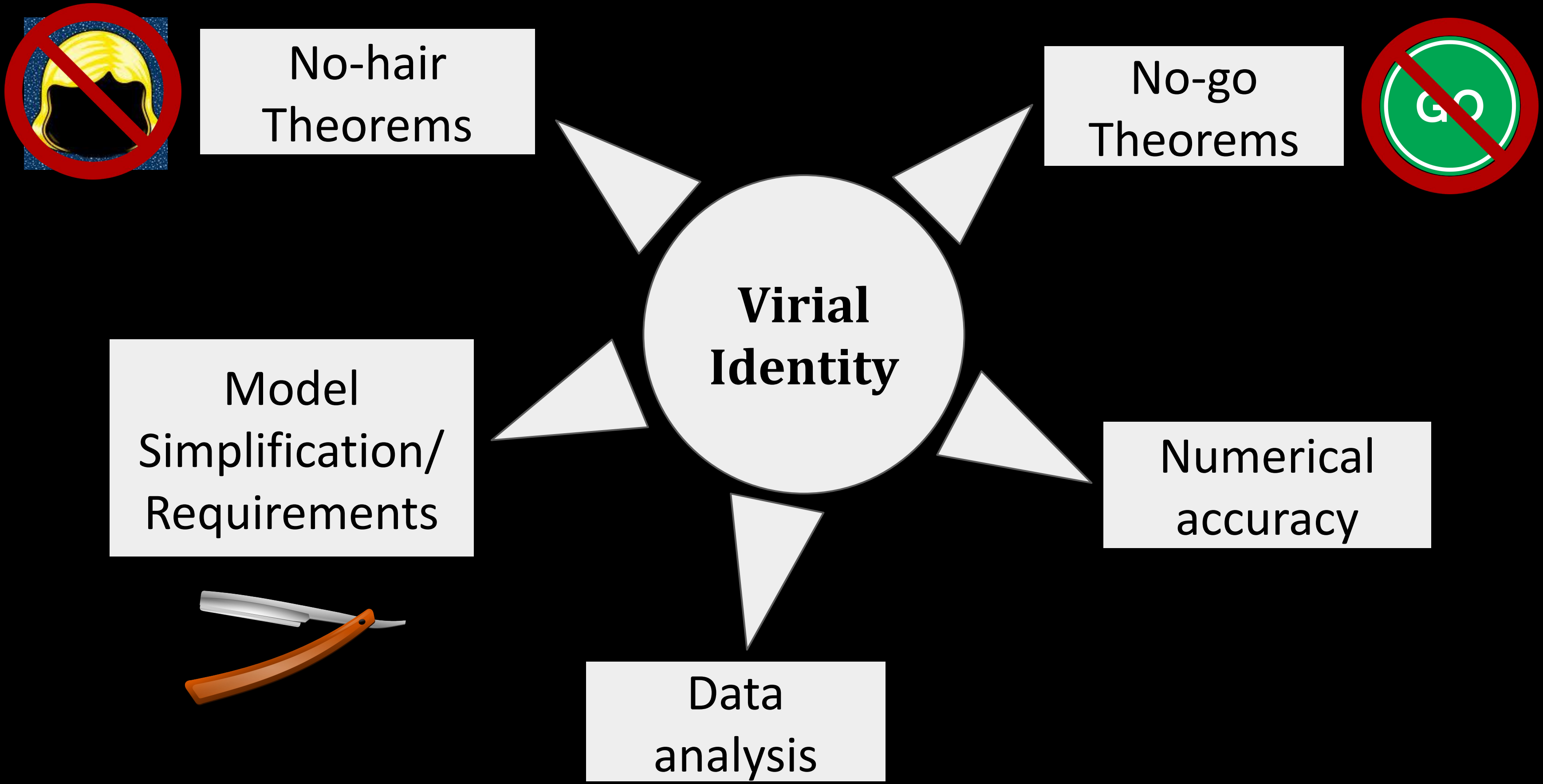
Importance


No-hair Theorems
No-go Theorems
Virial Identity
Model Simplification/ Requirements
Numerical accuracy
Data analysis

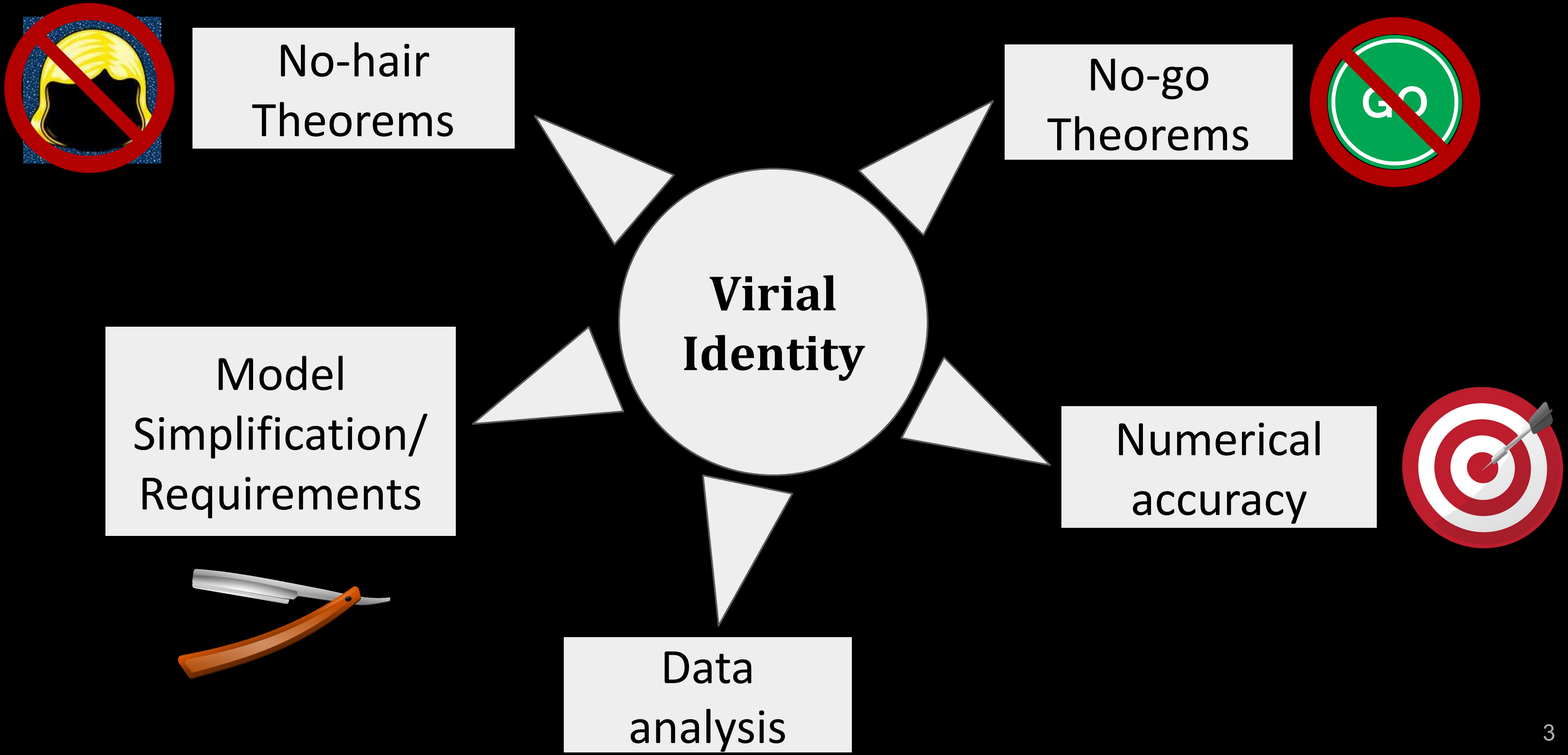
Importance


No-hair Theorems
No-go Theorems
GO
Virial Identity
Model Simplification/ Requirements
Numerical accuracy
Data analysis

Importance


No-hair Theorems
No-go Theorems
GO
Virial Identity
Model Simplification/ Requirements
Numerical accuracy
Data analysis

Importance


No-hair Theorems
No-go Theorems
GO
Virial Identity
Model Simplification/ Requirements
Numerical accuracy
Data analysis

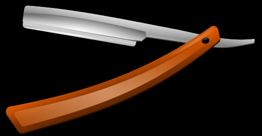
Importance

No-hair Theorems

No-go Theorems

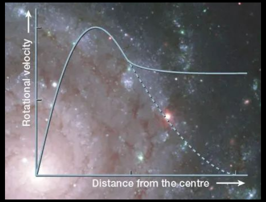
Virial Identity

Model Simplification/ Requirements



Numerical accuracy

Data analysis



Recipe

Recipe

Ingredients:

- Action S
- Metric ansatz $g_{\mu\nu}$
- Matter ansatz
- Gibbons-Hawking-York term (gravity)

Recipe

Ingredients:

- Action S
- Metric ansatz $g_{\mu\nu}$
- Matter ansatz
- Gibbons-Hawking-York term (gravity)

Material:

- Derrick's scaling argument
- Hamilton's principle
- Love and patience

Step-by-step:

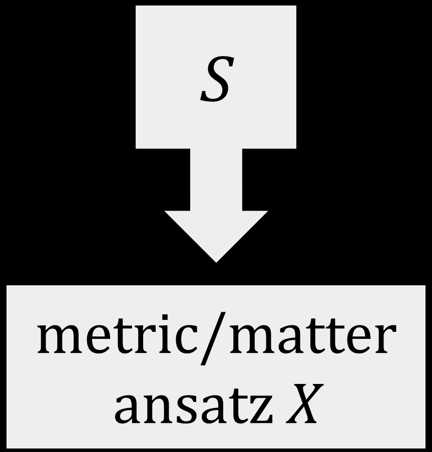
Step-by-step: Derrick's scaling argument

Model

S

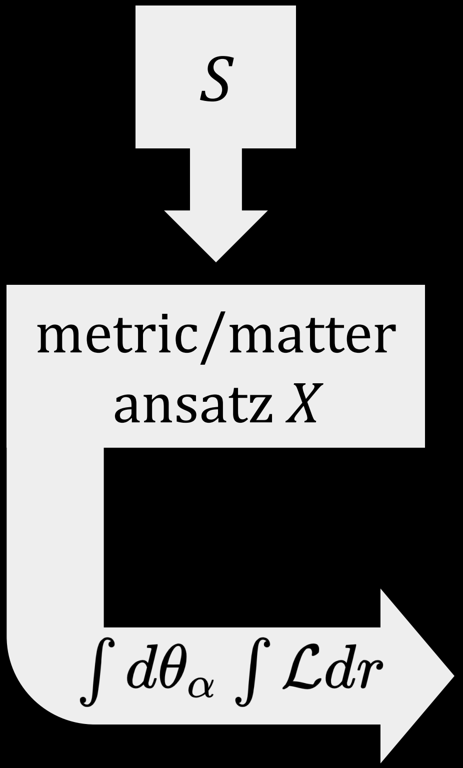
Step-by-step: Derrick’s scaling argument

Model



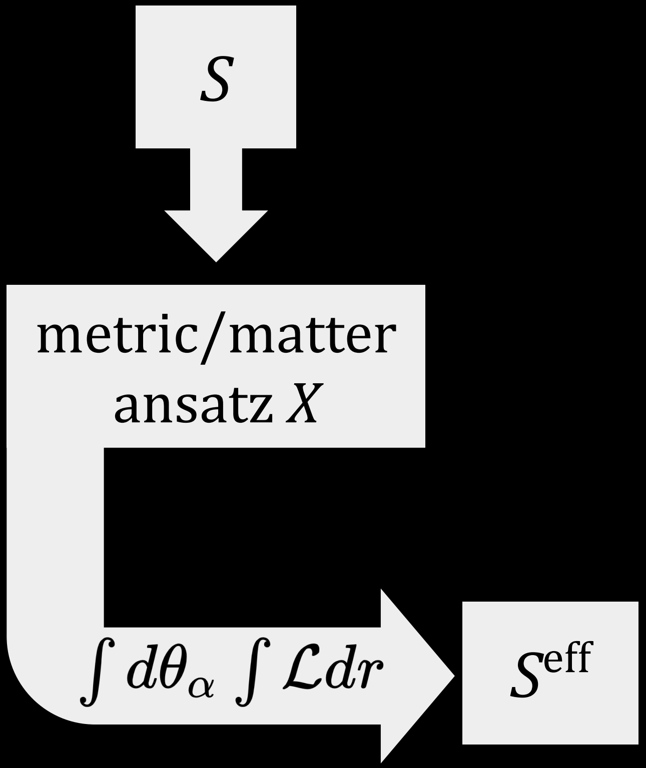
Step-by-step: Derrick's scaling argument

Model



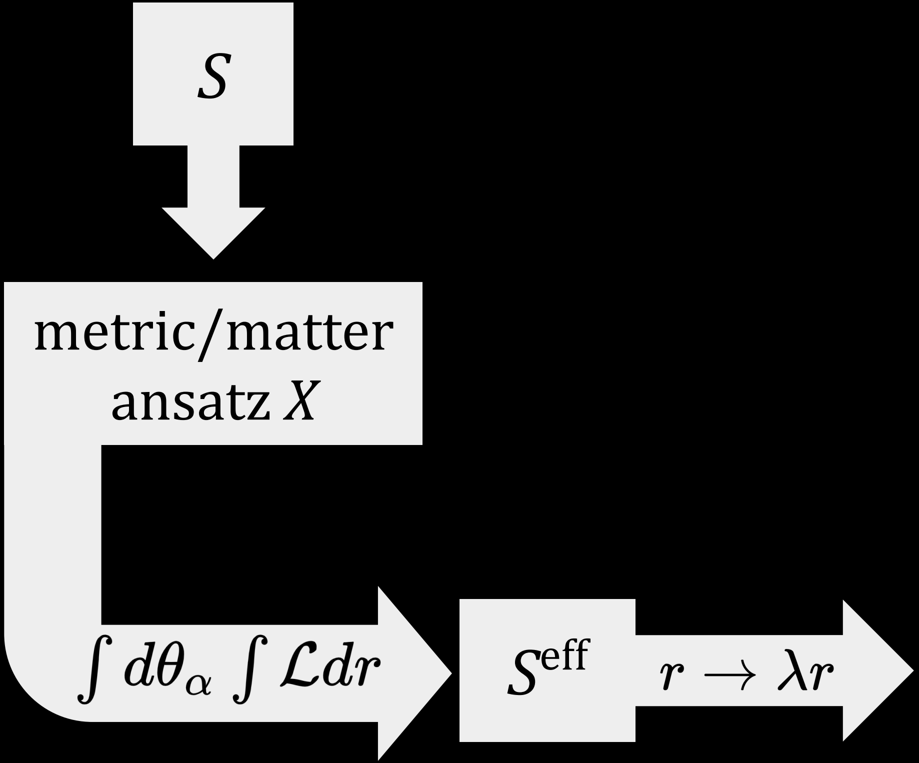
Step-by-step: Derrick’s scaling argument

Model



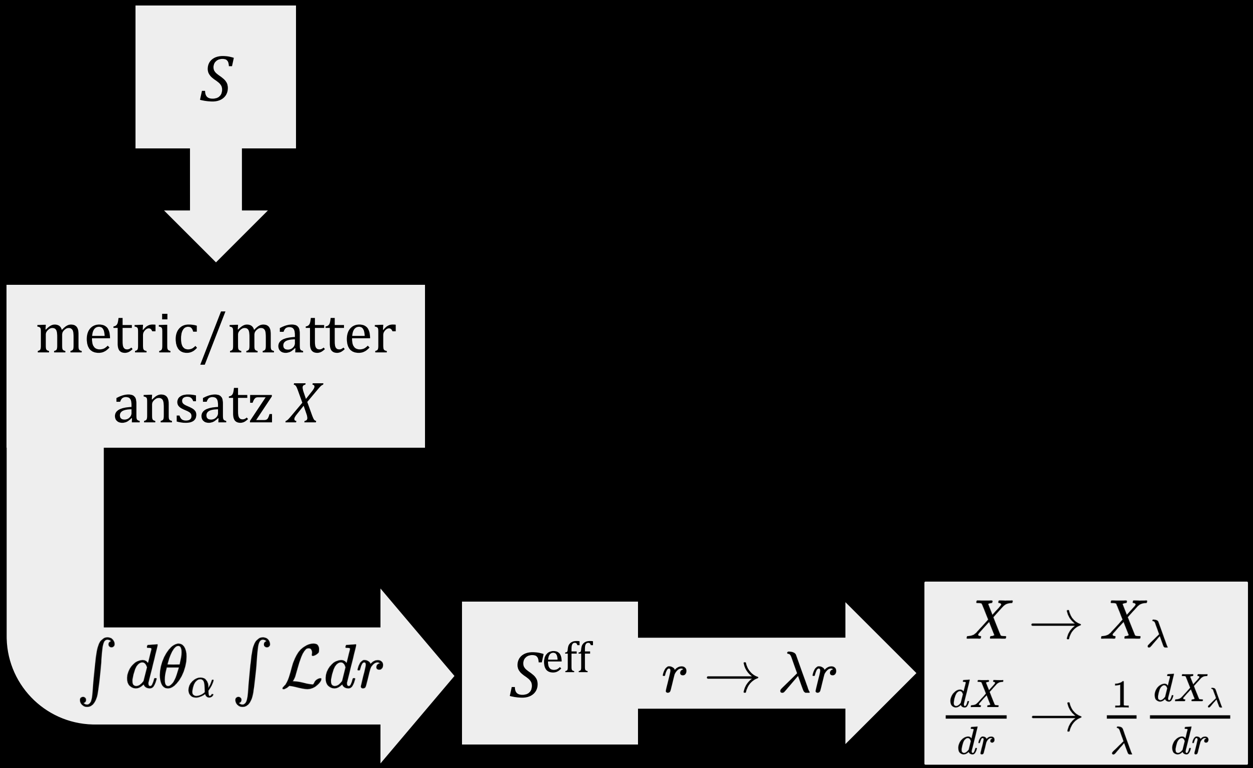
Step-by-step: Derrick's scaling argument

Model **Scaling**



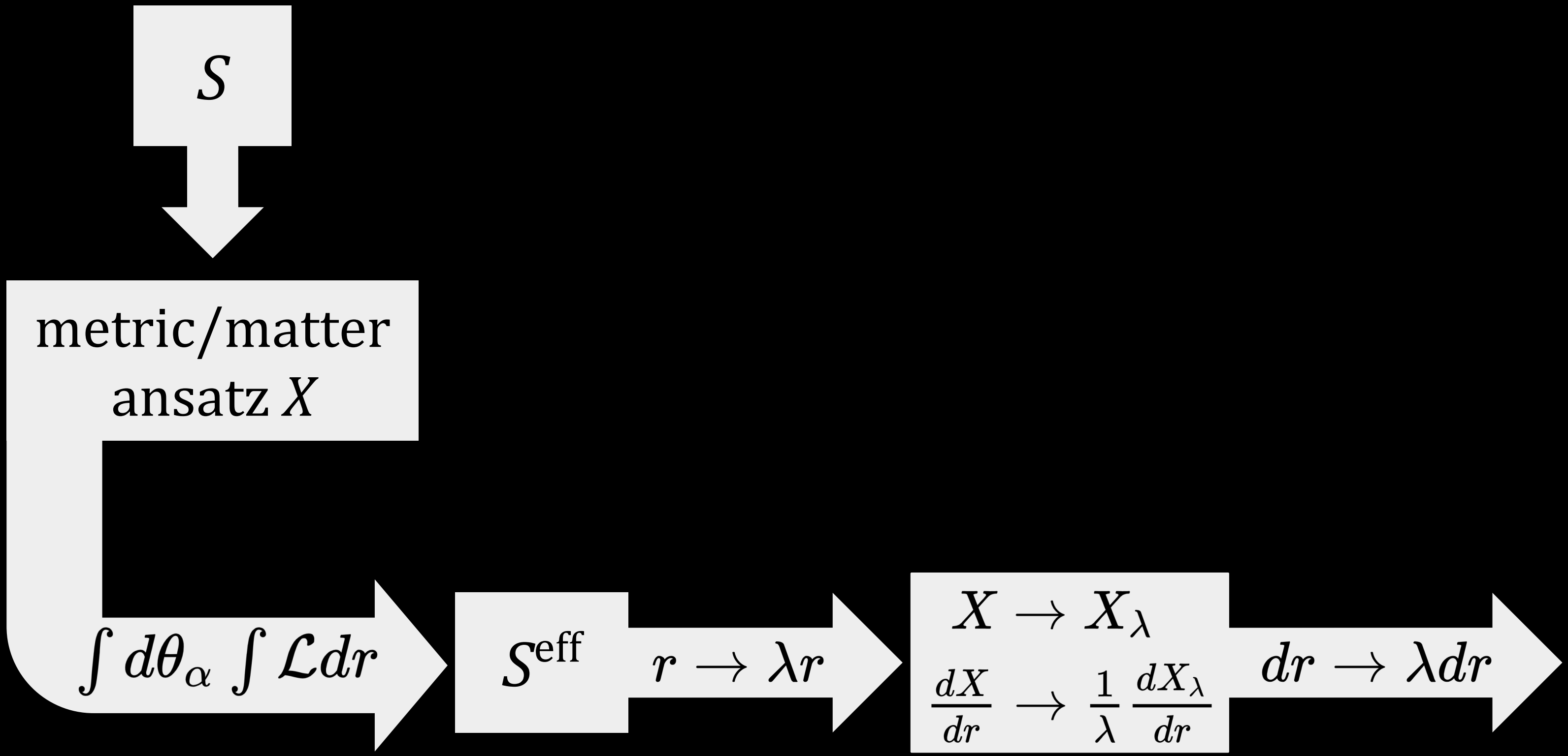
Step-by-step: Derrick's scaling argument

Model **Scaling**

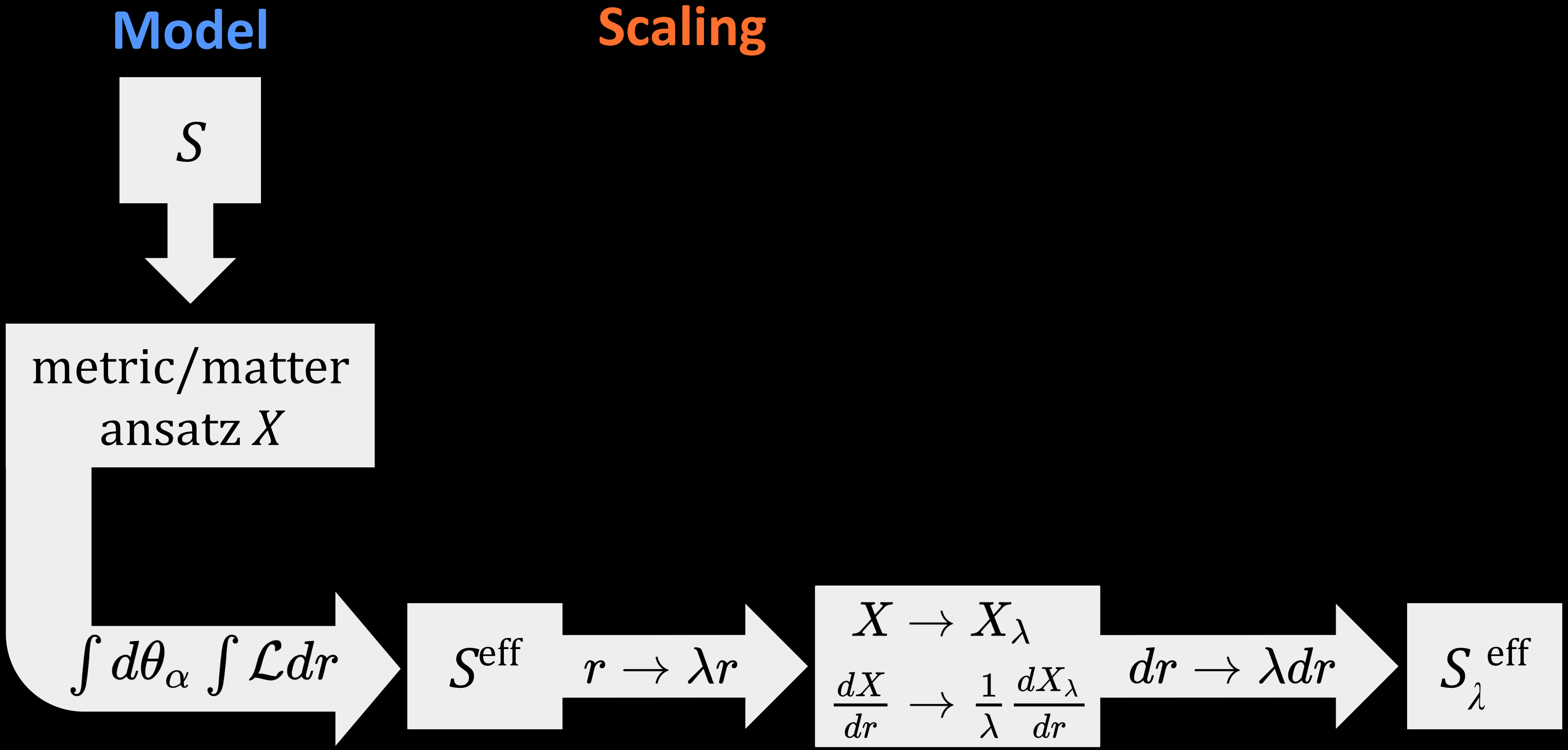


Step-by-step: Derrick's scaling argument

Model **Scaling**



Step-by-step: Derrick’s scaling argument

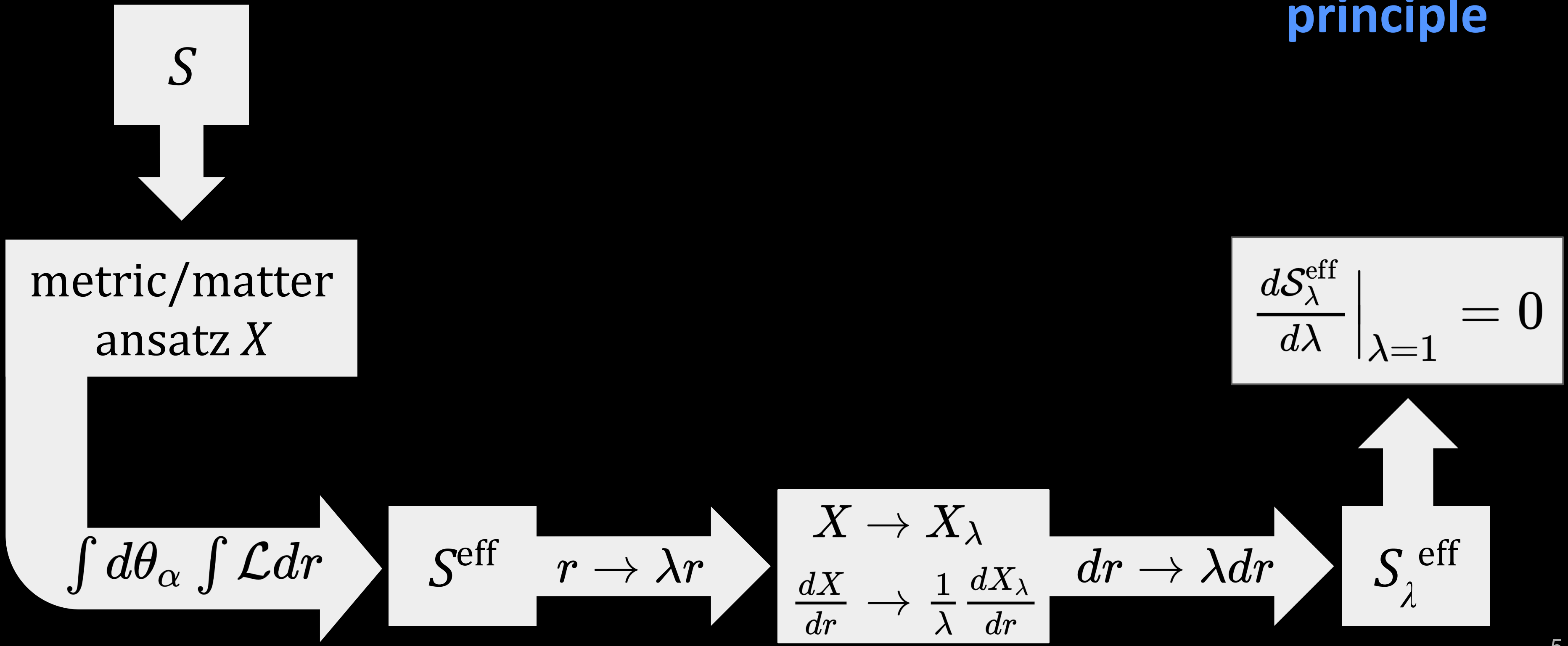


Step-by-step: Derrick's scaling argument

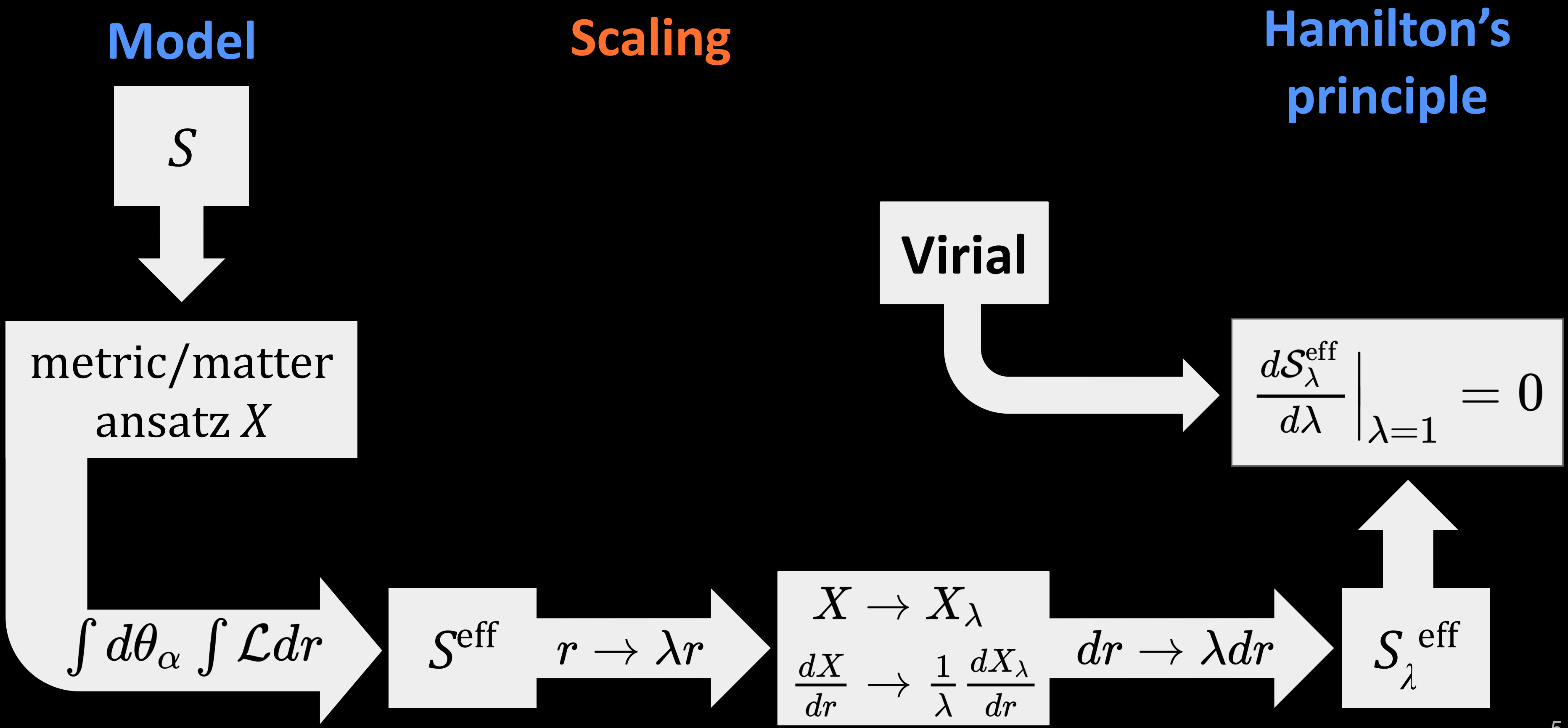
Model

Scaling

Hamilton's principle



Step-by-step: Derrick's scaling argument



Preparation

Derrick's argument

- The action of a real scalar field

Derrick's argument

- The action of a real scalar field

$$\mathcal{S}^{\Phi} = \int d^4x \Big[-\Phi_{,\mu}\Phi^{,\mu} - U \Big]$$

Derrick's argument

- The action of a real scalar field

$$\mathcal{S}^{\Phi} = \int d^4x \Big[- \Phi_{,\mu}\Phi^{,\mu} - U \Big]$$

$$S_0 \equiv \int d^3x\, \dot{\Phi}^2\,, \qquad S_1 \equiv \int d^3x (\nabla\Phi)^2\,, \qquad S_2 \equiv \int d^3x U$$

Derrick's argument

- The action of a real scalar field

$$\mathcal{S}^{\Phi} = \int d^4x \left[-\Phi_{,\mu}\Phi^{,\mu} - U \right]$$

$$S_0 \equiv \int d^3x\, \dot{\Phi}^2\,, \qquad S_1 \equiv \int d^3x (\nabla\Phi)^2\,, \qquad S_2 \equiv \int d^3x U$$

Derrick's argument

- The scaled configuration

$$r \to \tilde{r} = \lambda r$$

Derrick's argument

- The scaled configuration

$$r \to \tilde{r} = \lambda r$$

$$\Phi_\lambda(\mathbf{r}) = \Phi(\lambda \mathbf{r})$$

Derrick's argument

- The scaled configuration

$$r \to \tilde{r} = \lambda r$$

$$\Phi_\lambda(\mathbf{r}) = \Phi(\lambda \mathbf{r}) \qquad \Phi'_\lambda = \frac{\Phi'}{\lambda}$$

Derrick's argument

- The scaled configuration

$$r \to \tilde{r} = \lambda r$$

$$\Phi_\lambda(\mathbf{r}) = \Phi(\lambda \mathbf{r}) \qquad \Phi'_\lambda = \frac{\Phi'}{\lambda} \qquad dr \to \lambda dr$$

Derrick's argument

- The scaled configuration

$$r \to \tilde{r} = \lambda r$$

$$\Phi_\lambda(\mathbf{r}) = \Phi(\lambda \mathbf{r}) \qquad \Phi'_\lambda = \frac{\Phi'}{\lambda} \qquad dr \to \lambda dr$$

$$\left(\frac{dS_\lambda}{d\lambda}\right)\Big|_{\lambda=1} = 0$$

Derrick's argument

- The scaled configuration

$$r \rightarrow \tilde{r} = \lambda r$$

$$\Phi_\lambda(\mathbf{r}) = \Phi(\lambda \mathbf{r}) \qquad \Phi'_\lambda = \frac{\Phi'}{\lambda} \qquad dr \rightarrow \lambda dr$$

$$\left(\frac{dS_\lambda}{d\lambda}\right)\Big|_{\lambda=1} = 0$$

$$\int d^3x \, (\nabla\Phi)^2 = -3 \int d^3x \, U$$

Derrick's argument

- The scaled configuration

$$r \to \tilde{r} = \lambda r$$

$$\Phi_\lambda(\mathbf{r}) = \Phi(\lambda \mathbf{r}) \qquad \Phi'_\lambda = \frac{\Phi'}{\lambda} \qquad dr \to \lambda dr$$

$$\left(\frac{dS_\lambda}{d\lambda}\right)\Big|_{\lambda=1} = 0$$

$$\int d^3x\, (\nabla\Phi)^2 = -3 \int d^3x\, \boxed{U}$$

Derrick's argument

- The scaled configuration

$$r \to \tilde{r} = \lambda r$$

$$\Phi_\lambda(\mathbf{r}) = \Phi(\lambda \mathbf{r}) \qquad \Phi'_\lambda = \frac{\Phi'}{\lambda} \qquad dr \to \lambda dr$$

$$\left(\frac{dS_\lambda}{d\lambda}\right)\Big|_{\lambda=1} = 0$$

$$\int d^3x\,(\nabla\Phi)^2 \cancel{=} -3\int d^3x\,\boxed{U}$$

Plating

Axial Symmetry

arXiv:2206.02813

Derrick’s argument: Q-balls

- The action of a complex scalar field

Derrick's argument: Q-balls

- The action of a complex scalar field

$$\mathcal{S}^{\Phi} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2}g^{\mu\nu}\left(\Phi_{,\mu}\Phi^*_{,\nu} + \Phi^*_{,\mu}\Phi_{,\nu}\right) - U(|\Phi|)\Big]$$

Derrick’s argument: Q-balls

- The action of a complex scalar field

$$\mathcal{S}^{\Phi} = \int d^4x \sqrt{-g}\Big[-\frac{1}{2}g^{\mu\nu}\left(\Phi_{,\mu}\Phi^*_{,\nu} + \Phi^*_{,\mu}\Phi_{,\nu}\right) - U(|\Phi|)\Big]$$

$$\Phi = \phi(r,\theta)e^{-i\omega t + im\varphi}$$

Derrick's argument: Q-balls

- The action of a complex scalar field

$$\mathcal{S}^{\Phi} = \int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\left(\Phi_{,\mu}\Phi^*_{,\nu} + \Phi^*_{,\mu}\Phi_{,\nu}\right) - U(|\Phi|)\right]$$

$$\Phi = \phi(\boxed{r,\theta})e^{-i\omega t + im\varphi}$$

Derrick's argument: Q-balls

- The action of a complex scalar field

$$\mathcal{S}^{\Phi} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2}g^{\mu\nu}\left(\Phi_{,\mu}\Phi^*_{,\nu} + \Phi^*_{,\mu}\Phi_{,\nu}\right) - U(|\Phi|)\Big]$$

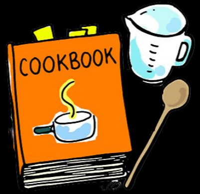
$$\Phi = \phi(r,\theta)e^{-i\omega t + im\varphi}$$

Derrick's argument: Q-balls

- The action of a complex scalar field

$$\mathcal{S}^{\Phi} = \int d^4x \sqrt{-g}\Big[-\frac{1}{2} g^{\mu\nu} \left(\Phi_{,\mu} \Phi^*_{,\nu} + \Phi^*_{,\mu} \Phi_{,\nu} \right) - U(|\Phi|) \Big]$$

$$\Phi = \phi(r,\theta) e^{-i\omega t + im\varphi}$$



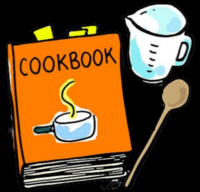
$$\int_0^\pi d\theta \int_0^{+\infty} dr\, r^2 \sin\theta \left[-3\omega^2 \phi^2 + \phi'^2 + \frac{\phi_{,\theta}^2}{r^2} + \frac{m^2 \phi^2}{r^2 \sin^2\theta} + 3U \right] = 0$$

Derrick's argument: Q-balls

- The action of a complex scalar field

$$\mathcal{S}^{\Phi} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2}g^{\mu\nu}\left(\Phi_{,\mu}\Phi^*_{,\nu} + \Phi^*_{,\mu}\Phi_{,\nu}\right) - U(|\Phi|)\Big]$$

$$\Phi = \phi(r,\theta)e^{-i\omega t + im\varphi}$$

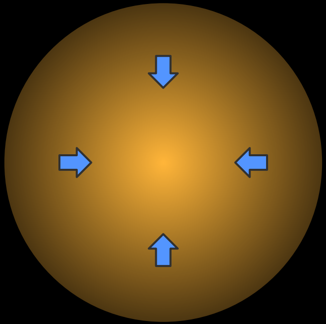


$$\int_0^{\pi} d\theta \int_0^{+\infty} dr\, r^2 \sin\theta\left[-3\omega^2\phi^2 + \phi'^2 + \frac{\phi_{,\theta}^2}{r^2} + \frac{m^2\phi^2}{r^2\sin^2\theta} + 3U\right] = 0$$

Derrick's argument: Q-balls

- The action of a complex scalar field

$$\mathcal{S}^{\Phi} = \int d^4x \sqrt{-g}\Big[-\frac{1}{2}g^{\mu\nu}\left(\Phi_{,\mu}\Phi^*_{,\nu} + \Phi^*_{,\mu}\Phi_{,\nu}\right) - U(|\Phi|)\Big]$$



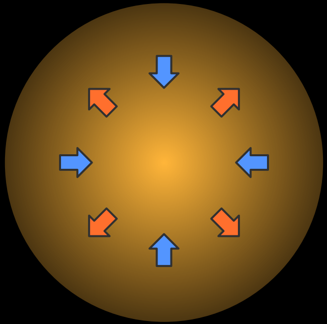
$$\Phi = \phi(r,\theta)e^{-i\omega t + im\varphi}$$

$$\int_0^{\pi} d\theta \int_0^{+\infty} dr\, r^2 \sin\theta \left[-3\omega^2\phi^2 + \phi'^2 + \frac{\phi_{,\theta}^2}{r^2} + \frac{m^2\phi^2}{r^2\sin^2\theta} + 3U\right] = 0$$

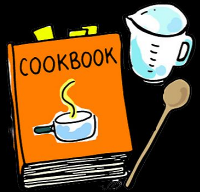
Derrick's argument: Q-balls

- The action of a complex scalar field

$$\mathcal{S}^{\Phi} = \int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\left(\Phi_{,\mu}\Phi^*_{,\nu} + \Phi^*_{,\mu}\Phi_{,\nu}\right) - U(|\Phi|)\right]$$



$$\Phi = \phi(r,\theta)e^{-i\omega t + im\varphi}$$

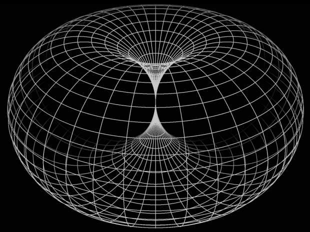


$$\int_0^{\pi} d\theta \int_0^{+\infty} dr\, r^2 \sin\theta \left[-3\omega^2\phi^2 + \phi'^2 + \frac{\phi_{,\theta}^2}{r^2} + \frac{m^2\phi^2}{r^2\sin^2\theta} + 3U\right] = 0$$

Derrick's argument: Q-balls

- The action of a complex scalar field

$$\mathcal{S}^{\Phi} = \int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\left(\Phi_{,\mu}\Phi^*_{,\nu} + \Phi^*_{,\mu}\Phi_{,\nu}\right) - U(|\Phi|)\right]$$



$$\Phi = \phi(r,\theta)e^{-i\omega t + im\varphi}$$

$$\int_0^{\pi} d\theta \int_0^{+\infty} dr\, r^2 \sin\theta \left[-3\omega^2\phi^2 + \phi'^2 + \frac{\phi_{,\theta}^2}{r^2} + \frac{m^2\phi^2}{r^2\sin^2\theta} + 3U\right] = 0$$

Derrick's argument: Gravity

- The gravitational action

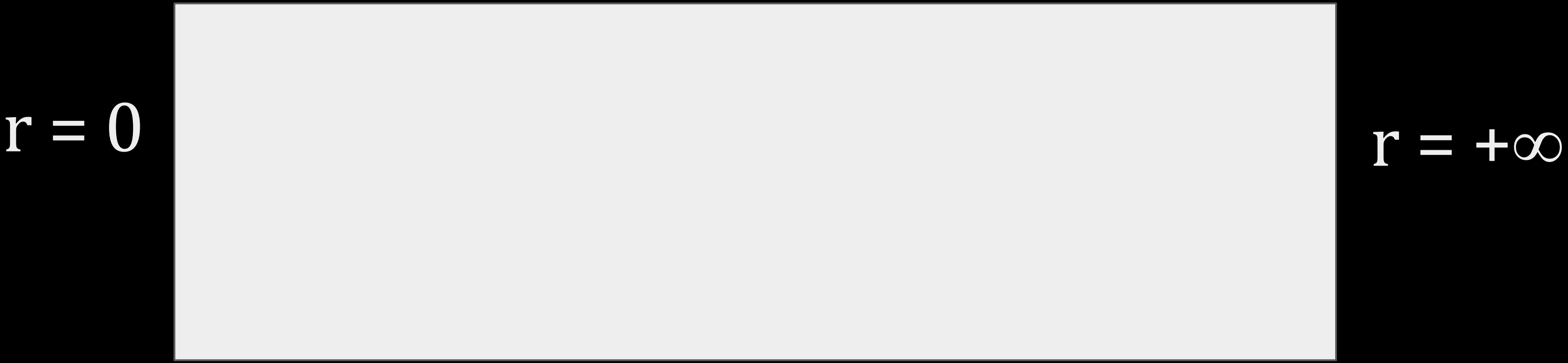
$$S_{grav} = S_{EH} + S_{GHY}$$

Derrick's argument: Gravity

- The gravitational action

$$S_{grav} = S_{EH} + S_{GHY}$$

$$= \frac{1}{4}\int_{\mathcal{M}} d^4x \sqrt{-g}R + \frac{1}{2}\int_{\partial\mathcal{M}} d^3x \sqrt{-\gamma}(K - K_0)$$



Derrick's argument: Gravity

- The gravitational action

$$S_{grav} = \boxed{S_{EH}} + S_{GHY}$$

$$= \frac{1}{4}\int_{\mathcal{M}} d^4x \boxed{\sqrt{-g}R} + \frac{1}{2}\int_{\partial\mathcal{M}} d^3x \sqrt{-\gamma}(K - K_0)$$

Derrick's argument: Gravity

- The gravitational action

$$S_{grav} = \boxed{S_{EH}} + \boxed{S_{GHY}}$$

$$= \frac{1}{4}\int_{\mathcal{M}} d^4x \boxed{\sqrt{-g}R} + \frac{1}{2}\int_{\partial\mathcal{M}} d^3x \boxed{\sqrt{-\gamma}(K - K_0)}$$

Derrick's argument: Gravity

- The gravitational action

$$S_{grav} = \boxed{S_{EH}} + \boxed{S_{GHY}}$$

$$= \frac{1}{4}\int_{\mathcal{M}} d^4x \boxed{\sqrt{-g}R} + \frac{1}{2}\int_{\partial\mathcal{M}} d^3x \boxed{\sqrt{-\gamma}(K - K_0)}$$

Derrick's argument: Gravity

- The gravitational action

$$S_{grav} = \boxed{S_{EH}} + \boxed{S_{GHY}}$$

$$= \frac{1}{4}\int_{\mathcal{M}} d^4x \boxed{\sqrt{-g}R} + \frac{1}{2}\int_{\partial\mathcal{M}} d^3x \boxed{\sqrt{-\gamma}(K - K_0)}$$

Derrick's argument: Gravity

- The gravitational action

$$S_{grav} = \boxed{S_{EH}} + \boxed{S_{GHY}}$$

$$= \frac{1}{4}\int_{\mathcal{M}} d^4x \boxed{\sqrt{-g}R} + \frac{1}{2}\int_{\partial\mathcal{M}} d^3x \boxed{\sqrt{-\gamma}(K - K_0)}$$

Derrick's argument: Gravity

- The gravitational action

$$S_{grav} = \boxed{S_{EH}} + \boxed{S_{GHY}}$$

$$= \frac{1}{4}\int_{\mathcal{M}} d^4x \boxed{\sqrt{-g}R} + \frac{1}{2}\int_{\partial\mathcal{M}} d^3x \boxed{\sqrt{-\gamma}(K - K_0)}$$

Derrick's argument: Gravity

- The gravitational action

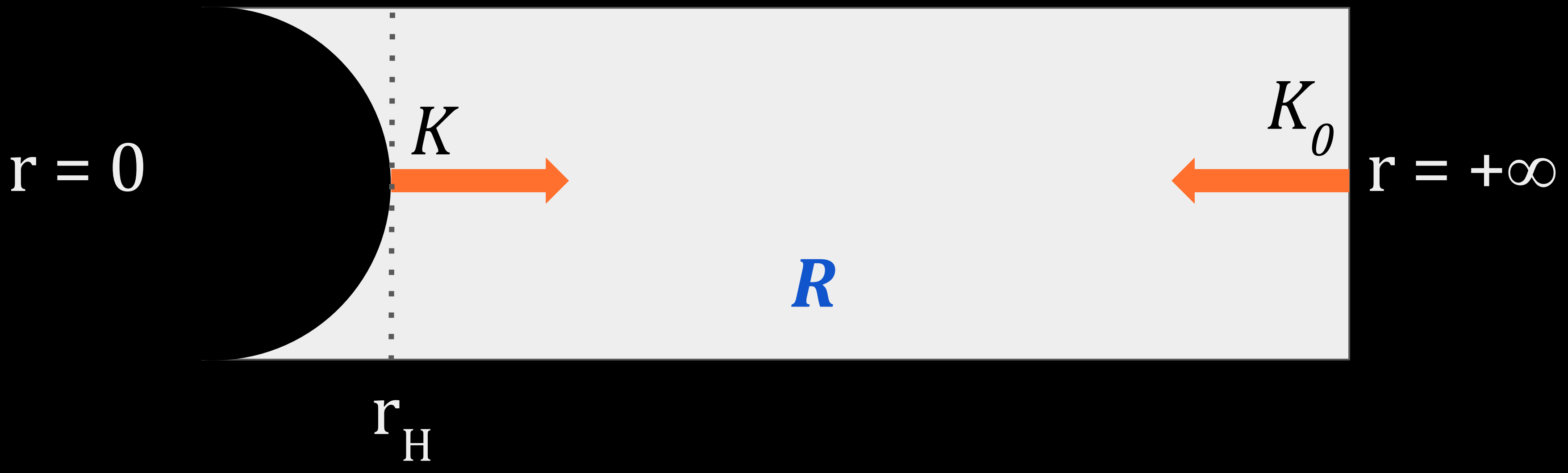
$$S_{grav} = \boxed{S_{EH}} + \boxed{S_{GHY}}$$

$$= \frac{1}{4}\int_{\mathcal{M}} d^4x \boxed{\sqrt{-g}R} + \frac{1}{2}\int_{\partial\mathcal{M}} d^3x \boxed{\sqrt{-\gamma}(K - K_0)}$$

The boundary term is needed since the gravitational Lagrangian density, *R*, contains second order derivatives of the metric tensor

Derrick's argument: Black hole

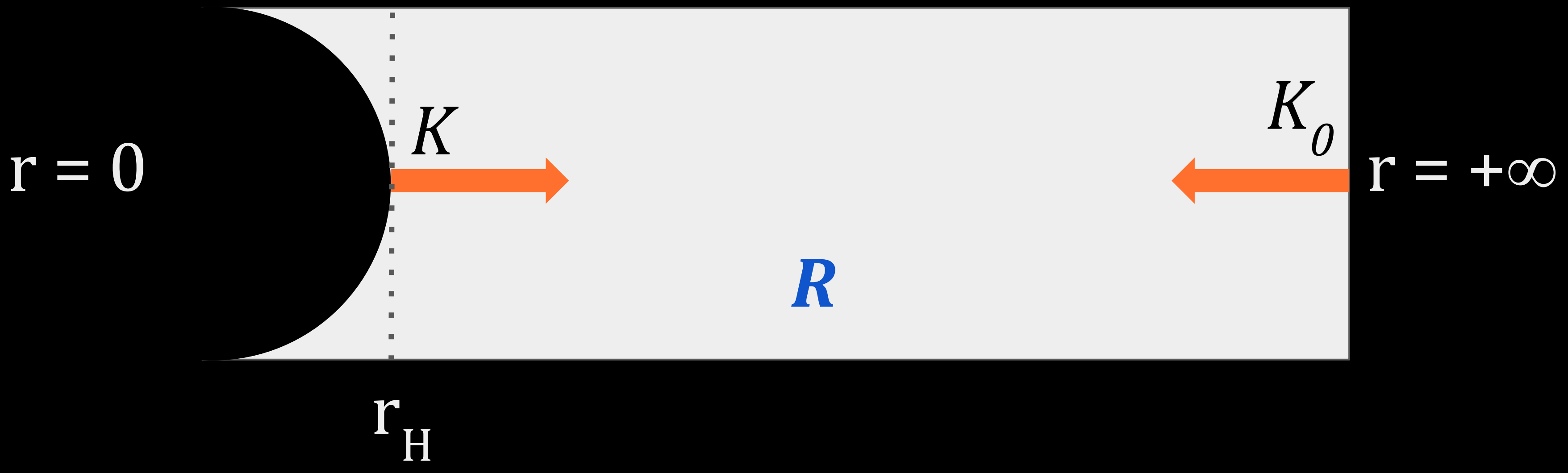
- In the presence of an horizon



Derrick's argument: Black hole

- In the presence of an horizon

$$r \to \tilde{r} = r_H + \lambda(r - r_H)$$



Derrick's argument: Kerr

- Let us pick the gravitational action again:

$$S_{grav} = S_{EH} + S_{GHY}$$

Derrick's argument: Kerr

- Let us pick the gravitational action again:

$$S_{grav} = S_{EH} + S_{GHY}$$

Derrick's argument: Kerr

- Let us pick the gravitational action again:

$$S_{grav} = S_{EH} + S_{GHY} \quad \Rightarrow \quad \int_0^{\pi} d\theta \int_{r_H}^{+\infty} dr\, I_R = I_{GHY}$$

Derrick's argument: Kerr

- Let us pick the gravitational action again:

$$S_{grav} = \boxed{S_{EH}} + S_{GHY} \quad \Rightarrow \quad \int_0^{\pi} d\theta \int_{r_H}^{+\infty} \boxed{dr\, I_R} = I_{GHY}$$

Derrick's argument: Kerr

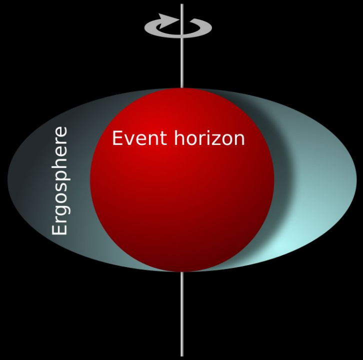
- Let us pick the gravitational action again:

$$S_{grav} = \boxed{S_{EH}} + \boxed{S_{GHY}} \Rightarrow \int_0^{\pi} d\theta \int_{r_H}^{+\infty} \boxed{dr\, I_R} = \boxed{I_{GHY}}$$

Derrick's argument: Kerr

- Let us pick the gravitational action again:

$$\mathcal{S}_{grav} = \mathcal{S}_{EH} + \mathcal{S}_{GHY} \quad \Rightarrow \quad \int_0^\pi d\theta \int_{r_H}^{+\infty} dr\, I_R = I_{GHY}$$

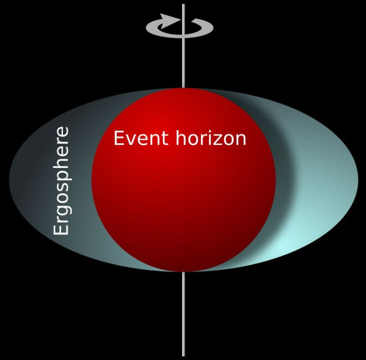


Derrick's argument: Kerr

- Let us pick the gravitational action again:

$$\mathcal{S}_{grav} = \mathcal{S}_{EH} + \mathcal{S}_{GHY} \quad \Longrightarrow \quad \int_0^{\pi} d\theta \int_{r_H}^{+\infty} dr\, I_R = I_{GHY}$$

$$ds^2 = -e^{2F_0} N dt^2 + e^{2F_1}\left(\frac{dr^2}{N} + r^2 d\theta^2\right) + e^{2F_2} r^2 \sin^2\theta (d\varphi - W dt)^2$$



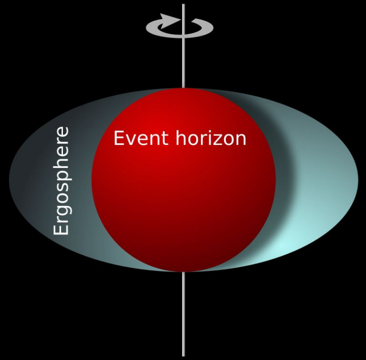
Derrick's argument: Kerr

- Let us pick the gravitational action again:

$$\mathcal{S}_{grav} = \mathcal{S}_{EH} + \mathcal{S}_{GHY} \quad \Longrightarrow \quad \int_0^{\pi} d\theta \int_{r_H}^{+\infty} dr\, I_R = I_{GHY}$$

$$ds^2 = -e^{2F_0} N dt^2 + e^{2F_1}\left(\frac{dr^2}{N} + r^2 d\theta^2\right) + e^{2F_2} r^2 \sin^2\theta (d\varphi - W dt)^2$$

$$N = 1 - \frac{r_H}{r}$$



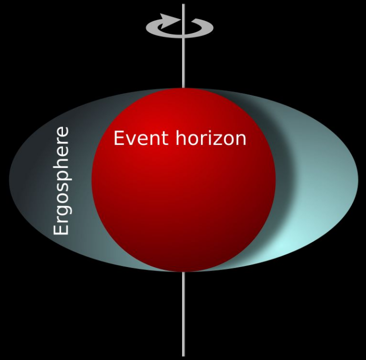
Derrick's argument: Kerr

- Let us pick the gravitational action again:

$$\mathcal{S}_{grav} = \mathcal{S}_{EH} + \mathcal{S}_{GHY} \quad \Longrightarrow \quad \int_0^{\pi} d\theta \int_{r_H}^{+\infty} dr\, I_R = \boxed{I_{GHY}}$$

$$ds^2 = -e^{2F_0} N dt^2 + e^{2F_1}\left(\frac{dr^2}{N} + r^2 d\theta^2\right) + e^{2F_2} r^2 \sin^2\theta (d\varphi - W dt)^2$$

$$N = 1 - \frac{r_H}{r}$$



Black Holes: Kerr

- The Gibbons-Hawking-York comes as

$$ds^2 = -e^{2F_0} N dt^2 + e^{2F_1}\left(\frac{dr^2}{N} + r^2 d\theta^2\right) + e^{2F_2} r^2 \sin^2\theta (d\varphi - W dt)^2$$

Black Holes: Kerr

- The Gibbons-Hawking-York comes as

$$ds^2 = -e^{2F_0} N dt^2 + e^{2F_1} \left(\frac{dr^2}{N} + r^2 d\theta^2 \right) + e^{2F_2} r^2 \sin^2\theta (d\varphi - W dt)^2$$

$$F_0 \approx \frac{c_t}{r} + \cdots \qquad F_1 = F_2 \approx -\frac{c_t}{r} + \cdots \qquad W \approx -\frac{c_t}{r^3} + \cdots$$

Black Holes: Kerr

- The Gibbons-Hawking-York comes as

$$ds^2 = -e^{2F_0} N dt^2 + e^{2F_1}\left(\frac{dr^2}{N} + r^2 d\theta^2\right) + e^{2F_2} r^2 \sin^2\theta (d\varphi - W dt)^2$$

$$F_0 \approx \frac{c_t}{r} + \cdots \qquad F_1 = F_2 \approx -\boxed{\frac{c_t}{r}} + \cdots \qquad W \approx -\frac{c_t}{r^3} + \cdots$$

Black Holes: Kerr

- The Gibbons-Hawking-York comes as

$$ds^2 = -e^{2F_0} N dt^2 + e^{2F_1}\left(\frac{dr^2}{N} + r^2 d\theta^2\right) + e^{2F_2} r^2 \sin^2\theta (d\varphi - W dt)^2$$

$$F_0 \approx \frac{c_t}{r} + \cdots \qquad F_1 = F_2 \approx -\frac{c_t}{r} + \cdots \qquad W \approx -\frac{c_t}{r^3} + \cdots$$

$$\sqrt{-\gamma} = e^{F_0+F_1+F_2}\sqrt{N}\, r^2 \sin\theta\,,$$

$$K = \frac{e^{-F_1}}{r\sqrt{N}}\left[\frac{rN'}{2} + 2N + Nr\left(F_0' + F_1' + F_2'\right)\right],$$

$$K_0 = 2\frac{e^{-F_1}}{r}\,,$$

Black Holes: Kerr

- The Gibbons-Hawking-York comes as

$$ds^2 = -e^{2F_0} N dt^2 + e^{2F_1}\left(\frac{dr^2}{N} + r^2 d\theta^2\right) + e^{2F_2} r^2 \sin^2\theta (d\varphi - W dt)^2$$

$$F_0 \approx \frac{c_t}{r} + \cdots \qquad F_1 = F_2 \approx -\frac{c_t}{r} + \cdots \qquad W \approx -\frac{c_t}{r^3} + \cdots$$

$$\sqrt{-\gamma} = e^{F_0+F_1+F_2}\sqrt{N}\, r^2 \sin\theta\,,$$

$$K = \frac{e^{-F_1}}{r\sqrt{N}}\left[\frac{rN'}{2} + 2N + Nr(F_0' + F_1' + F_2')\right],$$

$$K_0 = 2\frac{e^{-F_1}}{r}\,,$$

$$I_{GHY} = 4\,c_t$$

Black Holes: Kerr

- The Gibbons-Hawking-York comes as

$$ds^2 = -e^{2F_0} N dt^2 + e^{2F_1}\left(\frac{dr^2}{N} + r^2 d\theta^2\right) + e^{2F_2} r^2 \sin^2\theta (d\varphi - W dt)^2$$

$$F_0 \approx \frac{c_t}{r} + \cdots \qquad F_1 = F_2 \approx -\frac{c_t}{r} + \cdots \qquad W \approx -\frac{c_t}{r^3} + \cdots$$

$$\sqrt{-\gamma} = e^{F_0+F_1+F_2}\sqrt{N}\, r^2 \sin\theta\,,$$

$$K = \frac{e^{-F_1}}{r\sqrt{N}}\left[\frac{rN'}{2} + 2N + Nr\left(F_0' + F_1' + F_2'\right)\right],$$

$$K_0 = 2\frac{e^{-F_1}}{r}\,,$$

$$I_{GHY} = \boxed{4\, c_t}$$

Black Holes: Kerr

- The Einstein-Hilbert:

Black Holes: Kerr

- The Einstein-Hilbert:

$$I_R = \frac{\sin\theta}{2} e^{F_1 - F_0} \left[\; \cdots \; \right]$$

Black Holes: Kerr

- The Einstein-Hilbert:

$$I_R = \frac{\sin\theta}{2} e^{F_1 - F_0} \underbrace{\left[\ \cdots \ \right]}_{\textbf{Complicated}}$$

Black Holes: Kerr

- The Einstein-Hilbert:

$$I_R = \frac{\sin\theta}{2} e^{F_1 - F_0} \underbrace{\left[\;\ldots\; \right]}_{\textbf{Complicated}}$$

$$\int_0^{\pi} d\theta \int_{r_H}^{+\infty} dr\, I_R = 4\, c_t$$

Black Holes: Kerr

- The Einstein-Hilbert:

$$I_R = \frac{\sin\theta}{2} e^{F_1 - F_0} \underbrace{\left[\quad \cdots \quad \right]}_{\textbf{Complicated}}$$

$$\int_0^{\pi} d\theta \int_{r_H}^{+\infty} dr \, I_R = 4\, c_t$$

$$I_{GHY} = 4\, c_t$$

Black Holes: Kerr

- The Einstein-Hilbert:

$$I_R = \frac{\sin\theta}{2} e^{F_1 - F_0} \left[\ \cdots \ \right]$$

Complicated

$$\int_0^{\pi} d\theta \int_{r_H}^{+\infty} dr \, I_R = \boxed{4\, c_t}$$

$$I_{GHY} = 4\, c_t$$

Black Holes: Kerr

- The Einstein-Hilbert:

$$I_R = \frac{\sin\theta}{2} e^{F_1 - F_0} \left[\; \dots \; \right]$$

Complicated

$$\int_0^\pi d\theta \int_{r_H}^{+\infty} dr \, I_R = \boxed{4\, c_t}$$

$$I_{GHY} = \boxed{4\, c_t}$$

Black Holes: Kerr

- The Einstein-Hilbert:

$$I_R = \frac{\sin\theta}{2} e^{F_1 - F_0} \left[\cdots \right]$$

Complicated

$$\int_0^{\pi} d\theta \int_{r_H}^{+\infty} dr\, I_R = 4\, c_t$$

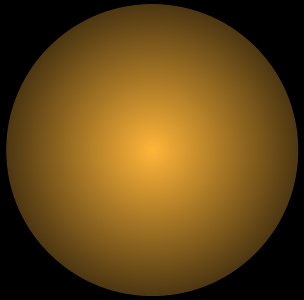
$$\|$$

$$I_{GHY} = 4\, c_t$$

Black Holes: Hairy Kerr

- Numerical metric ansatz:

$$\mathcal{S}^{\Phi} = \mathcal{S}_{grav} + \int d^4x \sqrt{-g}\left[-\frac{1}{2} g^{\mu\nu} \left(\Phi_{,\mu}\Phi^*_{,\nu} + \Phi^*_{,\mu}\Phi_{,\nu}\right) - U(|\Phi|)\right]$$



Black Holes: Hairy Kerr

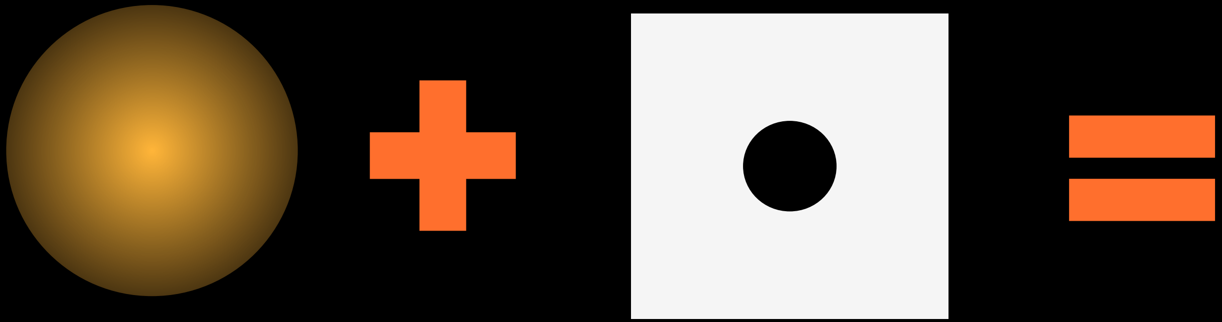
- Numerical metric ansatz:

$$\mathcal{S}^{\Phi} = \mathcal{S}_{grav} + \int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\left(\Phi_{,\mu}\Phi^{*}_{,\nu} + \Phi^{*}_{,\mu}\Phi_{,\nu}\right) - U(|\Phi|)\right]$$

Black Holes: Hairy Kerr

- Numerical metric ansatz:

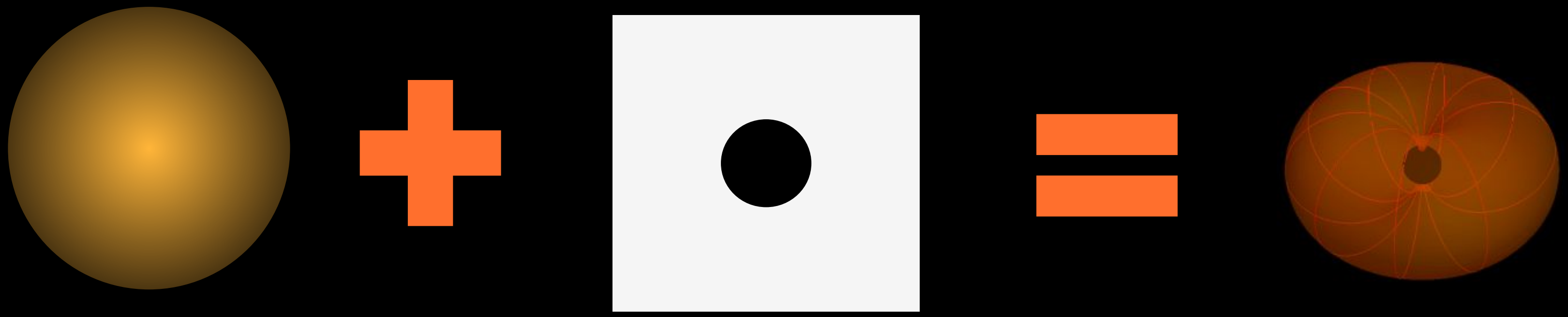
$$\mathcal{S}^{\Phi} = \mathcal{S}_{grav} + \int d^4x \sqrt{-g}\left[-\frac{1}{2} g^{\mu\nu} \left(\Phi_{,\mu} \Phi^*_{,\nu} + \Phi^*_{,\mu} \Phi_{,\nu} \right) - U(|\Phi|) \right]$$



Black Holes: Hairy Kerr

- Numerical metric ansatz:

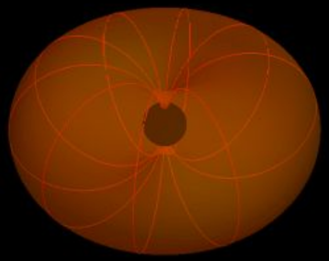
$$\mathcal{S}^{\Phi} = \mathcal{S}_{grav} + \int d^4x \sqrt{-g}\left[-\frac{1}{2} g^{\mu\nu} \left(\Phi_{,\mu} \Phi^*_{,\nu} + \Phi^*_{,\mu} \Phi_{,\nu} \right) - U(|\Phi|) \right]$$



Black Holes: Hairy Kerr

- Numerical metric ansatz:

$$\mathcal{S}^{\Phi} = \mathcal{S}_{grav} + \int d^4x \underbrace{\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\left(\Phi_{,\mu}\Phi^*_{,\nu} + \Phi^*_{,\mu}\Phi_{,\nu}\right) - U(|\Phi|)\right]}$$



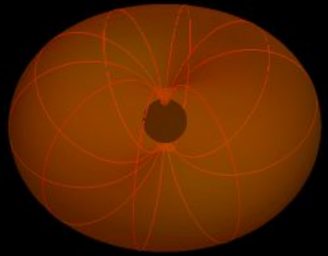
Black Holes: Hairy Kerr

- Numerical metric ansatz:

$$\mathcal{S}^{\Phi} = \mathcal{S}_{grav} + \int d^4x \underbrace{\sqrt{-g}\left[-\frac{1}{2} g^{\mu\nu} \left(\Phi_{,\mu} \Phi^*_{,\nu} + \Phi^*_{,\mu} \Phi_{,\nu} \right) - U(|\Phi|) \right]}$$

$$e^{F_0+F_2} \sin\theta \Big[r^2 N^2 \phi_{,r}^2 + \phi_{,\theta}^2 + e^{2(F_1 - F2)} \frac{m^2 \phi^2}{\sin^2\theta}$$

$$+ e^{2F_1} r^2 \left\{ \left(1 - \frac{2r_H}{3r} \right) U - e^{-2F_0} \left(\omega - mW \right)^2 \phi^2 \right\} \Big]$$



Black Holes: Hairy Kerr

- Numerical metric ansatz:

$$\mathcal{S}^{\Phi} = \mathcal{S}_{grav} + \int d^4x \sqrt{-g}\left[-\frac{1}{2} g^{\mu\nu} \left(\Phi_{,\mu} \Phi^*_{,\nu} + \Phi^*_{,\mu} \Phi_{,\nu} \right) - U(|\Phi|) \right]$$

$$e^{F_0+F_2} \sin\theta \Big[r^2 N^2 \phi_{,r}^2 + \phi_{,\theta}^2 + e^{2(F_1 - F2)} \frac{m^2 \phi^2}{\sin^2\theta}$$

$$+ e^{2F_1} r^2 \left\{ \left(1 - \frac{2r_H}{3r} \right) U - e^{-2F_0} \left(\omega - mW \right)^2 \phi^2 \right\} \Big]$$

$$I_R = -\frac{1}{2} \sin$$

$$\theta e^{F_2 - F_0} \Big(-2 r_H e^{2F_0} \left(2 \left((r - r_H) \left(F_0'' + F_1'' + F_2'' \right) + F_2' + (r - r_H) F_2'^2 \right) + F_0' \left(2(r - r_H) F_2' + 3 \right) + 2(r - r_H) F_0'^2 + F_1' \right)$$

$$+ 2e^{2F_0} \Big(2 \left(r(r - r_H) \left(F_0'' + F_1'' + F_2'' \right) + r(r - r_H) F_2'^2 + (3r - 2r_H) F_2 \right) - F_0' \left(-2r(r - r_H) F_2' - 4r + r_H \right) + 2\hat{\hat{F}}_0 + 2r(r - r_H) F_0'^2 + 2\hat{\hat{F}}_0$$

$$+ (2r - r_H) F_1' + 2\hat{\hat{F}}_2 + 2\hat{F}_2 \left(\hat{F}_2 + 2\cot\theta \right) \Big) + 4e^{2F_0} \hat{F}_0 \left(\hat{F}_2 + \cot\theta \right) + 4e^{2F_0} \hat{F}_0^{\,2} + r^3 r_H \sin^2\theta e^{2F_2} W'^2 - 3r^2 \sin^2\theta e^{2F_2} \left(r(r - r_H) W'^2 + \hat{W}^2 \right) \Big)$$

Black Holes: Hairy Kerr

- Numerical metric ansatz:

$$S^{\Phi} = S_{grav} + \ldots) - U(|\Phi|) \Big]$$

$$F2) \frac{m^2 \phi^2}{\sin^2 \theta}$$

$$- mW)^2 \phi^2 \Big\} \Big]$$

$$I_R = -\tfrac{1}{2}\sin$$

$$\theta e^{F_2 - F_0} \left(-2 r_H e^{2F_0} \left(2 \left((r - r_H) \right(\ldots \right)$$

$$+ 2e^{2F_0} \left(2 \left(r(r - r_H) \left(F_0'' + F_1'' \ldots 2r(r - r_H) F_0'^2 + 2\hat{\hat{F}}_0 \right.\right.$$

$$+ (2r - r_H) F_1' + 2\hat{\hat{F}}_2 + 2\hat{F}_2 \left(\hat{F}_2 \ldots \right. F_2 \left(r(r - r_H) W'^2 + \hat{W}^2 \right) \Big)$$

Plating

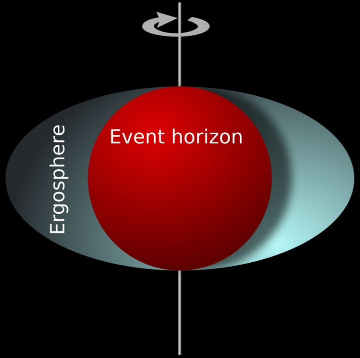
Convenient metric

arXiv:2207.12451

Derrick's argument: Black holes

- Let us introduce the a new metric ansatz:

$$ds^2 = -F_0^2 dt^2 + HF_1^2 dr^2 + (r - r_H)^2 F_1^2 d\theta^2 + F_2^2 (d\varphi - F_W dt)^2$$

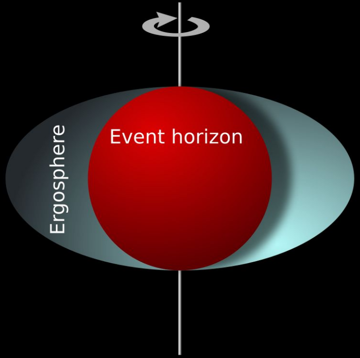


Derrick's argument: Black holes

- Let us introduce the a new metric ansatz:

$$ds^2 = -F_0^2 dt^2 + HF_1^2 dr^2 + (r - r_H)^2 F_1^2 d\theta^2 + F_2^2 (d\varphi - F_W dt)^2$$

$$S_{grav} = S_{EH} + S_{GHY}$$

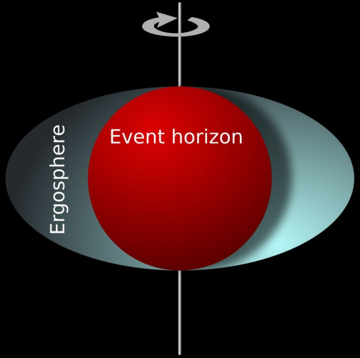


Derrick's argument: Black holes

- Let us introduce the a new metric ansatz:

$$ds^2 = -F_0^2 dt^2 + HF_1^2 dr^2 + (r - r_H)^2 F_1^2 d\theta^2 + F_2^2 (d\varphi - F_W dt)^2$$

$$S_{grav} = S_{EH} + S_{GHY}$$



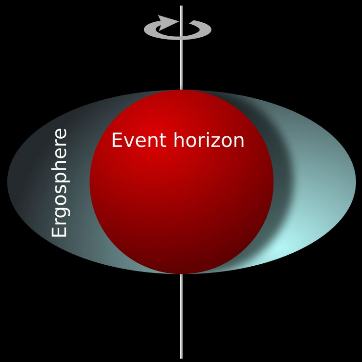
Derrick's argument: Black holes

- Let us introduce the a new metric ansatz:

$$ds^2 = -F_0^2\,dt^2 + H F_1^2\,dr^2 + (r - r_H)^2 F_1^2\,d\theta^2 + F_2^2\,(d\varphi - F_W\,dt)^2$$

$$S_{grav} = S_{EH} + S_{GHY}$$

$$\int_0^{\pi} d\theta \int_{r_H}^{+\infty} dr\, I_R = I_{GHY}$$



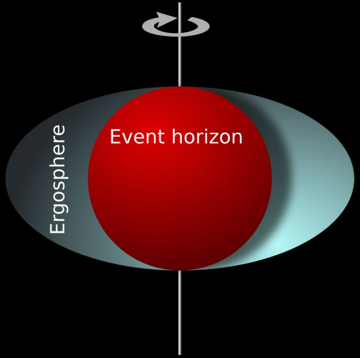
Derrick's argument: Black holes

- Let us introduce the a new metric ansatz:

$$ds^2 = -F_0^2 dt^2 + H F_1^2 dr^2 + (r - r_H)^2 F_1^2 d\theta^2 + F_2^2 (d\varphi - F_W dt)^2$$

$$S_{grav} = S_{EH} + S_{GHY}$$

$$\int_0^\pi d\theta \int_{r_H}^{+\infty} dr\, I_R = I_{GHY}$$



Derrick's argument: Black holes

- The Gibbons-Hawking-York term:

$$ds^2 = -F_0^2 dt^2 + H F_1^2 dr^2 + (r - r_H)^2 F_1^2 d\theta^2 + F_2^2 (d\varphi - F_W dt)^2$$

$$\sqrt{-\gamma} = \frac{1}{\sqrt{H}\, F_1} \,,$$

$$K = \frac{1}{(r-r_H) F_0 F_1^2 F_2 \sqrt{H}} \left[(r - r_H) F_1 F_2 F_0' + F_0 \Big((r - r_H) F_2 F_1' + F_1 \big(F_2 + (r - r_H) F_2' \big) \Big) \right] ,$$

$$K_0 = \frac{2}{(r-r_H) F_1} \,,$$

Derrick's argument: Black holes

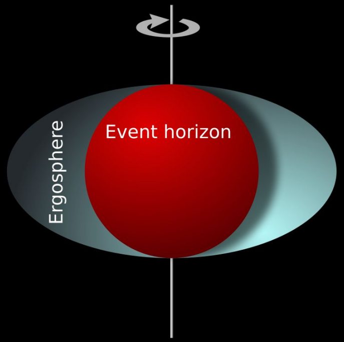
- The Gibbons-Hawking-York term:

$$ds^2 = -F_0^2 dt^2 + H F_1^2 dr^2 + (r - r_H)^2 F_1^2 d\theta^2 + F_2^2 (d\varphi - F_W dt)^2$$

$$\sqrt{-\gamma} = \frac{1}{\sqrt{H}\, F_1} \,,$$

$$K = \frac{1}{(r-r_H) F_0 F_1^2 F_2 \sqrt{H}} \Big[(r - r_H) F_1 F_2 F_0' + F_0 \Big((r - r_H) F_2 F_1' + F_1 \big(F_2 + (r - r_H) F_2' \big) \Big) \Big] \,,$$

$$K_0 = \frac{2}{(r-r_H) F_1} \,,$$



$$I_{GHY} = \int_0^{\pi} d\theta \; 0 \Big|_{r_H}^{\infty} = 0$$

Derrick's argument: Black holes

- The Gibbons-Hawking-York term:

$$ds^2 = -F_0^2 dt^2 + H F_1^2 dr^2 + (r - r_H)^2 F_1^2 d\theta^2 + F_2^2 (d\varphi - F_W dt)^2$$

$$\sqrt{-\gamma} = \frac{1}{\sqrt{H}\, F_1} \,,$$

$$K = \frac{1}{(r-r_H) F_0 F_1^2 F_2 \sqrt{H}} \left[(r - r_H) F_1 F_2 F_0' + F_0 \Big((r - r_H) F_2 F_1' + F_1 \big(F_2 + (r - r_H) F_2' \big) \Big) \right] ,$$

$$K_0 = \frac{2}{(r-r_H) F_1} \,,$$

$$I_{GHY} = \int_0^{\pi} d\theta \;\; 0 \Big|_{r_H}^{\infty} = 0$$

Derrick's argument: Black holes

- The Einstein-Hilbert part:

$$ds^2 = -F_0^2 dt^2 + H F_1^2 dr^2 + (r - r_H)^2 F_1^2 d\theta^2 + F_2^2 (d\varphi - F_W dt)^2$$

Derrick’s argument: Black holes

- The Einstein-Hilbert part:

$$ds^2 = -F_0^2\,dt^2 + H F_1^2\,dr^2 + (r - r_H)^2 F_1^2\,d\theta^2 + F_2^2\,(d\varphi - F_W\,dt)^2$$

$$\begin{aligned}
&\sqrt{-g}R \\
&= \frac{1}{2F_0F_1^2H^{3/2}(r-r_H)}\Bigg\{F_1^2F_2^2H\Big(H\hat{F}_W^2 + (r-r_H)^2F_W'^2\Big) + 2F_0F_1^2\Big[-2H^2\partial_\theta(\hat{F}_0F_2) + (r-r_H)\Big(-2\partial_r[F_0'F_2H(r-r_H)] + 3F_0'F_2H'(r-r_H)\Big)\Big] \\
&+ 2F_0^2\Big[2F_2H(H\hat{F}_1^2 + (r-r_H)F_1'^2) - 2H^2F_1(\hat{\hat{F}}_1F_2 + F_1\hat{\hat{F}}_2) + (r-r_H)\Big([(r-r_H)H' - 2H]F_1\partial_r(F_1F_2) - 2F_1H(r-r_H)(F_2\hat{\hat{F}}_2 + F_1F_2'') \\
&+ F_2F_1^2H'\Big)\Big]\Bigg\}
\end{aligned}$$

Derrick's argument: Black holes

- The Einstein-Hilbert part:

$$ds^2 = -F_0^2\,dt^2 + H F_1^2\,dr^2 + (r - r_H)^2 F_1^2\,d\theta^2 + F_2^2\,(d\varphi - F_W\,dt)^2$$

$$\begin{aligned}
&\sqrt{-g}R \\
&= \frac{1}{2F_0 F_1^2 H^{3/2}(r-r_H)}\Bigg\{F_1^2 F_2^2 H\Big(H\hat{F}_W^2 + (r-r_H)^2 F_W'^2\Big) + 2F_0 F_1^2\Big[-2H^2\partial_\theta(\hat{F}_0 F_2) + (r-r_H)\Big(-2\partial_r[F_0' F_2 H(r-r_H)] + 3F_0' F_2 H'(r-r_H)\Big)\Big] \\
&+ 2F_0^2\Big[2F_2 H(H\hat{F}_1^2 + (r-r_H)F_1'^2) - 2H^2 F_1(\hat{\hat{F}}_1 F_2 + F_1\hat{\hat{F}}_2) + (r-r_H)\Big([(r-r_H)H' - 2H]F_1\partial_r(F_1F_2) - 2F_1 H(r-r_H)(F_2\hat{\hat{F}}_2 + F_1 F_2'') \\
&+ F_2 F_1^2 H'\Big)\Big]\Bigg\}
\end{aligned}$$

For each radial derivative F_i' there is an $(r\text{-}r_H)$

Derrick's argument: Black holes

- The Einstein-Hilbert part:

$$ds^2 = -F_0^2 dt^2 + H F_1^2 dr^2 + (r - r_H)^2 F_1^2 d\theta^2 + F_2^2 (d\varphi - F_W dt)^2$$

$$\begin{aligned}
&\sqrt{-g}R \\
&= \frac{1}{2F_0 F_1^2 H^{3/2}(r-r_H)} \Big\{ F_1^2 F_2^2 H \Big(H \hat{F}_W^2 + (r - r_H)^2 F_W'^2 \Big) + 2F_0 F_1^2 \Big[-2H^2 \partial_\theta (\hat{F}_0 F_2) + (r - r_H) \Big(-2\partial_r [F_0' F_2 H (r - r_H)] + 3F_0' F_2 H' (r - r_H) \Big) \Big] \\
&+ 2F_0^2 \Big[2F_2 H \big(H \hat{F}_1^2 + (r - r_H) F_1'^2 \big) - 2H^2 F_1 (\hat{\hat{F}}_1 F_2 + F_1 \hat{\hat{F}}_2) + (r - r_H) \Big([(r - r_H) H' - 2H] F_1 \partial_r (F_1 F_2) - 2F_1 H (r - r_H)(F_2 \hat{\hat{F}}_2 + F_1 F_2'') \\
&+ F_2 F_1^2 H' \Big) \Big] \Big\}
\end{aligned}$$

For each radial derivative F_i' there is an $(r\text{-}r_H)$

$$\sqrt{-g_\lambda} R_\lambda = \frac{1}{\lambda} \sqrt{-g} R$$

Derrick's argument: Black holes

- The Einstein-Hilbert part:

$$ds^2 = -F_0^2\,dt^2 + H F_1^2\,dr^2 + (r-r_H)^2 F_1^2\,d\theta^2 + F_2^2\,(d\varphi - F_W\,dt)^2$$

$$\begin{aligned}
&\sqrt{-g}R \\
&= \frac{1}{2F_0F_1^2H^{3/2}(r-r_H)}\Bigg\{F_1^2F_2^2H\Big(H\hat{F}_W^2 + (r-r_H)^2F_W'^2\Big) + 2F_0F_1^2\Big[-2H^2\partial_\theta(\hat{F}_0F_2) + (r-r_H)\Big(-2\partial_r[F_0'F_2H(r-r_H)] + 3F_0'F_2H'(r-r_H)\Big)\Big] \\
&+ 2F_0^2\Big[2F_2H(H\hat{F}_1^2 + (r-r_H)F_1'^2) - 2H^2F_1(\hat{\hat{F}}_1F_2 + F_1\hat{\hat{F}}_2) + (r-r_H)\Big(\big[(r-r_H)H' - 2H\big]F_1\partial_r(F_1F_2) - 2F_1H(r-r_H)(F_2\hat{\hat{F}}_2 + F_1F_2'') \\
&+ F_2F_1^2H'\Big)\Big]\Bigg\}
\end{aligned}$$

For each radial derivative F_i' there is an $(r\text{-}r_H)$

$$\sqrt{-g_\lambda}R_\lambda = \frac{1}{\lambda}\sqrt{-g}R \quad \xrightarrow{\lambda\,dr}$$

Derrick's argument: Black holes

- The Einstein-Hilbert part:

$$ds^2 = -F_0^2\,dt^2 + H F_1^2\,dr^2 + (r-r_H)^2 F_1^2\,d\theta^2 + F_2^2\,(d\varphi - F_W\,dt)^2$$

$$\begin{aligned}
&\sqrt{-g}R \\
&= \frac{1}{2F_0F_1^2H^{3/2}(r-r_H)}\Big\{F_1^2F_2^2H\Big(H\hat{F}_W^2 + (r-r_H)^2F_W'^2\Big) + 2F_0F_1^2\Big[-2H^2\partial_\theta(\hat{F}_0F_2) + (r-r_H)\Big(-2\partial_r[F_0'F_2H(r-r_H)] + 3F_0'F_2H'(r-r_H)\Big)\Big] \\
&+ 2F_0^2\Big[2F_2H(H\hat{F}_1^2 + (r-r_H)F_1'^2) - 2H^2F_1(\hat{\hat{F}}_1F_2 + F_1\hat{\hat{F}}_2) + (r-r_H)\Big([(r-r_H)H' - 2H]F_1\partial_r(F_1F_2) - 2F_1H(r-r_H)(F_2\hat{\hat{F}}_2 + F_1F_2'') \\
&+ F_2F_1^2H'\Big)\Big]\Big\}
\end{aligned}$$

For each radial derivative F_i' there is an $(r\text{-}r_H)$

$$\sqrt{-g_\lambda}R_\lambda = \frac{1}{\lambda}\sqrt{-g}R \quad \xrightarrow{\lambda\,dr} \quad I_R = 0$$

Black Holes: Hairy Kerr

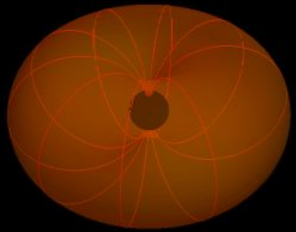
- Numerical metric ansatz:

$$\mathcal{S}^{\Phi} = \mathcal{S}_{grav} + \int d^4x \sqrt{-g}\left[-\frac{1}{2} g^{\mu\nu} \left(\Phi_{,\mu} \Phi^*_{,\nu} + \Phi^*_{,\mu} \Phi_{,\nu} \right) - U(|\Phi|) \right]$$

Black Holes: Hairy Kerr

- Numerical metric ansatz:

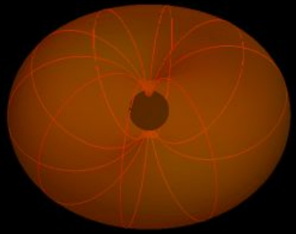
$$\mathcal{S}^{\Phi} = \mathcal{S}_{grav} + \int d^4x\sqrt{-g}\Big[-\frac{1}{2}g^{\mu\nu}\left(\Phi_{,\mu}\Phi^*_{,\nu} + \Phi^*_{,\mu}\Phi_{,\nu}\right) - U(|\Phi|)\Big]$$



Black Holes: Hairy Kerr

- Numerical metric ansatz:

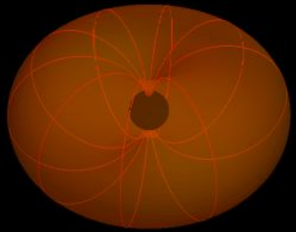
$$\mathcal{S}^{\Phi} = \mathcal{S}_{grav} + \int d^4x \sqrt{-g}\Big[-\frac{1}{2} g^{\mu\nu} \left(\Phi_{,\mu} \Phi^*_{,\nu} + \Phi^*_{,\mu} \Phi_{,\nu} \right) - U(|\Phi|) \Big]$$



Black Holes: Hairy Kerr

- Numerical metric ansatz:

$$S^{\Phi} = \mathcal{S}_{grav} + \int d^4x \sqrt{-g}\left[-\frac{1}{2} g^{\mu\nu} \left(\Phi_{,\mu} \Phi^*_{,\nu} + \Phi^*_{,\mu} \Phi_{,\nu} \right) - U(|\Phi|) \right]$$

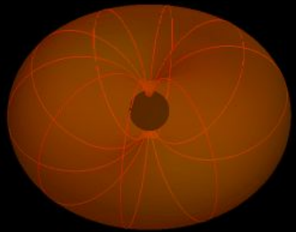


Black Holes: Hairy Kerr

- Numerical metric ansatz:

$$S^{\Phi} = S_{grav} + \int d^4x \sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\left(\Phi_{,\mu}\Phi^*_{,\nu} + \Phi^*_{,\mu}\Phi_{,\nu}\right) - U(|\Phi|)\right]$$

$$\int d^3x \; (r - r_H)F_1^2\left[\frac{\sqrt{H}}{F_0 F_2}\left(m^2 F_0^2 - F_2^2\left(\omega - m F_W^2\right)\right)\phi^2 + F_0 F_2 U(\phi)\right] = 0$$



Toppings

Master Identities: Hairy Kerr

- Matter part:

$$\mathcal{V}_{scalar} = \int d^3x\sqrt{-g}\Big\{\left(1-\frac{3r_H}{2r}\right)T^t_\varphi W - \Big[\left(1-\frac{r_H}{2r}\right)T^r_r + N\left(T^\theta_\theta + T^\varphi_\varphi\right) + \frac{r_H}{2r}T^t_t\Big]\Big\}$$

Master Identities: Hairy Kerr

- Matter part:

$$\mathcal{V}_{scalar} = \int d^3x \sqrt{-g} \left\{ \left(1 - \frac{3r_H}{2r}\right) T^t_\varphi W - \left[\left(1 - \frac{r_H}{2r}\right) T^r_r + N\left(T^\theta_\theta + T^\varphi_\varphi\right) + \frac{r_H}{2r} T^t_t \right] \right\}$$

$$E^\nu_\mu \equiv G^\nu_\mu - 8\pi T^\nu_\mu = 0$$

Master Identities: Hairy Kerr

- Matter part:

$$\mathcal{V}_{scalar} = \int d^3x\sqrt{-g}\left\{\left(1-\frac{3r_H}{2r}\right)T^t_\varphi W - \left[\left(1-\frac{r_H}{2r}\right)T^r_r + N\left(T^\theta_\theta + T^\varphi_\varphi\right) + \frac{r_H}{2r}T^t_t\right]\right\}$$

$$E^\nu_\mu \equiv G^\nu_\mu - 8\pi T^\nu_\mu = 0$$

$$\int d^3x\sqrt{-g}\left\{\left(1-\frac{3r_H}{2r}\right)E^t_\varphi W - \left[\left(1-\frac{r_H}{2r}\right)E^r_r + N\left(E^\theta_\theta + E^\varphi_\varphi\right) + \frac{r_H}{2r}E^t_t\right]\right\} = 0$$

Master Identities: Convenient gauge

- Matter part:

$$\int d^3x \, \sqrt{-g}\left[2\left(T_t^t - T_r^r - T_\theta^\theta\right) + \left(T_t^t - T_\varphi^\varphi\right)\left(1 - \frac{4\omega F_2(\omega - mF_W)}{m^2 F_0^2 + F_2^2(\omega^2 - m^2 F_W^2)}\right)\right] = 0$$

The stress energy tensor is multiplication parameters is matter dependent

Master Identities: Convenient gauge

- Matter part:

$$\int d^3x \sqrt{-g}\left[2\left(T^t_t - T^r_r - T^\theta_\theta\right) + \left(T^t_t - T^\varphi_\varphi\right)\left(1 - \boxed{\frac{4\omega F_2(\omega - mF_W)}{m^2F_0^2 + F_2^2(\omega^2 - m^2F_W^2)}}\right)\right] = 0$$

The stress energy tensor is multiplication parameters is matter dependent

Master Identities: Convenient gauge

- Matter part:

$$\int d^3x\ \sqrt{-g}\left[2\left(T^t_t - T^r_r - T^\theta_\theta\right) + \left(T^t_t - T^\varphi_\varphi\right)\left(1 - \boxed{\frac{4\omega F_2(\omega - mF_W)}{m^2F_0^2 + F_2^2(\omega^2 - m^2F_W^2)}}\right)\right] = 0$$

The stress energy tensor is multiplication parameters is matter dependent

Can be seen as a generalization of Desert's theorem as a sum of pressures

Conclusion

Summary

- We presented a generic recipe to compute virial identities in field theory
- The GHY term is required due to the presence of second-order derivatives of the metric
- One noticed that, for a generic metric, relations are too complex
- There is a special "gauge" choice that trivializes the gravitational contribution
- The identities can be recast as combinations of the equations of motion
- This has allowed us to obtain some master form identities

Summary

- We presented a generic recipe to compute virial identities in field theory
- The GHY term is required due to the presence of second-order derivatives of the metric
- One noticed that, for a generic metric, relations are too complex
- There is a special "gauge" choice that trivializes the gravitational contribution
- The identities can be recast as combinations of the equations of motion
- This has allowed us to obtain some master form identities

Summary

- We presented a generic recipe to compute virial identities in field theory
- The GHY term is required due to the presence of second-order derivatives of the metric
- One noticed that, for a generic metric, relations are too complex
- There is a special "gauge" choice that trivializes the gravitational contribution
- The identities can be recast as combinations of the equations of motion
- This has allowed us to obtain some master form identities

Summary

- We presented a generic recipe to compute virial identities in field theory
- The GHY term is required due to the presence of second-order derivatives of the metric
- One noticed that, for a generic metric, relations are too complex
- There is a special "gauge" choice that trivializes the gravitational contribution
- The identities can be recast as combinations of the equations of motion
- This has allowed us to obtain some master form identities

Summary

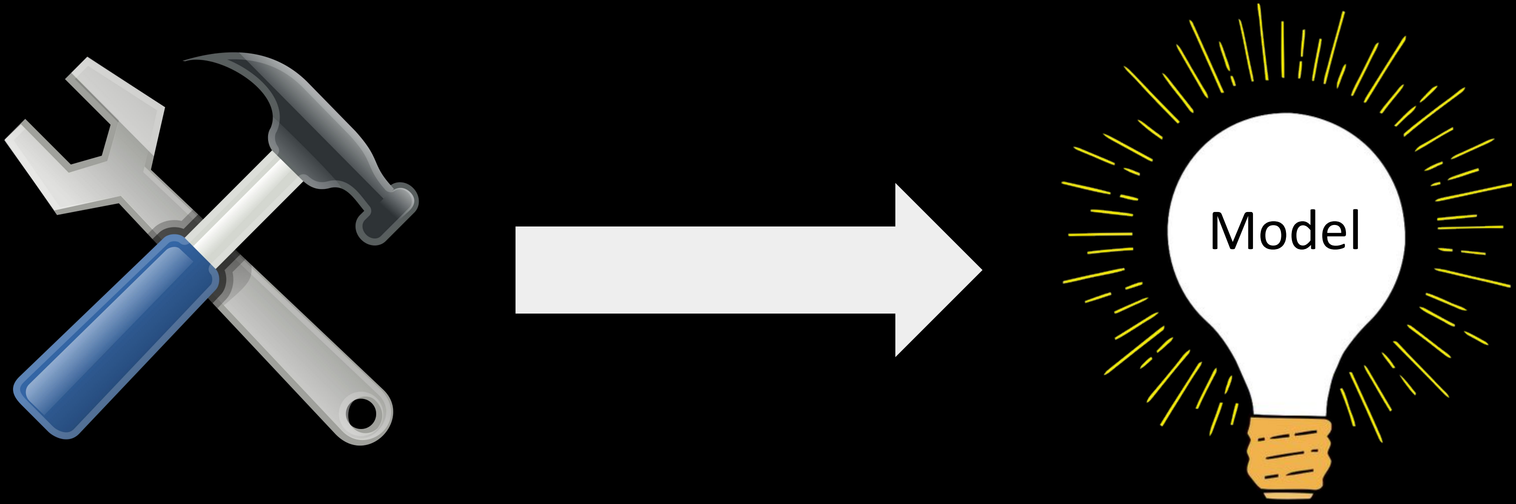
- We presented a generic recipe to compute virial identities in field theory
- The GHY term is required due to the presence of second-order derivatives of the metric
- One noticed that, for a generic metric, relations are too complex
- There is a special "gauge" choice that trivializes the gravitational contribution
- The identities can be recast as combinations of the equations of motion
- This has allowed us to obtain some master form identities

Summary

- We presented a generic recipe to compute virial identities in field theory
- The GHY term is required due to the presence of second-order derivatives of the metric
- One noticed that, for a generic metric, relations are too complex
- There is a special "gauge" choice that trivializes the gravitational contribution
- The identities can be recast as combinations of the equations of motion
- This has allowed us to obtain some master form identities

Conclusion

Virial identities are a helpful tool that can be used to have a better insight into the models



Virial **Insight**

Thanks

Obrigado!

FZU
ceico
Czech Academy of Sciences
GAČR
CZECH SCIENCE FOUNDATION

XV Black Holes Workshop
19–20 Dec 2022
ISCTE - University Institute of Lisbon

Virial identities in relativistic gravity

2109.05027
2206.02813
2207.12451

pombo@fzu.cz

Thank you!
Obrigado!

FZU ceico

Czech Academy of Sciences

GAČR
CZECH SCIENCE FOUNDATION

XV Black Holes Workshop

19–20 Dec 2022
ISCTE - University Institute of Lisbon

Virial identities in relativistic gravity

2109.05027
2206.02813
2207.12451

pombo@fzu.cz