Causal visualization of collapsing spacetime on hyperboloidal slices

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XV Black Holes Workshop, ISCTE Lisbon - 19th December 2022

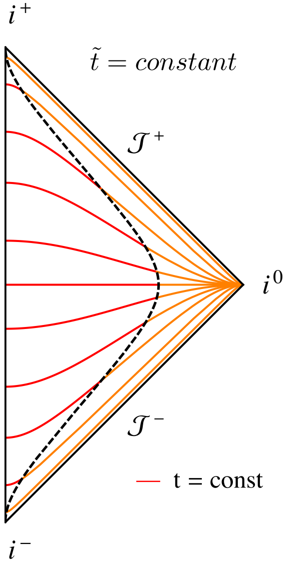
Visualisation of spacetime slices

Carter-Penrose diagrams:

- include the whole spacetime via a coordinate compactification,
- use a conformal transformation,

$$g_{ab} = \Omega^2 \tilde{g}_{ab},$$

- illustrate causal properties,
- show how the chosen coordinates (t, r) foliate spacetime.



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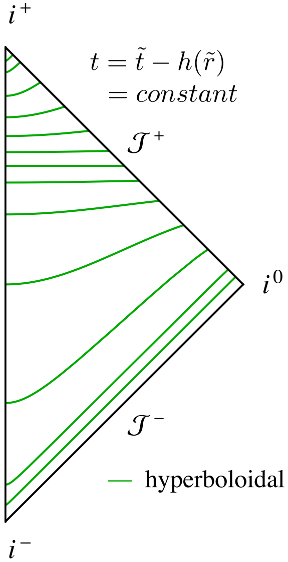
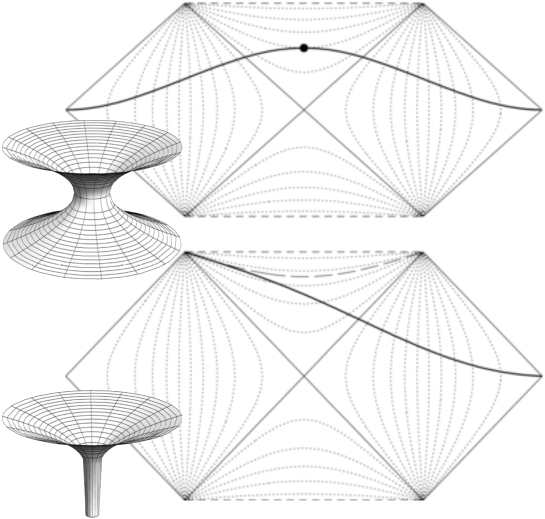
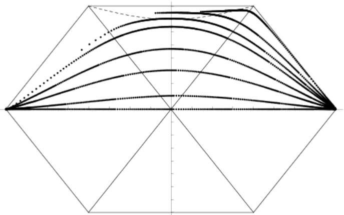


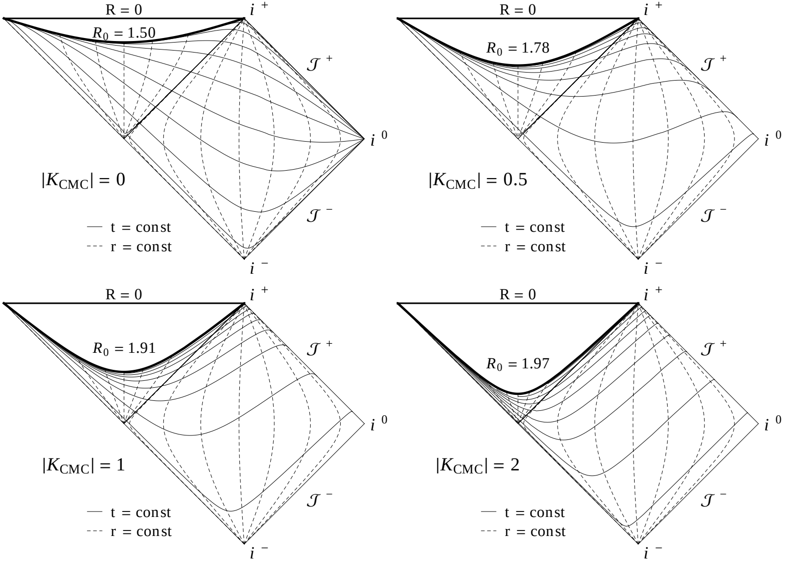
Wormhole to trumpet geometry in puncture evolution





Punctures, (Baumgarte, 2011, Class. Quantum Grav. 28 215003; Hannam et al, 2008, Phys.Rev. D78 064020).

Schwarzschild trumpet CMC slices



Evolve the Penrose diagram quantities

- The ingoing and outgoing null coordinates, $u = \tilde{t} - \tilde{r}\,, v = \tilde{t} + \tilde{r}$, satisfy the advection equations

$$\partial_t u = -c_+ \partial_r u\,, \quad \partial_t v = -c_- \partial_r v \quad \text{with} \quad c_\pm = \left(\pm\alpha\sqrt{\frac{\chi}{\gamma_{rr}}} - \beta^r \right)\,,$$

(from the Eikonal equations, $g^{uu} = g^{ab}\nabla_a u \nabla_b u = 0 = g^{vv}$.

- Evolve instead

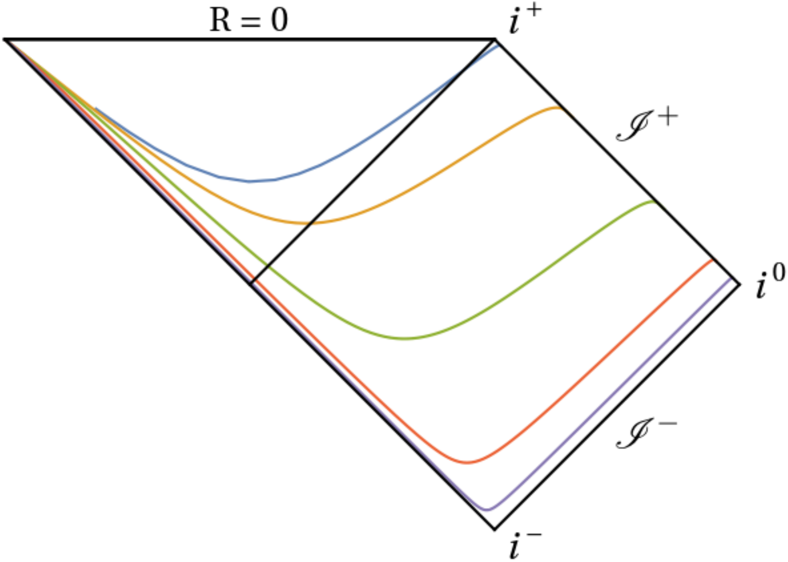
$$R = \frac{1}{2}(v - u)\,, \quad T = \frac{1}{2}(u + v)$$

for hyperboloidal compactified initial data:

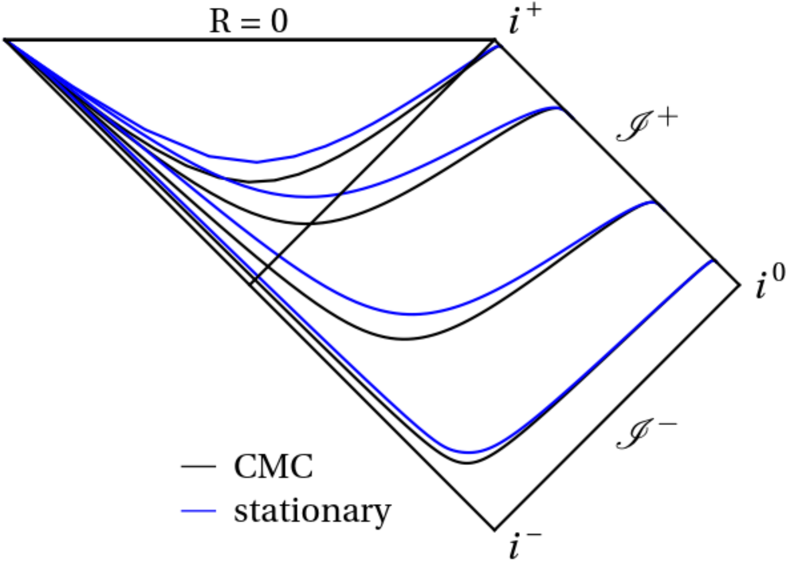
$$\partial_t R = \beta^r \partial_r R + \alpha\sqrt{\frac{\chi}{\gamma_{rr}}}\partial_r T\,, \qquad \partial_t T = \beta^r \partial_r T + \alpha\sqrt{\frac{\chi}{\gamma_{rr}}}\partial_r R\,.$$

- Plot T as a function of R in a Penrose diagram

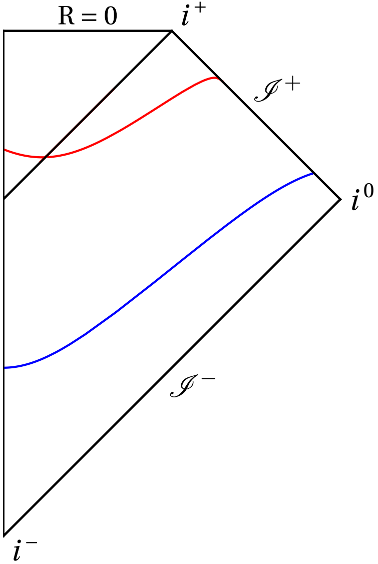
Evolution of hyperboloidal trumpet slices



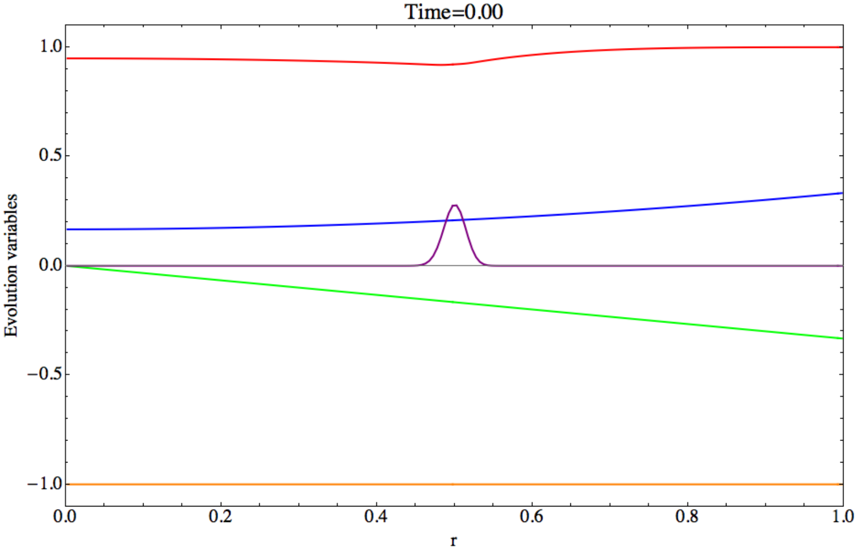
Comparison between CMC and relaxed slices



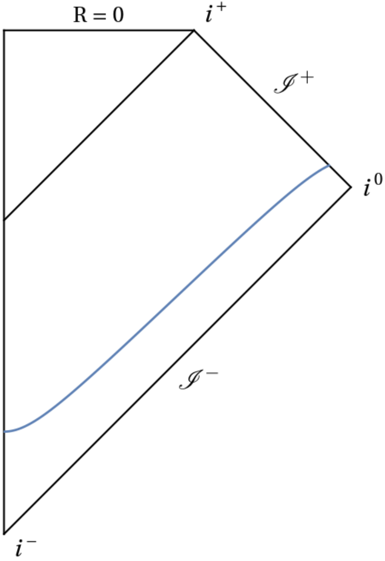
Collapse: change in slices



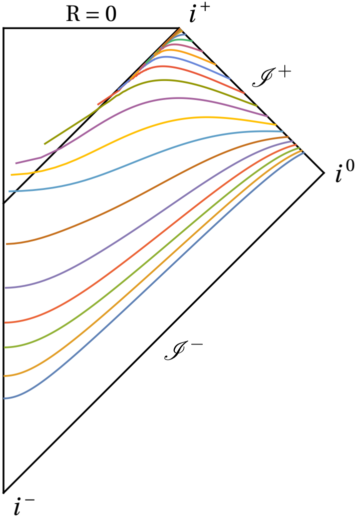
Evolution: $\chi, \tilde{K}, \alpha, \beta^r, \Phi/\Omega$ x

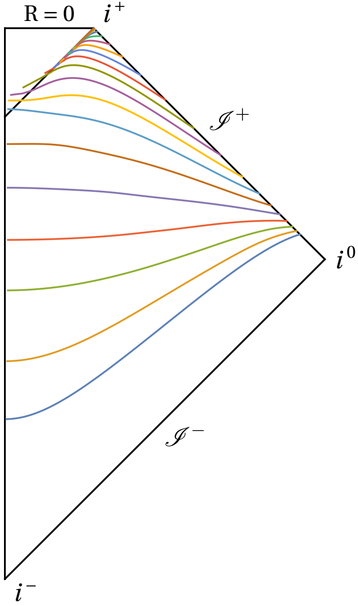


Evolution of collapse hyperboloidal slices x



Different initial slices





Summary

- Penrose diagrams are a very useful tool for the visualisation of spacetime foliations.
- Here they were used to represent hyperboloidal slices of Schwarzschild trumpet and collapsing spacetimes.
- Now able to depict the dynamical evolution of hyperboloidal spacetime slices due to gauge dynamics and collapse.
- Ongoing work:
    - Kerr hyperboloidal trumpet.

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Thank you for your attention!

Questions?