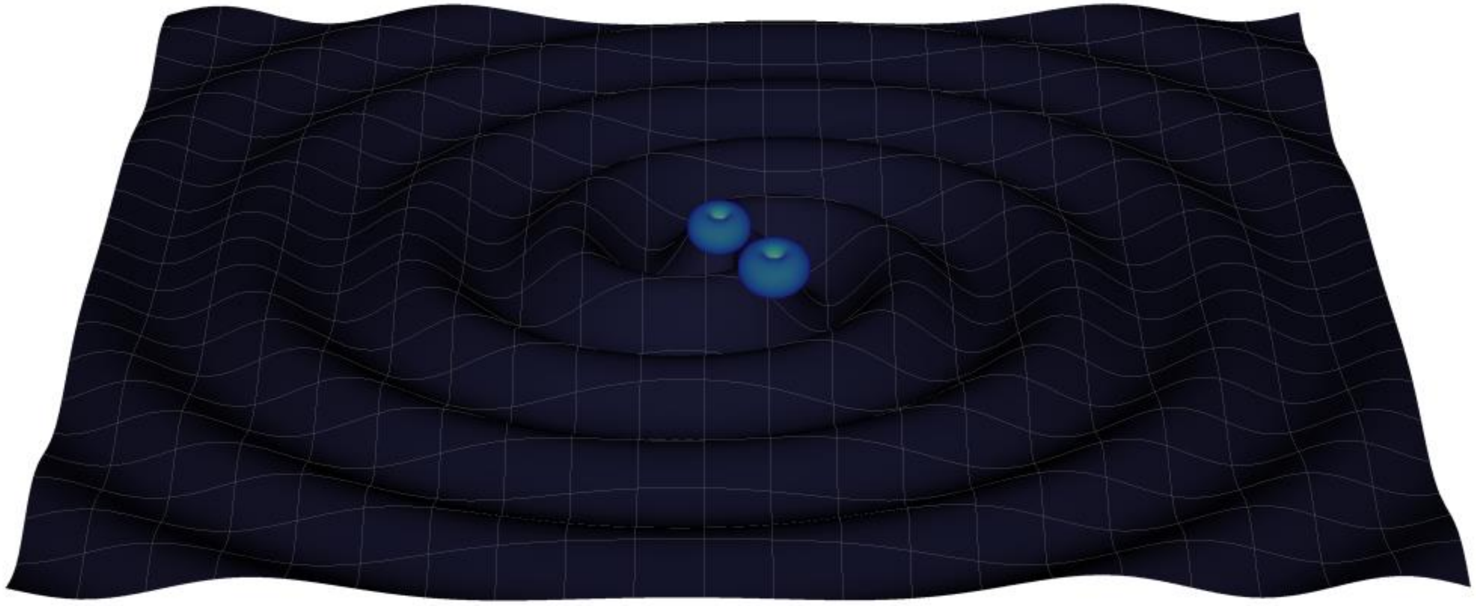
Parameter estimation on boson-star binary signals

(with a model-based coherent inspiral template)



XV Black Holes Workshop, Lisbon 19-12-2022

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SAPIENZA UNIVERSITÀ DI ROMA

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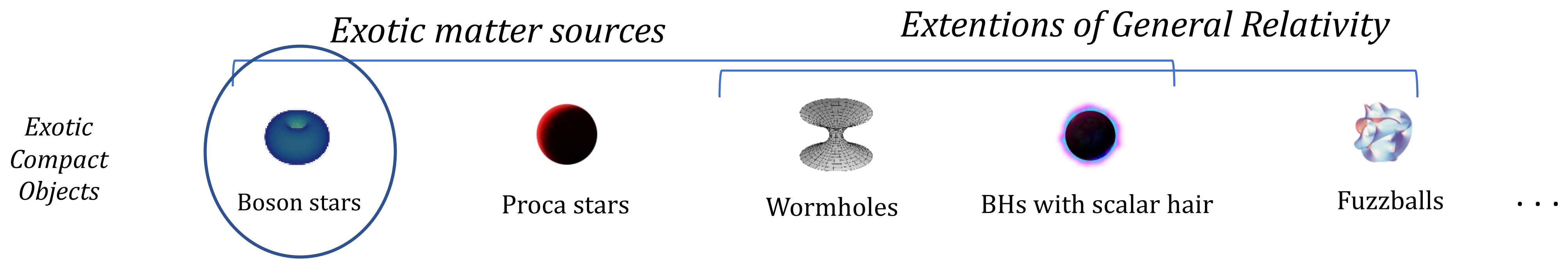
DarkGRA

Motivations for the work

- Gravitational waves allow to probe the nature of compact objects and to search for new physics

- The actual paradigm is that an astrophysical compact object, which is hevier than few solar masses, is a Black Hole (BH).



Main Idea: Build a coherent waveform for the inspiral of boson star binaries and test its ability to constrain their fundamental properties with observations from current and future interferometers

Work in collaboration with: *Costantino Pacilio (Sapienza University of Rome),*
Andrea Maselli (GSSI Institute, L'Aquila),
Paolo Pani (Sapienza University of Rome)

Properties of Boson Stars

- Boson stars are solutions of the Einstein gravity, minimally coupled to a complex scalar field:

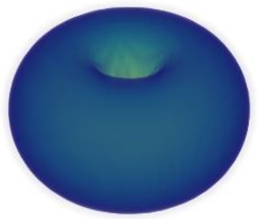
$$L = -\frac{1}{2} g^{\mu\nu} \phi^*_{,\mu} \phi_{,\nu} - \frac{1}{2} V(|\phi|^2) \xrightarrow{\text{Eqns.}} G_{ab} = 8\pi T_{ab}\,, \quad \frac{1}{\sqrt{-g}} \partial_a\left(\sqrt{-g}\, g^{ab} \partial_b \phi\right) = \frac{dV(|\phi|^2)}{d|\phi|^2} \phi$$

- Properties of **strongly** self-interacting BS with **quartic coupling** $V(|\phi|^2) = m^2|\phi|^2 + \frac{\lambda}{2}|\phi|^4 \;(\lambda \gg m^2)$

  – $M_{max} \sim 0.06 \frac{\lambda^{\frac{1}{2}}}{m^2} M_p^3$ — M_B reduced coupling

  – Smaller compactness compared to BHs: $C_{BS} \sim 0.16$ - $C_{BH} = 0.5$

  – Non-zero tidal deformability Λ and spin-induced multipole moments M_2, S_3 ...



3D energy-density plot of a boson star

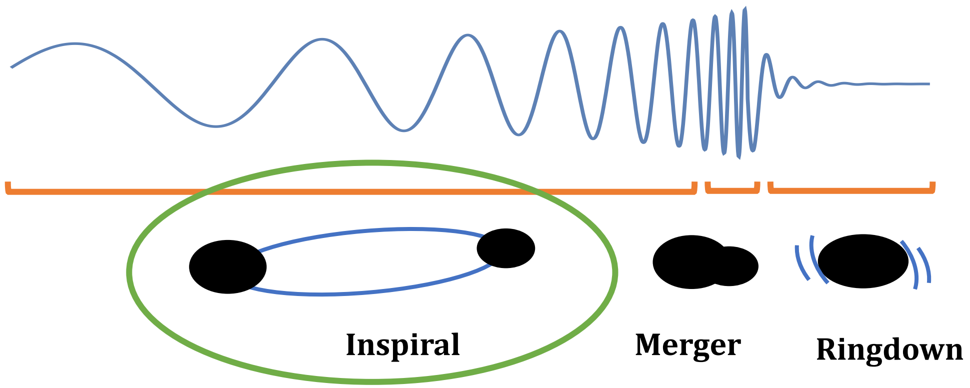
A coherent BS inspiral waveform model

- We want a coherent Post-Newtonian expanded waveform model in $v = (\pi M f)^{\frac{1}{3}}$ which consistently includes the corrections due to finite size effects:

$$\mathcal{A}(f) = \frac{M_t^2}{D_L}\sqrt{\frac{\pi\eta}{30}}\, v^{-7/6}$$

(0PN)

$$h \sim \mathcal{A}(f) e^{i\psi(f)}$$



$$\psi(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + v^{-5}\left(\sum_{n=0}^{7} \alpha_n v^n\right)$$

at 3.5PN — *C.K. Mishra et.al*, Phys. Rev. D, 93, 8 (2016), 084054 — Point particle +spin

+ quadrupole corrections at 2PN, 3PN and 3.5PN — *Krishnendu et.al*, Phys. Rev. Lett.,119,9 (2017) 091101

+ tidal corrections at 5PN and 6PN — *Lackey and L. Wade*, Phys. Rev. D, 91, (2015) 4 043002

Finite size effects

Quadrupole moment of boson stars (2PN)

- Multipole moments can be defined in General Relativity for asymptotically flat spacetimes (Geroch 1970, Hansen 1974, Thorne 1990…):

  Multipole moments in Newtonian theory ⟷ *Flatness of the Euclidean space*

- Stationary **axysimmetric** spacetime ⇒
  scalar mass moments M_0, M_2… and current moments S_1, S_3…

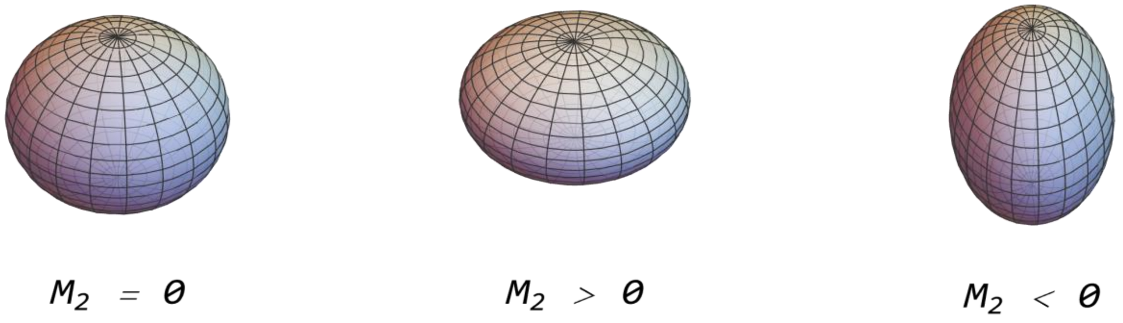
for a Kerr black hole

$$M_l + iS_l = M^{l+1}(i\chi)^l$$

$$\chi = \frac{J}{M^2}, \quad M = M_0$$

M_2 quadrupole moment

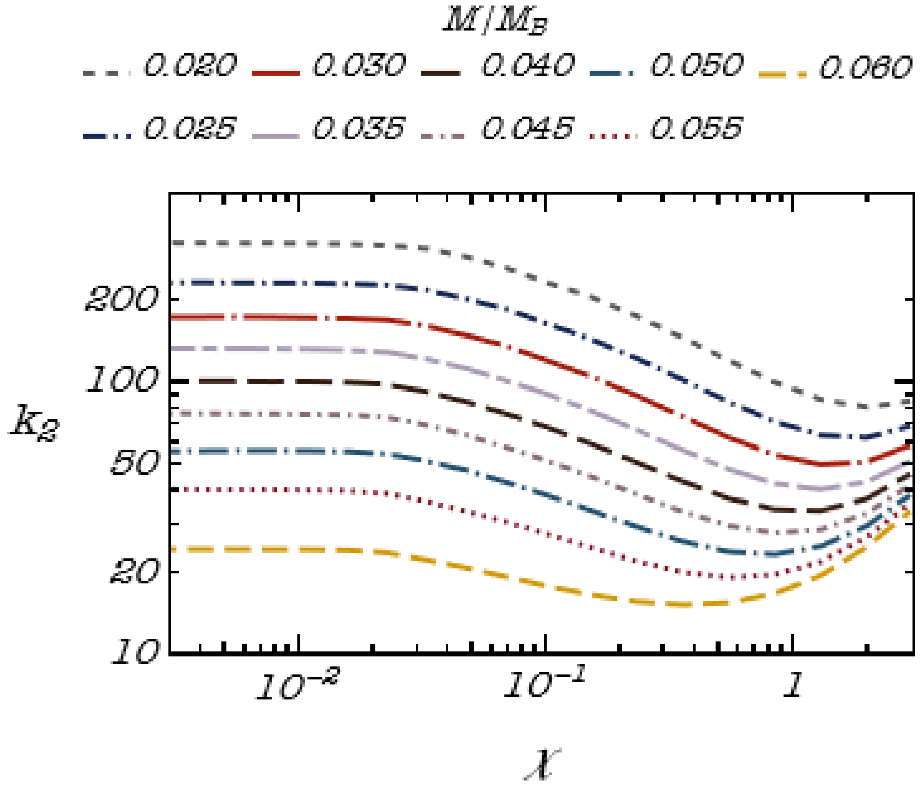
Not true for a generic compact object!



Quadrupole moment of boson stars (2PN)

- The plot shows $\kappa_2 = -\frac{M_2}{\chi^2 M^3}$ as a function of the dimensionless spin χ for different BS masses:

For a Kerr Black Hole
$k_2 = 1$



The multipolar structure of fast rotating boson stars:
Massimo Vaglio, Costantino Pacilio, Andrea Maselli, Paolo Pani, arXiv:2203.07442 (2022)

The expression for the quadrupole moment can be obtained as

$$M_2 = -\kappa_2(\chi, M/M_B)\chi^2 M^3$$

Tidal deformability of boson stars (5PN)

- The presence of the companion induces a quadrupole moment in the star as response to the external tidal field:

$$g_{00} = -1 + \frac{2M}{r} + \frac{3Q_{ij}}{r^3}\left(n_i n_j - \frac{\delta_{ij}}{3}\right) + O\left(\frac{1}{r^4}\right) - \varepsilon_{ij} x_i x_j + O(r^3)$$

$$Q_{ij} = -\lambda_T \varepsilon_{ij}$$

λ_T is the tidal deformability

- The tidal deformability for boson stars can be obtained exploiting the relation:

$$\frac{M}{M_B} = \frac{\sqrt{2}}{8\sqrt{\pi}}\left[-0.828 + \frac{20.99}{\log\Lambda} - \frac{99.1}{(\log\Lambda)^2} + \frac{149.7}{(\log\Lambda)^3}\right] \longrightarrow \boxed{\Lambda = \Lambda\left(\frac{M}{M_B}\right)}$$

where $\Lambda = \lambda_T / M^5$

N. Sennett et al., Phys. Rev. D, 96, 2 (2017) 024002

Parameter estimation - Setting

- To test our waveform, we performed parameter estimation on injected signals

prior — likelyhood

$$p(\vec{\theta}|d) = \frac{\pi(\vec{\theta})\mathcal{L}(d|\vec{\theta},\mathcal{H})}{\int d^m\theta\pi(\vec{\theta})\mathcal{L}(d|\vec{\theta},\mathcal{H})}$$

posterior — evidence

- In the analysis we fixed the extrinsic parameters:
  - $\boldsymbol{ra}, \boldsymbol{dec}$ sky localization angles
  - $\boldsymbol{\iota}$ system inclination angle
  - $\boldsymbol{\psi}$ wave polarization angle

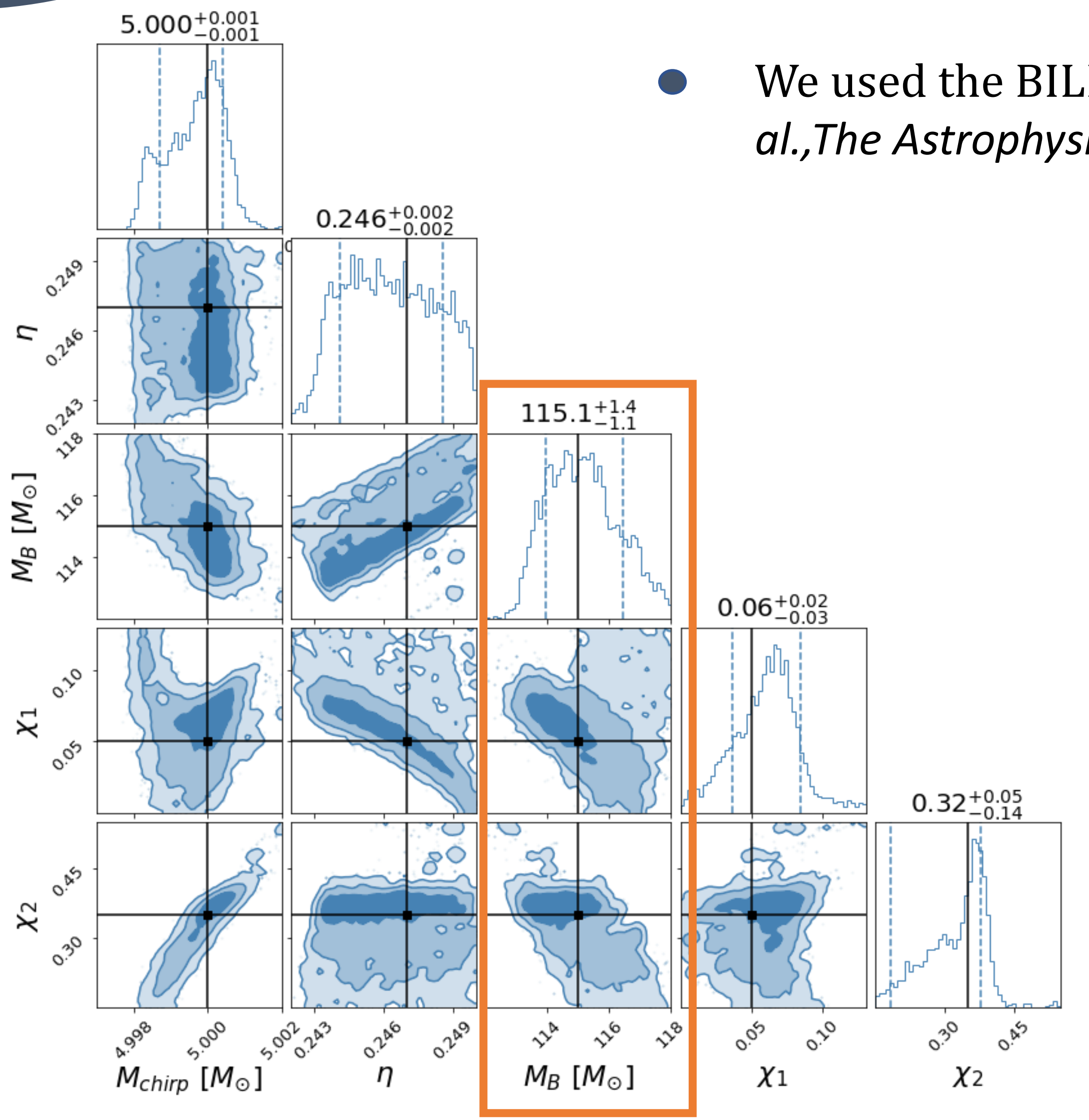
  and marginalize over:
  - $\boldsymbol{dL}$ Luminosity distance
  - $\boldsymbol{t_c}, \boldsymbol{\phi_c}$ time and phase at coalescence

- We considered only spins alligned or antialligned with $\vec{L}$:

$$\vec{\theta} = (\mathcal{M}, q, \chi_1, \chi_1, M_B)$$

$\mathcal{M} = (M_1M_2)^{\frac{3}{5}}/(M_1 + M_2)^{\frac{1}{5}}$ $\quad q = M_2/M_1$ $\quad \chi_{1/2} = (\vec{J}_{1/2}/M_{1/2}^2) \cdot \hat{L}$ $\quad M_B = (\lambda^{\frac{1}{2}}/m^2)\, M_p^3$

Parameter estimation - Results

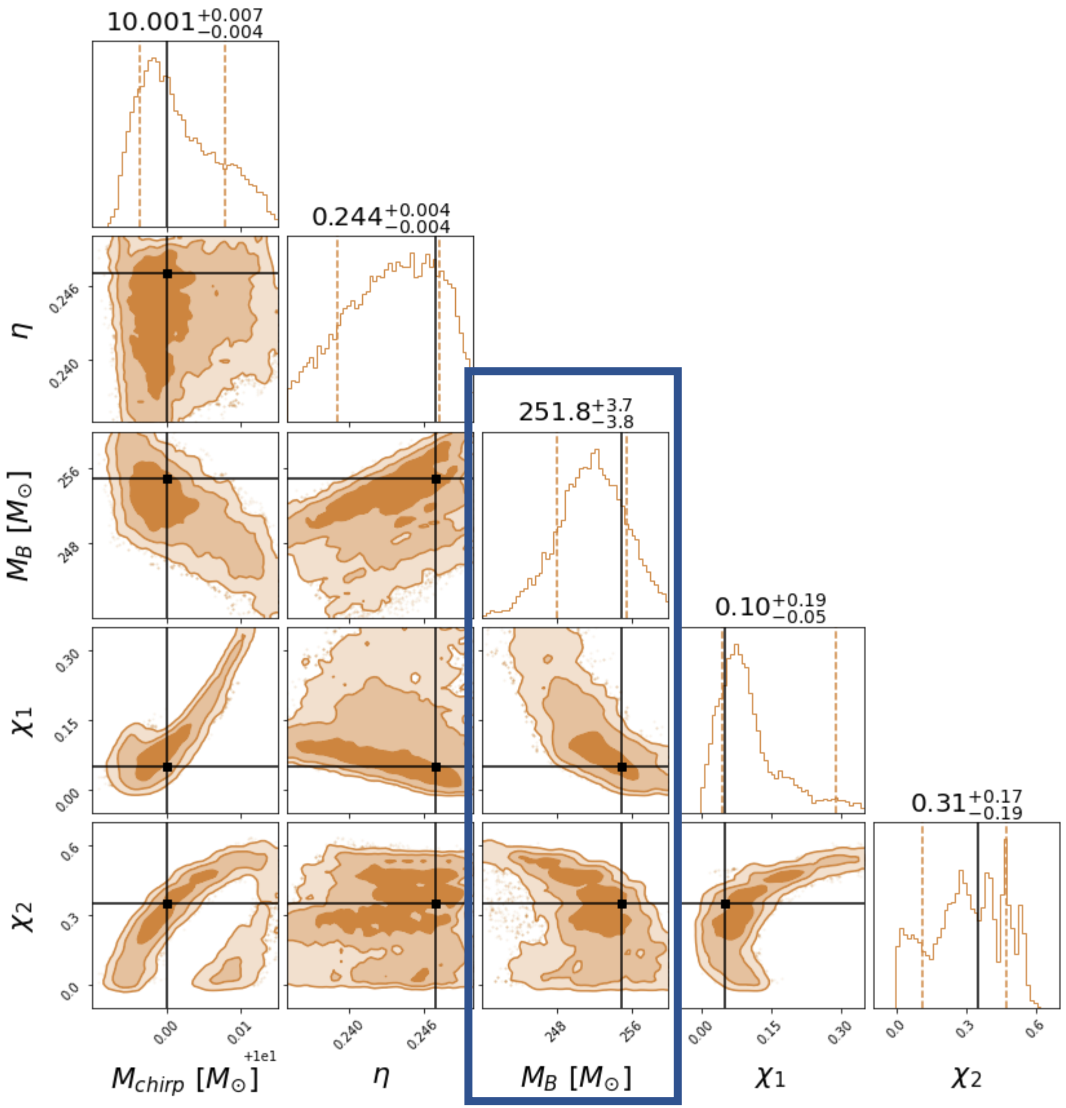


- We used the BILBY Bayesian Inference Library (Ashton, Gregory *et al.,The Astrophysical Journal Supplement Series, voln.241,2019*).

Injection and recovery of a signal with the Einstein Telescope ($SNR = 130$):

- $\mathcal{M} = 5M_{\odot}$
- $q = 0.8$
- $M_B = 115M_{\odot}$
- $\chi_1 = 0.05$
- $\chi_2 = 0.35$
- $f_{Roche} = 127Hz$

Parameter estimation - Results



Injection and recovery of a signal with the Einstein Telescope (SNR = 130):

- $\mathcal{M} = 10M_\odot$
- $q = 0.8$
- $M_B = 255M_\odot$
- $\chi_1 = 0.05$
- $\chi_2 = 0.35$
- $\boldsymbol{f_{Roche} = 50Hz}$ ←

Conclusions and perspectives

We developed a coherent waveform template for the inspiral of rotating self-interacting BSs in the strong coupling limit :

- There is a strong correlation between the mass ratio and the fundamental coupling $M_B = \sqrt{\lambda}/m^2$.
- With ET at SNR ~ 100 it is possible to constraint M_B with ~1% accuracy:

(m_1, m_2)	$\delta\mathcal{M}_{rel}$	$\delta\eta_{rel}$	$\delta M_{B_{rel}}$	$\delta\chi_{1_{rel}}$	$\delta\chi_{2_{rel}}$
$(6.4, 5.2)M_\odot$	0.015%	0.8%	1.0%	48%	37%
$(12.8, 10.3)M_\odot$	0.05%	1.7%	1.5%	92%	43%

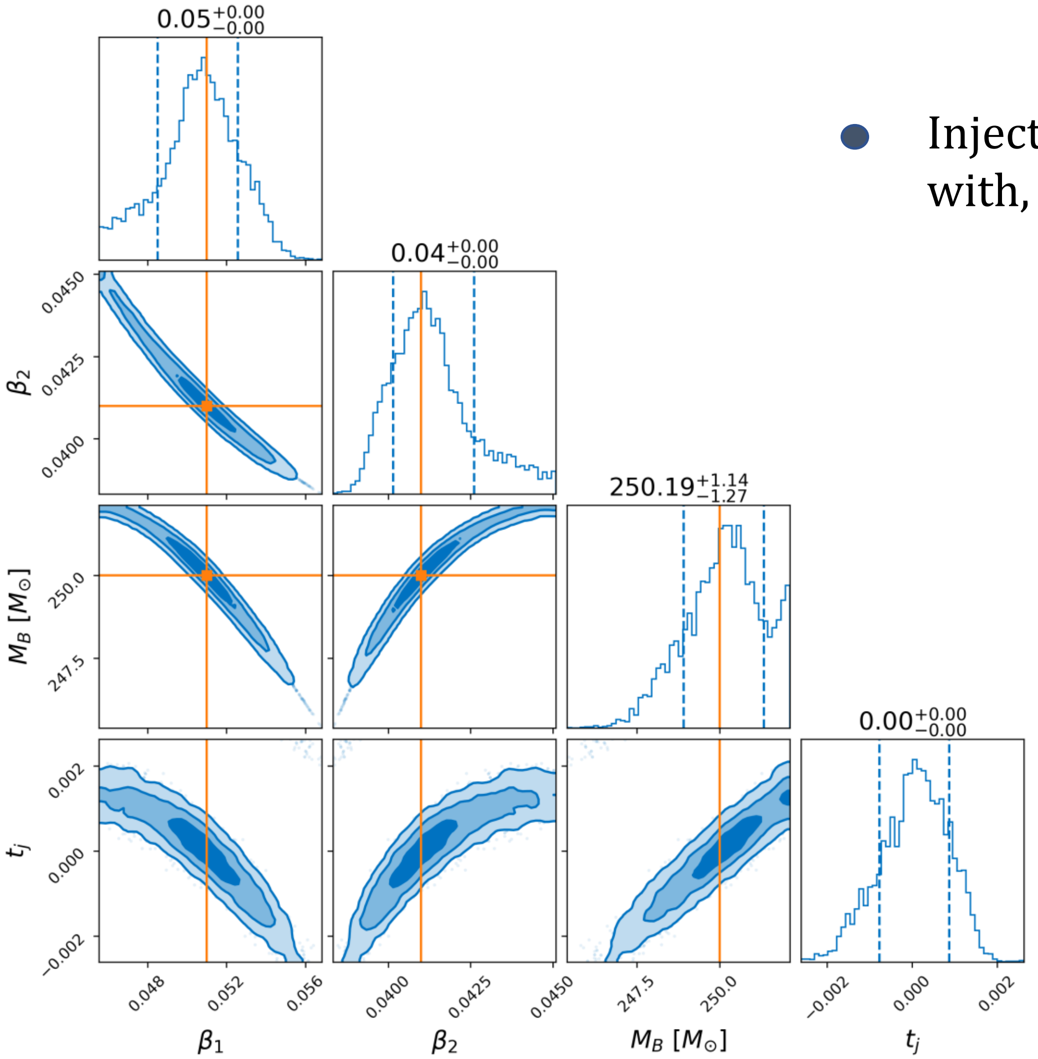
- Due to the low cutoff frequency, it is difficult to constraint binaries heavier than $\sim 10 M_\odot$.

That's why we need complete inspiral-merger-ringdown templates!

Next steps and future works:

- Generalization to other BS's models: change $V(|\phi|^2)$, Vector BSs, universal relations…
- Model selection between different boson star models

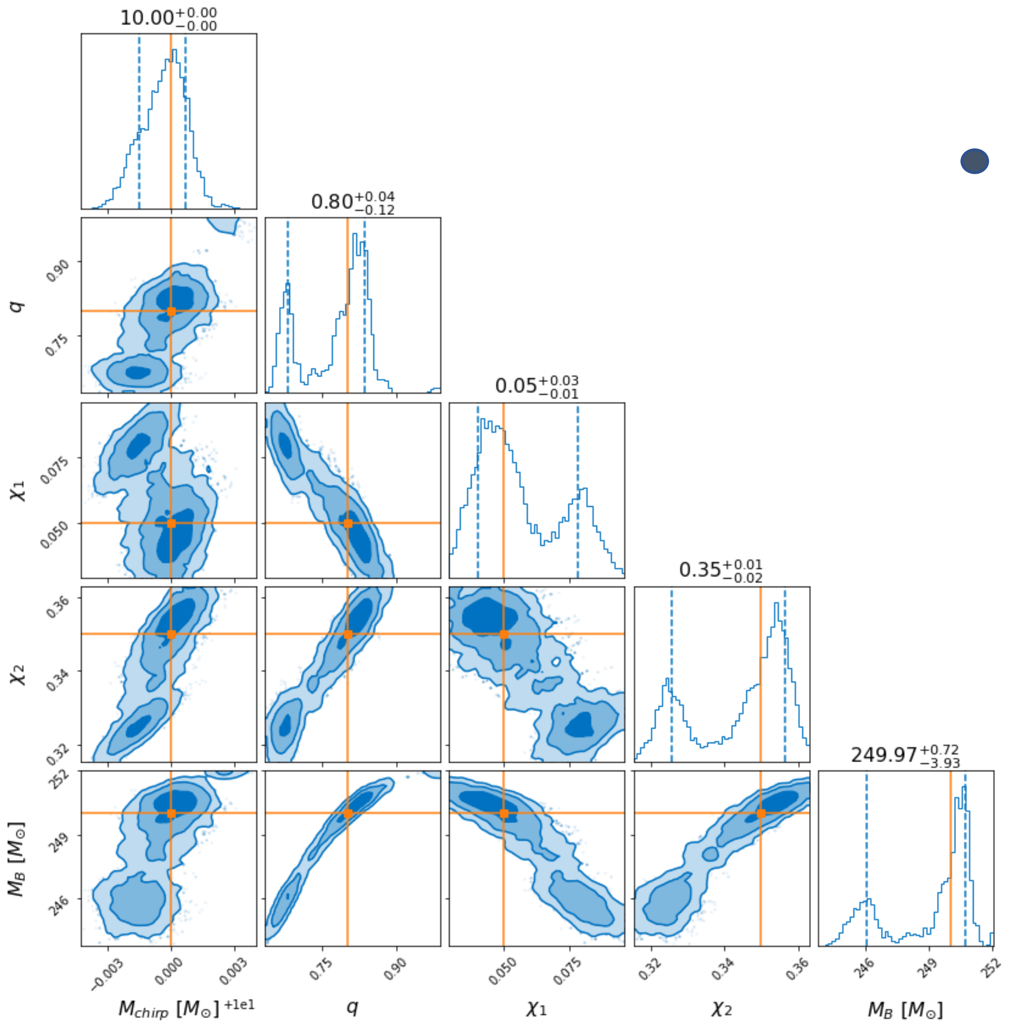
Backup Slide 1



- Injection and recovery of a simulated signal with, $\beta_1 = 0.051, \beta_2 = 0.041, M_B = 250.$

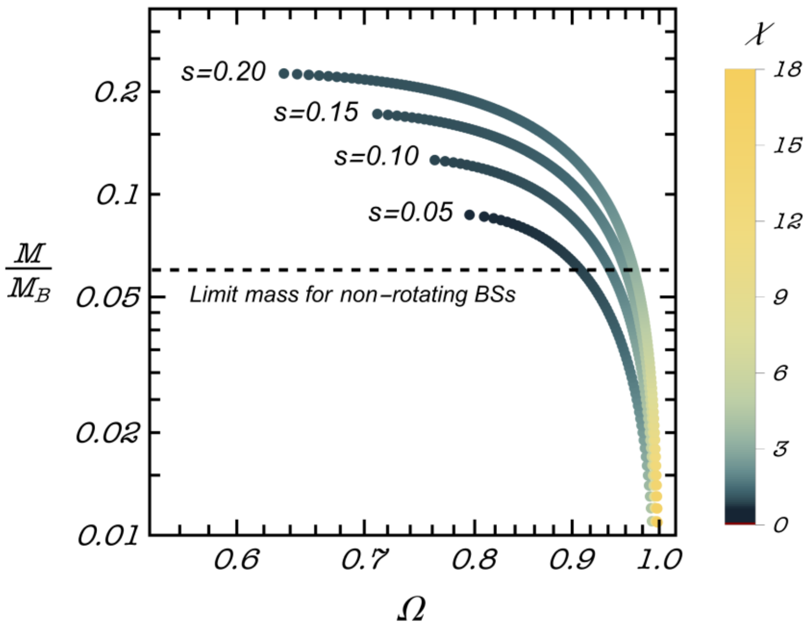
$$\beta = M/M_B$$

Backup Slide 1



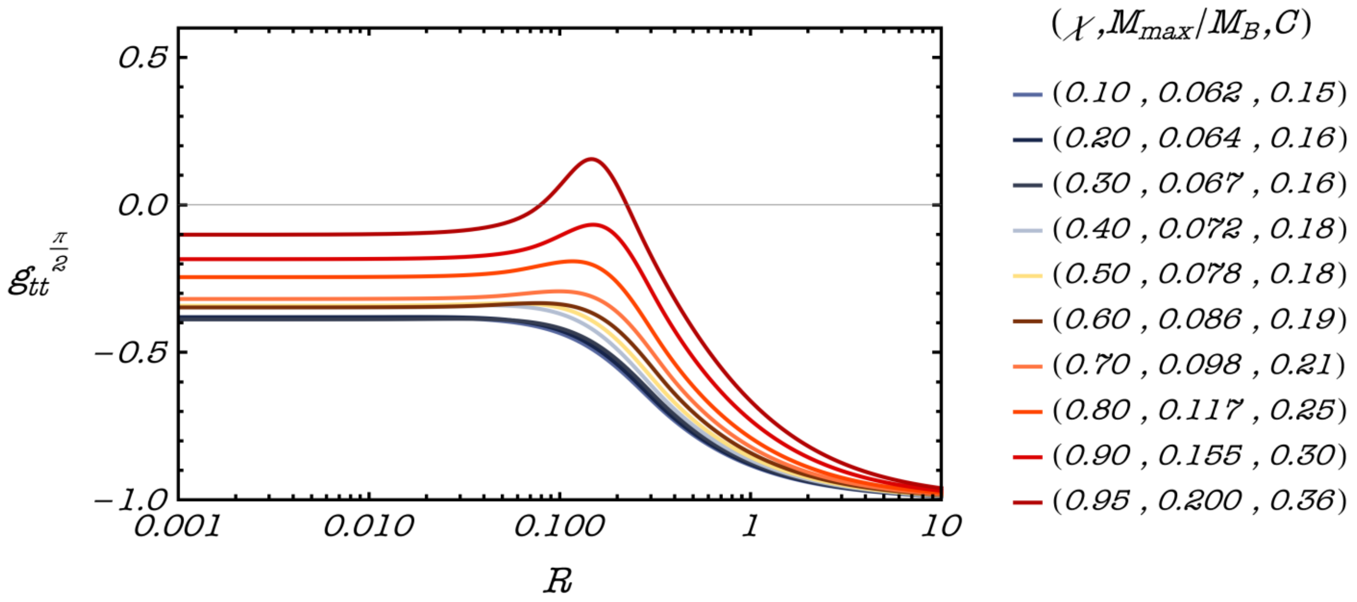
- Injection and recovery of a simulated signal with $\mathcal{M} = 10, q = 0.8$ and $\chi_1 = 0.05, \chi_2 = 0.35, M_B = 250$.

Maximum mass and ergoregions



Increasing the vaue of the winding number $s = \left(\frac{\sqrt{\lambda}}{m}\right)^{-1} \times n_r$, it is possible to exceed significantly the non-spinning maximum mass limit $M \sim 0.06 M_B$

The model allows for configurations featuring ergoregions in the (linearly) stable branch.



The multipolar structure of fast rotating boson stars: *Massimo Vaglio, Costantino Pacilio, Andrea Maselli, Paolo Pani,* arXiv:2203.07442 (2022)

Motivations for the work

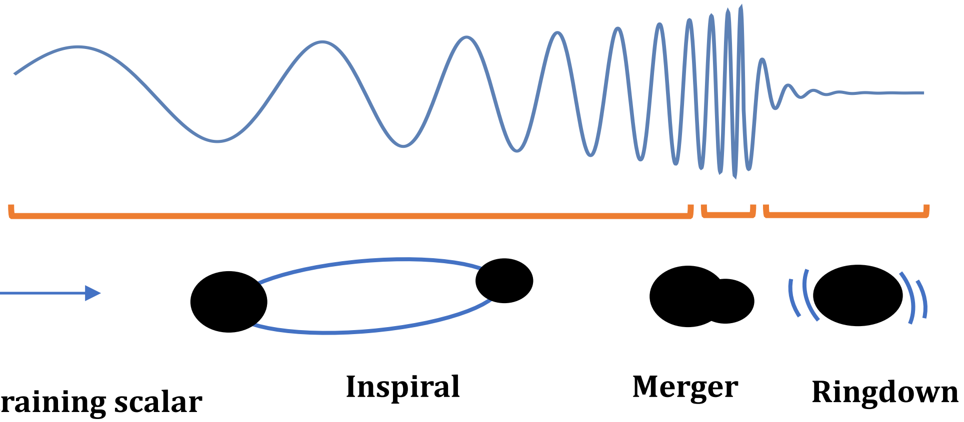
- Stationary axysimmetric spacetime $\Rightarrow$ scalar mass moments M_0, M_2... and current moments S_1, S_3...

for a Kerr black hole $$M_l + iS_l = M^{l+1}(i\chi)^l \quad \text{where} \quad \chi = \frac{J}{M^2}, \quad M = M_0$$

The multipolar structure affects the dynamics of binary systems and their gravitational wave emission

E.g. $$h \sim \mathcal{A}(f)e^{i(\psi_{BH}(f) + finite\ size\ corr\ )}$$

PN expanded waveform



Gravitational-wave detectors as particle-physics laboratories: Constraining scalar interactions with a coherent inspiral model of boson star binaries, *Costantino Pacilio, Massimo Vaglio, Andrea Maselli, Paolo Pani.* arXiv:2007.05264 *(2020)*

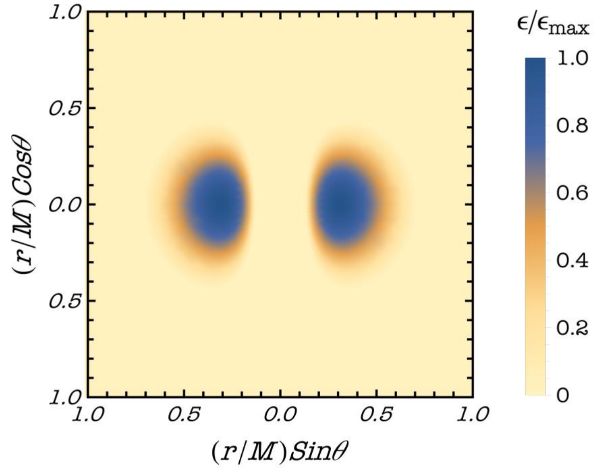
- The study of multipole moments can lead to the discovery of interesting properties (es: Love-Q relations)

Families of (rotating) Boson Stars

- Different families of BSs, correspond to different potenatials in the lagrangian:

(Neutron Stars: Equation Of State → *Boson Stars: Self-interactions* $V(|\phi|^2)$)

- Mini BSs $V(|\phi|^2) = m^2|\phi|^2$ $M_{max} \sim \frac{M_p^2}{m}$
- Massive BSs $V(|\phi|^2) = m^2|\phi|^2 + \lambda|\phi|^4$ $M_{max} \sim \frac{M_p^3}{m^2}\lambda^{\frac{1}{2}}$
- Solitonic BSs $V(|\phi|)^2 = m^2|\phi|^2\left(1 - \frac{2|\phi|^2}{\sigma^2}\right)$ $M_{max} \sim \frac{M_p^4}{m\sigma^2}$



Normalized energy-density of a BS in a transversal section

- To have stationarity and axysimmetry the field must satisfy:

azimuthal winding number

$$\phi = \phi_0(r,\theta)e^{i(n_r\varphi - \Omega t)}$$

frequency

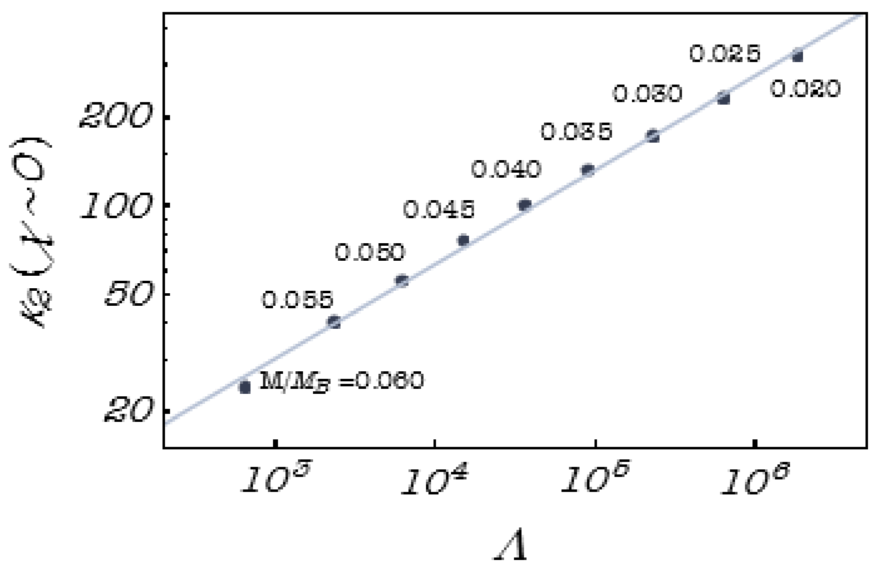
$J = n_r N$ *The angular momentum is quantized!*

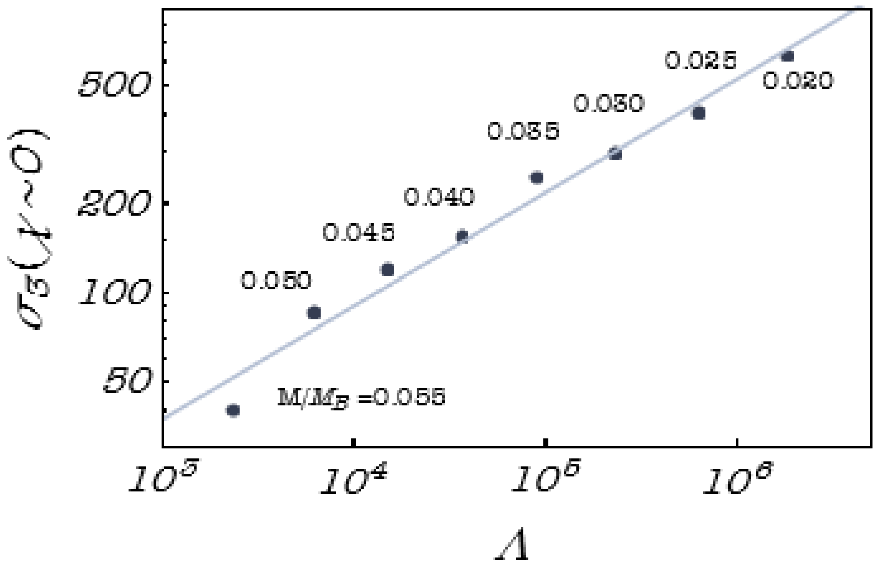
Universal Relations for Boson Stars?

- Neutron Stars feature simple relations linking their moment of inertia, the tidal deformability and the quadrupole moment which do not depend sensitively on the star's internal structure.

I-Love-Q Relations in Neutron Stars and their Applications to Astrophysics, Gravitational Waves and Fundamental Physics - *Kent Yagi and Nicolàs Yunes*

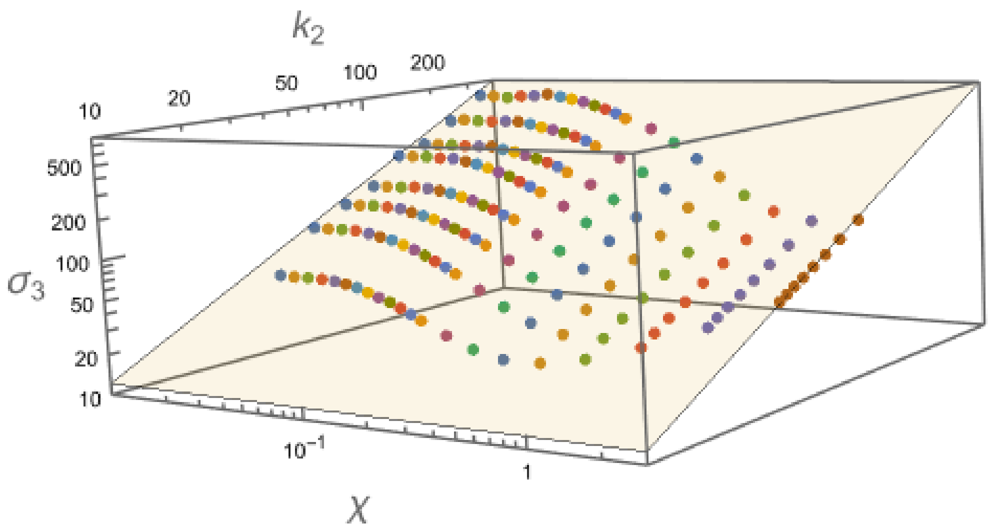
- We found the reduced quadrupole and octupole moments are simply connected to the tidal deformability of the boson star

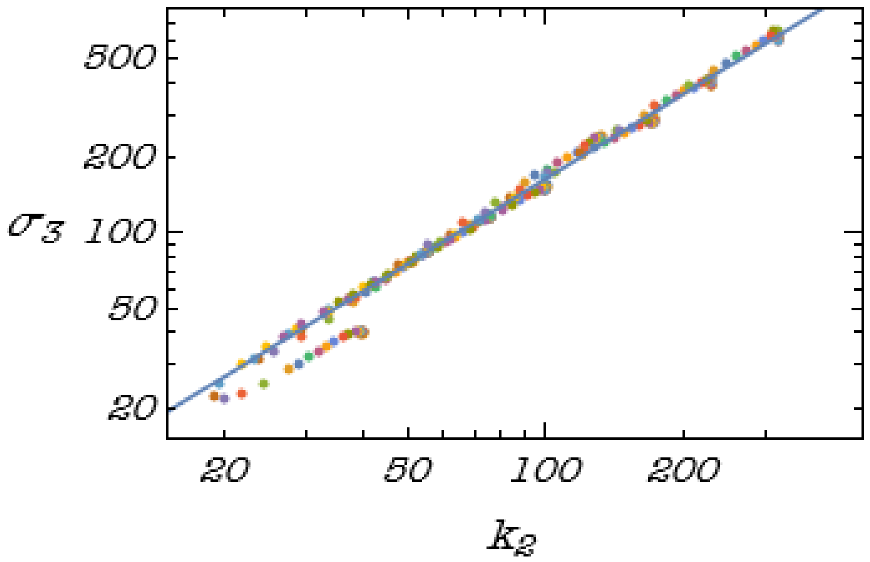




Universal Relations for Boson Stars?

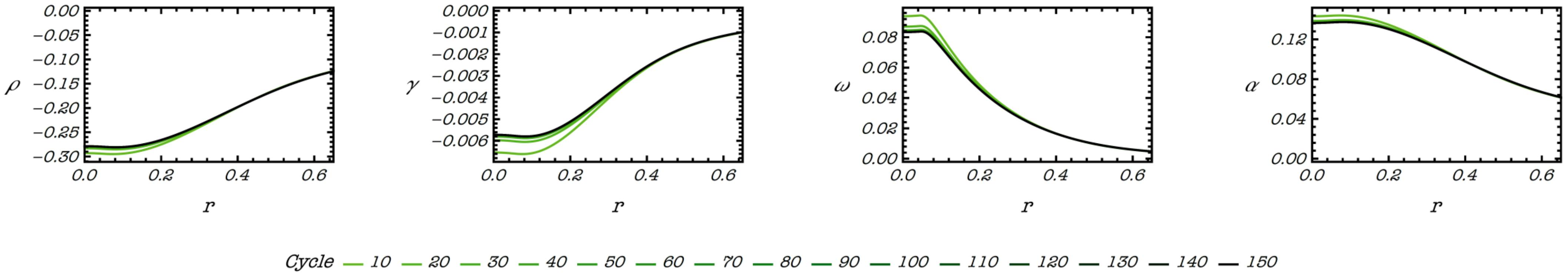
- The relation between κ_2 and σ_3 appears remarkably to be independent on the spin χ





- These relations have many applications and are especially useful to break degeneracies among parameters that characterize gravitational waveforms.

Integration and multipole moments

- 

Coordinates	$q = r/(1+r), \quad \mu = \cos\theta \quad q, \mu \in [0,1]$	Compactified
Grid	$n_q \times n_\mu$	Fixed equally spaced
Derivatives	–	Five points central
Integration	–	Trapezoidal rule

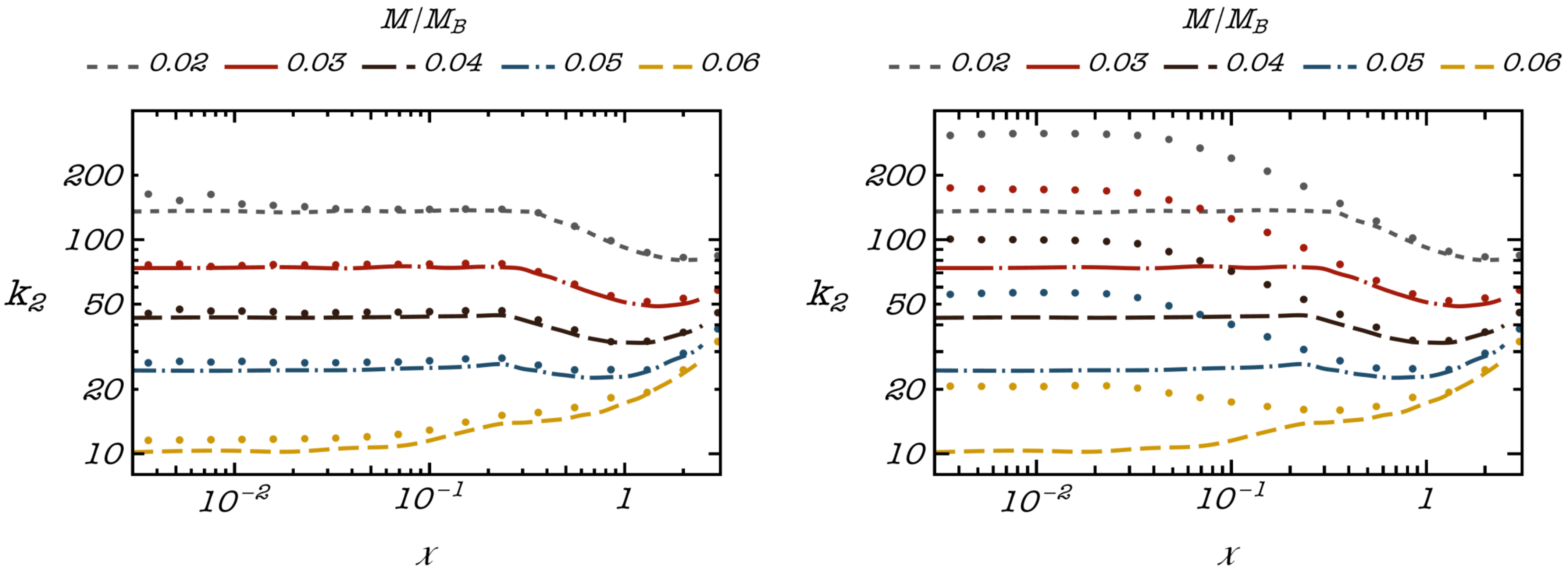
- Mass and current moments $\{M_0, M_2 \ldots\}$, $\{S_1, S_3 \ldots\}$ can be read off:

$$\rho(r,\mu) = \sum_{n=0}^{\infty} -2\frac{M_{2n}}{r^{2n+1}} P_2 n(\mu) + \text{higher orders}$$

$$\omega(r,\mu) = \sum_{n=1}^{\infty} -\frac{2}{2n-1}\frac{S_{2n-1}}{r^{2n+1}}\frac{P_{2n-1}^{1}(\mu)}{\sin\theta} + \text{higher orders}$$

Consistency with previous results

- Our findings about the quadrupole moments agree with previous results, when using the same grid $n_q \times n_\mu = 1600 \times 160$, but there is a deviation when n_μ is increased up to the saturation value $n_\mu \sim 20000$.



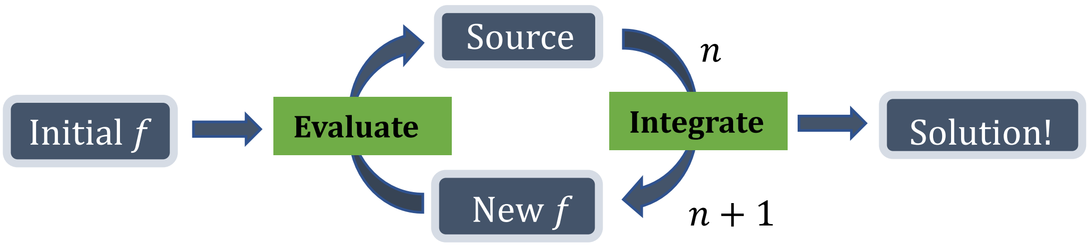
The dashed lines correspond to the values reported in ***F. D. Ryan, Phys. Rev. D 55, 6081 (1997)***

Self consistent field method

- The equations can be solved iteratively: $f(x) = \int G(x, x')S(f, \partial f, x')dx'$

(r', θ')

$(\rho, \gamma, \omega, \alpha)$



Es: $$\rho = -\frac{1}{4\pi} e^{-\frac{\gamma}{2}} \int_0^\infty dr' \int_{-1}^{1} d\mu' \int_0^{2\pi} d\phi' r' S_\rho(r', \mu') \frac{1}{|r - r'|}$$

Automatically satisfies aymptotic flatness conditions for reasonable sources!

Dependence on the integration grid

- Due to numerical erros, we found a non-zero value of $M_2^{(off)} \equiv M_2(\chi = 0)$

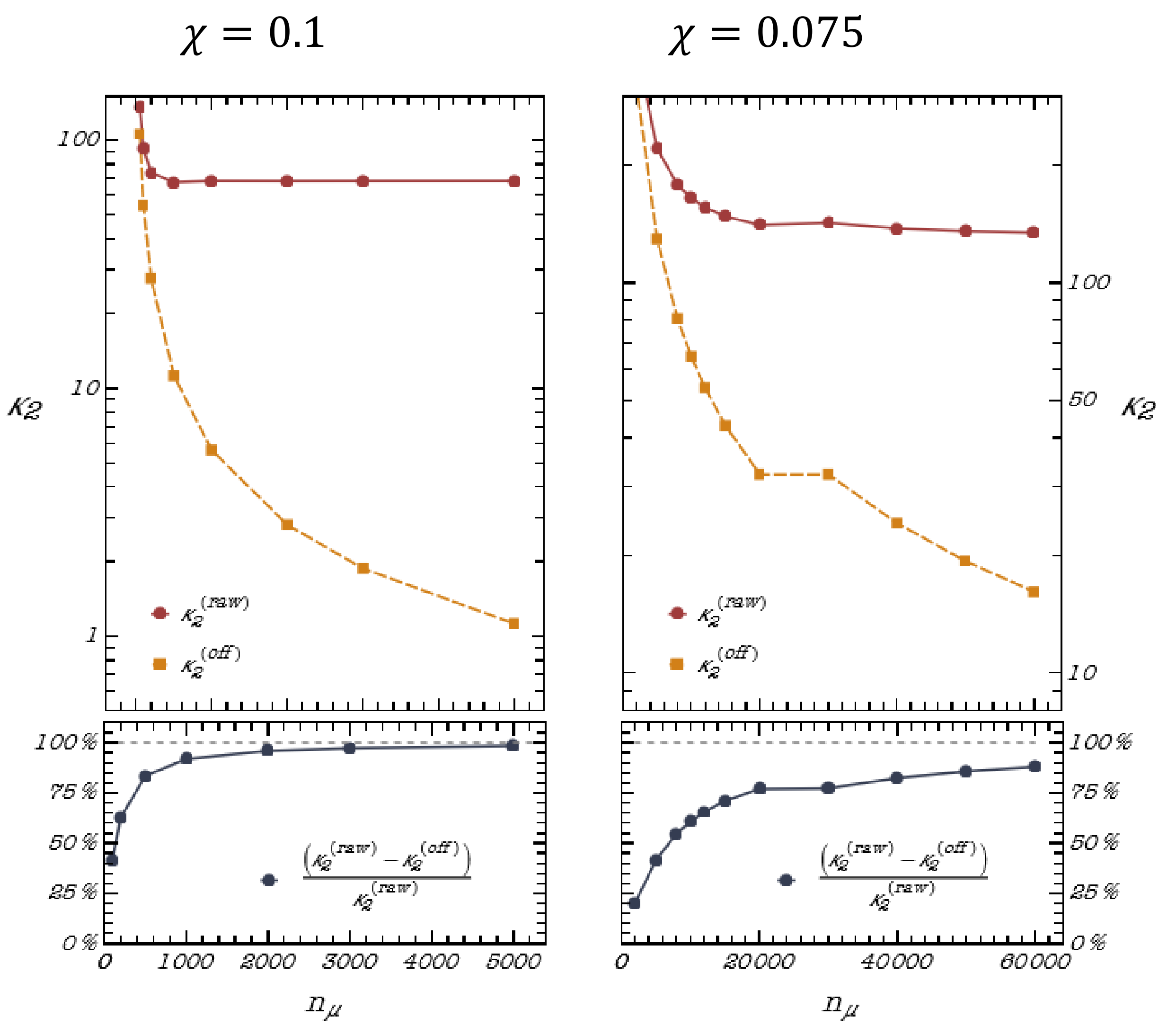
  In the plots (top panels):

  $$k_2^{(raw)} = M_2^{(raw)}/(\chi^2 M^3)$$

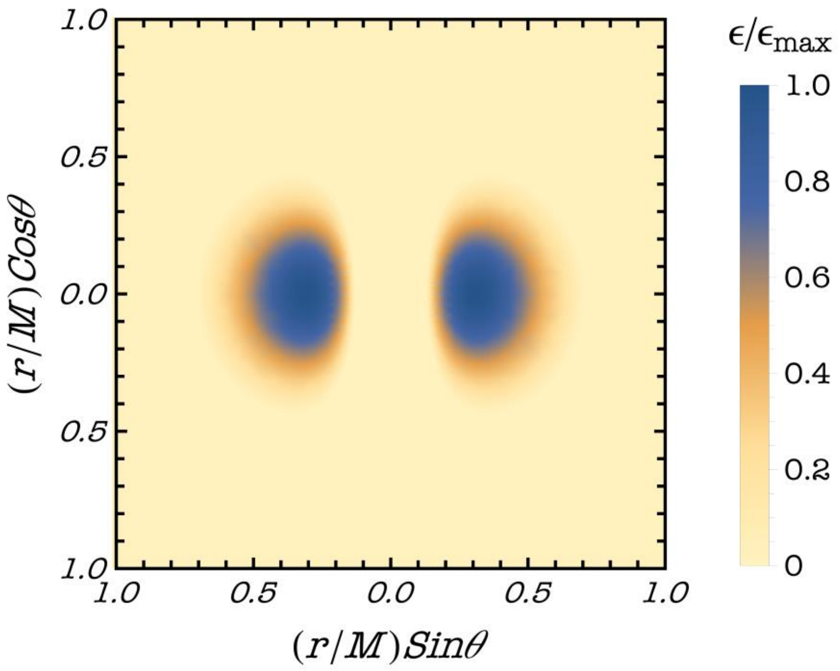
  $$k_2^{(off)} = M_2^{(off)}/(\chi^2 M^3)$$

  and their percentage difference (bottom panels), for fixed $M = 0.04 M_B$, $n_q = 1600$ and two values of χ.

- Extracting the quadrupole moments for slow spinning configurations requires more angular precision.



Energy-density plot



Normalized energy-density of a BS in a transversal section

Backup Slide – Scalar field

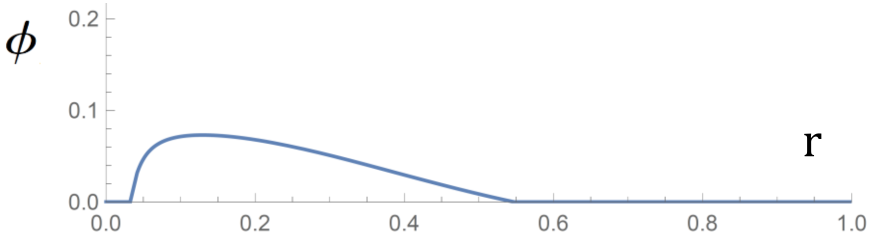
- The metric can be expressed in the Lewis-Papapetrou coordinates:

$$ds^2 = -e^{\gamma+\rho}dt^2 + e^{2\alpha}(dr^2 + r^2 d\theta^2) + e^{\gamma-\rho}r^2 \sin\theta^2\,(d\phi - \omega dt)^2$$

- The scalar field in the inner region satisfies: $(-g^{tt}\Omega^2 + 2g^{t\varphi}\Omega s - g^{\varphi\varphi}s^2 - m^2)\phi - \lambda|\phi|^2\phi = 0$

  But in the tail region $\phi \sim 0 \Rightarrow$ $|\phi|^2 = Max[0, (-g^{tt}\Omega^2 + 2g^{t\varphi}\Omega s - g^{\varphi\varphi}s^2 - m^2)/\lambda]$

- Substituting the metric coefficients: $$|\phi|^2 = Max\left[0, \frac{1}{\lambda}\left(\frac{(\Omega - s\omega)^2}{e^{\gamma+\rho}} - \frac{e^{\gamma-\rho}s^2}{r^2\sin\theta^2} - m^2\right)\right]$$



Rotating BSs are shaped like doughnuts!

Backup Slide – Coordinate rescaling

- It is possible to get rid of the coupling constants trought the following rescalings:

$$t = \frac{\lambda^{\frac{1}{2}}}{m^2}\tilde{t} \quad s = \frac{\lambda^{\frac{1}{2}}}{m}\tilde{s} \quad r = \frac{\lambda^{\frac{1}{2}}}{m^2}\tilde{r} \quad \Omega = m\tilde{\Omega} \quad \epsilon = \frac{m^4}{\lambda}\tilde{\epsilon} \quad \omega = \frac{m^2}{\lambda^{\frac{1}{2}}}\tilde{\omega} \quad P = \frac{m^4}{\lambda}\tilde{P} \quad |\phi|^2 = \frac{m^2}{\lambda}|\tilde{\phi}|^2$$

- Consequently we have the following change in the relevant expressions:

$$\tilde{P} = \frac{1}{4}|\tilde{\phi}|^4 \qquad \tilde{\epsilon} = |\tilde{\phi}|^2 + \frac{3}{4}|\tilde{\phi}|^4 \qquad |\tilde{\phi}|^2 = Max\left[0, \frac{(\tilde{\Omega} - \tilde{s}\tilde{\omega})^2}{e^{\gamma+\rho}} - \frac{e^{\gamma-\rho}\widetilde{s^2}}{\widetilde{r^2}\sin\theta^2} - m^2\right]$$

$$d\widetilde{s^2} = -e^{\gamma+\rho}d\widetilde{t^2} + e^{2\alpha}\left(d\widetilde{r^2} + \widetilde{r^2}d\theta^2\right) + e^{\gamma-\rho}\widetilde{r^2}\sin\theta^2\,(d\phi - \tilde{\omega}d\tilde{t})^2$$

- Physical quantities can be derived multiplying the rescaled ones by: $\frac{\lambda^{\frac{1}{2}}}{m^2} \equiv M_B$

Backup Slide – The equations

- The Einstein equations can be rewritten as:

$$\triangle\left(\rho e^{\frac{\gamma}{2}}\right) = S_\rho(r,\mu) \qquad \left(\triangle + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}\mu\frac{\partial}{\partial\mu}\right)\gamma e^{\frac{\gamma}{2}} = S_\gamma(r,\mu) \qquad \left(\triangle + \frac{2}{r}\frac{\partial}{\partial r} - \frac{2}{r^2}\mu\frac{\partial}{\partial\mu}\right)\omega e^{\frac{(\gamma-2\rho)}{2}} = S_\omega(r,\mu)$$

where $\mu = \cos\theta$ and I removed the 'tilde'.

- The first can be easily inverted:

$$\rho = -\frac{1}{4\pi}e^{-\frac{\gamma}{2}}\int_0^\infty dr'\int_{-1}^{1} d\mu'\int_0^{2\pi} d\phi' r' S_\rho(r',\mu')\frac{1}{|r-r'|}$$

Automatically satisfies aymptotic flatness conditions for reasonable sources!

Backup Slide – The equations

- Expanding the $1/|r - r'|$ term and repeating for the other equations:

$$\rho(r,\mu) = -e^{-\gamma/2}\sum_{n=0}^{\infty} P_{2n}(\mu)\left[\frac{1}{r^{2n+1}}\int_0^r dr'(r')^{2n+2}\int_0^1 d\mu' P_{2n}(\mu')S_\rho(r',\mu') + r^{2n}\int_r^\infty dr'\frac{1}{(r')^{2n-1}}\int_0^1 d\mu' P_{2n}(\mu')S_\rho(r',\mu')\right]$$

$$\gamma(r,\mu) = -\frac{2}{\pi}e^{-\gamma/2}\sum_{n=1}^{\infty}\frac{\sin[(2n-1)\theta]}{(2n-1)\sin\theta}\left[\frac{1}{r^{2n}}\int_0^r dr'(r')^{2n+1}\int_0^1 d\mu' \sin[(2n-1)\theta']\, S_\gamma(r',\mu') + r^{2n-2}\int_r^\infty dr'\frac{1}{(r')^{2n-3}}\int_0^1 d\mu' \sin[(2n-1)\theta']\, S_\gamma(r',\mu')\right]$$

$$\omega(r,\mu) = -e^{\rho-\gamma/2}\sum_{n=1}^{\infty}\frac{P^1_{2n-1}(\mu)}{2n(2n-1)\sin\theta}\left[\frac{1}{r^{2n+1}}\int_0^r dr'(r')^{2n+2}\int_0^1 d\mu' \sin\theta' P^1_{2n-1}(\mu')S_\omega(r',\mu') + r^{2n-2}\int_r^\infty dr'\frac{1}{(r')^{2n-3}}\int_0^1 d\mu' \sin\theta' P^1_{2n-1}(\mu')S_\omega(r'\mu')\right]$$

Backup Slide – The equations

- The sources are complicated expressions of the the metric functions and their derivatives :

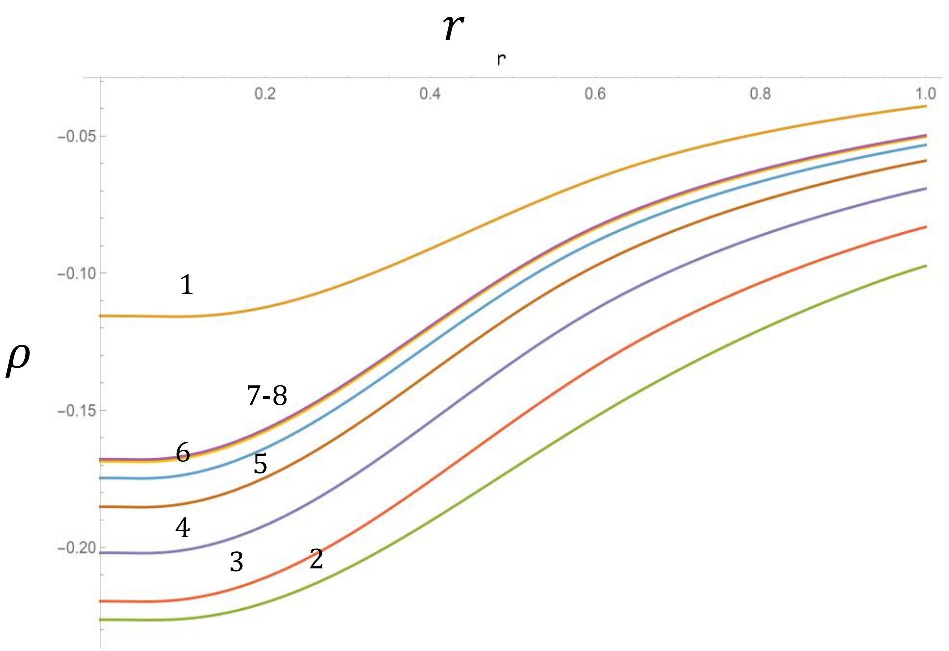
$$S_\rho(r,\mu) = e^{\gamma/2}\left(8\pi e^{2\alpha}(\epsilon+P)\frac{1+v^2}{1-v^2} + r^2(1-\mu^2)e^{-2\rho}\left(\omega_{,r}^2 + \frac{1-\mu^2}{r^2}\omega_{,\mu}^2\right) + \frac{1}{r}\gamma_{,r} - \frac{\mu}{r^2}\gamma_{,\mu} + \frac{1}{2}\rho\left[16\pi e^{2\alpha}P - \gamma_{,r}\left(\frac{1}{2}\gamma_{,r}+\frac{1}{r}\right) - \frac{1}{r^2}\gamma_{,\mu}\left(\frac{1-\mu^2}{2}\gamma_{,\mu} - \mu\right)\right]\right)$$

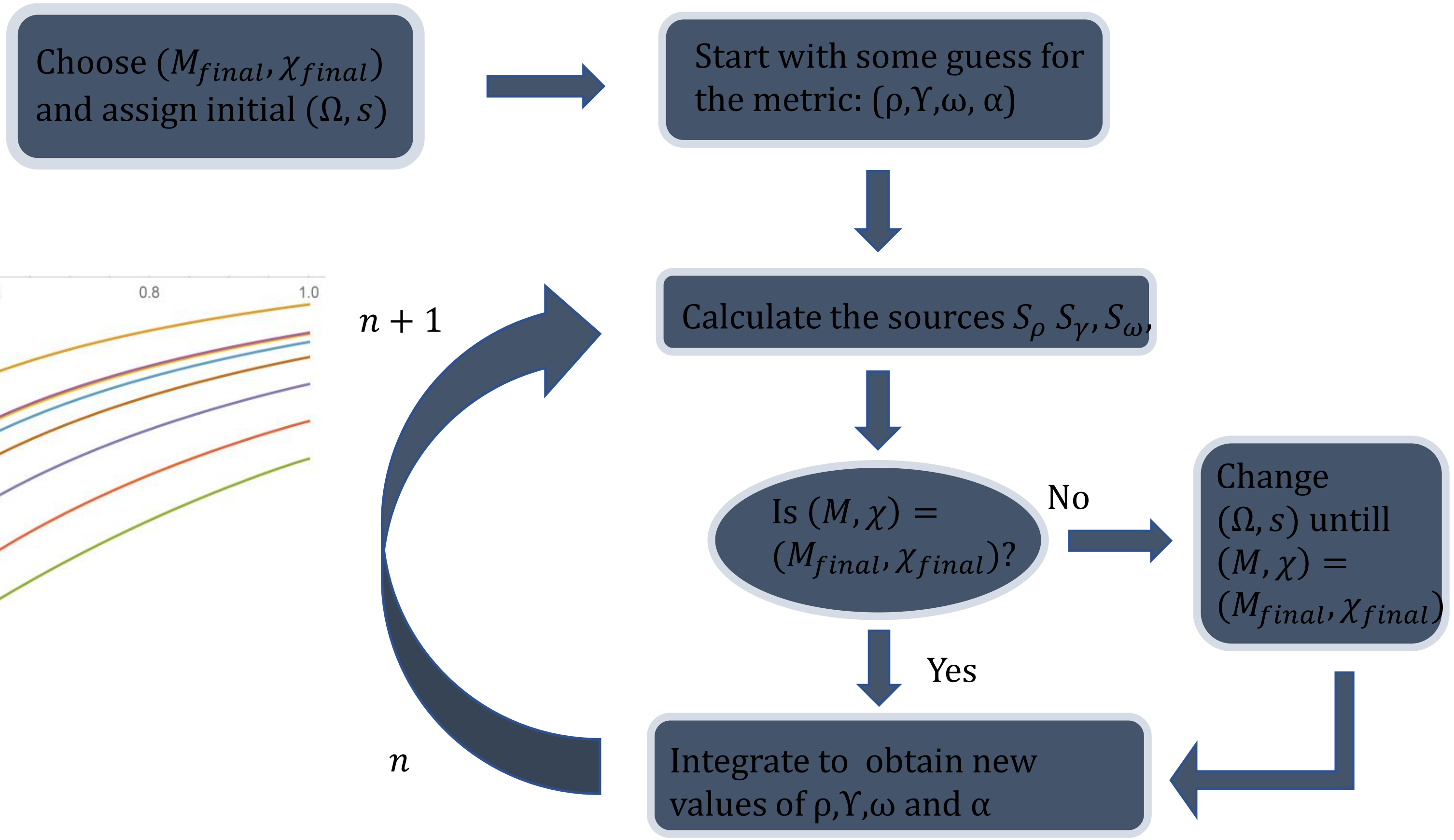
$$S_\gamma(r,\mu) = e^{\gamma/2}\left[16\pi e^{2\alpha}P + \frac{\gamma}{2}\left(16\pi e^{2\alpha}P - \frac{1}{2}\gamma_{,r}^2 - \frac{1-\mu^2}{2r^2}\gamma_{,\mu}^2\right)\right]$$

$$S_\omega(r,\mu) = e^{\gamma/2-\rho}\left(-16\pi e^{2\alpha+\rho}\frac{v(\epsilon+P)}{(1-v^2)r\sin\theta} + \omega\left[-8\pi e^{2\alpha}\frac{(1+v^2)\epsilon+2v^2P}{1-v^2} - \frac{1}{r}\left(2\rho_{,r}+\frac{1}{2}\gamma_{,r}\right) + \frac{\mu}{r^2}\left(2\rho_{,\mu}+\frac{1}{2}\gamma_{,\mu}\right) + \rho_{,r}^2 - \frac{1}{4}\gamma_{,r}^2 \right.\right.$$
$$\left.\left. + \frac{1-\mu^2}{r^2}\left(\rho_{,\mu}^2 - \frac{1}{4}\gamma_{,\mu}^2\right) - r^2(1-\mu^2)e^{-2\rho}\left(\omega_{,r}^2 + \frac{1-\mu^2}{r^2}\omega_{,\mu}^2\right)\right]\right)$$

where: $v = \frac{\tilde{s}}{\tilde{\Omega} - \tilde{s}\tilde{\omega}}\frac{e^\rho}{\tilde{r}\sin\theta}$ is the proper velocity with respect to the ZAMO

Backup Slide – The algorithm

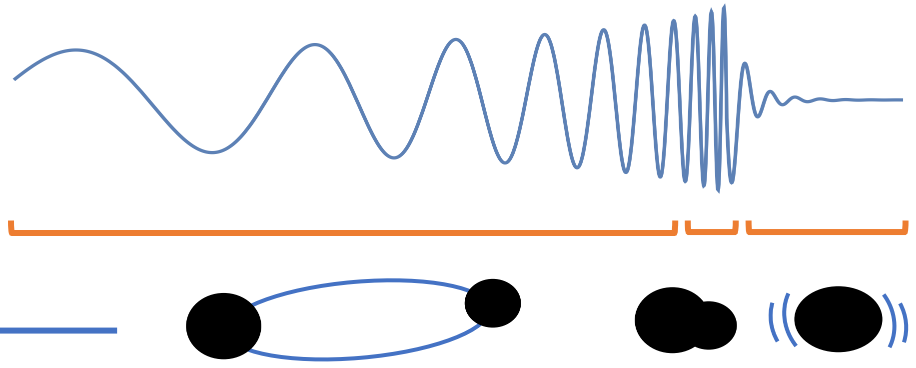




Change in the ρ function after the first iterations of the method

Binary Boson Star signal

- Multipole moments enter in the PN expansion in $v = (\pi M f)^{\frac{1}{3}}$ of the inspiral signal:



$$h \sim \mathcal{A}(f) e^{i\psi(f)}$$

$$\psi(f) = \psi_{BH}(f) + \psi_{\kappa_2}(f) + \psi_{\Lambda}(f)$$

Leading order

$$\psi_{\kappa_2} = -\frac{75}{64}\frac{(\kappa_2^1 M_1^2 \chi_1^2 + \kappa_2^2 M_2^2 \chi_2^2)}{M_1 M_2}(\pi M_t f)^{-1/3}$$

Reduced Quadrupole

Gravitational-wave detectors as particle-physics laboratories: Constraining scalar interactions with a coherent inspiral model of boson star binaries, *Costantino Pacilio, Massimo Vaglio, Andrea Maselli, Paolo Pani. Phys.Rev. D* 102 (2020) 8, 083002

Backup Slide – Mass scale

- We want to explore the possibility of constraining the BS coupling with future observations:

$$M_{max} \approx 0.06(1 + 0.76\chi^2)M_B \quad \Rightarrow$$

$$M_{max}(\chi \sim 0) \approx 0.06 M_B \approx 0.06\frac{\sqrt{\lambda}}{m^2} \approx 0.06\frac{\sqrt{\lambda\hbar}}{m_S^2}M_P^3 \approx 10^5 M_\odot \sqrt{\lambda\hbar}\left(\frac{\text{MeV}}{m_S}\right)^2$$

We can cover the whole spectrum of sources for LISA and ET varying λ and m_s

Backup Slide – Parameter Estimation

- The expression for the quadrupole moment as a funtion of mass, spin of the BS:

$$Q = -\kappa(\chi, M/M_B)\chi^2 M^3$$

can be used within parameter estimation to measure directly the effective coupling from GWs observation of BS binaries :

$$\vec{\theta} = (\mathcal{A}, t_c, \phi_c, \log \mathcal{M}\,, \log \eta\,, \chi_s, \chi_a, M_B)$$

- We used a Fisher matrix approach and a Post Newtonian expanded waveform to estimate the uncertainty with which M_B can be measured by LISA and ET in the following scenario:

Individual masses

$$(M_1, M_2) \sim (0.05M_B, 0.06M_B)$$

Mass scale

$$0.06M_B = \begin{cases} 1 - 100M_\odot & ET \\ 10^4 - 10^6 M_\odot & LISA \end{cases}$$

Spins

$$(\chi_1, \chi_2) = \begin{cases} (0.1,0) \\ (0.6,0.3) \\ (0.9,0.8) \end{cases}$$

Backup Slide – The Waveform

- Post Newtonian expansion in $v = (\pi M f)^{\frac{1}{3}}$

$$\mathcal{A}(f) = \frac{M_t^2}{D_L}\sqrt{\frac{\pi\eta}{30}}(\pi M_t f)^{-7/6}$$ Newtonian approx

$$\psi(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + v^{-5}\left(\sum_{n=0}^{7}\alpha_n v^n\right)$$ at 3.5PN

C.K. Mishra et.al, Phys. Rev. D, 93, 8 (2016), 084054

+ quadrupole corrections at 2PN, 3PN and 3.5PN

Krishnendu et.al, Phys. Rev. Lett.,119,9 (2017) 091101

+ tidal corrections at 5PN and 6PN

Lackey and L. Wade, Phys. Rev. D, 91, (2015) 4 043002

- $\psi(f) = \psi_{BH}(f) + \psi_\kappa(f) + \psi_\Lambda(f)$

Leading order

$$\psi_\kappa = -\frac{75}{64}\frac{(\kappa_1 M_1^2\chi_1^2 + \kappa_2 M_2^2\chi_2^2)}{M_1 M_2}(\pi M_t f)^{-1/3}$$

$$\psi_\Lambda = -\frac{117}{256\eta}\widetilde{\Lambda}(\pi M_t f)^{5/3}$$

Backup Slide – Tidal deformability

- To include the tidal deformability in the waveform we exploited the relation:

$$\frac{M}{M_B} = \frac{\sqrt{2}}{8\sqrt{\pi}}\left[-0.828 + \frac{20.99}{\log\Lambda} - \frac{99.1}{(\log\Lambda)^2} + \frac{149.7}{(\log\Lambda)^3}\right]$$

N. Sennett et al., Phys. Rev. D, 96, 2 (2017) 024002

where $\Lambda = \lambda_T/M^5$ and λ_T is defined as $\mathcal{Q}_{ij} = -\lambda_T \varepsilon_{ij}$

- Λ will affect the waveform through an effective combination of the values of each BS

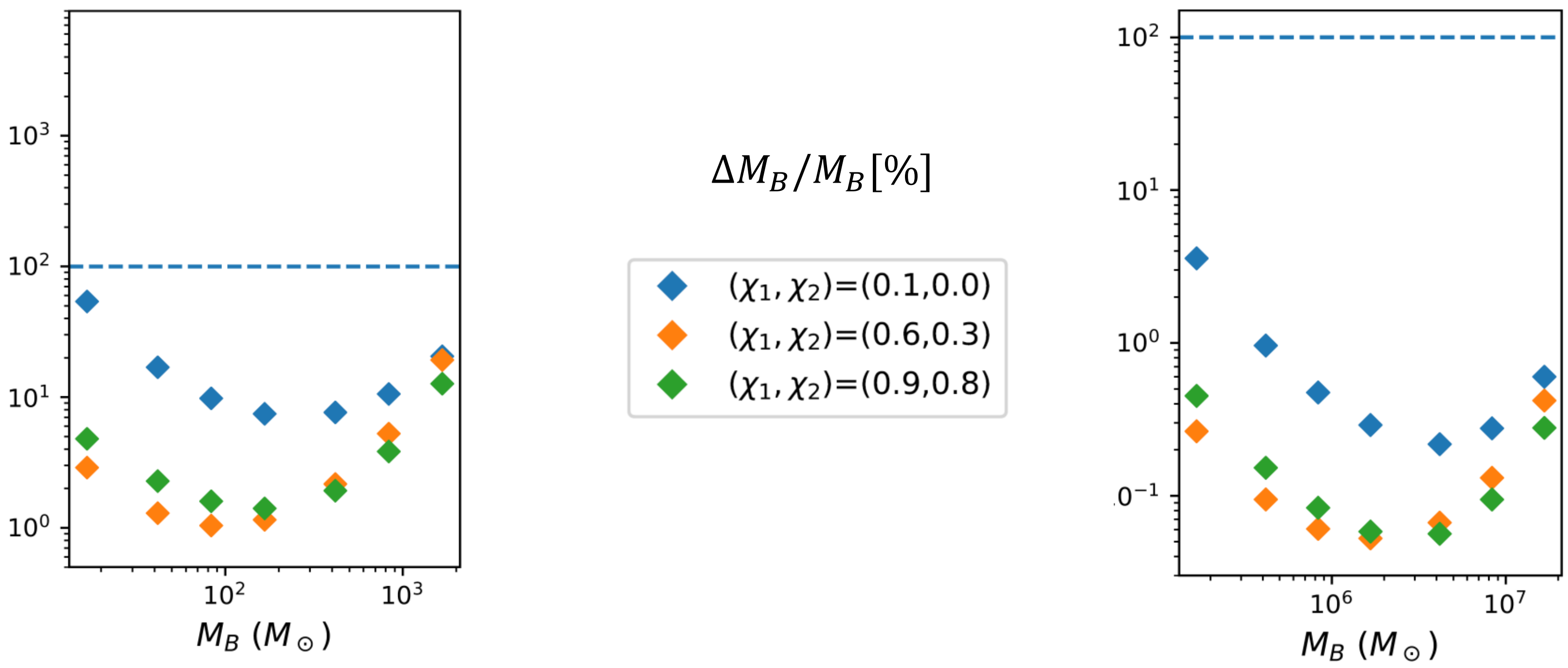
$$\tilde{\Lambda} = \frac{16}{13}\left[\left(1 + \frac{12}{q}\right)\frac{M_1^5}{M_t^5}\Lambda_1 + (1 + 12q)\frac{M_2^5}{M_t^5}\Lambda_2\right]$$

Backup Slide – Constraining scalar interactions

- The errors on M_B for ET and LISA are at the percent and sub-percent level in the most optimistic configurations:

ET ($D_L = 500\ Mpc$)

LISA ($D_L = 1\ Gpc$)



Backup Slide – The initial data

- An obvious initial guess for ρ,ϒ,ω and α is a solution for a non-spinning BS with the same mass.

$$\widetilde{ds^2} = -e^{\gamma+\rho}\widetilde{dt^2} + e^{2\alpha}\left(\widetilde{dr^2} + \widetilde{r^2}d\theta^2\right) + e^{\gamma-\rho}\widetilde{r^2}\sin\theta^2\,(d\phi - \widetilde{\omega}d\tilde{t})^2$$

- In the non-spinning limit one has:

$$\widetilde{\omega} \to 0 \qquad \gamma(\tilde{r},\theta),\rho(\tilde{r},\theta),\alpha(\tilde{r},\theta) \to \gamma(\tilde{r}),\rho(\tilde{r}),\alpha(\tilde{r}) \qquad \text{and} \qquad e^{\gamma-\rho} = e^{2\alpha}$$

- The metric beocmes: $\widetilde{ds^2} = -e^{2(\rho(\tilde{r})+\alpha(\tilde{r}))}\widetilde{dt^2} + e^{2\alpha(\tilde{r})}\left(\widetilde{dr^2} + \widetilde{r^2}d\theta^2 + \widetilde{r^2}\sin\theta^2\,d\phi^2\right)$

This is not the common choice when dealing with spherically symmetric problems!

$$ds^2 = -e^{v(r)}dt^2 + e^{u(r)}dr^2 + r^2d\theta^2 + r^2\sin\theta^2\,d\phi^2$$

Backup Slide – The initial data

- Comparing the two metrics:
$$\begin{cases} d\widetilde{s^2} = -e^{2(\rho(\tilde{r})+\alpha(\tilde{r}))}d\widetilde{t^2} + e^{2\alpha(\tilde{r})}\left(d\widetilde{r^2} + \widetilde{r^2}d\theta^2 + \widetilde{r^2}\sin\theta^2\, d\phi^2\right) \\ ds^2 = -e^{v(r)}dt^2 + e^{u(r)}dr^2 + r^2 d\theta^2 + r^2 \sin\theta^2\, d\phi^2 \end{cases}$$

one finds: 1) $e^{2\alpha(\tilde{r})}d\widetilde{r^2} = e^{u(r)}dr^2$ 2) $e^{2\alpha(\tilde{r})}\widetilde{r^2} = r^2$ and dividing term by term:

$$\frac{d\tilde{r}}{r} = \frac{e^{\frac{u(r)}{2}}}{r}dr \quad \Rightarrow \quad \boxed{\tilde{r}(r) = exp\left[\int_{r_0}^{r} \frac{e^{\frac{u(r')}{2}}}{r'}dr'\right]\cdot c}$$

- Finally: $\alpha(\tilde{r}) = \log\frac{r(\tilde{r})}{\tilde{r}}$ $\gamma(\tilde{r}) = \rho(\tilde{r}) + 2\alpha(\tilde{r})$ $\rho(\tilde{r}) = v\left(r(\tilde{r})\right) - \frac{1}{2}\alpha(\tilde{r})$

Backup Slide – Multipole moments

$$\rho(r,\mu) = \sum_{n=0}^{\infty} -2\frac{M_{2n}}{r^{2n+1}} P_2 n(\mu) + \text{higher orders} \quad \omega(r,\mu) = \sum_{n=1}^{\infty} -\frac{2}{2n-1}\frac{S_{2n-1}}{r^{2n+1}}\frac{P^1_{2n-1}(\mu)}{\sin\theta} + \text{higher orders}$$

$$\Rightarrow$$

$$M_{2n} = \frac{1}{2}\int_0^r dr'(r')^{2n+2}\int_0^1 d\mu' P_{2n}(\mu')S_\rho(r',\mu')$$

$$S_{2n-1} = \frac{1}{4n}\int_0^r dr'(r')^{2n+2} \times \int_0^1 d\mu' \sin\theta' P^1_{2n-1}(\mu')S_\omega(r',\mu')$$

- Correction factors to correctly match the Geroch-Hansen multipole moments

χ	κ_2	κ_2^{new}	$corr[\%]$
0.1	22.4	22.1	-1.4%
0.2	15.7	15.6	-0.5%
0.5	15.2	15.3	$\lesssim +0.1\%$
0.8	16.4	16.4	$\lesssim +0.1\%$
1,0	17.4	17.5	$\lesssim +0.1\%$
1.3	19.3	19.4	$\lesssim +0.1\%$
2.0	24.6	24.6	$\lesssim +0.05\%$

Table 1: Reduced quadrupole moment correction factors for different value of the spin χ and $M = 0.06$.